(fixed, that is, under the little group operations) which is allowed to vanish at the end of the calculations. The Mandelstam invariants in such a case are six in number and are given by

$$s=(p^{(1)}+p^{(2)})^2,\quad s'=(p^{(3)}+p^{(4)})^2 \tag{B12}$$

and similar equations for t, t', u, u'. They are subject to the one constraint

$$s+s'+t+t'+u+u'=\xi^2+8m^2. \tag{B13}$$

The angles φ_i can now be imbedded in this system if we set

$$z_1=2^{-1/2}(s^{1/2}+is'^{1/2})=2^{-1/2}(s+s')^{1/2}e^{i\varphi_1} \tag{B14}$$

and likewise for z_2, z_3. The group $SU(3)$ is allowed to act on (z_1,z_2,z_3) in the standard way [cf. Eq. (2.4) and Ref. 5]. When ξ_μ goes to zero, ξ^2 goes to zero, s', t', u' go, respectively, to s, t, u, and (B13) reduces to the familiar constraint (B.2a).

The situation can be expressed in a picturesque form by saying that our group $SU(3)$ is the remaining ghost group of the five-particle problem when one of the particles collpases to the vacuum.

We conclude with a minor explanation. The three differential operators in (B.10a) are *a priori* defined in their corresponding physical regions and the latter are mutually disjoint. In writing (B.10b) (and in many other remarks in this paper), the analytic continuation of these operators to suitable domains is understood.

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Scalar Density Terms, $K\pi$ and KK Scattering Lengths, and a Symmetry-Breaking Parameter*

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An estimation of s-wave $K\pi$ and KK scattering lengths is made, and a discussion is given of the relation of these quantities to scalar density matrix elements and the relevant symmetry-breaking parameter.

INTRODUCTION

USING low-energy theorems from the $SU(2)\times SU(2)$ current algebra and Adler's partial conservation of axial-vector current (PCAC) self-consistency conditions, Weinberg[1] has estimated the π-π scattering lengths, assuming that a linear expansion of the amplitude into Mandelstam invariants is approximately valid up to threshold. In order to determine some of the coefficients in the above expansion, the isospin properties had to be specified of the scalar density matrix element, or "σ term" $\langle\pi|[A^i,\partial^\mu A_\mu{}^j]|\pi\rangle$, which involves the commutator of an axial-vector charge and divergence of the axial-vector current (i,j are isospin indices or, more generally, unitary spin indices). It turns out that, within the parametrization of Ref. 1, specification alone of the pion vector-charge matrix element and the requirement of no $I=2$ contributions in $[A^i,\partial^\mu A_\mu{}^j]$ completely determine[2] the value of this matrix element:

$$i\langle\pi^l|[A^i,\partial^\mu A_\mu{}^j]|\pi^k\rangle=(m_\pi{}^2)\delta_{ij}\delta_{kl}. \tag{1}$$

On the other hand, Khuri[3] arrived at the same value by applying a low-energy limit directly to the scalar density vertex defined by (1) upon using [the $SU(2)\times SU(2)$ version of] the algebra of scalar and pseudoscalar densities[4] which transform as $(3,3^*)+(3^*,3)$ under $SU(3)\times SU(3)$.

In this paper, an estimation of s-wave $K\pi$ and KK scattering lengths will be made using similar current-algebra techniques, and the relation of these quantities to scalar density terms will be noted. In particular, the scalar density matrix elements appearing in the KK case are estimated by established $SU(3)\times SU(3)$ commutators and PCAC principles, just as in Weinberg's $\pi\pi$ estimation described above. For the $K\pi$ case, additional assumptions are needed and the scalar density matrix elements are determined by: (a) applying a low-energy limit to the scalar density vertex, and (b) using an (approximate) determining relation for the relevant symmetry-breaking parameter κ. The scattering-length determination for the KK case is consistent with this last procedure. Finally, the consistency of our use of the relation for κ itself, Eq. (15), is discussed.

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[1] S. Weinberg, Phys. Rev. Letters **17**, 616 (1966).

[2] Reference 1; Eqs. (16) and (20) determine the coefficient of $\delta_{ab}\delta_{cd}$. In the text, we use the state normalization $\langle p|p\rangle=(2\pi)^3 2p_0\delta(0)$.

[3] N. Khuri, Phys. Rev. **153**, 1477 (1967).

[4] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962); Physics **1**, 63 (1964).

