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Nucleon-Nucleon Models, Charge Splitting, and the Nucleon-Nucleon Data near 210 MeV*

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Phase-shift analyses have been made of the nucleon-nucleon scattering data near 210 MeV. Several nuclear potential models are compared with the results and are found to be clearly distinguishable by differences in their isospin-singlet phase shifts, particularly by the coupling parameter ϵ_1 which is of importance for nuclear physics. There is no evidence of charge splitting in the $^1S_0$ state.

I. INTRODUCTION

THERE have recently been a large number of neutron-proton and proton-proton scattering measurements made with good precision at Rochester. In addition, there has been a major effort to reevaluate data previously measured in the vicinity of 210 MeV. Since the previous data were insufficient to decide between some of the widely differing phase parameters of the current potential models, it was felt that comparison of the models with the results of analyses including the new data was highly desirable.

II. DATA SELECTION

As shown in Table I, there are 73 proton-proton and 65 neutron-proton scattering data available in the energy range 197–225 MeV. Some of these were rejected for reasons discussed below. The final data selection is shown in Table II, where it is compared with the data used in a previously published analysis of the older *pp* and *np* data at this energy.[1] Note that the new *pp* data do not include any new types of experiment, whereas the new *np* data include two new types of measurements, in addition to more precise values for previously measured quantities.

According to Chauvenet's criterion, in a data set of this size one should reject any datum with a χ^2 contribution of about 8 or more.[2] In our first analysis, which included all available data, there were six data in this category. As shown in Table III, two of these belong to the set of 215-MeV *np* relative cross sections. Since another, more recent, set of relative cross sections is available at 199 MeV with about the same angular distribution and much higher precision,[3] and because the total χ^2 contribution for the nine 215-MeV data was 37.2, the entire older cross section was rejected.

The two $R'R$ data shown in Table III had been rejected in a previous analysis,[4] but were included in our initial data selection, since it was thought that the additional number of data included in this analysis might make these two data tolerable. This proved not to be the case and so the two data were again rejected.

In treating the deuteron data there might be an uncertainty about the direction of the unit scattering vector $\hat{n}$ used in Table I of Ref. 5, resulting in uncertainty in the sign of C in the equations in that table. Table IV shows the correct relationship between the parameters in Table I of Ref. 5 and the M-matrix elements as defined by Stapp *et al.*[6] Predictions of P_t, D_t, and R_t using these equations were found to agree with similar calculations made by Thorndike using the same values for the M-matrix elements.[7]

The phase shifts are not constant over the range of energies covered by the data. However, they are very nearly linear in this region, so that the energy dependence of the data can be accounted for by using the first derivatives of the phase shifts with respect to energy obtained from an energy-dependent phase-shift analysis. In this analysis the slopes were fixed at the values obtained in the energy-dependent analysis of AM-IV.[1]

Charge independence was assumed for all $T=1$ phase shifts and was later tested for the $^1S_0$ state as described in Sec. V of this paper.

III. SELECTION OF FREE PHASES

The longest-range part of the nucleon-nucleon interaction is firmly believed to be due to the exchange of single pions, so that the higher-angular-momentum phase shifts can be calculated directly from the one-

* Supported in part by the U. S. Atomic Energy Commission.
[1] R. A. Arndt and M. H. MacGregor, Phys. Rev. **141**, 873 (1966), hereafter referred to as AM-IV. The results of the AM-IV analysis were used only to show the effect of the new data on the phase shifts and their error bars. For a more recent 210-MeV $pp+np$ analysis of the Livermore group, see MAW-IX (Ref. 8), in which the new data are included.
[2] L. G. Parratt, *Probability and Experimental Errors in Science* (John Wiley & Sons, Inc., New York, 1961), especially p. 176. See also Ref. 4.
[3] A. R. Thomas, D. Spalding, and E. H. Thorndike, Phys. Rev. **167**, 1240 (1968).
[4] P. S. Signell, N. R. Yoder, and J. E. Matos, Phys. Rev. **135**, B1128 (1964).
[5] N. W. Reay, E. H. Thorndike, D. Spalding, and A. R. Thomas, Phys. Rev. **150**, 801 (1966).
[6] H. P. Stapp, T. J. Ypsilantis, and N. Metropolis, Phys. Rev. **105**, 302 (1957).
[7] E. H. Thorndike (private communication).

