Representation Mixing and SU_3 Symmetry Breaking in the $\frac{1}{2}^-$ Baryon Octet*

PETER N. DOBSON, JR.
University of Hawaii, Honolulu, Hawaii 96822
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Widths and coupling-constant products are obtained for members of the $\frac{1}{2}^-$ baryon resonance octet by fitting the η-production data near threshold with a direct-channel resonance model. An interpretation of the fitted parameters in terms of representation mixing is shown to lead to large singlet-octet and decuplet-octet mixing. Quark-model modifications of the vertices, as proposed by Mitra and Ross, are also considered, with much the same results. An alternative interpretation in terms of an SU_3 symmetry-breaking interaction based on a suggestion by Pakvasa is given.

I. INTRODUCTION

RECENTLY, much attention has been focused by theorists on the properties of the low-lying negative-parity baryon resonances. The study of these resonances has received considerable impetus from the predictions of the quark model of the baryons,[1–6] in which such states can occur naturally as excitations of the three-quark system. In any event, the SU_3 structure of these states must be deduced in order to make a detailed comparison with any dynamical model. Hendry has made an analysis of the $\frac{3}{2}^-$ baryons[7] under the hypothesis of singlet-octet-decuplet mixing. Section III of this paper considers the same hypothesis applied to the $\frac{1}{2}^-$ baryon resonances associated with the η-production thresholds.

The existence of a complete set of these states is still a matter of controversy. The N^* member has been inferred both from recent phase-shift analyses[8,9] and from resonance-model fits to the η-production data.[10–12] The case for the Y_0^* member is less convincing, but it has at least earned a place in the Rosenfeld compilations.[13] The Y_1^* member has not even received this much recognition and there is presently no evidence at all for the Ξ^*. This paper will not address itself to this question. Instead, we shall make the hypothesis that the observed effects on η production near the thresholds are indeed due to nearby resonances. In Sec. II, we shall fit the data under this hypothesis and obtain the relevant parameters of the assumed resonances. Section IV will give an interpretation of these parameters in terms of a simple model of SU_3 symmetry breaking.

II. PARAMETERS OF THE RESONANCES

Data for the production of η mesons by pions[14,15] and kaons[16–18] on nucleons are fitted under the assumption that the process is dominated near threshold by the diagram of Fig. 1, where the internal baryon line is assigned a complex mass $M=M_r-\frac{1}{2}i\Gamma$. The resulting formula for the total η-production cross section is then

$$\sigma(s)=\frac{(g_1g_2)^2}{4\pi}\frac{k_2}{k_1}\left(\frac{E_1E_2}{m_1m_2}\right)^{1/2}\times\left\{\frac{(s+|M|^2)(E_1E_2+m_1m_2)+2s^{1/2}Mr(E_1m_2+E_2m_1)}{s|s-M^2|^2}\right\},\tag{1}$$

where E_i is the c.m. energy of baryon B_i with mass m_i, k_1 and k_2 are the incoming and outgoing meson c.m. momenta, and s is the total c.m. energy. M_r, Γ,

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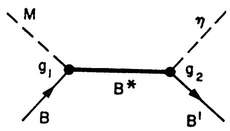

Fig. 1. Diagram for η-production reactions.

and the coupling-constant product g_1g_2 are varied to produce a fit to the data near threshold. The use of Eq. (1) is substantially in accord with the resonance models that have been used to fit the $\pi^-p\to\eta n$ data.[10–12] It differs from some in the detailed shape provided by a Feynman-diagram interpretation of Fig. 1. It seems reasonable that this shape is as likely to be correct as any of the usual ones, and in fact the resulting N^* parameters are quite similar to those of Ref. 11.

Figure 2 shows the fit obtained to the $\pi^-p\to\eta n$ data[14,15] with the N^* parameters taken to be

$$M_r=1536\text{ MeV},\quad \Gamma=168\text{ MeV},\quad g_{N\pi}g_{N\eta}=\pm2.49.$$

Figure 3 is a fit to the $K^-p\to\eta\Lambda$ data[16] with the parameters of the Y_0^* taken to be

$$M_r=1665\text{ MeV},\quad \Gamma=17\text{ MeV},\quad g_{N\bar{K}}g_{\Lambda\eta}=\pm0.30.$$

Finally, Figs. 4 and 5 show the fits to the data for $K^-n\to\eta\Sigma^-$ [17,18] and $K^-p\to\eta\Sigma^0$,[18] assuming

$$M_r=1746\text{ MeV},\quad \Gamma=110\text{ MeV},\quad g_{N\bar{K}}g_{\Sigma\eta}=\pm1.15$$

for the Y_1^*. The coupling constants obtained in this way are for unmodified point vertices in Fig. 1. Mitra and Ross[4] have suggested that the baryon s-wave decays may be dominated by "recoil" effects in the quark model, which modify the relevant vertices. The quark-model vertices may be approximated by replacing the coupling constant g by $g(\omega/M)$, where ω is the c.m.