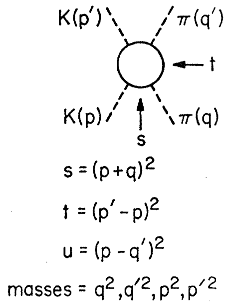

FIG. 1. Off-mass-shell pseudoscalar scattering amplitude.

$K\pi$ SYSTEM

Here there are the two independent s-wave scattering lengths in the $I=\frac{1}{2}$ and $\frac{3}{2}$ states: $a^{1/2}$ and $a^{3/2}$ (the $\bar{K}\pi$ system is related to the $K\pi$ system by a charge-conjugation operation or t-channel crossing symmetry). A linear expansion of the amplitude $A^{I}(s,t,u;\, q^2,q'^2,p^2,p'^2)$ in terms of invariants s, t, u, q^2, q'^2, p^2, and p'^2 is extrapolated to threshold, assuming that there are no $J^P=0^+$ bound states and that unitarity effects do not lead to rapid variations of the amplitude at low energies. (See Fig. 1 for kinematical definitions.)

We write the definite isospin s-channel amplitudes in terms of amplitudes with definite t-channel charge-conjugation properties $\mathfrak{C}=\pm$ satisfying crossing relations.

$$A^{(3/2)}=A^{(+)}-A^{(-)},\qquad A^{(1/2)}=A^{(+)}+2A^{(-)},\tag{2}$$

where

$$A^{(\pm)}(s,t,u;\, q^2,q'^2,p^2,p'^2)=\pm A^{(\pm)}(u,t,s;\, q'^2,q^2,p^2,p'^2)=\pm A^{(\pm)}(u,t,s;\, q^2,q'^2,p'^2,p^2).\tag{3}$$

Using t-channel crossing symmetry (3) and the kinematical condition $s+t+u=q^2+q'^2+p^2+p'^2$, we write the linear expansions with constant coefficients,

$$A^{(+)}(s,t,\cdots)=A+B(s+u)+Ct+D(p^2+p'^2),\qquad A^{(-)}(s,t,\cdots)=A'(s-u).\tag{4}$$

In order to evaluate the coefficients A, B, C, D, and A', we consider low-energy limits in current-commutator identities of the form

$$F^{ij}(s,t,\cdots)=q'^{\mu}q^{\nu}T_{\mu\nu}{}^{ij}-q'^{\mu}\int d^4x\, e^{iq'\cdot x}\times\langle p'|\,\delta(x_0)[A_\mu{}^i(x),A_0{}^j(0)]\,|p\rangle+i\int d^4x\, e^{iq'\cdot x}\times\langle p'|\,\delta(x_0)[A_0{}^i(x),\partial^\nu A_\nu{}^j(0)]\,|p\rangle,\tag{5}$$

where

$$F^{ij}=-i\int d^4x\, e^{iq'\cdot x}\langle p'|\,T\{\partial^\mu A_\mu{}^i(x),\partial^\nu A_\nu{}^j(0)\}\,|p\rangle,$$
$$T_{\mu\nu}{}^{ij}=-i\int d^4x\, e^{iq'\cdot x}\langle p'|\,T\{A_\mu{}^i(x),A_\nu{}^j(0)\}\,|p\rangle,\tag{6}$$

are Fourier transforms of axial-vector current and divergence time-ordered products. Observing the normalization condition $\langle\pi^i|\,\partial^\mu A_\mu{}^i|0\rangle=m_i^2f_i/\sqrt{2}$ ($f_\pi=\sqrt{2}MG_A/g_{\pi NN}$), the following connection will be assumed between the off-mass-shell boson-boson amplitude A^{ij} and the amplitude F^{ij} for low-energy applications:

$$A^{ij}(s,t,\cdots)=\frac{(q^{2\prime}-m_i^2)(q^2-m_j^2)}{(m_i^2f_i/\sqrt{2})(m_j^2f_j/\sqrt{2})}F^{ij}(s,t,\cdots).\tag{7}$$

We now take various low-energy limits in (5) for, say, $A^{(3/2)}=A(K^+\pi^+\to K^+\pi^+)$.

