CP Violation and Coherent Decays of Kaon Pairs*

HARRY J. LIPKIN†
Palmer Physical Laboratory, Princeton University, Princeton, New Jersey
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The possible experimental use of correlations between two neutral-kaon decays is discussed. Kaon pairs produced either in proton-antiproton annihilation from an s state or in φ decay are in a p wave, which is forbidden by Bose statistics for identical bosons. The observance of a particular decay mode, i.e., $\pi^+\pi^-$, for one kaon implies that the other kaon is forbidden to decay at the same time in this mode. Thus a kaon beam can be constructed that is just that coherent linear combination of K_S and K_L whose amplitudes for decay into this mode cancel one another. The existence of $\Delta I=\frac{3}{2}$ transitions leading to the $I=2$ final state in K^0 decay can be tested by looking for double decays where one of the kaons decays into $\pi^+\pi^-$ and the other into $\pi^0\pi^0$. This double decay is allowed at the same time only if both $\Delta I=\frac{1}{2}$ and $\Delta I=\frac{3}{2}$ amplitudes are present in the kaon decay. Detailed calculations and experimental implications are discussed.

I. INTRODUCTION

WHEN a pair of neutral kaons is produced in a given reaction, the subsequent decays of the two kaons are not independent. Correlations between the two decays appear because of the quantum-mechanical coherence effects of the type first pointed out by Einstein, Podolsky, and Rosen.[1] In this paper, we discuss these correlations and their possible use in experimental studies of CP violation.

We first review the Einstein-Podolsky-Rosen effect in proton-antiproton annihilation into two neutral kaons without CP violation. If the initial state is an s state, only the K_1K_2 final state is allowed[2]; the states K_1K_1 and K_2K_2 are forbidden. This result is most simply derived from parity conservation and Bose statistics. The initial state has odd parity, while Bose statistics forbids odd-parity states for two identical spinless bosons. The K_1K_1 and K_2K_2 systems can only have symmetric spatial wave functions that have even parity.

The K_1 decays into two pions, whereas the K_2 does not. Thus if one of the two kaons decays into two pions, the other had better not, or it would violate either Bose statistics or parity conservation in strong interactions. However, how does the second kaon know that the first one has really decayed into two pions or that it has decayed at all? It might have interacted with a proton and turned into a K^+ in a charge-exchange scattering. In that case, the second kaon had better be a $\bar{K}^0$, or it would violate strangeness conservation in strong interactions. As a $\bar{K}^0$ it would be an equal mixture of K_1 and K_2 with a probability of about 50% for decay into two pions. Thus in order to know whether or not it should decay into two pions, the second kaon must know whether the other kaon has decayed or has interacted far away. Somehow it does, and these correlations must always be remembered dealing with annihilation and decay products. They are not indicative of any "final-state interaction" or long-range dynamical forces, just quantum mechanics.

Now that we have CP violation,[3] we have the short- and long-lived kaons K_S and K_L rather than K_1 and K_2, and both the K_S and K_L are observed to decay into two pions. However, the essential physics of the above derivation requires only parity conservation and Bose statistics and does not depend on CP. The conclusion is then still valid. We cannot observe two identical bosons in a p wave. If one kaon decays into $\pi^+\pi^-$, the other cannot. However, these kaon states now have new names in terms of K_S and K_L. The state that is observed to decay into two pions is not specified as K_S or K_L, because both are allowed to decay in this way. However, the other kaon, which cannot decay into $\pi^+\pi^-$, is now specified as a definite coherent mixture, $\alpha|K_S\rangle+\beta|K_L\rangle$, with coefficients α and β chosen to make the decay matrix elements of the two terms to the state $|\pi^+\pi^-\rangle$ cancel one another. Thus the process of proton-antiproton annihilation at rest allows us to construct a "coherent neutral kaon beam," which is a particular linear combination of K_S and K_L with well-defined coefficients and relative phase. The same would be true for any other process, such as φ-meson decay,[4] which leads to a zero-strangeness, odd-parity two-kaon state. By constructing an apparatus that looks at both kaon decays and selects a particular decay mode on one side, one makes a coherent kaon beam on the other side, which is just that particular mixture of K_S and K_L forbidden to decay into the mode selected for the first decay. Such coherent K-beam experiments might be useful in the future. In the remainder of this paper, we express this verbal argument formally and obtain expressions that can be generalized to take into account various factors that are neglected in the simple argu-

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† On leave of absence from the Weizmann Institute of Science, Rehovoth, Israel.