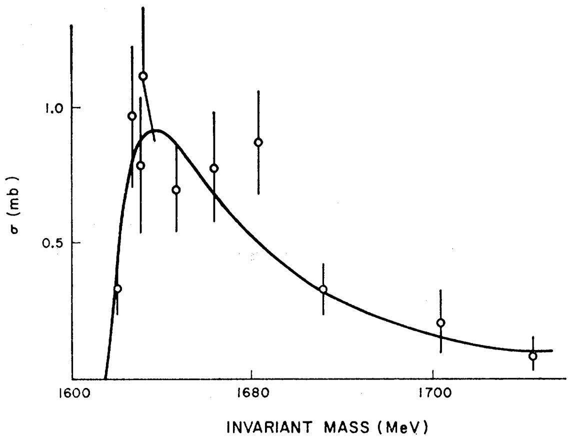

Fig. 3. Fit to the reaction $K^-p\to\eta\Lambda$.

energy of the meson at the vertex and M is related to the quark mass. In the absence of any scale to fix the absolute values of the coupling constants, we may take M to be any convenient normalization mass. In the following, we shall use the mass of the η meson for this purpose. For the quark-model case, then, we find the coupling-constant products to be ±2.24 for the N^*, ±0.26 for the Y_0^*, and ±0.91 for the Y_1^*.

III. REPRESENTATION MIXING

If we assume that πN and ηN are the only significant decay modes of the $N^*(1536)$, then the width and coupling-constant product are sufficient to determine its couplings to these modes. For the partial widths, we use the expression

$$\Gamma=\frac{g^2}{4\pi}\frac{k}{M_r}(E+m),\qquad(2)$$

where M_r is the resonance mass, k is the c.m. momentum

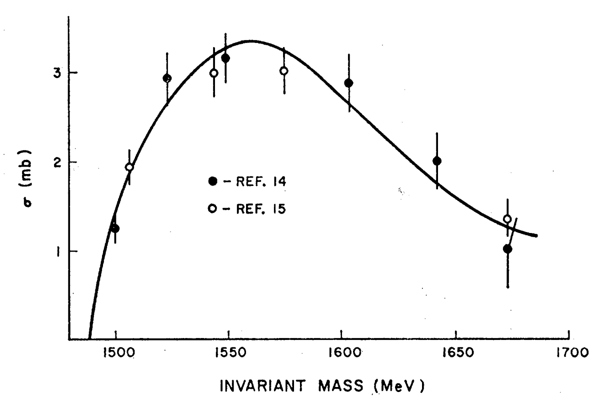

Fig. 2. Fit to the reaction $\pi^-p\to\eta n$.

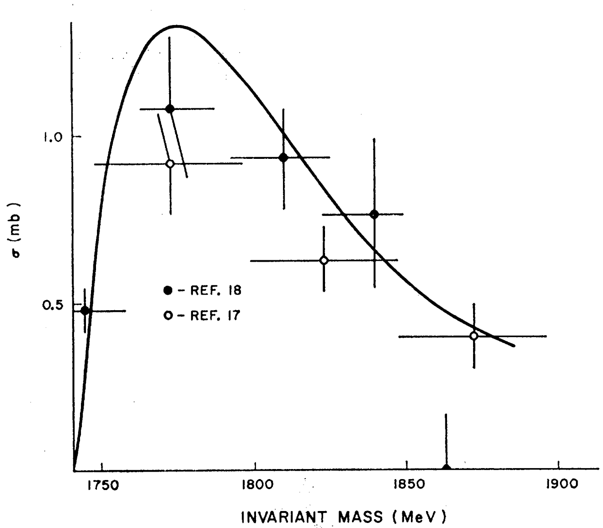

Fig. 4. Fit to the reaction $K^-n\to\eta\Sigma^-$.