pion-exchange (OPE) functional forms.[8] Without knowing the two-pion-exchange contributions, however, one has no precise way of deciding upon that angular momentum above which the phases can be taken as purely OPE. The method used here for determining that dividing line was similar to that used in a previously reported analysis.[4] In order to obtain a rough idea of which phase shifts should be non-OPE, a series of analyses was made using all of the 210-MeV pp and np data. All phases with orbital angular momentum $L>L_{\max}$ were fixed at their OPE values, while those with $L\leq L_{\max}$ were simultaneously varied in order to obtain a least-squares fit to the data. The variation of the minimized χ^2 with $L_{\max}$ is shown in Fig. 1. Clearly, all phases through the F waves must be released from their OPE values, and probably some of the G and H waves as well. It was finally decided that, in addition to all the S, P, D, and F phase shifts, the ${}^1G_4$, ${}^3G_3$, and ${}^3H_4$ phase shifts should be released. The other G and H phase shifts appeared to be near enough to their OPE values in previous analyses so that even if they were released, there would not be a significant reduction in χ^2. This was verified by our analysis B, which was identical to analysis A in Table V except that the other G and H phase shifts were released. Although χ^2 decreased slightly when the additional phase shifts were released, the ratio of χ^2 to its expected value actually increased, so that the decrease in χ^2 was less than that which would be expected if the phase shifts had been released from values given by arbitrary functional forms.

TABLE I. Data considered in this analysis. The normalization N is for the data on the following line. Polarizations and cross sections not preceded by a normalization are relative only. Note that the effective number of relative data in a set is one less than the number shown.

Particles	E_{lab} (MeV)	Type	Number	Reference
pp	210	N	1	a
		P	6	a,b
	213	σ	7	c
		N	1	d
		σ	13	d
		N	1	a
		P	13	d
		D	7	e
		R	6	f
		R	1	g
		AR	5	f
		A	2	g
		$R'R$	7	h
	217	N	1	i
		P	6	b
	225	σ_{int}	1	j
np	197	D_t	3	k
	199	σ	8	l
		N	1	l
		P	8	l
	200	σ_{tot}	1	m
		σ	6	m
		σ	15	m
	203	R_t	5	n
	212	D	5	o
	215	σ	10	p
	217	N	1	b
		P	6	b
	220	σ_{tot}	1	q
		σ_{tot}	1	r

[a] E. H. Thorndike, Rev. Mod. Phys. **39**, 513 (1967).
[b] J. H. Tinlot and R. E. Warner, Phys. Rev. **124**, 890 (1961).
[c] A. Konradi, Ph.D. thesis, Rochester, 1961 (unpublished); datum from R. Wilson, *Nucleon-Nucleon Interaction* (Interscience Publishers, Inc., New York, 1963).
[d] J. F. Marshall, C. N. Brown, and F. Lobkowicz, Phys. Rev. **150**, 1119 (1966).
[e] K. Gotow, F. Lobkowicz, and E. Heer, Phys. Rev. **127**, 2206 (1962).
[f] A. C. England, W. A. Gibson, K. Goton, E. Heer, and J. Tinlot, Phys. Rev. **124**, 561 (1961).
[g] K. Gotow and F. Lobkowicz (unpublished); datum from Ref. a.
[h] K. Gotow and F. Lobkowicz, Phys. Rev. **136**, B1345 (1964).
[i] Reference 7.
[j] O. Chamberlain, G. Pettengill, E. Segrè, and C. Wiegand, Phys. Rev. **93**, 1424 (1954).
[k] D. Spalding, A. R. Thomas, and E. H. Thorndike, Phys. Rev. **158**, 1338 (1967); data from Ref. a.
[l] Reference 3.
[m] Yu. M. Kazarinov and Yu. N. Simonov, Zh. Eksperim. i Teor. Fiz. **43**, 35 (1962) [English transl.: Soviet Phys.—JETP **16**, 24 (1963)].
[n] Reference 5 (data from Ref. a).
[o] R. E. Warner and J. H. Tinlot, Phys. Rev. **125**, 1028 (1962).
[p] G. L. Guernsey, G. Mott, and B. K. Nelson, Phys. Rev. **88**, 15 (1952).
[q] J. DeJuren and B. J. Moyer, Phys. Rev. **81**, 919 (1951).
[r] G. Mott, G. L. Guernsey, and B. K. Nelson, Phys. Rev. **88**, 9 (1952).

