From (55) and (56) we therefore have

$$A_- \leqslant A \leqslant A_+ \quad \text{if} \quad p(r) > 0\,, \tag{61}$$

$$A_- \leqslant A \quad \text{if} \quad (p+K) \text{ is positive definite}\,, \tag{62}$$

where

$$A_- = \lim_{k\to 0} \frac{2}{k^2} F(\Phi,0)\,, \quad A_+ = \lim_{k\to 0} \frac{2}{k^2} F(\Psi,1)\,. \tag{63}$$

Equation (61) thus provides upper and lower bounds for the scattering length, subject to the condition $p(r)>0$ (repulsive potentials). The lower bound in (62) may be satisfied for $p(r)<0$ (attractive potentials) provided that $(p+K)$ is positive definite. The lower bound A_- is equivalent to the Schwinger result, while the upper bound A_+ appears to be new.

From (57) and (58)

$$A \leqslant A_+' \quad \text{if} \quad (-p-K) \text{ is positive definite}\,, \tag{64}$$

and

$$A_-' \leqslant A \leqslant A_+' \quad \text{if} \quad (-p-K) \text{ is strictly positive}\,, \tag{65}$$

where

$$A_-' = \lim_{k\to 0} \frac{2}{k^2} F\left(\Psi, \frac{-p}{\omega}\right)\,, \quad A_+' = \lim_{k\to 0} \frac{2}{k^2} F(\Phi,0) = A_-\,. \tag{66}$$

The bounds in (64) and (65) have possible applications only for the $p<0$ (attractive potential) case. The lower bound A_-' appears to be new, while A_+' leads to the Schwinger upper bound in this case.

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Can the Parity of the Ω^- Be Measured?*

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The Ω^- is unusual in that it can only decay weakly, and in that, for purposes of measuring its parity, it can only be produced in reactions with (at least) three-body final states. These circumstances imply that, with one possible exception, it may be so difficult to measure the parity of the Ω^- that such a measurement is a practical impossibility for some time to come.

If it should turn out that the spin parity of the Ω^- were not $\frac{3}{2}^+$, a good deal of our confidence in the validity of SU(3) as a particle symmetry might evaporate. At the very least, we would hope to be able to measure these quantities soon. Unfortunately, the following considerations appear to indicate that a measurement of the parity of the Ω^- lies far in the future.

To see this, we first observe that all decays of the Ω^- are weak, parity-nonconserving ones. Thus, no experimental[1] information about its parity can be obtained in its decay.

One is thus led to ask whether information can be obtained in experiments with more than two final particles. The simplest such process with only two nonzero-spin particles is $\bar{K}+N \to K+K+\Omega^-$. Any other process will have additional particles in the final state, and the following remarks will always be applicable. A process such as $\bar{p}p \to \bar{\Omega}^+\Omega^-$ cannot be used to determine the Ω^- parity, because of the simultaneous presence of both Ω^- and $\bar{\Omega}^+$, while one such as $\Lambda\bar{p} \to \Omega^-\bar{\Xi}^0$ has four particles with spin, in addition to being rather impractical.

We now apply the theorem[2] that in the absence of dynamical information it is (essentially[2]) impossible to measure the parity of a particle produced in a reaction with more than two particles in the final state.

Crudely speaking, the argument is as follows. For a reaction $a+b \to c+d$, the scattering amplitude can be expanded into a set of invariant tensors, each multiplied by an unknown function of the scalar variables which one can form out of the momenta. The scalar variables, written in terms of the initial ($\mathbf{q}$) and final ($\mathbf{q}'$) center-of-mass three-momenta,[3] are $\mathbf{q}^2$, $\mathbf{q}'^2$, and $\mathbf{q}\cdot\mathbf{q}'$. They are all quadratic, and are all scalars under parity. Consequently, the unknown coefficients of the invariants are

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[1] It is conceivable that when we have a reliable theory of the dynamics of parity violation in weak decays, the Ω^- parity can be deduced from such information. We would like to thank Professor Y. P. Yao for emphasizing this point.

[2] P. S. Csonka, M. J. Moravcsik, and M. D. Scadron, Phys. Rev. Letters 14, 861 (1965).

[3] Although we use a noncovariant language, all of the arguments given are fully relativistic (see Ref. 1).

even under parity. One can look experimentally to see whether the invariants present are even or odd under parity, thus determining the product of the intrinsic parities of the particles involved. If three intrinsic parities are known, the fourth is determined.

To determine the parity of a particle by such a process, it is necessary[4] to measure some information about the polarization state of all particles involved. For instance, assuming that a and c are spinless particles, and that b and d have spin $\frac{1}{2}$, it is sufficient[4] first to measure the polarization of d in one experiment, then to produce d from a target of polarized b's and measure the d asymmetry, and then to compare the polarization and asymmetry at the same energy and angle. For reactions with four nonzero-spin particles, in order to determine the parity of one of the participating particles, one has to measure something about the polarization of all the particles.

