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Interference, Mixing, and Angular Correlations in Decays of Boson Resonances*

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Two new features of boson resonances are discussed. (1) Interference and mixing in decays of different bosons having common decay channels, and (2) competition between partial-wave amplitudes when more than one is allowed. Several examples are considered. Interference between $f^0 \to K\bar{K}$ and $A_2^0 \to K\bar{K}$ is discussed. The axial-vector nonets are discussed in detail, as well as the application to higher-spin resonances.

WE wish to point out and discuss two new features of decays of higher-boson resonances which are not found in the lowest-lying pseudoscalar and vector multiplets: (a) the possibility of interference and mixing in the decays of different bosons having common decay channels, and (b) the dynamical significance of decays where more than one partial wave is allowed and the relative magnitudes and phases of these partial waves can be measured. If these effects are not properly taken into account they can cause considerable confusion in the analysis of boson decays. If they are properly understood, they can be utilized to provide new methods for analysis and interpretation of data.

I. INTERFERENCE AND MIXING IN A SIMPLE NONET EXAMPLE

As a simple example, consider the vector mesons ρ, ω, and φ in the hypothetical case where the masses are nearly degenerate and well above 1 GeV, so that the $K\bar{K}$ decay mode is allowed and has appreciable phase space. Although much of this discussion is not relevant to the low-mass physical vector nonet where the $K\bar{K}$ channel is closed for the ρ and ω, it applies to any higher boson nonet which is allowed by angular momentum and parity conservation to decay into two pseudoscalar mesons.

Interference in K-Pair Decays of Neutral Mesons

Consider a reaction in which a neutral vector-meson is produced and is observed in the K^+K^- or $K^0\bar{K}^0$ decay modes. Since the observation of the final state does not determine the particular vector-meson intermediate states, the contributions from the ρ^0, ω, and φ are coherent. A mass plot of the K-pair system through the region of the ρ, ω, and φ can therefore be very complicated. Instead of a superposition of three Lorentzian curves with different centers and widths, there is a superposition of three *amplitudes* whose relative magnitude and relative phase are determined by the production mechanism.[1]

This coherence and interference can be observed between the isovector ρ and the isoscalar ω and φ, even though the states have different isospin and G parity, because the final states K^+K^- and $K^0\bar{K}^0$ individually are not eigenstates of isospin nor of G parity. However, if the neutral and charged decay rates are added, the interference terms between the ρ and the isoscalar mesons cancel, because the charged and neutral contributions are equal and opposite. This can be seen explicitly as follows:

Consider the decay of a neutral vector-meson state which is some coherent linear combination of ρ^0, ω, and φ,

$$|V\rangle=\alpha|\rho^0\rangle+\beta|\omega\rangle+\gamma|\varphi\rangle, \tag{1}$$

where α, β, and γ are constants which define the particular mixture and are determined in a given experiment by the production mechanism. The decay amplitudes for the K-pair decay modes are then given by

$$\langle K^+K^-|V\rangle=\alpha\langle K^+K^-|\rho^0\rangle+\beta\langle K^+K^-|\omega\rangle +\gamma\langle K^+K^-|\varphi\rangle, \tag{2a}$$

$$\langle K^0\bar{K}^0|V\rangle=\alpha\langle K^0\bar{K}^0|\rho^0\rangle+\beta\langle K^0\bar{K}^0|\omega\rangle +\gamma\langle K^0\bar{K}^0|\varphi\rangle. \tag{2b}$$

However, the charged and neutral decay amplitudes for each individual meson are related by isospin conservation, with opposite relative phase for the isoscalar and isovector states

$$\langle K^+K^-|\rho^0\rangle=-\langle K^0\bar{K}^0|\rho^0\rangle, \tag{3a}$$

$$\langle K^+K^-|\omega\rangle=\langle K^0\bar{K}^0|\omega\rangle, \tag{3b}$$

$$\langle K^+K^-|\varphi\rangle=\langle K^0\bar{K}^0|\varphi\rangle. \tag{3c}$$

We thus find

$$\langle K^0\bar{K}^0|V\rangle=-\alpha\langle K^+K^-|\rho^0\rangle+\beta\langle K^+K^-|\omega\rangle +\gamma\langle K^+K^-|\varphi\rangle. \tag{4}$$