[8] P. Cziffra, M. H. MacGregor, M. J. Moravcsik, and H. P. Stapp, Phys. Rev. **114**, 880 (1959).

TABLE II. Number of data of various types used here, compared with that of a previous analysis (Ref. 1).

	pp data		np data	
Kind of data	Current	Old	Current	Old
σ	20	7	27	20
P	25	14	14	6
D	7	7	5	5
R	7	7		
A	2	5		
AR	5			
$R'R$	5			
D_t			3	
R_t			5	
R'		4		
Total	71	44	54	31

IV. ANALYSIS RESULTS

The preferred phase shifts are the ones obtained from analysis A of Table V, with 21 free phase parameters. These phase shifts are in good agreement with the recent values of MacGregor, Arndt, and Wright[9] (here designated MAW-IX) from a data set similar to the one used here. For purposes of comparing the effects of the new data on the phase shifts, analysis B with 25 free parameters should be compared with the old analysis labeled AM-IV in Table V. The most obvious difference

TABLE III. Data not used in the final analysis.

E_{lab} (MeV)	Type	Angle	χ^2 in initial analysis	Reference
213	pp $R'R$	60°	15.0	a
213	pp $R'R$	70°	13.2	a
200	np σ	165°	8.4	b
200	np σ	180°	8.5	b
215	np σ	All	37.2 for nine data	c

[a] K. Gotow and F. Lobkowicz, Phys. Rev. **136**, B1345 (1964).
[b] Yu. M. Kazarinov and Yu. N. Simonov, Zh. Eksperim. i Teor. Fiz. **43**, 35 (1962) [English transl.: Soviet Phys.—JETP **16**, 24 (1963)].
[c] G. L. Guernsey, G. Mott, and B. K. Nelson, Phys. Rev. **88**, 15 (1952).

[9] M. A. MacGregor, R. A. Arndt, and R. M. Wright, Phys. Rev. **173**, 1272 (1968).

TABLE IV. Relationship between parameters in Table I of Ref. 5 and the M-matrix elements defined in Ref. 6. θ is the c.m. scattering angle of incoming proton and $\theta_{\mathrm{lab}}{}^n$ is the lab angle of the outgoing neutron.

$$A=\tfrac{1}{4}(2M_{11}+M_{00}+M_{ss})$$
$$B=\tfrac{1}{4}(-2M_{1-1}+M_{00}-M_{ss})$$
$$C=i\tfrac{1}{4}\sqrt{2}(M_{10}-M_{01})$$
$$g=\tfrac{1}{4}(M_{11}+M_{1-1}-M_{ss})$$
$$h=(\sqrt{2}/2\sin\theta)(M_{10}+M_{01})$$
$$E=g-h$$
$$F=g+h$$
$$\sin\theta_{\mathrm{lab}}{}^n=\cos\tfrac{1}{2}\theta$$
$$\cos\theta_{\mathrm{lab}}{}^n=\sin\tfrac{1}{2}\theta$$

between the results of the new analyses and AM-IV is the large reduction in the uncertainties on the isospin-singlet phase shifts.[9] As shown in Table V, the uncertainties in the isospin-singlet phase shifts in analysis A are smaller than those of AM-IV by at least a factor of 2 and in most cases by a factor of 3 or more. In an attempt to determine whether or not any one particular set of new data was responsible for the large increase in precision with which the phase shifts could be determined, several analyses were made in a manner identical to that of analysis A, except that each was missing a different subset of the np scattering data. From the results of those analyses it appeared that all of the new np data shared fairly equally in determining the $T=0$ phase shifts.

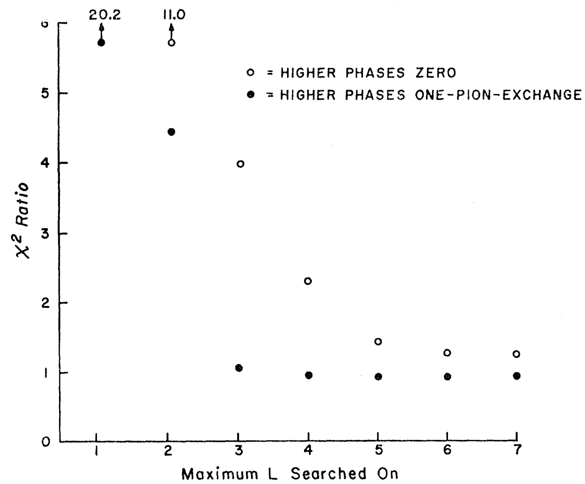

FIG. 1. Ratio of the goodness-of-fit parameter χ^2 to its expected value, as a function of the maximum orbital angular momentum of the phases being fixed by the data. The difference between the open and dark circles is a measure of the evidence for the OPE mechanism.