TABLE I. Octet-singlet mixing solutions for the $Y_0^*(1665)$.

Vertex type	Mixing angle (deg)	α	Coupling $N\bar{K}$	Coupling $\Lambda\eta$	Coupling $\Sigma\pi$	Fraction (%) $N\bar{K}$	Fraction (%) $\Lambda\eta$	Fraction (%) $\Sigma\pi$	Width of $Y_0^*(1405)$ (MeV)
Unmod.	60.5	2.17	−0.28	1.04	0.49	18	18	64	63
Unmod.	68.7	2.17	−0.63	0.47	−0.15	90	7	3	36
Unmod.	83.6	−1.67	−0.48	−0.62	−0.40	52	6	42	186
Unmod.	63.5	−1.67	−0.23	−1.29	0.48	12	27	61	193
Quark	56.0	2.15	−0.22	1.18	0.64	14	23	63	3
Quark	69.1	2.15	−0.55	0.46	−0.10	95	4	1	5
Quark	58.5	−1.65	−0.19	−1.38	0.62	11	31	58	24
Quark	84.9	−1.65	−0.47	−0.54	−0.43	68	5	27	23

of the decaying particles, and E is the c.m. energy of the daughter baryon of mass m. We find

$$g_{N\pi}=0.98 \text{ and } g_{N\eta}=\pm 2.53$$

for unmodified vertices, and

$$g_{N\pi}=0.94 \text{ and } g_{N\eta}=\pm 2.38$$

for quark-model vertices. Both cases give branching fractions of 32% for πN and 68% for ηN, in agreement with accepted values. If we write the N^* coupling constants in the form

$$\begin{aligned} g_{N\pi}&=\sqrt{3}g_8\,,\\ g_{N\eta}&=\tfrac{1}{3}\sqrt{3}g_8(4\alpha-1)\,, \end{aligned} \tag{3}$$

where the parameter $\alpha=F/(F+D)$, then we find $g_8=0.57$, $\alpha=2.17$ or -1.67 for unmodified vertices, and $g_8=0.54$, $\alpha=2.15$ or -1.65 for quark-model vertices.

Assuming that the $Y_0^*(1665)$ is a mixture of singlet and octet, we may write its coupling constants in the form

$$\begin{aligned} g_{N\bar{K}}&=-\tfrac{1}{3}(\sqrt{6})g_8(2\alpha+1)\cos\theta-\tfrac{1}{2}g_1\sin\theta\,,\\ g_{\Lambda\eta}&=-\tfrac{2}{3}\sqrt{3}g_8(1-\alpha)\cos\theta-\tfrac{1}{4}\sqrt{2}g_1\sin\theta\,,\\ g_{\Sigma\pi}&=2g_8(1-\alpha)\cos\theta-\tfrac{1}{4}(\sqrt{6})g_1\sin\theta\,. \end{aligned} \tag{4}$$

We assume in (4) that the mixing may be described in terms of a single mixing angle.[19] From (4) it follows that

$$\cos\theta=(2g_{\Lambda\eta}-\sqrt{2}g_{N\bar{K}})/2g_{N\eta} \tag{5}$$

and

$$g_{\Sigma\pi}=\sqrt{3}[3g_{\Lambda\eta}-2\sqrt{2}(1-\alpha)g_{N\bar{K}}]/(4\alpha-1)\,. \tag{6}$$

Using knowledge of $g_{N\bar{K}}g_{\Lambda\eta}$ to eliminate $g_{N\bar{K}}$ and Eq. (6) to eliminate $g_{\Sigma\pi}$, we can write an expression for the total width that reduces to a quadratic equation for $g_{\Lambda\eta}^2$. Then Eq. (5) is used to find θ. The complete set of real solutions is given in Table I. The last column of the table contains the predicted width of the $Y_0^*(1405)$ on the assumption that it is the other state involved in the mixing.

For unmodified vertices, the solution with $\alpha=2.17$ and $\theta=68.7°$ is especially interesting. It predicts a width of 36 MeV for the $Y_0^*(1405)$, which is reasonably close to the accepted 50 MeV, particularly since experimental values around 30 MeV have been reported.[20,21] It also has a relatively small branching ratio into $\Sigma\pi$, which is in accord with present experimental information.[22] Both of the quark-model solutions with $\alpha=-1.65$ are fairly acceptable; however, both give somewhat smaller values of the $Y_0^*(1405)$ width and substantially larger values of the $\Sigma\pi$ branching ratio than desired.