The decay rates for the coherent state $|V\rangle$ into charged and neutral K pairs are proportional to the squares of the matrix elements (2a) and (4), respectively. The $\rho\omega$ interference terms are equal and opposite for the two cases and similarly for $\rho\varphi$, while all other terms are equal. Thus if we measure the total decay rates of the V states into both charged and neutral K pairs,

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[1] G. Goldhaber, Phys. Rev. Letters 19, 976 (1967).

the $\rho\omega$ and $\rho\varphi$ interference terms drop out, while the $\omega\varphi$ interference term remains.

$$\Gamma(V\rightarrow KK)\propto|\langle K^+K^-|V\rangle|^2+|\langle K^0\bar{K}^0|V\rangle|^2 =2|\alpha\langle K^+K^-|\rho^0\rangle|^2+2|\beta\langle K^+K^-|\omega\rangle +\gamma\langle K^+K^-|\varphi\rangle|^2. \quad (5)$$

Missing-mass spectrometer experiments automatically sum over all possible decay modes and cancel out interference effects between states having different eigenvalues of conserved quantities, like the $\rho\omega$ and $\rho\varphi$ interference terms in this example. However, counter and bubble-chamber experiments generally look at *either* the charged *or* the neutral K pairs and these interference effects should be observed.

Mixing in a Single Nonet

Because of the difference in isospin and G parity between the isovector ρ and the isoscalar ω and φ, there is no mixing between them by strong interactions which conserve isospin. Isospin and G-parity conservation also provide selection rules for decays into final states which, unlike $K\bar{K}$, are eigenstates of G parity. For example, the G-parity eigenstate 2π is allowed for the ρ and forbidden for ω and φ, while the $\rho\pi$ state is allowed for ω and φ and forbidden for ρ.

No such selection rule prohibits the mixing of the ω and φ by strong interactions. If $SU(3)$ were an exact symmetry, the $SU(3)$ singlet and octet states would not mix and would correspond to the physically observed mesons. However, $SU(3)$ is known to be broken very badly in the classification of the vector-meson nonet; the observed mesons are mixtures which are far from the pure singlet and octet states.

Summary of Interference and Mixing

The two effects which appear in this simple example can be summarized as follows:

(a) *Interference*. Any two overlapping resonances with a common decay mode can contribute coherently to this mode. If the resonances have different eigenvalues of conserved quantum numbers (e.g., G parity or isospin), the interference can only be observed in decay modes which are not eigenstates of these conserved quantities. The interference effects then disappear when the total decay rate into all channels is considered. For example, interference between states having different isospin or G parity, is observable by looking at final states involving K mesons in a particular charged mode, which are neither isospin nor G-parity eigenstates. In principle, interference is also observable between resonances having different spins and parities by looking at decays at definite angles. These interference effects, however, disappear when the total decay rate over all angles is considered and are considerably blurred at a given angle by finite angular resolution.

(b) *Mixing*. Two neighboring resonances having the same quantum numbers for all quantities conserved in strong interactions and differing only by $SU(3)$ quantum numbers can be mixtures of $SU(3)$ eigenstates as in the case of ω and φ. The decay modes of these particles should all be the same, as any mode allowed for one is also allowed for the other. Thus both mixing and the interference effects discussed above are present in this case.