[1] For a description of the Einstein-Podolsky-Rosen effect, see D. Bohm, *Quantum Theory* (Prentice-Hall, Inc., Engelwood Cliffs, N. J., 1951), p. 611. The application to positronium annihilation is discussed by A. S. Wightman, Phys. Rev. **74**, 1813 (1948) to kaon pairs by T. B. Day, *ibid.* **121**, 1204 (1961).

[2] B. d'Espagnat, Nuovo Cimento **20**, 1217 (1961); G. A. Snow, Phys. Letters **1**, 213 (1962).

[3] J. M. Christensen, J. W. Cronin, V. L. Fitch, and R. Turlay, Phys. Rev. Letters **13**, 138 (1964). For a review of CP violation and a detailed bibliography see L. B. Okun' and C. Rubbia, in *Proceedings of the Heidelberg International Conference on Elementary Particles*, edited by H. Filthuth (North-Holland Publishing Co., Amsterdam, 1968), p. 301.

[4] J. J. Sakurai, Phys. Rev. Letters **9**, 472 (1962).

ment above, such as the time dependence of the decays and the presence of background.

Much of the material in Secs. II and IV should be immediately obvious to anyone who is intimately familiar with the subtleties of the Einstein-Podolsky-Rosen effect, and has appeared in earlier treatments.[5] It is included for the convenience of those readers who do not find everything in the previous paragraph trivially obvious.

II. DECAY CORRELATIONS WITHOUT TIME DEPENDENCE

Consider two neutral kaons produced either in the decay of a φ meson, or in proton-antiproton annihilation from an s state. The wave function for this two-kaon state a very short time after production, before any K-decays can take place, is

$$|i\rangle=\tfrac{1}{2}\sqrt{2}[\,|K^0(z);\bar{K}^0(-z)\rangle-|K^0(-z);\bar{K}^0(z)\rangle],\quad (1)$$

where we have chosen the z axis as the direction of the momenta of the kaons in the c.m. system; (z) means that the particle is moving in the positive z direction and $(-z)$ means that the particle is moving in the negative z direction.

The following physical features of the wave function (1) are important for our discussion: The kaon moving in the positive z direction may be either a K^0 or $\bar{K}^0$ with equal possibilities. The same is true for the kaon moving in the negative z direction. However, these probabilities are not independent. They are correlated as a result of conservation laws which are valid in the strong interactions which produce the kaon pair.

(a) Since the initial state has zero strangeness, it follows from conservation of strangeness that if one of the kaons is a K^0 the other kaon must be a $\bar{K}^0$.

(b) Since the initial state has odd parity, parity conservation in the strong decay or annihilation process requires the negative sign between the two terms in (1). Note that, for a system of two spinless particles, odd parity implies odd angular momentum and odd behavior under permutations. Although parity is not conserved in the subsequent decay of the kaon, both angular momentum and permutation symmetry are conserved. Thus the odd relative phase of the two terms in (1) is not changed in the subsequent decays.

The presence of two terms in the wave function (1) with a definite and well-defined phase between them gives rise to correlation effects in the decays that we now discuss in detail. We first neglect the effect of the decay on the wave function (1); i.e., we consider decays that take place in a very short time after the production of the kaon pair. Consider the amplitude describing the following decay: The kaon moving in the positive z direction decays into a state D_1; the kaon moving in the negative z direction decays into a state D_2. The final states D_1 and D_2 are each completely specified; e.g., $\pi^+\pi^-$, $\pi^0\pi^0$, $\pi\pi$ in a state of definite isospin, or $\pi^+\pi^-\pi^0$ in a relative s state with all momenta specified. The transition amplitude is then

$$\langle D_1(z);D_2(-z)|T|i\rangle=\tfrac{1}{2}\sqrt{2}(\langle D_1|T|K^0\rangle\langle D_2|T|\bar{K}^0\rangle - \langle D_1|T|\bar{K}^0\rangle\langle D_2|T|K^0\rangle),\quad (2)$$

where $\langle D_1|T|K^0\rangle$ is the transition matrix element for the decay of K^0 in the decay mode D_1, etc.