The main feature of all the solutions is the large mixing angle, which was to be expected in view of the small width of the $Y_0^*(1665)$. In the limit of no mixing, $g_{\Lambda\eta}$ and $g_{\Sigma\pi}$ are proportional to one another and there is a large phase-space bias in favor of the $\Sigma\pi$ mode. Strong mixing is required to suppress the $\Sigma\pi$ decay while leaving the coupling to $\Lambda\eta$ large enough to account for the η-production data.

Turning now to the $Y_1^*(1746)$, we assume it to be a mixture of octet and decuplet, and write

$$\begin{aligned} g_{N\bar{K}}&=-\sqrt{2}g_8(2\alpha-1)\cos\phi+\tfrac{1}{6}(\sqrt{6})g_{10}\sin\phi\,,\\ g_{\Sigma\eta}&=\tfrac{2}{3}\sqrt{3}g_8(1-\alpha)\cos\phi+\tfrac{1}{2}g_{10}\sin\phi\,,\\ g_{\Lambda\pi}&=\tfrac{2}{3}\sqrt{3}g_8(1-\alpha)\cos\phi-\tfrac{1}{2}g_{10}\sin\phi\,,\\ g_{\Sigma\pi}&=2\sqrt{2}g_8\alpha\cos\phi+\tfrac{1}{6}(\sqrt{6})g_{10}\sin\phi\,. \end{aligned} \tag{7}$$

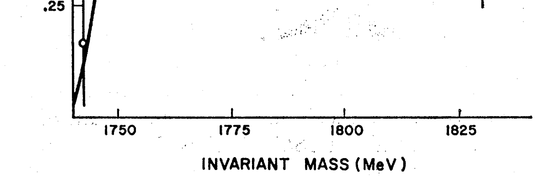

FIG. 5. Fit to the reaction $K^-p\to\eta\Sigma^0$.

[19] The possible failure of this description has been pointed out by S. Coleman and H. S. Schnitzer [Phys. Rev. **134**, B863 (1964)] and more recently by N. M. Kroll, T. D. Lee, and B. Zumino [*ibid.* **157**, 1376 (1967)].

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TABLE II. Octet-decuplet mixing solutions for the $Y_1^*(1746)$.

Vertex type	α	Mixing angle (deg)	Couplings $N\bar{K}$	$\Sigma\eta$	$\Lambda\pi$	$\Sigma\pi$	Fraction (%) $N\bar{K}$	$\Sigma\eta$	$\Lambda\pi$	$\Sigma\pi$	$N\pi$ partial width of $\Delta(1640)$ (MeV)
Unmod.	2.17	83.8	−1.81	−1.06	0.89	−0.41	47	6	39	8	102
Unmod.	2.17	71.5	−1.35	−0.85	0.37	0.61	72	4	7	17	44
Unmod.	−1.67	85.7	1.05	1.09	−0.83	0.58	44	7	34	15	99
Unmod.	−1.67	71.2	1.35	0.85	0.28	−0.64	73	4	4	19	9
Quark	2.15	85.2	−0.94	−0.96	0.84	−0.46	56	5	31	8	86
Quark	2.15	74.7	−1.15	−0.78	0.40	0.39	84	4	7	5	40
Quark	−1.65	87.0	0.91	0.99	−0.81	0.60	53	6	29	12	87
Quark	−1.65	73.5	1.18	0.77	0.17	−0.48	88	4	1	7	10

From (7) it follows that

$$\cos\phi=[2g_{\Sigma\eta}-(\sqrt{6})g_{N\bar{K}}]/2g_{N\eta}, \quad (8)$$

$$g_{\Sigma\pi}=(\sqrt{6})g_{\Sigma\eta}-2g_{N\bar{K}}, \quad (9)$$

and

$$g_{\Lambda\pi}=[(5-8\alpha)g_{\Sigma\eta}-2(\sqrt{6})(1-\alpha)g_{N\bar{K}}]/(4\alpha-1). \quad (10)$$

As in the case of the Y_0^*, we use (9), (10), and $g_{N\bar{K}}g_{\Sigma\eta}$ to eliminate all coupling constants except $g_{\Sigma\eta}$ in the expression for the total width, which then reduces to a quadratic in $g_{\Sigma\eta}^2$. Equation (8) provides the mixing angle ϕ once the couplings have been determined.