(a) $q\to 0$ (or $q'\to 0$) with the other three particles on their mass shells (Adler's PCAC consistency condition).

$$A^{(3/2)}\to A^{(3/2)}(m_K^2,m_\pi^2,m_K^2;\,0,m_\pi^2,m_K^2,m_K^2)=0$$

or

$$A+2m_K^2(B+D)+m_\pi^2C=0.\tag{8}$$

(b) q', $q\to 0$ with the kaons on mass shell.

$$\begin{aligned}A^{(3/2)}&\to A^{(3/2)}(m_K^2+2p\cdot q,\ 0,\ m_K^2-2p\cdot q;\ 0,0,m_K^2,m_K^2)\\&=[A+2m_K^2(B+D)]-4(p\cdot q)A'+O(q^2,q^{2\prime},q\cdot q'),\\&=(2i/f_\pi^2)\langle K^+(p)|\,[A^{\pi^-},\partial^\mu A_\mu{}^{\pi^+}]\,|K^+(p)\rangle\\&\qquad+2(p\cdot q)/f_\pi^2+O(q\cdot q'),\end{aligned}$$

and

$$A'=-1/2f_\pi^2,\tag{9}$$

$$A+2m_K^2(B+D)=(2/3f_\pi^2)(\sqrt{2}+\kappa)\times\langle K^+(p)|\sqrt{2}u_0+u_8|K^+(p)\rangle.\tag{10}$$

In the last expression, the quantity

$$\langle K^+|\,[A^{\pi^-},\partial^\mu A_\mu{}^{\pi^+}]\,|K^+\rangle$$

has been re-expressed using the (quark) algebra of scalar and pseudoscalar densities[4] together with a particular energy density given by

$$\theta_{00}=\theta_{00}{}^{\times}+u_0+\kappa u_8,$$

where $\theta_{00}{}^{\times}$ is defined to be the $SU(3)\times SU(3)$ invariant piece and κ is the symmetry-breaking parameter. Using

$$[A^i,v^j]=id_{ijk}u_k\quad\text{and}\quad[A^i,u^j]=-id_{ijk}v_k,\tag{11}$$

we have

$$i\left[\int d^3x(\theta_{00}{}^{\times}+u_0+\kappa u_8),A^i\right]=\frac{1}{i}\left[A^i,\int d^3x(u_0+\kappa u_8)\right]=\int d^3x\,\partial^\mu A_\mu{}^i,$$

so that

$$\partial^\mu A_\mu{}^i=-(d_{ii0}+\kappa d_{ii8})v_i,\quad i\neq 0,8.\tag{12}$$

So, for example,

$$[A^{\pi^-},\partial^\mu A_\mu{}^{\pi^+}]=-\tfrac{1}{6}\sqrt{3}(\sqrt{2}+\kappa)[A^{1-i2},v^{1+i2}]$$
$$=-\tfrac{1}{3}i(\sqrt{2}+\kappa)(\sqrt{2}u_0+u_8),$$

to give (10). Suppose we now use a low-energy limit to evaluate the right-hand side of (10),

$$\langle K^+(p')|\sqrt{2}u_0+u_8|K^+(p)\rangle$$
$$=i\sqrt{2}\frac{(m_K{}^2-p'^2)}{m_K{}^2f_K}\int d^4x\, e^{ip'\cdot x}$$
$$\times\langle 0|T\{\partial^\mu A_\mu{}^{K^-}(x)(\sqrt{2}u_0+u_8)\}|K^+(p)\rangle \xrightarrow[(p'\to 0)]{}$$
$$-i\sqrt{2}f_K{}^{-1}\langle 0|[A^{K^-},\sqrt{2}u_0+u_8]|K^+(p)\rangle=\frac{\tfrac{3}{2}m_K{}^2}{(\sqrt{2}-\tfrac{1}{2}\kappa)}.$$

The right-hand side becomes

$$\left(\frac{\sqrt{2}+\kappa}{\sqrt{2}-\tfrac{1}{2}\kappa}\right)\frac{m_K{}^2}{f_\pi{}^2}. \qquad (13)$$

However, this last expression can be greatly simplified by use of a determining relation for the parameter κ,

$$\frac{\langle 0|\partial^\mu A_\mu{}^{\pi^+}|\pi^-\rangle}{\langle 0|\partial^\mu A_\mu{}^{K^+}|K^-\rangle}=\left(\frac{m_\pi{}^2}{m_K{}^2}\right)\left(\frac{f_\pi}{f_K}\right)=\left(\frac{\sqrt{2}+\kappa}{\sqrt{2}-\tfrac{1}{2}\kappa}\right)\frac{\langle 0|v^{\pi^+}|\pi^-\rangle}{\langle 0|v^{K^+}|K^-\rangle}. \qquad (14)$$