The transition matrix element (2) vanishes if the two decay modes D_1 and D_2 are the same, because the two terms of the right-hand side of (2) cancel one another,

$$\langle D_1(z);D_1(-z)|T|i\rangle=0.\quad (3)$$

If CP is conserved in the kaon decay, this result is trivial. The K_1 and K_2 are the linear combinations of K^0 and $\bar{K}^0$, which are eigenstates of CP, and CP conservation allows a transition from the φ or $\bar{p}p$ only into K_1K_2 and not into K_1K_1 or K_2K_2. If CP is conserved in the kaon decay, the states K_1 and K_2 are not allowed to decay into the same decay mode, and the selection rule (3) follows directly from CP conservation.

However, we have obtained the selection rule (3) from Bose statistics *without the use of CP conservation.* Any state D_1 that is allowed by angular-momentum conservation to be a final state in K^0 decay must have zero total angular momentum. Two identical states of this type must behave like two identical spinless bosons, which cannot be in an antisymmetric spatial state.

III. POSSIBLE EXPERIMENTAL TEST OF THE $\Delta I=\frac{1}{2}$ RULE

The selection rule (3) forbids the decay of both kaons into two charged pions even if CP is violated. The decay of both kaons into two neutral pions is similarly forbidden. This consequence of quantum mechanics and Bose statistics tells us nothing about kaon physics. However, we now consider the possibility of observing two charged pions in the positive z direction and two neutral pions in the negative z direction. We choose D_1 to be the zero-isospin two-pion state. The selection rule (3) implies that we cannot observe the simultaneous decay of both kaons into a *zero-isospin* two-pion state. Thus, if the zero-isospin state *is the only one allowed for the decay* (e.g., as a result of some general principle like the $\Delta I=\frac{1}{2}$ rule),[6] *the simultaneous observation of two-pion*

[5] C. P. Enz and R. R. Lewis, Helv. Phys. Acta 38, 873 (1965); V. L. Lyuboshitz and E. O. Okonov, Yadern. Fiz. 4, 1194 (1966) [English transl. Soviet J. Nucl. Phys. 4, 859 (1967)]. The author is indebted to V. L. Telegdi for calling the latter reference to his attention.

[6] For a review of the present status of the $\Delta I=\frac{1}{2}$ rule, see L. Wolfenstein, in *Proceedings of the Heidelberg International Conference on Elementary Particles*, edited by H. Filthuth (North-Holland Publishing Co., Amsterdam, 1968), p. 289. Since the $\Delta I=\frac{1}{2}$ rule is probably violated to about 5% in K_S decays, the discussion in Sec. III of this paper is slightly inaccurate. However, at this time it appears to be extremely difficult to obtain an observable effect even if the K_S decay is pure $\Delta I=\frac{1}{2}$ and the K_L decay is pure $\Delta I=\frac{3}{2}$. Any corrections which are of the order of 5% of this effect are rather academic. If precise measurements should become possible at some future time, the results should be compared with the precise prediction, given by Eq. (9). This shows that the effect is proportional to the quantity $(\eta_{+-}-\eta_{00})$ in the conventional notation, and vanishes if $\eta_{+-}=\eta_{00}$.

decays for both kaons is forbidden regardless of charge. One would then not observe two charged pions in one decay and two neutral pions in the other. However, if kaon decays are allowed into the $I=2$ state as well as the $I=0$ state, two-kaon decays into both these states would be allowed and would be observed as a decay of one kaon into two charged pions and a decay of the other into two neutral pions. This result now gives information about kaon physics, not just about conservation laws and quantum mechanics.

This simple example illustrates the basic principles behind the use of decay correlations to give information about kaon physics in the light of *CP* violation. The presence of a two-pion final state with $I=2$ in the *CP*-violating mode can be measured *directly* by an experimental quantity which gives a null result if this final state is absent. The presence of both $I=0$ and $I=2$ final states would be indicated by the observation of coincidences between charged decays on one side and neutrals on the other. If the $I=2$ state is absent, no such coincidences between two charged or two neutral decays can be observed. Thus, although the experiment might be very difficult to carry out, the results once obtained would have a very direct interpretation, in contrast to measurements of the branching ratio into neutral and charged modes.