Table II gives the complete set of real solutions. The last column gives the $N\pi$ partial width for the decay of the $\Delta(1640)$, assuming it to belong to the decuplet involved in the mixing. The unmodified vertex solution with $\alpha=2.17$ and $\phi=71.5°$ is favored. Its value of α is the same as for the best solution for the $Y_0^*(1665)$, and the value of 44 MeV for the $\Delta\to N\pi$ partial width compares favorably with the accepted value of about 54 MeV. Of the two quark-model solutions with $\alpha=-1.67$, that with $\phi=87°$ is not unreasonable, although its $\Delta\to N\pi$ partial width of 87 MeV is not as good as was obtained with unmodified vertices. In any case, the octet-decuplet mixing angle is quite large.

In view of the apparently large mixing required for both the $Y_0^*(1665)$ and the $Y_1^*(1746)$, it is appropriate at this point to examine the arguments of Pakvasa and Tuan[23] to the effect that mixing should be absent in these states. These arguments are motivated by the observation that the masses of the negative-parity baryon resonances appear to fit the Gell-Mann–Okubo mass sum rules quite well. For the states under consideration here, of course, this assertion cannot really be justified until the missing Ξ^* is identified. However, the regular association of these resonances with η-production thresholds provides at least a strong presumption that the Ξ^* mass should lie near the sum of the Ξ and η masses, at about 1865 MeV, which would provide agreement with the assertion. However, even if this should be the case, we cannot be certain that mixing is absent. All three states Y_0^*, Y_1^*, and Ξ^* would be mixtures, and the agreement with the mass sum rule could be due to whatever dynamical mechanism associates them with the η-production thresholds rather than a direct consequence of their own SU_3 purity.

[23] S. Pakvasa and S. F. Tuan, Nucl. Phys. (to be published).

To examine this possibility, let us assume a mixing angle of 68.7° for the $Y_0^*(1665)$ and $Y_0^*(1405)$. Then the mass of the pure octet state will be 1439 MeV, while the pure singlet will have a mass of 1631 MeV. The low value for the Λ member of the octet requires a large Σ mass if we are to get reasonable masses for the Ξ^* resonances. Let us choose the $Y_1^*(2250)$ as the state mixed with the $Y_1^*(1746)$ and take the mixing angle to be 71.5°. We obtain a value of 2200 MeV for the pure octet mass and 1796 MeV for the pure decuplet mass. Now the mass sum rules provide us with the masses of the Ξ members; these are 1724 MeV for the octet and 1952 MeV for the decuplet. Note that the decuplet spacing is 156 MeV, which is a plausible value for this parameter. If we now assume that the $\Xi^*(1815)$ is one of the states participating in the mixing, we find the other at 1861 MeV, which is indeed very near the $\eta\Xi$ threshold.

Without more experimental information, the foregoing can hardly be regarded as more than an exercise in apologetics, especially since the $\Xi^*(1815)$ is usually taken to be a $\frac{3}{2}^-$ state. For this reason, it is of interest to consider the possibility that the decays of these states can be understood in terms of a model that avoids mixing. We shall explore this question in the next section.

IV. SYMMETRY-BREAKING MODEL

In this section we shall assume that the states are essentially pure SU_3 states, but that the Hamiltonian describing the decay contains terms that break the symmetry. We shall consider only interactions transforming like the $I=0$, $Y=0$ member of an octet such as are involved in the mass splittings. The main feature to be explained for the states under consideration is their large coupling to the η-baryon system. In order to get as little additional symmetry breaking as possible, we shall use the quark-model modifications of the vertices; these already provide some relative enhancement of the η-baryon couplings. Further help along this line is provided by a symmetry-breaking interaction of the form

$$G\,\mathrm{tr}(\bar{B}^*B)P_3{}^3, \quad (11)$$

where $P_3^3 = -\frac{1}{3}(\sqrt{6})\eta$ in the notation of Pais.[24] This interaction supplies the desired modification of the η-baryon couplings; however, it is not sufficient to account for all the properties of the states.