For a reaction $a+b \to c+d+e$, we can again make a decomposition into invariants multiplied by scalar functions. The unknown coefficient functions can depend on all the rank-zero tensors we can construct. We construct them,[3] for example, out of the initial center-of-mass momentum $\mathbf{q}$ and two final relative momenta $\mathbf{q}_1$ and $\mathbf{q}_2$. Then the rank-zero tensors are all the bilinear products of these, plus the pseudoscalar $\mathbf{q}\cdot(\mathbf{q}_1\times\mathbf{q}_2)$. Since the unknown coefficient functions can be even or odd under parity now, because they can also depend on the pseudoscalar $\mathbf{q}\cdot(\mathbf{q}_1\times\mathbf{q}_2)$, all of the invariant tensors will be present. There is thus no way, in general, to determine the product of the parities of the particles involved.

The simplest experiment that can, in general, be used to determine the Ω^- parity can be deduced from the above discussion. One must first choose a kinematical situation such that $\mathbf{q}\cdot(\mathbf{q}_1\times\mathbf{q}_2)$ vanishes for all events; e.g., only coplanar final states can be used. Then one must, as in the four-particle process above, obtain information about both the proton and Ω^- polarizations. Thus, even in the simplest case, one must obtain on a polarized proton target sufficient three-body coplanar final states containing an Ω^- to determine the polarization of the Ω^- from its decays. Or, as above, two experiments to determine the Ω^- polarization and asymmetry could be combined. It would appear that, at best, such experiments will not be feasible for some time to come, if ever.

There is, however, one possible situation which would allow one to decrease the above difficulty. If the actual Ω^- production should be to a significant extent through a channel where the final state is effectively a two-body one, such as $K^-+p \to K^{++}+\Omega^-$ or $K^-+p \to K^++\Xi^{*-}$, $\Xi^{*-} \to K^0\Omega^-$, then conventional techniques could be used in the foreseeable future to obtain the Ω^- spin and parity, although even in this case some polarization information must be obtained about all nonzero-spin particles in the reaction.[4] (K^{++} and Ξ^{*-} are used to denote possible meson resonances of strangeness $+2$ and baryon resonances of strangeness -2, respectively.) It is then particularly amusing to note that it has been pointed out[5] by the authors of the Argonne National Laboratory Ω^- experiment that their three Ω^- events, and the first Brookhaven Ω^- event, and the first Birmingham-Glasgow-London (I.C.)-Munich-Oxford-Rutherford Laboratory collaboration Ω^- event (five events total), all have the property that the effective mass of all final particles except one K^+ is approximately 2700 MeV. That is, if we lump events into the description $K^-+p \to K^++\Omega^-+\mathrm{MM}$, where MM stands for the remaining missing mass, then the effective mass of the Ω^--plus-MM combination is always about 2700 MeV. Thus, it may be that a significant number of Ω^-'s will be produced through the process $K^-+p \to K^++\Xi^{*-}(2700)$, $\Xi^{*-}(2700) \to \Omega^-+K^0$. One qualification is necessary: If the Ξ resonance should have spin $\frac{1}{2}$,[6] all of its decays would be isotropic and nothing would be gained. Otherwise, it might eventually be possible to obtain the spin parity of the Ξ resonance, and ultimately that of the Ω^-.

In summary, we have argued that, barring production of an Ω^- mainly through a Ξ resonance, it will require sufficiently difficult experiments to measure the parity of the Ω^- that such a measurement verges on being a practical impossibility for a long time to come. Such a circumstance leads one to raise interesting questions concerning the relationship between our beliefs in the validity of a theory and experimental tests of that theory.

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[4] P. L. Csonka, M. J. Moravcsik, and M. D. Scadron, Physics Letters **15**, 353 (1965).

[5] P. F. Schultz *et al.*, Phys. Rev. **168**, 1509 (1968). The masses given were 2.73, 2.70, 2.76, 2.69, and 2.68 in GeV/c^2. One of us (G.L.K.) is grateful to Professor D. Mortera for bringing this information to his attention.

[6] Such a resonance has been suggested, on the basis of dynamical arguments that lead one to expect that a $J^P=\frac{1}{2}^+$ baryon $\overline{10}$ $SU(3)$ multiplet might exist, with a mass formula that gave an isospin-$\frac{3}{2}$ Ξ^* at a mass of about 2.75 GeV. The isospin-$\frac{3}{2}$ requirement may vitiate the relevance of this calculation. See J. J. Brehm and G. L. Kane, Phys. Rev. Letters **17**, 764 (1966); G. L. Kane, *ibid.* **17**, 719 (1966).