Possible Classification of Exchange-Degenerate Trajectories*

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(Received 26 June 1968)

A possible classification of nearly all the known mesons in terms of exchange-degenerate trajectories is given. This classification is consistent with the quark model and is also in general agreement with the behavior of the trajectories inferred from high-energy phenomenology. Predictions are made for the quantum numbers of some resonances. Arguments for the Lorentz-pole classification of these trajectories are also given.

I. INTRODUCTION

WITH the population explosion of resonances on the one hand and the large number of Regge parameters on the other, any reasonably approximate symmetry which relates these quantities to each other is highly desirable. Some two years ago, a classification of vector and tensor trajectory octets (and singlets) together with their residues, based on $SU(3)$ symmetry and exchange degeneracy was given by one of the authors.[1] This formalism was applied to high-energy phenomenology and numerous relations among various reactions were obtained which are in good agreement with the corresponding data.[1,2] Based on the same model, numerous other relations have been obtained by various authors and are generally in agreement with the existing data, although the extent of agreement also depends upon the additional assumptions made.[3]

In the present article, in addition to the exchange-degenerate vector-tensor case, six more octets (and singlets) of pairwise exchange-degenerate trajectories are considered. A brief account of certain phenomenological aspects of this model has been given by one of the authors in a previous letter,[4] and further detailed study will be given in later publications. In Sec. 2, we present the $I=1$ and $I=\frac{1}{2}$ members of the eight octets of pairwise exchange-degenerate trajectories. Section 3 gives a quark-model description which is closely related to the work of Sinanoğlu.[5] Finally, in Sec. 4 we discuss the Lorentz-pole classification.

II. EIGHT FUNDAMENTAL OCTETS

For our purposes, the following nomenclature seems to be convenient. Let us designate the vector-meson nonet (octet plus a singlet with arbitrary mixing angle) of trajectories by a^-, where the minus sign denotes the signature. Similarly, the tensor-meson nonet is designated by a^+, and the exchange-degenerate vector-tensor nonet is simply referred to as a. We use the index $(2I+1)$ to denote the isomultiplet member of the octet. Thus, a_3^- denotes the ρ trajectory, a_3^+ the A_2 trajectory, and a_3 the exchange-degenerate ρ-A_2 trajectory. Similarly, a_2 is the K^*-$K^{*\prime}$ trajectory and a_1 is the V_8-T_8 trajectory.[1] The $SU(3)$ singlet member of this nonet is given the index zero, a_0. In a like manner, we consider the two exchange-degenerate nonets of trajectories to which the $A_1(1080)$ belongs. This is called b. Furthermore, the exchange-degenerate pseudoscalar-axial vector trajectories to which π and $B(1210)$ belong are designated as c. Finally, the two exchange-degenerate nonets to which $\pi_V(1016)$ belongs are called d. The order is chosen to agree with the corresponding intercepts of the isotriplet trajectories. The nonet a is the highest, the b next, and then c and d as inferred from the masses of the resonances as described below.

In what follows we shall only consider the isotriplet and isodoublet members of these octets. The isosinglet members are not discussed here primarily because of the lack of information on the singlet-octet mixing angles.

The $I=1$ members are shown in Fig. 1.

These trajectories have already been discussed by Sutherland.[6] There are, however, some minor differences between our trajectory assignments of the $I=1$ resonances.

a_3 *trajectory*. This trajectory was first given in Ref. 1 based on the ρ and A_2 masses. The intercept thus obtained was utilized in a phenomenological study of high-energy scattering.[1,2] Numerous $I=1$ resonances were later obtained by Focacci *et al.*,[7] and it has often been pointed out[6,8–10] that in addition to ρ and A_2, the

* Supported in part by the Air Force Office of Scientific Research, Office of Aerospace Research, United States Air Force, under AFOSR grant No. 1294-67.