The $T=1$ phase shifts at 210 MeV were quite well

TABLE V. Comparison of analyses A, B, C, and AM-IV. The preferred analysis A has 21 non-OPE phases, analysis B has 25, and analysis C is identical to A except that the np $^1S_0$ and pp $^1S_0$ were allowed to vary separately. Phase shifts in parentheses were fixed at the OPE values shown. All phase parameters are nuclear bar, in degrees.

	$^1S_0$	$^1D_2$	$^1G_4$	$^3P_0$	$^3P_1$
Analysis A	5.44±0.44	7.03±0.17	1.10±0.08	−1.25±0.49	−22.02±0.24
Analysis B	5.55±0.45	7.02±0.24	1.08±0.09	−1.28±0.48	−22.11±0.27
Analysis C	5.43±0.44 pp 2.13±4.50 np	7.03±0.17	1.10±0.08	−1.29±0.49	−22.03±0.24
AM-IV	5.18±0.60	7.02±0.32	1.04±0.16	−0.79±0.60	−21.59±0.61
	$^3P_2$	ϵ_2	$^3F_2$	$^3F_3$	$^3F_4$
Analysis A	15.49±0.16	−2.84±0.12	1.32±0.24	−2.78±0.16	2.09±0.12
Analysis B	15.41±0.18	−2.84±0.14	1.47±0.25	−2.68±0.18	2.19±0.13
Analysis C	15.48±0.16	−2.85±0.12	1.34±0.24	−2.79±0.16	2.10±0.12
AM-IV	15.89±0.27	−2.78±0.20	1.58±0.37	−2.58±0.22	2.32±0.22
	ϵ_4	$^3H_4$	$^3H_5$	$^3H_6$	$^1P_1$
Analysis A	−0.97±0.08	0.15±0.08	(−0.91)	(0.15)	−21.47±1.62
Analysis B	−0.97±0.08	0.10±0.21	−0.99±0.19	0.06±0.12	−23.39±2.51
Analysis C	−0.98±0.08	0.15±0.08	(−0.91)	(0.15)	−22.33±1.96
AM-IV	−0.94±0.09	0.47±0.40	−0.64±0.21	0.41±0.30	−23.36±7.43
	$^1F_3$	$^3S_1$	ϵ_1	$^3D_1$	$^3D_2$
Analysis A	−4.23±0.67	15.05±1.20	6.64±0.69	−18.28±1.27	27.47±1.12
Analysis B	−3.66±1.00	15.86±1.58	6.52±0.74	−19.11±1.71	26.27±2.00
Analysis C	−4.52±0.74	15.07±1.22	6.57±0.70	−18.33±1.28	27.17±1.23
AM-IV	−5.49±2.47	18.53±3.57	2.91±3.21	−23.41±4.72	23.01±4.94
	$^3D_3$	ϵ_3	$^3G_3$	$^3G_4$	$^3G_5$
Analysis A	4.53±0.90	7.08±0.35	−1.58±0.60	(5.23)	(0.89)
Analysis B	4.65±1.01	6.97±0.53	−1.63±0.61	4.76±1.05	− 0.37±0.64
Analysis C	4.37±0.91	7.04±0.36	−1.85±0.71	(5.23)	(0.89)
AM-IV	2.71±1.98	7.08±1.20	−0.40±1.49	4.53±3.93	0.17±1.95

	$\chi_{pp}{}^2$	$\chi_{np}{}^2$	$\chi_{\mathrm{expt}}{}^2$	Number of data
Analysis A	36	35	0.87	125
Analysis B	54	35	0.88	125
Analysis C	56	34	0.88	125
AM-IV	28	25	1.06	75

determined[4] by the old pp scattering data, and the new data caused no major changes in either those phase shifts or their error bars.