If now one assumes the pseudoscalar states to be nearly $SU(3)$ symmetric,[5] as motivated, for example, by success of the Gell-Mann–Okubo mass formula, the algebraic operators v connect the vacuum to states with unit norm, so that

$$\left(\frac{\sqrt{2}+\kappa}{\sqrt{2}-\tfrac{1}{2}\kappa}\right)=\left(\frac{m_\pi{}^2}{m_K{}^2}\right)\left(\frac{f_\pi}{f_K}\right). \qquad (15)$$

We note the relation between the proximity of the value of κ to $-\sqrt{2}$ and the smallness of the ratio $m_\pi{}^2/m_K{}^2$. Inserting (13) and (15) into (10),

$$A+2m_K{}^2(B+D)=m_\pi{}^2/f_\pi f_K. \qquad (10')$$

Similarly, keeping the pions on mass shell and using low-energy limits for the kaons,

(c) $p\to 0$ (or $p'\to 0$):

$$A^{(3/2)}\to A^{(3/2)}(m_\pi{}^2,m_K{}^2,m_\pi{}^2;\,m_\pi{}^2,m_\pi{}^2,m_K{}^2,0)=0$$

and

$$A+2m_\pi{}^2B+m_K{}^2(C+D)=0. \qquad (16)$$

(d) $p,\,p'\to 0$:

$$A^{(3/2)}\to A^{(3/2)}(m_\pi{}^2+2p\cdot q,\,0,\,m_\pi{}^2-2p\cdot q;\,m_\pi{}^2,m_\pi{}^2,0,0)$$
$$=A+2m_\pi{}^2B-4(p\cdot q)A',$$
$$=(2/3f_K{}^2)(\sqrt{2}-\tfrac{1}{2}\kappa)\langle\pi^+(q)|\sqrt{2}u_0-\tfrac{1}{2}u_8+\tfrac{1}{2}\sqrt{3}u_3|\pi^+(q)\rangle+2(p\cdot q)/f_K{}^2$$

and

$$A'=-1/2f_K{}^2, \qquad (17)$$

$$A+2m_\pi{}^2B=m_K{}^2/f_\pi f_K. \qquad (18)$$

Again, in Eq. (18), a low-energy limit has been used to evaluate the scalar density term together with the determining relation for κ.

From (9) and (17), simultaneous pion and kaon low-energy limits would require the (approximately valid) equality, $f_\pi=f_K\equiv f$. Using the independent equations (8), (9), (10′), (16), and (18) to determine[6] the five constants A', A, B, C, and D, we find

$$A'=-1/2f^2,\quad A=(m_K{}^2+m_\pi{}^2)/f^2,$$
$$B=-1/2f^2,\quad C=-1/f^2,\quad D=0. \qquad (19)$$

The s-wave scattering lengths are given by

$$-8\pi(m_K+m_\pi)a^I=A^I\langle(m_K+m_\pi)^2,0,(m_K-m_\pi)^2;\,m_\pi{}^2,m_\pi{}^2,m_K{}^2,m_K{}^2\rangle.$$

At threshold, $A_{\rm th}{}^{(+)}=A+2(m_K{}^2+m_\pi{}^2)B+2m_K{}^2D=0$ and $A_{\rm th}{}^{(-)}=4m_Km_\pi A'=-2m_Km_\pi/f^2$, so that, using (2),

$$a^{1/2}=2\left(\frac{m_K}{m_K+m_\pi}\right)L\simeq 2L\simeq 0.22m_\pi{}^{-1},$$
$$a^{3/2}=-\left(\frac{m_K}{m_K+m_\pi}\right)L\simeq -L\simeq -0.11m_\pi{}^{-1}, \qquad (20)$$

where $L=m_\pi/4\pi f^2\simeq 0.11m_\pi{}^{-1}$.

These results for $a^{1/2,3/2}$ are identical to the ones obtained from Weinberg's "heavy-target" formula in which $A_{\rm th}{}^{(+)}$ is omitted. In fact, if only (a) and (b) are used, $A_{\rm th}{}^{(+)}/A_{\rm th}{}^{(-)}=O(m_\pi/m_K)$. The use of all the equations ($f_\pi=f_K$), however, gives $A_{\rm th}{}^{(+)}=0$.