[1] A. Ahmadzadeh, Phys. Rev. Letters **16**, 952 (1966).
[2] A. Ahmadzadeh and C. H. Chan, Phys. Letters **22**, 692 (1966).
[3] See for example, A. Ahmadzadeh, Phys. Letters **22**, 669 (1966); R. C. Arnold, Phys. Rev. **153**, 1506 (1967); C. A. Levinson, N. S. Wall, and H. T. Lipkin, Phys. Rev. Letters **17**, 1122 (1966); Jong-Ping Hsu and S. Okubo, Phys. Letters **24B**, 179 (1967); A. Ahmadzadeh and R. J. Jacob, Phys. Rev. **160**, 1359 (1967); Yuval Ne'eman and John D. Reichert, Phys. Rev. Letters **18**, 1226 (1967); A. Borgese, F. Buccella, M. Colocci, and E. Celeghini, Nuovo Cimento **49**, 199 (1967).
[4] A. Ahmadzadeh, Phys. Rev. Letters **20**, 1125 (1968).
[5] Oktay Sinanoğlu, Phys. Rev. Letters **16**, 207 (1966); Phys. Rev. **145**, 1205 (1966).
[6] D. G. Sutherland, Nucl. Phys. **B2**, 157 (1967).
[7] M. N. Focacci *et al.*, Phys. Rev. Letters **17**, 890 (1966).
[8] G. Goldhaber, in *Proceedings of the Thirteenth Annual International Conference on High-Energy Physics, Berkeley, California 1966* (University of California Press, Berkeley, 1967), p.103; also University of California Lawrence Radiation Laboratory Report No. UCRL-17696, 1967 (unpublished).
[9] R. H. Dalitz, in *High Energy Physics* (Gordon and Breach Science Publishers, Inc., New York, 1966), p. 253; in *Proceedings of the Thirteenth Annual International Conference on High Energy Physics, Berkeley, California 1966* (University of California Press, Berkeley 1967), p. 215—see this article also for a number of other references.
[10] D. Cline, Nuovo Cimento **45A**, 750 (1966).

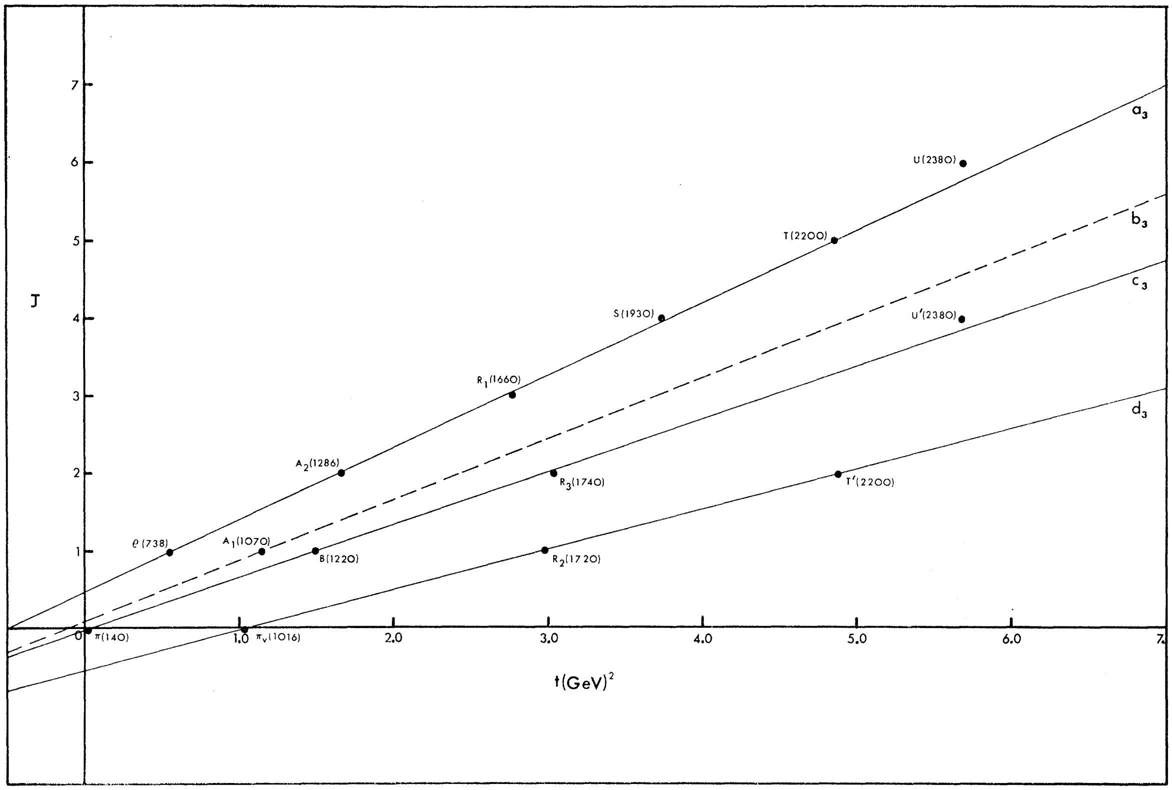

FIG. 1. Isotriplet trajectories.