Pakvasa[25] has suggested the form

$$(\bar{B}^*B)_3{}^jP_j{}^3 + (\bar{B}^*B)_j{}^3P_3{}^j,$$

in which the B^* and B octets are coupled to an octet tensor by almost pure F-type coupling. This conjecture is based on the observation that a vertex structure of the form

$$(\bar{B}B)_3{}^jP_j{}^2 + \text{(Hermitian conjugate)}$$

can account very well for the weak parity-violating s-wave hyperon decays. The strong-interaction analog of these decays is suggested to be decays involving $\frac{1}{2}^- \to \frac{1}{2}^+$ for the baryons involved. Both SU_6 and the quark model predict F-type coupling of the B octets for the weak decays, and a small admixture of D is found sufficient to give excellent agreement with the experimental data. A term of the form (11) would be compatible with the conjecture, since its weak-interaction analog would modify only the couplings to K^0 and $\bar{K}^0$ and would have no observable effect.

The precise form that we shall use is

$$\begin{aligned}\gamma_F(\bar{B}^*{}_i{}^jB_3{}^i - \bar{B}^*{}_3{}^iB_i{}^j)P_j{}^3 + \gamma_D(\bar{B}^*{}_i{}^jB_3{}^i + \bar{B}^*{}_3{}^iB_i{}^j)P_j{}^3 \\ + \gamma_F(\bar{B}^*{}_j{}^iB_i{}^3 - \bar{B}^*{}_i{}^3B_j{}^i)P_3{}^j \\ + \gamma_D(\bar{B}^*{}_j{}^iB_i{}^3 + \bar{B}^*{}_3{}^iB_i{}^j)P_3{}^j. \quad (12)\end{aligned}$$

With the contributions of (11) and (12) added to the SU_3 symmetric interaction, the N^* couplings become

$$\begin{aligned} g_{N\pi} &= \sqrt{3}g_8\,, \\ g_{N\eta} &= \tfrac{1}{3}\sqrt{3}g_8(4\alpha-1) - \tfrac{1}{3}(\sqrt{6})(2\gamma_F + 2\gamma_D + G)\,. \end{aligned} \quad (13)$$

Similarly, for the Y_0^* we have

$$\begin{aligned} g_{N\bar{K}} &= -\tfrac{1}{3}(\sqrt{6})g_8(2\alpha+1) + \tfrac{1}{3}\sqrt{3}(3\gamma_F - \gamma_D)\,, \\ g_{\Lambda\eta} &= -\tfrac{2}{3}\sqrt{3}g_8(1-\alpha) - \tfrac{1}{9}(\sqrt{6})(8\gamma_D + 3G)\,, \\ g_{\Sigma\pi} &= 2g_8(1-\alpha)\,. \end{aligned} \quad (14)$$

Finally, the Y_1^* couplings are

$$\begin{aligned} g_{N\bar{K}} &= -\sqrt{2}g_8(2\alpha-1) + \gamma_D + \gamma_F\,, \\ g_{\Sigma\eta} &= \tfrac{2}{3}\sqrt{3}g_8(1-\alpha) - \tfrac{1}{3}(\sqrt{6})G\,, \\ g_{\Lambda\pi} &= \tfrac{2}{3}\sqrt{3}g_8(1-\alpha)\,, \\ g_{\Sigma\pi} &= 2\sqrt{2}g_8\alpha\,. \end{aligned} \quad (15)$$

Inserting the values $g_8 = 0.55$, $\alpha = 0.75$, $G = -3.51$, $\gamma_F = 1.41$, and $\gamma_D = 0.03$ into Eqs. (13)–(15), we obtain the results of Table III. The total widths are in quite good agreement with the values of Sec. II, as are the coupling-constant products

$$g_{N\pi}g_{N\eta} = 2.21\,,$$
$$g_{N\bar{K}}g_{\Lambda\eta} = 0.24\,,$$

and

$$g_{N\bar{K}}g_{\Sigma\eta} = 1.00\,.$$

The value of 90% for the branching fraction of the $Y_0^*(1665)$ into $\Lambda\eta$ agrees with the estimate made by Tripp *et al.*[26] on the basis of a two-channel assumption. The ratio γ_D/γ_F of about 0.02 agrees very well with Pakvasa's conjecture.

TABLE III. Predictions of the symmetry-breaking interaction described in the text.