IV. TIME DEPENDENCE OF THE DECAY CORRELATION

We now consider decays which take place an appreciable time after the formation of the kaon pair in annihilation or φ decay. This is probably of greater experimental interest because of the presence of large background effects in the region of space and time close to the production of the kaon pair. The previous treatment is easily extended by introducing the linear combinations of K^0 and $\bar{K}^0$, which have definite lifetimes, K_L and K_S. These are conventionally defined by the relations[3]

$$|K_L\rangle = p|K^0\rangle - q|\bar{K}^0\rangle, \tag{4a}$$

$$|K_S\rangle = p|K^0\rangle + q|\bar{K}^0\rangle. \tag{4b}$$

The wave function (1) represents the state of the two-kaon system at time $t=0$. We express this state in terms of the kaon states K_L and K_S by substituting Eq. (4),

$$|i(t=0)\rangle = (1/2\sqrt{2}pq)[\,|K_L(z);K_S(-z)\rangle - |K_L(-z);K_S(z)\rangle], \tag{5}$$

where Bose statistics has allowed us to use the identity

$$|K_L(z);K_S(-z)\rangle \equiv |K_S(-z);K_L(z)\rangle. \tag{6}$$

This can be justified rigorously by expressing the states (6) in a second-quantized notation with creation operators for the two-kaon states operating on the vacuum, and using boson commutation rules. With fermions, there would be additional phase factors. Although the states K_L and K_S are not necessarily orthogonal, this does not affect our treatment.

We now calculate the decay amplitude in a straightforward manner, as in the previous case. The decay amplitude for the states K_L and K_S appear in our results rather than the amplitudes for the states K^0 and $\bar{K}^0$. The time dependence of the decays is completely taken into account by noting that the individual states K_L and K_S decrease exponentially in time and change their relative phase in a manner described by complex coefficients Γ_L and Γ_S. Thus if we are interested in the transition amplitude for observing the decay mode D_1 at a time t_1 in the positive z direction and the decay mode D_2 at a time t_2 in the negative z direction, we obtain the result

$$2\sqrt{2}pq\langle D_1(t_1,z);D_2(t_2,-z)|T|i\rangle = (e^{-(\Gamma_L t_1+\Gamma_S t_2)/2}\langle D_1|T|K_L\rangle\langle D_2|T|K_S\rangle - e^{-(\Gamma_L t_2+\Gamma_S t_1)/2}\langle D_1|T|K_S\rangle\langle D_2|T|K_L\rangle). \tag{7}$$

This result can be rewritten in a more convenient form:

$$2\sqrt{2}pq\langle D_1(t_1,z);D_2(t_2,-z)|T|i\rangle = e^{-(\Gamma_L+\Gamma_S)(t_1+t_2)/4} \times\{(\langle D_1|T|K_L\rangle\langle D_2|T|K_S\rangle - \langle D_1|T|K_S\rangle \times\langle D_2|T|K_L\rangle)\cosh[\tfrac{1}{4}(\Gamma_S-\Gamma_L)(t_1-t_2)] + (\langle D_1|T|K_L\rangle\langle D_2|T|K_S\rangle + \langle D_1|T|K_S\rangle \times\langle D_2|T|K_L\rangle)\sinh[\tfrac{1}{4}(\Gamma_S-\Gamma_L)(t_1-t_2)]\}. \tag{8}$$

For the particular case where both decays are observed with the same delay, $t_1=t_2=t$, this expression reduces to the simple form

$$2\sqrt{2}pq\langle D_1(t,z);D_2(t,-z)|T|i\rangle = e^{-(\Gamma_L+\Gamma_S)t/2}(\langle D_1|T|K_L\rangle\langle D_2|T|K_S\rangle - \langle D_1|T|K_S\rangle\langle D_2|T|K_L\rangle). \tag{9}$$

This result for $t_1=t_2$ differs from the result (2) for the simple case only by the multiplicative time-dependent factor and reduces to the simple case for $t=0$. We also obtain a generalization of the selection rule (3)

$$\langle D_1(t,z);D_1(t,-z)|T|i\rangle = 0. \tag{10}$$

Thus all the discussion of two-pion decays based on the selection rule (3) holds as well for the case where both decays are measured with equal time delays. However, additional information is obtained by measuring the decays at different times. For the particular case where the same decay is measured on both sides, Eq. (8) reduces to

$$\sqrt{2}pq\langle D_1(t_1,z);D_1(t_2,-z)|T|i\rangle = e^{-(\Gamma_L+\Gamma_S)(t_1+t_2)/4}\sinh[\tfrac{1}{4}(\Gamma_S-\Gamma_L)(t_1-t_2)] \times\langle D_1|T|K_L\rangle\langle D_1|T|K_S\rangle. \tag{11}$$

This amplitude goes through zero at $t_1=t_2$ but its variation as a function of t_1-t_2 can be used to measure quantities of interest in kaon physics. In particular, $\Gamma_S-\Gamma_L$ is complex and the imaginary part is related to the K_L-K_S mass difference.