four resonances $R_1(1660)$, $S(1930)$, $T(2200)$, and $U(2380)$ may constitute the next four occurrences on this trajectory. Their masses all fit remarkably well on a straight line. Crennell *et al.*,[11] have discussed the quantum numbers of the R_1 meson. The G parity and the tentative spin and parity which they report are in agreement with this assignment of R_1. Except for the isospin, the quantum numbers of S, T, and U are not known. One can only speculate that future determinations of them will be in agreement with the present assignment based upon their positions on the a_3 trajectory.

b_3 trajectory. So far, the only resonance in this sequence that has been identified is the $A_1(1070)$. The status of A_1 as a resonance has improved recently due to arguments by Chew and Pignotti.[12] Experimentally, there seems to be no other known resonance on the b_3 trajectory. In Fig. 1, we draw a dashed line through A_1 only to indicate that such a trajectory should be expected to fill in from future experimental data with roughly the slope and intercept shown.

c_3 trajectory. This is the trajectory passing through the $\pi(140)$ and the $B(1220)$ mesons. Extrapolating through higher values of J, we find that $R_2(1720)$ and $R_3(1750)$ are in the right mass region to qualify as the first π recurrence. However, R_2 is known to have positive G parity while the G parity of R_3 appears to be negative as it has been observed to decay into an odd number of charged tracks only.[13] We have therefore chosen R_3 to be the Regge recurrence of the π meson implying that its spin and parity should be $J^P=2^-$. We should also mention that the $U(2380)$ mass fits quite well on this, as well as on the a_3 trajectory described above. In fact, the U peak may actually be a superposition of two resonances.

d_3 trajectory. Here we take the trajectory to pass through $\pi_V(1016)$ and $R_2(1720)$. The first π_V recurrence should be at 2210 MeV which is very close to the mass of the $T(2200)$ resonance which we placed on the a_3 trajectory. Again, the possibility of the superposition of two resonances is not ruled out.

The slopes (in GeV^{-2}) and intercepts of the above trajectories are given by the following equations:

$$\begin{aligned} a_3&:\ \alpha(t)=0.45+0.94t,\\ b_3&:\ \alpha(t)=?,\\ c_3&:\ \alpha(t)=-0.013+0.68t,\\ d_3&:\ \alpha(t)=-0.54+0.52t. \end{aligned} \tag{1}$$

Let us now consider the $I=\frac{1}{2}$ members of these octets. Under exact $SU(3)$ symmetry, the isodoublet

[11] D. J. Crennell *et al.*, Phys. Rev. Letters **18**, 323 (1967).
[12] G. F. Chew and A. Pignotti, Phys. Rev. Letters **20**, 1078 (1968).
[13] C. Baltay *et al.*, Phys. Rev. Letters **20**, 887 (1968).