Dynamical Mixing and Selection Rules

Even in the case of the physical vector mesons, there is one decay mode which is in principle allowed for two states. The 3π decay mode is allowed for both ω and φ. However, a dynamical selection rule[2] forbids the 3π (or $\rho\pi$) decay mode of the φ. Although this is obtainable from symmetry[3] and quark-model[4] arguments, there is also a dynamical argument suggesting that when mixing is possible, *the mixing angle may be just that one which forbids the dominant common decay mode.*[2] This occurs when the mixing is primarily due to transitions through the dominant decay mode ($\rho\pi$ in the particular case of $\omega\varphi$ mixing) as the intermediate state. The diagonalization of the 2×2 mixing matrix then leaves one of the two eigenstates nearly decoupled from the dominant decay mode. The 2^+ nonet has a similar selection rule decoupling the f^* from the dominant 2π mode. It would be interesting to see whether such dynamical selection rules exist for higher resonances, particularly in cases where there are no alternative quark-model or symmetry derivations.

Possible Interference in the 2^+ Nonet

Although the physical masses of the ρ and ω mesons do not allow the $K\bar{K}$ decay mode, similar arguments hold for any meson nonet in which the masses are sufficiently high to allow this decay. The first such case is the 2^+ nonet, where interference is to be expected between the f^0 and A_2 decays in the $K\bar{K}$ mode. This interference may account for the difficulty in measuring this decay mode and establishing the branching ratio.[5]

II. MIXING AND INTERFERENCE BETWEEN TWO OCTETS: AXIAL-VECTOR MESONS

Further interference and mixing possibilities exist when there are two nonets of mesons having the same spin and parity. The 1^+ nonets which include the A_1 and B mesons are examples of such a pair. The quark model[6] predicts a series of such nonet pairs with the

[2] A. Katz and H. J. Lipkin, Phys. Letters **7**, 44 (1963).
[3] F. Gürsey, A. Pais, and L. A. Radicati, Phys. Rev. Letters **13**, 299 (1964); H. J. Lipkin, *ibid.* **13**, 590 (1964); **14**, 513 (1965); H. J. Lipkin and S. Meshkov, *ibid.* **14**, 670 (1965).
[4] W. Thirring, Acta. Phys. Austriaca Suppl. **III**, 294 (1966). The equivalence of the quark model and $SU(6)_W$ derivations is discussed by H. J. Lipkin, Nucl. Physics **B1**, 597 (1967).
[5] G. Goldhaber (private communication).
[6] R. H. Dalitz, in *Proceedings of the Thirteenth International Conference on High-Energy Physics, Berkeley 1966* (University of California Press, Berkeley, 1967), p. 215.

spin and parity assignments 1^+, 2^-, 3^+, etc. Since these states are all forbidden to decay into two pseudoscalar mesons by angular-momentum and parity conservation, we consider the vector-pseudoscalar decay mode.

Nonstrange Decays of Nonstrange Mesons

Consider first the nonstrange axial-vector mesons and those decay modes which involve only nonstrange particles, and therefore are eigenstates of G parity. For simplicity, we consider only $SU(3)$ octet states and neglect octet-singlet mixing. The isovector members of the two octets are the A_1 and B mesons. We denote the corresponding nonstrange isoscalars by η_A and η_B, respectively. These four states have different isospin and G-parity assignments and therefore cannot be mixed by strong interactions. However, interference can still occur in decays *even to G-parity eigenstates*. The A_1 and η_B have the same G parity although they have different isospin and opposite behavior under charge conjugation, and similarly for the B and η_A. The $\rho^+\pi^-$ and $\rho^-\pi^+$ decay modes are allowed for both the A_1 and the η_B because the individual charge states are eigenstates of G but not of I nor of C. However, the observed interference must have opposite signs in the $\rho^+\pi^-$ and $\rho^-\pi^+$ states as a result of either I or C invariance, and thus cancel in the total decay rate. The explicit relations analogous to Eqs. (1)–(5) are easily written for the decay of a coherent mixture of A_1 and η_B.