The essential physical feature of these phenomena can be summarized as follows: The observation of a particular decay mode D_1 at a time t_1 in the positive z direction imposes constraints on the kaon beam observed in coincidence in the negative z direction, which force it to be a definite linear combination of K_L and K_S. It is just that linear combination which at time t_1 is forbidden to decay into the particular mode D_1. Thus we have a mechanism for producing a neutral kaon beam which is just that linear combination of K_S and K_L whose contributions to the decay mode D_1 are equal and opposite at time t_1, and therefore cancel one another. This kind of experiment can be considered as a mechanism for producing a neutral kaon beam with peculiar coherence properties determined by the decay mode and the time in the other channel. One might design all kinds of experiments in which different decay modes are observed at different times and regenerators are put in various places. If such experiments are at all feasible they offer many possibilities for ingenious and interesting measurements.

V. EFFECTS OF BACKGROUND

Unfortunately, one never observes the decay of a φ meson. One observes many events in which kaon pairs exhibit a mass spectrum peaked around the φ mass. Thus if one selects kaon pairs in a given mass region, one always has a background of kaon pairs which are not from φ decay and which do not have the wave function (1). In annihilation there might be a background from annihilation in the p state, which has even parity. However, the kaons are still not completely uncorrelated, since they must still conserve strangeness. It is therefore possible to take into account and minimize the effects of background by a judicious choice of the time delay of the experiment.

The background of kaon pairs in the region of the φ mass peak or from p-state annihilation consists of states which may have odd or even parity, but total strangeness zero. All odd-parity background states must have the desired form (1). Their decays lead to exactly the same results as considered above and cause no complications. The even-parity background has a wave function of the form

$$|b\rangle=\tfrac{1}{2}\sqrt{2}(\,|K^0(z);\bar{K}^0(-z)\rangle+|K^0(-z);\bar{K}^0(z)\rangle)\,, \quad (12a)$$

$$=(1/2\sqrt{2}pq)\times(\,|K_S(z);K_S(-z)\rangle-|K_L(z);K_L(-z)\rangle). \quad (12b)$$

In the background term, both kaons are either K_S or K_L. This allows experiments to be designed which minimize the effect of the background. The background term would be much more troublesome if it had the same form as the desired contribution (5) but with a positive relative phase of the two terms. Such a term is excluded by strangeness conservation, since it consists exclusively of states of strangeness ± 2.

The decay of the background term (12) can be calculated in a straightforward manner, and leads to the result

$$2\sqrt{2}pq\langle D_1(t_1,z);D_2(t_2,-z)|T|b\rangle = e^{-\Gamma_S(t_1+t_2)/2}\langle D_1|T|K_S\rangle\langle D_2|T|K_S\rangle + e^{-\Gamma_L(t_1+t_2)/2}\langle D_1|T|K_L\rangle\langle D_2|T|K_L\rangle. \quad (13)$$

This expression (13) exhibits characteristics of the background that enable a choice of experimental parameters to minimize its effects. Consider, for example, the case where both decays, like the two-pion decays, are allowed for K_S and forbidden for K_L in the limit of CP conservation. The matrix elements $\langle D_1|T|K_L\rangle$ and $\langle D_2|T|K_L\rangle$ are therefore much smaller than $\langle D_1|T|K_S\rangle$ and $\langle D_2|T|K_S\rangle$. If the background is of the same order of magnitude as the desired contribution, we see by comparing Eqs. (8) and (13) that the background dominates at very short times because of the first term in (13), which has two large matrix elements, and also at very long times because of the second term, which has the slowest exponential decay factors. The desired decays are largest compared to background in an intermediate time region. One would hope to choose K^0 pairs in an experiment so that the background is minimal to begin with, although probably not small enough to enable it to be neglected at short times. The proper choice of experimental parameters would require a time long enough to reduce the background to manageable proportions, but probably not long enough to maximize the signal-to-noise ratio, because the signal itself decreases with time.

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