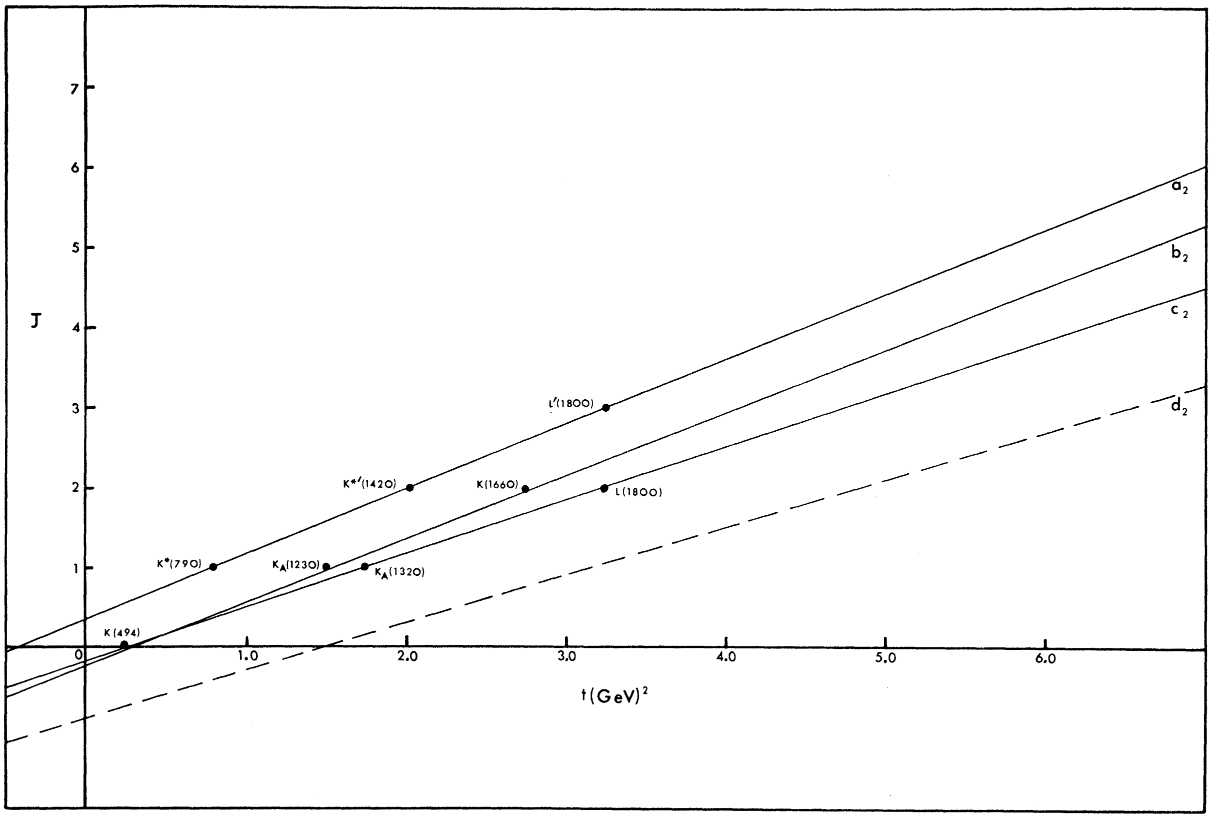

FIG. 2. Isodoublet trajectories.

and the corresponding isotriplet trajectories would coincide. However, the mass splittings dictate trajectory splittings. The isodoublet trajectories as shown in Fig. 2 are as follows:

a_2 trajectory. This trajectory was first described in Ref. 1 on the basis of the $K^*(890)$ and $K_V(1420)$ mesons. It has already been pointed out[14] that this trajectory passes through 1800 MeV which is the mass of the L meson.[15,16] However, from its decay mode the spin and parity assignment 2^- is favored.[16] We would like to suggest that the $L(1800)$ peak may actually consist of a $J^P=3^-$ as well as a $J^P=2^-$ resonance. This idea fits well with L clusters in the quark model as discussed by Goldhaber.[8]

b_2 trajectory. The $K_A(1230)$ and the newly observed $K^*(1660)$ [17] combine to form this trajectory. It would appear that this trajectory also predicts a spin-zero meson at about the same mass as $K(494)$. However, a glance at the quark model discussed in Sec. 3 shows that this point on the trajectory is a $^3S_0$ quark-antiquark system. Therefore, the trajectory chooses nonsense at this point.

c_2 trajectory. The masses of $K(494)$, $K_A(1320)$, and $L(1800)$ fit beautifully on a straight line approximating this trajectory. This trajectory is remarkably parallel to its isotriplet companion given in Fig. 1.

d_2 trajectory. We have found no experimentally observed meson belonging to this trajectory. A dashed line has been drawn in Fig. 2 in the hope of future discoveries.