KK SYSTEM

There is only one s-wave scattering length $a^{(I=1)}$ since $a^{(0)}$ vanishes by Bose statistics. Applying to the $I=0$, 1 s-channel amplitudes (i) Bose statistics in the s channel and (ii) PT invariance, which says invariance under

[5] For discussion relating to the validity of this statement, see M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. (to be published). See also Eq. (30) in the text.

[6] It should perhaps be pointed out that if one were to take limits p', $q'\to 0$ while leaving the initial pion and kaon on mass shell (without taking the necessary additional energy-conserving limit $p^2\to m_\pi{}^2$ or, alternatively, $q^2\to m_K{}^2$), two additional equations would be obtained whose right-hand sides are zero owing to the supposed absence of $I=\tfrac{5}{2}$ components [$SU(3)$ **27** transformation contributions] in the relevant commutators. These equations turn out to be consistent with solutions (19) by substitution. Taking three particles off the mass shell for energy conservation, one again has to evaluate scalar density terms. The variation from $p^2=m_K{}^2$ to $m_\pi{}^2$ involves the momentum transfer to one of these scalar terms, which, if it is slowly varying, explains the former naive limit where energy conservation was ignored.

$(s,t,u;\, q^2,q^{2\prime},p^2,p^{2\prime}) \leftrightarrow (s,t,u;\, q^{2\prime},q^2,p^{2\prime},p^2)$, or (ii′) t-channel crossing symmetry, one finds

$$A^{(1)} = a + b(u+t) + cs\,, \quad A^{(0)} = a'(u-t)\,. \tag{21}$$

Using low-energy limits in commutator identities to determine the coefficients, one finds

(a) $q \to 0$ (consistency condition):

$$A^{(1)} \to A^{(1)}(m_K^2, m_K^2, m_K^2;\, 0, m_K^2, m_K^2, m_K^2) = 0$$

or

$$a + 2m_K^2 b^- + m_K^2 c = 0\,. \tag{22}$$

(b) $q', q \to 0$ (or $p', p \to 0$):

$$\begin{aligned} A^{(1)} &\to A^{(1)}(m_K^2 + 2p\cdot q,\, 0,\, m_K^2 - 2p\cdot q;\, 0,0,m_K^2,m_K^2) \\ &= a + m_K^2(b+c) + 2(p\cdot q)(c-b)\,, \\ &= (2i/f_K^2)\langle K^+(p)|[A^{K^-},\, \partial^\mu A_\mu{}^{K^+}]|K^+(p)\rangle \\ &\quad + [4(p\cdot q)/f_K^2]\,, \end{aligned}$$

or

$$c - b = 2/f_K^2\,, \tag{23}$$

$$a + m_K^2(b+c) = (2/3f_K^2)(\sqrt{2} - \tfrac{1}{2}\kappa)\langle K^+(p)|\sqrt{2}u_0 - \tfrac{1}{2}u_8 + \tfrac{1}{2}\sqrt{3}u_3|K^+(p)\rangle\,. \tag{24}$$

Also,

$$\begin{aligned} A^{(0)} &\to m_K^2 a' - 2(p\cdot q)a' \\ &= (2i/f_K^2)\{2\langle K_0|[A^{K^-},\, \partial^\mu A_\mu{}^{K^+}]|K_0\rangle \\ &\quad - \langle K^+|[A^{K^-},\, \partial^\mu A_\mu{}^{K^+}]|K^+\rangle\} + O(p\cdot q) \end{aligned}$$

or

$$a' = 0\,, \tag{25}$$

$$a' = (2/3f_K^2)(\sqrt{2} - \tfrac{1}{2}\kappa)\langle K^+(p)|\sqrt{2}u_0 - \tfrac{1}{2}u_8 - \tfrac{3}{2}\sqrt{3}u_3|K^+(p)\rangle\,. \tag{26}$$

(c) $p', q' \to 0$ (or $p, q \to 0$):

$$\begin{aligned} A^{(1)} &\to A^{(1)}(0,\, m_K^2 + 2p\cdot q',\, m_K^2 - 2p\cdot q';\, m_K^2, 0, m_K^2, 0) \\ &= a + 2m_K^2 b \\ &= (2i/f_K^2)\langle 0|[A^{K-},\, \partial^\mu A_\mu{}^{K-}]|K^+(p)K^+(-p)\rangle = 0\,. \end{aligned}$$