$$|M_A\rangle=\alpha|A_1\rangle+\beta|\eta_B\rangle. \tag{6}$$

From either charge conjugation or isospin conservation, we have

$$\langle\rho^+\pi^-|A_1\rangle=+\langle\rho^-\pi^+|A_1\rangle, \tag{7a}$$

$$\langle\rho^+\pi^-|\eta_B\rangle=-\langle\rho^-\pi^+|\eta_B\rangle. \tag{7b}$$

Thus,

$$|\langle\rho^+\pi^-|M_A\rangle|^2+|\langle\rho^-\pi^+|M_A\rangle|^2=2|\alpha\langle\rho^+\pi^-|A_1\rangle|^2 +2|\beta\langle\rho^+\pi^-|\eta_B\rangle|^2. \tag{8}$$

The neutral decay mode $\rho^0\pi^0$ is allowed for η_B but forbidden for A_1 by either isospin conservation or charge conjugation. There is therefore no interference in this channel.

Interference effects might account for difficulties in observing the neutral A_1 and the η_B. They would not be present in looking at the charged A_1 because there is no charged counterpart of the η_B.

The analogous interference between the B and η_A states would require a common decay mode, like πA_1, into two *different isovector bosons* having the same G parity, in order to construct an even-G state which is not automatically also an eigenstate of C or of isospin. Such states do not seem to exist below the mass of the B.

Strange Decay Modes of Nonstrange States

Interference between the A_1 and B states cannot occur for final states which are G-parity eigenstates because any decay allowed as a final state for one is forbidden for the other. The situation would be different if the $K^*\bar{K}$ decay were allowed energetically, and would be analogous to the $K\bar{K}$ decays discussed above for the ρ and ω. For the charged meson decays both initial and final states are isospin eigenstates, but the final states are not eigenstates of G parity. As in the other cases, when the two initial states have different eigenvalues of conserved quantum numbers, the interference term must cancel out in the total decay rate. Here the cancellation comes about because of the two channels which go into one another under G parity. From G invariance,

$$\frac{\langle K^{*+}\bar{K}^0|A_1^+\rangle}{\langle\bar{K}^{*0}K^+|A_1^+\rangle}=-\frac{\langle K^{*+}\bar{K}^0|B^+\rangle}{\langle\bar{K}^{*0}K^+|B^+\rangle}=1. \tag{9}$$

The total decay rate of a mixed state is proportional to

$$|\langle K^{*+}\bar{K}^0|\alpha A_1^+ +\beta B^+\rangle|^2+|\langle\bar{K}^{*0}K^+|\alpha A_1^+ +\beta B^+\rangle|^2 =2|\alpha\langle K^{*+}\bar{K}^0|A_1^+\rangle|^2+2|\beta\langle K^{*+}\bar{K}^0|B^+\rangle|^2. \tag{10}$$

Mixtures of A_1 and B mesons could be produced in reactions from initial states like πN or KN which are not eigenstates of G parity. Effects of coherence and interference would then be seen in the $K^*\bar{K}$ decay mode which would have coherent contributions from both the A_1 and B intermediate states. These effects are not observed because the masses of the particles are such that the $K^*\bar{K}$ decay channel is closed by energy conservation.

Decays of Strange Mesons

A different situation arises in the case of the strange isodoublet K^* mesons[1,7–9] We denote the strange members (K^*) of the A_1 and B octets by K_A and K_B, respectively. The dominant decay modes $K^*\pi$ and ρK are allowed for both K_A and K_B states. These decays are analogous to the $K^*\bar{K}$ decay for the A_1 and B. In the limit of $SU(3)$ symmetry, conserved "parities" G_U and G_V analogous to G parity can be defined by replacing isospin by U spin or V spin in the definition of G parity.[9] The neutral and charged K's are eigenstates of G_U and G_V, respectively. However, the charged ρ and π mesons are not eigenstates of either of these parities, just as the K mesons are not eigenstates of G parity. Thus there is no selection rule forbidding $K^*\pi$ and ρK final states for either of these decays. If the K_A and K_B are produced coherently in some experiment, they contribute coherently to the ρK and $K^*\pi$ final states.