The slopes and intercepts of the $I=\frac{1}{2}$ trajectories are given by the following equations:

$$\begin{aligned} a_2&: \alpha(t)=0.35+0.82t,\\ b_2&: \alpha(t)=-0.21+0.80t,\\ c_2&: \alpha(t)=-0.16+0.67t,\\ d_2&: \alpha(t)=?. \end{aligned} \tag{2}$$

As will be discussed in Sec. III, all the trajectories that we have considered couple to quark-antiquark systems. They also couple at a $B\bar{B}$ vertex, where B and $\bar{B}$ are members of the baryon and antibaryon octets, respectively. We should also mention that we have found possible trajectory assignments for nearly all the known resonances with the following exceptions. We have not considered the $\delta(956)$ whose existence is in doubt[18] and whose quantum numbers are unknown.

[14] A. Ahmadzadeh, Nuovo Cimento **46A**, 415 (1966).
[15] See for example, Aachen-Berlin-CERN-London-Vienna-Collaboration, Phys. Letters **22**, 357 (1966).
[16] W. Majerotto *et al.*, Nucl. Phys. **B2**, 449 (1967).
[17] M. Jobes *et al.*, Phys. Letters, **26B**, 49 (1967).
[18] For references for and against the δ, see Arthur H. Rosenfeld *et al.*, Rev. Mod. Phys. **40**, 77 (1968).

TABLE I. Trajectory assignments for isotriplet and isodoublet boson resonances. Masses are given in MeV. Quantum numbers given as $I^G(J^P)^C$ for isotriplets and $I(J^P)$ for isodoublets.

a_3	a_2	b_3	b_2	c_3	c_2	d_3	d_2
$\rho(765)$	$K^*(890)$	$A_1(1070)$	$K_A(1230)$	$\pi(140)$	$K(494)$	$\pi_V(1016)$	[a]
$1^+(1^-)^-$	$\frac{1}{2}(1^-)$	$1^-(1^+)^+$	$\frac{1}{2}(1^+)$	$1^-(0^-)^+$	$\frac{1}{2}(0^-)$	$1^-(0^+)^+$	
$A_2(1305)$	$K_V(1420)$	[a]	$K^*(1660)$	$B(1220)$	$K_A(1320)$	$R_2(1720)$	
$1^-(2^+)^+$	$\frac{1}{2}(2^+)$		$\frac{1}{2}(2^-)$	$1^+(1^+)^-$	$\frac{1}{2}(1^+)$	$1^+(1^-)^-$	
$R_1(1660)$	$L(1800)$[b]		(2000)[c]	$R_3(1740)$	$L(1800)$[b]	$T(2210)$[d]	
$1^+(3^-)^-$	$\frac{1}{2}(3^-)$		$\frac{1}{2}(3^+)$	$1^-(2^-)^+$	$\frac{1}{2}(2^-)$	$1^-(2^+)^+$	
$S(1930)$	(2110)[c]		(2290)[c]	(2100)	(2180)[c]	(2610)[c]	
$1^-(4^+)^+$	$\frac{1}{2}(4^+)$		$\frac{1}{2}(4^-)$	$1^+(3^+)^-$	$\frac{1}{2}(3^+)$	$1^+(3^-)^-$	
$T(2200)$[d]	(2380)[c]		(2550)[c]	$U(2430)$[d]	(2500)[c]	(2960)[c]	
$1^+(5^-)^-$	$\frac{1}{2}(5^-)$		$\frac{1}{2}(5^+)$	$1^-(4^-)^+$	$\frac{1}{2}(4^-)$	$1^-(4^+)^+$	
$U(2380)$[d]	(2630)[c]		(2780)[c]	(2710)[c]	(2780)[c]	(3260)[c]	
$1^-(6^+)^+$	$\frac{1}{2}(6^+)$		$\frac{1}{2}(6^-)$	$1^+(5^+)^-$	$\frac{1}{2}(5^+)$	$1^+(5^-)^-$	

[a] Slope and intercept of trajectory unknown.
[b] C_2 trajectory assignment preferred by tentative quantum number measurement (see text). Possibility exists of two resonances within peak.
[c] Approximate masses are predicted for as yet unobserved peaks.
[d] The quantum numbers assigned here have not been established. Possibility exists of two resonances within peak.

The $\pi_A(1650)$ would have been a candidate for the b_3 trajectory but it seems to have the wrong G parity. A third exception is the recently observed $K^*(1280)$ [19] which seems unlikely to have $J^P=0^+$ according to its decay modes. A 0^+ particle with this mass is needed for the d_2 trajectory.