Therefore, if there is no $Y=2$ component in the commutator $[A^i, \partial^\mu A_\mu{}^j]$,[4]

$$a + 2m_K^2 b = 0\,. \tag{27}$$

We note that Eqs. (23), the kaon vector-charge matrix element, and (27), the requirement of no $Y=2$ scalar density, determine the scalar density matrix element in (24): The right-hand side of (24) equals $2m_K^2/f_K^2$. On the other hand, using the low-energy limit to evaluate the matrix element in (24) by taking one kaon four-momentum to zero yields this same answer as well as confirming the equivalence of (25) and (26). Of course, this observation is entirely similar to one made previously for the $\pi\pi$ system.

Equations (22), (23), (25), and (27) give

$$a' = 0\,, \quad a = 4m_K^2/f_K^2\,, \quad b = -2/f_K^2\,, \quad c = 0\,. \tag{28}$$

So $A^{(1)}(s,t,u,\cdots) = (2/f_K^2)[2m_K^2 - (u+t)]$ [while $A^{(0)}(s,t,\cdots) = 0 +$ quadratic terms in invariants] and, using

$$\begin{aligned} -32\pi m_K a^{(1)} &= A_{\rm th}{}^{(1)} \\ &= A^{(1)}(4m_K^2, 0, 0;\, m_K^2, m_K^2, m_K^2, m_K^2)\,, \\ a^{(1)} &= -m_K/8\pi f_K^2 \approx -\tfrac{1}{6}m_\pi{}^{-1}\,. \end{aligned} \tag{29}$$

A linear expansion (21) for the KK amplitudes $A^{(0),(1)}$ is plausible because for this $Y=2$ system there are no unphysical threshold effects. However, such an expansion for the $Y=0$ $K\bar{K}$ system (which can be related by crossing to KK) would not be expected to be valid, since there are a considerable number of unphysical thresholds for both $I=0$ and $I=1$ lying below the elastic one at $s_{\rm th} = 4m_K^2 = 0.98$ BeV^2. Small $J^{PG} = 0^{++}$, 0^{+-} effects may permit a calculation of s wave $K\bar{K}$ scattering lengths. But the experimental situation reveals significant $I=0$, 1 $K\bar{K}$ effects although there exist a variety of interpretations in the fits[7]: nonresonance (with positive real scattering length), bound system (with complex scattering length), or resonance just above threshold.

Finally, we note that the ratio of the scalar density terms coming from (1), $\frac{1}{3}(\sqrt{2}+\kappa)\langle\pi|\sqrt{2}u_0 + u_8|\pi\rangle = m_\pi{}^2$, and from (24), $\frac{1}{3}(\sqrt{2} - \frac{1}{2}\kappa)\langle K^+|\sqrt{2}u_0 - \frac{1}{2}u_8 + \frac{1}{2}\sqrt{3}u_3|K^+\rangle = m_K^2$, together with $SU(3)$ symmetry for pseudoscalar states, can be evaluated without the necessity of a low-energy limit as

$$\begin{aligned} &\frac{(\sqrt{2}+\kappa)}{(\sqrt{2}-\frac{1}{2}\kappa)} \frac{\langle\pi|\sqrt{2}u_0 + u_8|\pi\rangle}{\langle K^+|\sqrt{2}u_0 - \frac{1}{2}u_8 + \frac{1}{2}\sqrt{3}u_3|K^+\rangle} \\ &\quad = \left(\frac{\sqrt{2}+\kappa}{\sqrt{2}-\frac{1}{2}\kappa}\right)\left(\frac{\alpha + \frac{1}{3}\sqrt{3}\beta}{\alpha - \frac{1}{2}(-\frac{1}{6}\sqrt{3})\beta + \frac{1}{2}\sqrt{3}(\frac{1}{2})\beta}\right) \\ &\quad = \left(\frac{\sqrt{2}+\kappa}{\sqrt{2}-\frac{1}{2}\kappa}\right) = \frac{m_\pi{}^2}{m_K{}^2}\,. \end{aligned} \tag{30}$$

Equation (30) agrees ($f_\pi = f_K$) with our previously derived (15). This observation, together with the fact that (15) involves merely a vacuum to single-particle state transition matrix element, serves to sharpen the derivation of a similar equation, independently derived, in Ref. 5 (where κ is denoted by c).

ACKNOWLEDGMENT

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