[7] H. J. Lipkin, in *Proceedings of the Heidelberg International Conference on Elementary Particles*, edited by H. Filthuth (North-Holland Publishing Co., Amsterdam, 1968), p. 253.
[8] R. Gatto and L. Maiani, Phys. Letters **26B**, 95 (1967).
[9] G. L. Kane and H. S. Mani, Phys. Rev. **171**, 1533 (1968).

If $SU(3)$ is broken, G_U and G_V parities are not conserved. There can then be mixing, analogous to $\omega\varphi$ mixing, between the K_A and K_B states, even though G parity remains conserved and prevents mixing of the corresponding nonstrange states.

We now consider the decay of a mixed state

$$|M_K\rangle=\alpha|K_A\rangle+\beta|K_B\rangle. \quad (11)$$

For the $K^*\pi$ and ρK decay modes the branching ratio is unity in the $SU(3)$ limit except for differences in kinematic (phase space) factors for the two final states. However, because the two octets have opposite charge-conjugation behavior, the A_1-octet decay is described with D coupling and the B-octet decay with F coupling. The relative phases of the $K\rho$ and $K^*\pi$ decay amplitudes are thus opposite for the two cases

$$\langle K\rho|K_A\rangle=\langle K^*\pi|K_A\rangle, \quad (12a)$$

$$\langle K\rho|K_B\rangle=-\langle K^*\pi|K_B\rangle, \quad (12b)$$

the decay amplitudes for the mixed state (11) are then

$$\langle K^*\pi|M_K\rangle=\alpha\langle K^*\pi|K_A\rangle+\beta\langle K^*\pi|K_B\rangle, \quad (13a)$$

$$\langle K\rho|M_K\rangle=\alpha\langle K^*\pi|K_A\rangle-\beta\langle K^*\pi|K_B\rangle. \quad (13b)$$

The decay rates are proportional to the squares of these amplitudes. Thus the interference term must have opposite phase for the two decay modes.

Information about the relative phases of the mixed state produced in a given experiment can be deduced from assumptions about the production mechanism. For example, a reasonable assumption suggested by Goldhaber[1] for production in peripheral reactions is the combination of vector and Pomeron exchange. The relative amounts and relative phase of K_A and K_B depend upon the relative amounts of vector and Pomeranchukon exchange, which vary with energy. One can therefore attempt to describe the shapes of the resonance curves using these theoretical models. The extreme variation in possible shapes has been described in detail by Goldhaber.

In $\bar{p}p$ annihilation, the initial state is an isospin mixture of $I=0$ and $I=1$. If the two isospin channels annihilate into different mixtures of $K_A\bar{K}$ and $K_B\bar{K}$, the mixing observed in $\bar{p}p$ annihilation will be different in the different charge states. For annihilation at rest, where all these decays must come from the $^3S_1$ state, there should be two different mixed states of the form (11) produced. One mixture should appear in all the charged final states, the other in all the neutral final states. The two mixtures should have different values of α and β. Both the $K^*\pi$ and $K\rho$ decay modes can be observed and the large variety of charge states of the $KK\pi\pi$ system formed after the vector meson decay can all be studied. The set of final states can be divided into four groups according to (a) whether the initial axial-vector meson was charged or neutral and (b) according to whether its decay mode was $K^*\pi$ or $K\rho$. The mass plot of the $KK\pi\pi$ system should be the same for all final states within a given one of the four groups. The charged axial-vector meson and neutral axial-vector meson groups should be completely independent, while the $K^*\pi$ and $K\rho$ group within either the charged or neutral group should show opposite interference. Since the initial state is an eigenstate of charge conjugation, final states related by charge conjugation show the same behavior; i.e., any mixed state (11) and its antiparticle are produced with the same value of the parameters α and β.