The discussion in this section is summarized in Table I.

III. QUARK CONTENT OF TRAJECTORIES

Here we give a brief account of the Regge trajectories discussed above in terms of bound states of (perhaps purely mathematical) quark-antiquark systems. There have been numerous discussions on quark models. In addition to the contributions of Zweig[20] and Dalitz,[9] an early effort has been made by Sinanoğlu to unravel the quark chemistry of mesons.[5] Although Sinanoğlu's work does not seem to have been well publicized, he has given an extensive discussion of the meson masses in terms of the energy levels of the $q\bar{q}$ system bound by various spin-orbit, central, tensor, etc., forces. Our brief discussion here is closely similar to Sinanoğlu's work. More recently it has been observed by Collins, Johnson, and Squires[21] that it is not possible to produce, for example, the ρ trajectory as a resonance state of two pions if the trajectory is to be a real analytic function of (energy)^2, with only a right-hand cut. But, if such a trajectory is a bound state of a very heavy quark and an antiquark, then they find that a straight-line trajectory in the observed region is quite possible. Furthermore, as first pointed out in Ref. 1, if we take the quark model seriously, the exchange degeneracy would be a natural consequence of a quark-antiquark bootstrap model. Since the u channel would consist of two quarks, and as there are no known systems consisting of two quarks, the u-channel contributions must be very weak.

The trajectories a through d that we have considered have the following spectroscopic notation in the quark model:

$$\begin{aligned} &a:\ {}^3S_1,\ {}^3P_2,\ {}^3D_3,\ {}^3F_4,\ \cdots \quad (J=L+S,\ S=1),\\ &b:\ {}^3P_1,\ {}^3D_2,\ {}^3F_3,\ {}^3G_4,\ \cdots \quad (J=L,\ S=1),\\ &c:\ {}^1S_0,\ {}^1P_1,\ {}^1D_2,\ {}^1F_3,\ \cdots \quad (J=L,\ S=0),\\ &d:\ {}^3P_0,\ {}^3D_1,\ {}^3F_2,\ {}^3G_3,\ \cdots \quad (J=L-S,\ S=1). \end{aligned} \tag{3}$$

The parity, G parity, and charge-conjugation quantum numbers of these sequences are given by

$$P=(-1)^{L+1},\quad G=(-1)^{L+S+I},\quad \text{and}\quad C=(-1)^{L+S}, \tag{4}$$

where $I=0$, 1 is the isospin of the multiplet. Of course, the $I=\frac{1}{2}$ members are not eigenstates of G parity and charge conjugation. We can however define the G parity and charge conjugation of an octet to be that of its $I=0$ or $I=1$ members. These quantum numbers are related to the spin of the particle on the trajectory in each case by the relations

$$\begin{aligned} &a:\ P=(-1)^J,\quad G=(-1)^{J+1},\quad C=(-1)^J;\\ &b:\ P=(-1)^{J+1},\quad G=(-1)^J,\quad C=(-1)^{J+1};\\ &c:\ P=(-1)^{J+1},\quad G=(-1)^{J+1},\quad C=(-1)^J;\\ &d:\ P=(-1)^J,\quad G=(-1)^{J+1},\quad C=(-1)^J. \end{aligned} \tag{5}$$

We should remark that the mesons on the b and c trajectories which have the same spin and parity have opposite G parity and charge conjugation. This is easily seen from Eqs. (5). On the other hand, as the same equations show, mesons of the same spin and parity on a and d trajectories also have the same G

[19] M. Jobes *et al.*, Phys. Letters **26B**, 30 (1967).
[20] G. Zweig, *Symmetries in Elementary Particle Physics* (Academic Press Inc., New York, 1965), p. 192.
[21] P. D. B. Collins, R. C. Johnson, and E. J. Squires, Phys. Letters **26B**, 223 (1968).

parity and charge conjugation. Thus, aside from the difference in masses, only the orbital angular momentum is different between the two trajectories. But, whereas the G parity and charge conjugation quantum numbers of these mesons can be determined, there is no simple way of observing the orbital angular momentum L. Thus, for example, aside from its mass, there seems to be no way of deciding whether the $\rho(760)$ meson is a $^3S_1$ belonging to the a trajectory or a $^3D_1$ belonging to d.