Resonance	Total width (MeV)	Decay mode	Coupling	Fraction (%)
$N^*(1536)$	164	$N\pi$	0.95	34
		$N\eta$	2.33	66
$Y_0^*(1665)$	22	$N\bar{K}$	0.09	2
		$\Lambda\eta$	2.68	90
		$\Sigma\pi$	0.27	8
$Y_1^*(1746)$	119	$N\bar{K}$	0.33	6
		$\Sigma\eta$	3.03	50
		$\Lambda\pi$	0.16	1
		$\Sigma\pi$	1.17	43

V. DISCUSSION

In Secs. III and IV, we have given two interpretations of the $\frac{1}{2}^-$ baryons associated with the η-production thresholds. The first of these regards them as mixtures of SU_3 representations, the second as pure SU_3 states. The former has the disadvantage that the mass spectrum of these states is probably no longer simply related to SU_3 symmetry breaking, although it may be possible to account for it in a consistent way by an appropriate choice of mixing partners. It has the advantage that it can successfully account for the parameters of other resonances; in particular, we find reasonable values for the width of the $Y_0^*(1405)$ and for the $N\pi$ partial width of the $\Delta(1640)$. In fairness, however, note that an examination of Tables I and II shows that we could have claimed a modest success for quite a few values of these parameters. We find fair fits both for unmodified vertices and for vertices modified by quark-model considerations. In the latter case, we can probably also expect that the octet part of our states is really a mixture of the two $\frac{1}{2}^-$ octets predicted by the quark model.[27] This mixing has already been estimated by Faiman and Hendry.[6] In the presence of octet-octet mixing, the value of α is only an effective value and the intrinsic F/D ratios of the two octets must await further information on branching ratios of the $N^*(1710)$.

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In the case of unmodified vertices, we are free to regard the value of α as intrinsic, and try to place the state in a $U(6)\otimes U(6)$ classification, although the quark model is by no means ruled out. The F/D ratio corresponding to $\alpha=2.17$ is -1.85, which could be interpreted to favor a classification in the 70 of (210,6*) with a tendency to 70 dominance of the decay amplitudes.[28]

The alternative interpretation in terms of a symmetry-breaking decay interaction has the advantage that the presumed mass spectrum has a natural interpretation. It further has the appealing feature that the interaction may be treated in a unified fashion with the weak-interaction hyperon decays. Presumably, both mixing and direct symmetry breaking of the interaction are present. Our investigation here leaves it rather a matter of taste how much of each to use.

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Two-Pion-Exchange Contributions to Nucleon-Nucleon Scattering*

W. R. Wortman†

Department of Physics, Texas A&M University, College Station, Texas 77843

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Contributions to N-N phase shifts are found, using Feynman diagrams through fourth order in the coupling constant in pseudoscalar-meson theory. The amplitudes are evaluated using Cutkosky's method to express them in dispersion-relation form, and geometric unitarization is applied to calculate phase shifts. In addition to pion and nucleon intermediate states, particles in the baryon and psuedoscalar octets are allowed, subject to SU_3 symmetry. The results indicate that there are features of the phase shifts, particularly in higher partial waves, that are due to two-pion-exchange effects. The addition of the SU_3-restricted intermediate states gives no improvement.

1. INTRODUCTION

A PROGRAM for the application of Feynman diagrams to the nucleon-nucleon interaction[1] suggested that the long-range effects be accounted for by one-pion exchange (OPE). It was further suggested that the intermediate-range effects could be described by appropriate higher-order diagrams.

The first part of this program has shown remarkable success. Indeed, peripheral N-N scattering is very well described by the OPE effect, and this fact is used in virtually every model of N-N scattering as well as in phase-shift analyses.[2,3]

In the intermediate range, potentials have been generated from the two-pion-exchange (TPE) amplitude.[4,5] These early efforts suffered from calculational difficulties and led to potentials bearing little relation to phenomenological potentials. Recently, interest in these simple TPE potentials has been revived because of suggested relationships between TPE and σ exchange.[6] The need for the σ boson in one-boson-exchange (OBE) models[7,8] is firmly established. With the σ, this model does a good job of describing the intermediate-range effects.

The TPE calculations mentioned above achieved a unitary amplitude by reduction to a potential and subsequent use of the Schrödinger equation. This is a somewhat arbitrary and necessarily nonrelativistic means of unitarization. Geometric unitarization, meaning setting the real part of the partial-wave amplitude equal to an appropriate expression involving the phase shift, has been applied to the OBE model with much the same results as in the potential model.[7]

Gupta, Haracz, and Kaskas[9] have attacked the relativistic problem by evaluating all Feynman diagrams through fourth order. They found the contributions to the full amplitudes and concluded that the TPE effects

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† NASA Predoctoral Trainee. Present address: Los Alamos Scientific Laboratory, Los Alamos, N. M.

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