III. DECAY CORRELATIONS FOR DECAYS WITH TWO PARTIAL WAVES

The states 1^+, 2^-, 3^+, etc. are not allowed to decay into two pseudoscalar mesons but have two independent couplings allowed for the decay into vector-pseudoscalar states. For the 1^+ case, these correspond to s and d waves in the final state. For the general case of a meson with spin J and parity $(-1)^{(J+1)}$, these correspond to orbital angular momenta $L=J+1$ and $L=J-1$ in the final state. The two decay amplitudes can also be labeled by the helicity of the outgoing vector meson in the Lorentz frame where the initial state is at rest. By parity conservation, the amplitudes for helicity $+1$ and -1 are equal. There are therefore two independent amplitudes: longitudinal (helicity$=0$) or transverse (helicity$=\pm1$). The helicity amplitudes are, of course, linear combinations of the two allowed orbital angular-momentum states.

In a simple description where all of the bosons are considered as elementary fields, one would expect the contributions to the decay from different orbital waves to differ by kinematic factors depending upon the final-state momentum, because different centrifugal barriers are present for the two cases (or because the simplest couplings have different powers of momentum). In descriptions based on the quark model[4] on $SU(6)_W$ invariance,[10] or on superconvergent sum rules, it is natural to use helicity states rather than those of definite orbital angular momentum. Predictions are made in which the final state is dominated by either the longitudinal or transverse helicity states of the vector meson.[11] A pure helicity state is a linear combination of the two orbital waves with a momentum-independent mixing determined by Clebsch-Gordan coefficients. These predictions are thus qualitatively different from the conventional case where the ratio of s- to -d contributions depends upon momenta.

The experimental determination of the ratio of the two types of couplings is therefore of interest in choosing between different dynamical theories. A direct measurement of the vector-meson alignment is made by studying

[10] See, e.g., J. J. Coyne, S. Meshkov, and G. B. Yodh, Phys. Rev. Letters **17**, 666 (1966); P. G. O. Freund, A. N. Maheshwari, and E. Schonberg, Phys. Rev. **159**, 1232 (1967).

[11] See, e.g., F. J. Gilman and H. Harari, Phys. Rev. **165**, 1803 (1968).

the angular distribution of the vector-meson decay in the c.m. system of the vector meson, relative to the direction of the pseudoscalar meson emitted in the previous decay of the axial-vector meson.[12] If this angle is denoted by θ, the transverse vector-meson decay mode gives a $\sin^2\theta$ distribution while the longitudinal mode gives $\cos^2\theta$. This test depends only on the dynamics of the decay process and is independent of any production mechanism or alignment of the axial-vector meson. In the case where two 1^+ meson octets are present and mixing phenomena can arise, one may find interesting effects by dividing the events according to different ranges of the value of θ and plotting mass distributions for the different sets.

Detailed Properties of Helicity Amplitudes for Axial-Vector-Meson Decays

Consider the decay

$$A \to V+P\,, \tag{14}$$

where A, V, and P denote the axial-vector, vector, and pseudoscalar mesons. We choose a coordinate system in which the initial axial-vector meson is at rest and the z axis is in the direction of the final-state momenta. From conservation of angular momentum,

$$J_z = J_{Az} = J_{Vz}\,, \tag{15}$$

where J_z, J_{Az}, and J_{Vz} are the z components of the total angular momentum, and of spins of the axial vector and vector meson, respectively. The polarization state of the outgoing vector meson is thus completely determined by the polarization of the initial state. Since parity conservation relates the amplitudes for helicity $+1$ and -1, the decay (14) is described in terms of two independent helicity amplitudes for the outgoing vector meson, one for longitudinal polarization ($J_z=0$) and one for transverse polarization ($J_z=\pm 1$).