V. CHARGE SPLITTING

Although it has been standard procedure to assume charge independence in phase-shift analyses which include both pp and np data, one should show that the experimental data do not contradict this assumption. There is evidence that charge splitting would be observed if sufficiently accurate data were available. For example, in a previous paper[10] an estimate of the charge splitting due to the difference in the quantum masses in the np and pp interactions was obtained from the Saylor-Bryan-Marshak model at 213 MeV. In the $^1S_0$ state the splitting was calculated to be 5.8°. Also, the large difference in the singlet scattering lengths for the np and pp interactions is an indication of charge splitting near zero energy.[11] On the other hand, Breit[12] found no statistically significant charge splitting between the pion-nucleon coupling constant g^2 obtained from an analysis of combined np and pp data and that obtained from an analysis of pp data alone. The method that he used was to assume charge independence for the non-OPE lower-angular-momentum phase shifts and search on values for those phases and on the value of g^2 in order to fit all of the 0–330-MeV data. About 1000 pp data were used to obtain $(g^2)_{pp}$, and another thousand np data were added in order to obtain $(g^2)_{pp+np}$. The good agreement found between $(g^2)_{pp}$ and $(g^2)_{pp+np}$ would seem at first glance to be evidence for charge independence, at least for the OPE potential. However, the comparison for charge independence should be between $(g^2)_{pp}$ and $(g^2)_{np}$, and not between $(g^2)_{pp}$ and $(g^2)_{pp+np}$. Since the pp data are much more accurate than the np data, $(g^2)_{pp+np}$ would necessarily lie much closer to $(g^2)_{pp}$ than to $(g^2)_{np}$, even if charge independence were grossly violated. For example, suppose that the $(g^2)_{pp}$ and $(g^2)_{np}$ values really differed by five standard deviations. The main effect in the above analysis would be to raise the χ^2 for the 2000 combined data by about 25 over what it would have been if the same 2000 pp and np data had been analyzed separately, allowing for splitting of $(g^2)_{pp}$ and $(g^2)_{np}$. Since the pp and np data were never analyzed separately, there was no way of knowing whether the value of χ^2 actually found contained a part produced by violation of charge independence. In any case, it would seem preferable to examine directly the lower-angular-momentum phase shifts for charge splitting rather than to look for splitting of the OPE phases caused by a possibly false assumption of charge independence for the lower-angular-momentum phases.

[10] P. Signell, Phys. Letters 8, 73 (1964).
[11] See L. Heller, P. Signell, and N. R. Yoder [Phys. Rev. Letters 13, 577 (1964)] for a discussion of charge splitting and its possible origins.
[12] G. Breit, Rev. Mod. Phys. 39, 560 (1967).

TABLE VI. Comparison of the χ^2 values for analysis A (Table V) and several potential models (see text).

	χ_{pp}^2	χ_{np}^2	Total
Analysis A[a]	56	35	91
Bressel-Kerman[b]	170	87	257
Hamada-Johnston[c]	316	82	398
Yale[d]	320	209	529
Lomon-Feshbach[e]	305	298	603
Reid[f]	494	265	759
HJ+BK($^3P_2$)[a]	195	···	···

[a] Present work. [b] Reference 13. [c] Reference 14. [d] Reference 15. [e] Reference 16. [f] Reference 17.

In the present analysis an attempt was made to determine what charge splitting, if any, is demanded for the $^1S_0$ state by the current data. Analysis C of Table V was identical to analysis A, except that the np and pp $^1S_0$ phase shifts were allowed to be independent. The splitting between the two phases turned out to be 3.3°±4.5°, which is consistent with no splitting at all. In MAW-IX an attempt was made to measure the charge splitting in the $^1S_0$ state by determining the difference between the $^1S_0$ phase shift obtained from an analysis of the pp data alone and the $^1S_0$ phase shift obtained from an analysis including both the pp and the np data. The MAW-IX result of 0.06°±0.74° is also consistent with no splitting, but limits the magnitude of any possible splitting to less than 0.8°, whereas we found a splitting of as much as 7.8° to be consistent with the current data. This discrepancy clearly emphasizes the point made in the previous paragraph, that quantitative evidence for or against charge splitting can only be obtained by allowing for charge splitting of the phase shifts in the analysis.

From Table III it can be seen that the error on the np $^1S_0$ is larger by a factor of 10 than that on the pp $^1S_0$. Additional np scattering data would be valuable in reducing this difference and thus helping to determine the presence or absence of charge splitting in the $^1S_0$ state. The pertinent data could be determined by additional analyses if further np experiments at this energy are contemplated.