Finally, let us emphasize that it would indeed be interesting if the above classification would close the chapter on the resonances. It is seen here that the classification of the trajectories given in the previous section is in agreement with the quark model. There may also be resonances on the daughter trajectories of a, b, c, and d which could be interpreted as quark-antiquark systems. We must look to future experiments for an answer to the interesting question: Is this the whole story, or are there still more resonances which in fact do not couple to $q\bar{q}$ (or $B\bar{B}$) systems?

IV. LORENTZ-POLE CLASSIFICATION

A brief account of the Lorentz-pole classification of the c trajectories together with some corresponding phenomenological implications has already been given in a previous letter.[4] Here we give a possible Lorentz-pole classification of all the trajectories considered in the previous sections without discussing their phenomenological applications.

It is well known that the a trajectories contribute to the nucleon-nucleon total cross sections,[22] and therefore belong to the type I classification described by Freedman and Wang.[23] By this, we mean that the two exchange-degenerate nonets, a^+ and a^-, together with their daughters are associated with two nonets of exchange-degenerate Lorentz poles, $A^\pm$, at $t=0$. Note that we speak of $SU(3)$ and exchange degeneracy as though [aside from $SU(3)$ mass splittings] they were exact symmetries. This need not be the case, and if there are significant symmetry breakings, they can easily be incorporated. We do of course expect that both $SU(3)$ and exchange degeneracy will hold to a reasonable approximation and, indeed, there are numerous phenomenological tests in support of this optimism.[1–3]

The two exchange-degenerate nonets of b trajectories presumably belong to the type II classification. That is, the $b^\pm$ nonets together with their even daughters plus conspiring $b'^\pm$ nonets (with trajectory intercepts one unit below those of b) and their even daughters are associated with two exchange-degenerate Lorentz poles, $B^\pm$, at $t=0$.

In agreement with the Lorentz-pole classification considered for the pion trajectory by numerous authors,[24] we take the c trajectories to be of type III. Again, this means that the two nonets $c^\pm$ with their corresponding parity doublets $c'^\pm$ (which have the same intercepts at $t=0$), and a third pair of exchange-degenerate nonets $c''^\pm$ (with intercepts one unit below), form a class III conspiracy. This is the conspiring class considered in Ref. 4 where a number of phenomenological implications have been suggested. In this case, each member of the c nonet together with its even daughters and the corresponding members of c' and c'' and their even daughters are associated with an exchange-degenerate Lorentz-pole nonet C.

The classification of the d trajectories will have to be determined from future analysis. For example, if the difference between the π^-p and π^+p total cross sections should require exchange of another trajectory, (in addition to the ρ trajectory, a_3^-) with the ρ quantum numbers, then the d nonet should be classified as type I. Otherwise, it is type III.

Note added in proof. Recently evidence for spin and parity assignment of $J^P=3^-$ for $R_1(1660)$ (or the g meson) has been reported by Johnston *et al.*[25]—in agreement with our expectation. Also recent experimental data on $K_A(1320)$ and $L(1800)$ has been reported by Bartsch *et al.*[26]; their analysis gives spin parity $J^P=1^+$ (with 2^- much less probable) for the $K_A(1320)$ and $J^P=1^+$, 2^-, 3^+—for the L meson. The assignment $J^P=1^+$ for $K_A(1320)$ is of course in agreement with our predicted assignment. As for the L meson, our prediction is of course $J^P=2^-$ and we also require another resonance in this mass region with spin parity $J^P=3^-$.

22 See for example, A. Ahmadzadeh and E. Leader, Phys. Rev. **134**, B1058 (1964).

23 D. F. Freedman and J. M. Wang, Phys. Rev. **160**, 1560 (1967); A. Sciarino and M. Toller, University of Rome, Report No. 108, 1967 (unpublished).

24 See for example, F. Arbab and J. Dash, Phys. Rev. **163**, 1603 (1967).

25 T. F. Johnston, J. D. Prentice, N. R. Steenberg, T. S. Yoon, A. F. Garfinkel, R. Morse, B. Y. Oh, and W. D. Walker, Phys. Rev. Letters **20**, 1414 (1968).

26 J. Bartsch *et al.*, Aachen-Berlin-CERN-London-Vienna Collaboration (unpublished).