Thus the two independent amplitudes for the decay can be described as either s- and d-wave amplitudes or as longitudinal and transverse polarization amplitudes. There is a third description which is relevant when the 1^+, 2^-, etc., meson states are described as quark-antiquark pairs in spin singlet or triplet states[6]: $S=0$ or 1 with orbital angular momentum L and total angular momentum $J=L$. For the axial-vector case the initial state has $L=J=1$. The final state has no internal orbital angular momentum in either the V or P states but may have relative orbital angular momentum between the two mesons. We can thus rewrite the angular-momentum conservation condition (15) in terms of orbital and spin parts as

$$J_z = L_{Az} + S_{Az} = S_{Vz}\,, \tag{16a}$$

$$L_{Az} = S_{Vz} - S_{Az}\,, \tag{16b}$$

[12] This technique for measuring vector-meson alignment was used in the measurement of the K^* spin by W. Chinowsky, G. Goldhaber, S. Goldhaber, W. Lee, and T. O'Halloran, Phys. Rev. Letters **9**, 330 (1962).

TABLE I. Relation between helicity amplitudes and $\Delta L=0$ or ± 1 transitions.

Triplet-spin state	$S=1,\ J=L$	$[{}^3P_1(A)$ for $J^p=1^+]$
	$\Delta L_z=-L_{Az}=$even,	$\lvert S_{Az}\rvert=\lvert S_{Vz}\rvert=1$,
	$\Delta L_z=-L_{Az}=\pm 1$,	$S_{Vz}=$either ± 1 or 0.
Singlet-spin state	$S=0,\ J=L$	$[{}^1P_1(B)$ for $J^p=1^+]$
	$\Delta L_z=-L_{Az}=0$,	$S_{Az}=S_{Vz}=0$
	$\Delta L_z=-L_{Az}=\pm 1$,	$S_{Az}=0,\ S_{Vz}=\pm 1$.

where S_A and S_V denote the contribution of the total quark spin to the angular momentum of the meson. From Eq. (16b), L_{Az} can be 0, ± 1, or ± 2. We can describe the two types of transitions as follows:

(a) $\Delta L_z = \Delta S_z =$ even ,

$$(L_{Az}=0 \text{ or } 2,\quad |S_{Az}|=|S_{Vz}|) \tag{17a}$$

(b) $\Delta L_z = \pm 1 = -\Delta S_z$,

$$(L_{Az}=\mp 1,\quad S_{Az}=S_{Vz}\pm 1)\,. \tag{17b}$$

For the axial-vector case, $L=1$, $L_z=\pm 2$ is excluded, and the case $\Delta L_z=$ even reduces to $\Delta L_z=0$.

This classification into $\Delta L_z=0$ or ± 1 transitions is relevant in several simple models where only one type of transition is allowed and the other is forbidden. In the simple quark-model picture,[4] the transition (14) is described as the emission of a pion by a quark or antiquark which is initially at rest. There cannot be any spin flip of the quark and only the $\Delta L_z=0$ transitions allowed. The same selection rule results from invariance under the collinear W-spin group. On the other hand, if one uses the electrodynamics analogy to the transition (14) with the vector meson replaced by photon, one has an electric dipole transition ($1^+ \to 0^- + \gamma$), which is clearly of the type $\Delta L_z=\pm 1$ for real photons. Although quark-model calculations tend to use the simple model in which the quark emits a pion, arguments of vector dominance of the electromagnetic interaction would lead to the other model. It is not clear *a priori* which description is correct, i.e., which final-state meson is properly considered as a quark-antiquark pair and which as a field quantum.

The relation between the $\Delta L=$ even or ± 1 transitions and the helicity amplitudes is obtained by examining the angular-momentum couplings in the initial axial-vector-meson state. These are conveniently summarized in Table I for the two axial-vector states ${}^3P_1$ and ${}^1P_1$ which correspond to the A_1 and B nonets. Thus, in a model which allows only $\Delta L_z=0$ and $\Delta L_z=2$ transitions, the vector mesons from triplet decays are all transverse while those from singlet decays are longitudinal.[13] For $\Delta L_z=\pm 1$ transitions, all helicities are allowed for the triplet decay, but only the transverse helicity states are allowed for the singlet decay.