VI. COMPARISON WITH MODELS

As shown in Table VI, the Bressel-Kerman[13] (BK) and Hamada-Johnston[14] (HJ) potentials give the better fits to the data among current potential models. In the last row, labeled "HJ+BK($^3P_2$)," the BK phase shift for the $^3P_2$ state has been combined with the HJ phases for all other states. One sees from Tables VI and VII that the 0.83° difference between the HJ and HJ+BK($^3P_2$) phase shifts causes a pp χ^2 difference of 130. In fact, the major difference between the pp χ^2 values of the HJ and BK models is due to this small difference in their $^3P_2$ phase shifts.

[13] C. B. Bressel and A. K. Kerman (private communication), quoted in P. C. Bhargava and D. W. L. Sprung, Ann. Phys. (N.Y.) 42, 222 (1967).
[14] T. Hamada and I. D. Johnston, Nucl. Phys. 34, 382 (1962).

TABLE VII. Comparison of nuclear-bar phase parameters from analysis A and from the potential models of Table VI.

	$^1S_0$	$^1D_2$	$^1G_4$	$^3P_0$	$^3P_1$	$^3P_2$	ϵ_2	$^3F_2$	$^3F_3$	$^3F_4$	$^1P_1$	$^1F_3$	$^3S_1$	ϵ_1	$^3D_1$	$^3D_2$	$^3D_3$	ϵ_3	$^3G_3$
Analysis A[a]	5.44	7.03	1.10	−1.25	−22.02	15.49	−2.84	1.32	−2.78	2.09	−21.5	−4.2	15.1	6.6	−18.3	27.5	4.5	7.1	−1.6
	±0.44	±0.17	±0.08	±0.49	±0.24	±0.16	±0.12	±0.24	±0.16	±0.12	±1.6	±0.7	±1.2	±0.7	±1.3	±1.1	±0.9	±0.4	±0.6
Bressel-Kerman[b]	5.57	7.75	0.98	−0.95	−21.43	16.54	−2.53	1.59	−2.88	1.68	−22.2	−4.2	17.6	4.2	−22.2	23.0	3.2	6.7	−2.9
Hamada-Johnston[c]	5.54	7.87	0.94	−2.05	−20.75	17.37	−2.51	1.47	−2.82	1.55	−22.1	−4.0	19.9	4.8	−22.1	22.8	3.0	6.5	−2.7
Yale[d]	1.07	8.54	1.21	−1.75	−21.62	16.74	−2.42	0.85	−3.46	1.15	−23.5	−2.0	21.9	6.5	−20.1	23.0	3.3	5.9	−2.3
Lomon-Feshbach[e]	3.05	8.03	1.14	−1.50	−21.07	17.04	−2.87	2.12	−3.11	1.43	−14.3	−4.5	17.3	0.4	−23.1	25.7	2.4	6.6	−3.3
Reid[f]	5.11	7.15	(0.72)	−0.95	−21.62	15.98	−2.64	0.96	(−3.46)	(0.60)	−29.2	(−3.7)	18.1	5.0	−21.2	24.1	(−4.3)	(7.0)	(−2.3)

[a] Present work. [b] Reference 13. [c] Reference 14. [d] Reference 15. [e] Reference 16. [f] Reference 17.

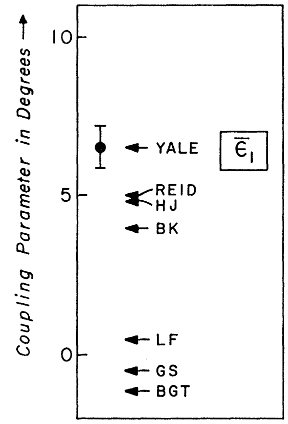

FIG. 2. Values of the $J=1$ nuclear-bar coupling parameter ϵ_1 for a number of models (see text). The datum is from analysis A of Table V.