Some insight into the nature of the $\Delta L_z=\pm 1$ transitions may be gained from the observation that such a

[13] J. Uretsky, in *High Energy Theoretical Physics*, edited by Hadi Aly (Beirut, to be published); H. J. Lipkin, Phys. Rev. **159**, 1303 (1967).

transition must be described by an operator which transforms like the x or y component of a vector under rotations in L space. The electric dipole moment is clearly such a vector. However, both in the simplified quark model which neglects quark motion perpendicular to the z direction and in the collinear W-spin description, there are no vectors perpendicular to the z direction, and the $\Delta L_z=\pm 1$ transitions must be forbidden. Conversely, if such transitions do, in fact, take place, the transition matrix elements involve the matrix elements of such a vector operator, like the electric dipole moment or the relative momentum of the quark-antiquark pairs. Any model for the meson structure used to describe such transitions must be sufficiently detailed to include such a transverse vector. Note that for the particular case where the vector is the electric dipole moment, the triplet-singlet spin transition is forbidden and there will be no $\Delta L_z=\pm 1$ decays for the triplet state.

Helicity Amplitudes for Higher Spin-Meson Decays

The treatment given above for axial-vector-meson decays is directly applicable to the decays of 2^-, 3^+, etc., mesons in the vector-pseudoscalar decay mode. Equations (14)–(17) are directly applicable to the cases where A is one of these higher spin bosons as well as an axial-vector meson. For any particular spin J of the initial state, the two independent decay amplitudes are $L=J-1$ and $L=J+1$. These reduce to s and d waves for the case $J=1$. In the quark model the meson is described as a quark-antiquark pair with orbital angular momentum $L=J$ and two possible spin states $S=0$ or 1. The discussion of $\Delta L_z=$ even or ± 1 transitions is applicable directly to the general case, because no explicit use was made of the fact that $L=1$ in the treatment of the axial-vector-meson case. Thus the properties of the triplet and singlet states in Table I hold for any L, as well as the prediction that in a model which allows only $\Delta L_z=0$ or $\Delta L_z=$ two transitions, the vector mesons emitted in decays of the triplet-spin states are all transversely polarized while those from the singlet-spin states are longitudinal.

Combined Effects of Mixing and Decay Polarizations

The analysis of the decay of a mixed state described in Eqs. (11)–(13) apply separately to any given decay coupling, either s or d wave, or alternatively either to longitudinal or transverse polarization. Particularly interesting effects are predicted by models which require a $\Delta L_z=0$ transition as discussed above. The vector-meson is emitted in a state of *pure longitudinal* polarization from the ^1P decay and in a state of *pure transverse* polarization from a ^3P decay. In this case, the interference effects between the ^3P and ^1P will not be observed if no decay-correlation measurements are made. The measurement gives an incoherent sum over the longitudinal and transverse polarization states of the vector meson and the interference term cancels out. If, however, events are selected which lie in a given range of the decay angle, a coherent linear combination of the longitudinal and transverse states is observed and interference effects can again be seen. This can be seen explicitly by writing expressions for the transition matrix element for the decay of the axial-vector-meson states into the $K^*\pi$ mode with a subsequent K^* decay at an angle θ with respect to the direction of the axial vector-meson decay. Suppose that

$$\langle K^*\pi,\theta|K_A\rangle = A\sin\theta\,, \tag{18a}$$

$$\langle K^*\pi,\theta|K_B\rangle = B\cos\theta\,, \tag{18b}$$

where A and B are parameters specifying the strength of the decay coupling. The corresponding transition amplitude for the mixed state (1) is then

$$\langle K^*\pi,\theta|M\rangle = \alpha A\sin\theta+\beta B\cos\theta\,. \tag{19}$$

Thus at any angle θ there is an interference term proportional to $\sin\theta\cos\theta$. If the decay is averaged over all angles θ, the interference term goes out.