Four of the five potential models listed in Table VII have values of the $J=1$ coupling parameter $\bar{\epsilon}_1$ which are too low. These are plotted in Fig. 2, along with the analysis-A and Yale[15] potential values, which are in agreement. The value of $\bar{\epsilon}_1$ is a very important one for nuclear physics, since it is strongly correlated with the relative proportions of central and tensor potentials in the $^3S_1$-$^3D_1$ states. The binding energy per nucleon (B.E./N) in nuclear matter in particular is strongly influenced by this "central/tensor ratio." For example. the Brueckner-Gammel-Thaler[16] (BGT) and HJ potentials have very different proportions of central and tensor potentials, yet give almost identical $^3S_1$ and $^3D_1$ phase shifts from 0 to 320 MeV. Their values of $\bar{\epsilon}_1$ are very different, however, as can be seen in Fig. 2. It has been estimated[17] that there is at least a 4–5-MeV/nucleon difference in the nuclear-matter B.E./N for the $^3S_1$-$^3D_1$ state contributions between the HJ and BGT potentials. Unfortunately, the BGT potential, which gives the better B.E./N, gives the worse $\bar{\epsilon}_1$. The same can be said of that particular Green-Sawada[18] (GS) potential which was used by McCarthy and Kohler[19]; its $\bar{\epsilon}_1$ is seen in Fig. 2 to be about 10 standard deviations below the "experimental" analysis-A value. The Reid, HJ, and BK values of $\bar{\epsilon}_1$ could probably be made to agree with the experimental value by making small changes in appropriate model parameters, but the Lomon-Feshbach[20] (LF) model is apparently a different case.[21]

Note that all of the model $\bar{\epsilon}_1$ values were compatible with the very much larger errors found in analyses made

[15] K. E. Lassila, M. H. Hull, Jr., H. M. Ruppel, F. A. MacDonald, and G. Breit, Phys. Rev. **126**, 881 (1962).
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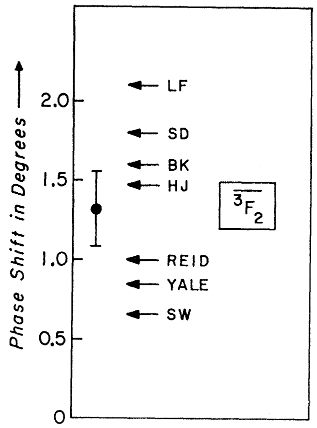

FIG. 3. Values of the nuclear-bar $^3F_2$ phase shift for a number of models (see text). The datum is from analysis A of Table V.

prior to the publication of the current np data set. This can be easily seen by comparing the model values in Table VII with those of AM-IV in the last column of Table V.

The extreme difference between the values of the $^1P_1$ and $\bar{\epsilon}_1$ phase shifts of the LF model and the values obtained in analysis A made it desirable to check for the possible presence of another χ^2 minimum in the neighborhood of the LF values. To do this, an analysis was made with ϵ_1 kept fixed at the value predicted by the LF model. All other phase shifts were released, but from the values predicted by the LF model. The phase shifts corresponding to the minimized χ^2 of this analysis were then used as the starting phase shifts in a second analysis. In this one, $\bar{\epsilon}_1$ was now also released. The final phase shifts were the same as those of analysis A. There is no χ^2 minimum near the phases produced by the LF model.

The Reid potential[22] will require more than minor changes in parameters in order to fit the data earn 210 MeV. This potential fixes many of the phase shifts at their OPE values when they are in fact very different from those values. In particular, the $^3D_3$ phase used by the Reid potential is the OPE value of $-4.3°$, which is almost 10 standard deviations from the value required by the data.

Another interesting phase is the $^3\bar{F}_2$, shown in Fig. 3. The predictions of two semitheoretical models are shown, those of LF[20] and of Scotti and Wong[23] (SW). The former used a phenomenological modification of pre-baryon-resonance perturbation theory, while the latter used a Reggeized-N/D-one-boson-exchange model. Both the LF and SW $^3\bar{F}_2$ values are an unacceptable three standard deviations away from the experimental analysis-A value, and are on opposite sides of it. This is rather surprising, since one would expect that the centrifugal barrier would make this $L=3$ phase shift sensitive only to the intermediate and long-range parts of the interaction, and these are just the parts of the interaction that a peripheral theory should give best. One would at least hope for agreement between theoretical models. The $(1\pi+2\pi)$-exchange value of Signell and Durso[24] (SD) is also shown in Fig. 3. It is two standard deviations too high, but one would expect that the addition of $3\pi(\omega)$ exchange would bring it into agreement with the experimental value. However, neither the sign nor the magnitude of the *uncorrelated* 3π-exchange contribution is known at this time.

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