of the branching ratio as well as of the polarization be made.

The comparison of the value of Reξ obtained from the branching ratio with Reξ obtained from a measurement of $K_{\mu 3}$ alone is based on the assumption of μ-e universality. Lee and Wu (Ref. 5, p. 491), using earlier data which were not as precise, drew the conclusion that there was evidence for μ-e universality in the K decays. Until one understands the present discrepancy, the conclusion that there is μ-e universality in K decays is not valid.

ACKNOWLEDGMENTS

We wish to acknowledge the advice and assistance of Dr. D. W. Carpenter, particularly during the analysis of this experiment. We would like to thank the staff of the Argonne Zero Gradient Synchrotron for their cooperation. We are very indebted to C. Smock, L. Seward, H. Barton, P. Mantsch, and E. Harris for their help in the construction of the apparatus and in the running of the experiment. We are grateful to Mrs. P. Martin for her careful supervision of the scanning and measuring.

PHYSICAL REVIEW VOLUME 176, NUMBER 5 25 DECEMBER 1968

Relativistic Energy Loss by Ionization in Nuclear Emulsions

F. J. CONGEL* AND P. J. MCNULTY

Physics Department, Clarkson College of Technology, Potsdam, New York 13676

(Received 1 July 1968)

The variation in grain density in the tracks of 5-, 8-, 12-, and 24-GeV/c protons and 5-GeV/c pions has been investigated as a function of velocity in the same plate of an Ilford K-5 nuclear emulsion. About 80 000 blobs were counted for each beam. The pion-to-proton ratio of grain densities at 5 GeV/c is 1.114 ±0.006. The results are in agreement with the Sternheimer formula using a mean ionization potential for AgBr of 434 eV and a cutoff energy T_0 of 2–5 keV. Comparison of these data with an earlier experiment in a G-5 emulsion shows no dependence of the rate of the relativistic rise in grain density on type of emulsion. The combined data from the two experiments are in excellent agreement with Sternheimer's formula with $I=434$ eV and $T_0=5$ keV with no correction due to secondary ionization. A correction for secondary ionization corresponding to a 10% contribution due to secondary ionization at minimum ionization gives excellent agreement with $I=434$ eV and $T_0=2$ keV.

I. INTRODUCTION

THERE have been many attempts to measure the relativistic rise of the grain density in nuclear emulsions, and much evidence[1–14] has been accumulated to support a proportionality between the grain density and the restricted energy loss given by the Sternheimer formula.[15–19] In comparing their data with the Sternheimer formula, the parameters I (mean ionization potential of emulsion atoms) and T_0 (allowed energy transferred in individual collisions) have been varied by experimenters to obtain a best fit yielding, however, a wide fluctuation in the values of both constants. In most of the previous experiments, the relativistic rise was calculated from the ratio of grain density measurements at large values of γ [$\gamma=(1-v^2/c^2)^{-1/2}>200$] and at minimum ionization ($3\leq\gamma\leq4$). Objections can be raised to this method of measurement, however. Theoretical and experimental studies by Patrick and Barkas[9] on the effect of secondary grain densities (i.e., due to δ rays) would seem to indicate that a shift of as much as 6% might be expected in the ratio of the plateau to minimum grain densities from that predicted by Sternheimer's formula, while the second-order radiative correction predicted by Tsytovitch[20] would introduce another correction of 4–5% in the asymptotic region.

---

* National Science Foundation Trainee. Present address: Argonne National Laboratory, Argonne, Ill.

[1] E. Pickup and L. Voyvodic, Phys. Rev. **80**, 89 (1950).

[2] M. M. Shapiro and B. Stiller, Phys. Rev. **87**, 682 (1952).

[3] R. Michaelis and C. Violet, Phys. Rev. **90**, 723 (1953).

[4] A. Morrish, Phys. Rev. **91**, 423 (1953).

[5] J. Fleming and J. Lord, Phys. Rev. **92**, 511 (1953).

[6] B. Stiller and M. M. Shapiro, Phys. Rev. **92**, 735 (1953).

[7] G. Alexander and R. Johnston, Nuovo Cimento **5**, 363 (1957).

[8] B. Jongejans, Nuovo Cimento **26**, 626 (1960).

[9] J. Patrick and W. Barkas, Nuovo Cimento Suppl. **23**, 1 (1962).

[10] B. Stiller, in *Proceedings of the Fourth International Conference on Corpuscular Photography*, edited by H. Freiser and G. Heiman (Munich, 1963), p. 542.

[11] F. R. Buskirk, J. N. Dyer, H. D. Hanson, R. Seng, and R. H. Weidman, CERN Report No. 65-4 (unpublished).

[12] A. J. Herz and B. Stiller, in *Proceedings of the Fifth International Conference on Corpuscular Photography*, edited by E. Dahl-Jensen (CERN, Geneva, 1965).

[13] Z. V. Anzon, A. Vinitskii, Z. S. Takaibaev, I. Y. Chasnikov, and T. I. Shakova, Zh. Eksperim. i Teor. Fiz. **47**, 2051 (1964) [English transl.: Soviet Phys.—JETP **20**, 1378 (1965)].

[14] C. A. Nicoletta, P. J. McNulty, and P. L. Jain, Phys. Rev. **164**, 1693 (1967).

[15] R. M. Sternheimer, Phys. Rev. **88**, 851 (1952).

[16] R. M. Sternheimer, Phys. Rev. **91**, 256 (1953).

[17] R. M. Sternheimer, Phys. Rev. **103**, 511 (1956).

[18] R. M. Sternheimer, Phys. Rev. **145**, 247 (1966).

[19] R. M. Sternheimer, Phys. Rev. **164**, 349 (1967).

[20] V. N. Tsytovitch, Dokl. Akad. Nauk SSSR **144**, 310 (1962) [English transl.: Soviet Phys.—Doklady **7**, 411 (1962)].

If either of these effects is sizable, it is obvious that experiments measuring $G_{\text{plateau}}/G_{\min}$ would give misleading values of I and T_0. Furthermore, since both of these theories predict a flat plateau over most of the asymptotic region, accurate determinations of I and T_0, independent of measurements on the Fermi plateau, would seem essential to a study of possible deviations from Sternheimer's formula in the plateau region.

It is important to note that secondary ionization would cause deviations from Sternheimer's formula (i.e., misleading values of I and T_0) in the region of the relativistic rise in grain density as well as the Fermi plateau, while the effect predicted by Tsytovitch would be restricted to the plateau. We became interested, therefore, in making careful measurements in the region of the relativistic rise ($5<\gamma<40$) which would give an independent estimate of the effect of secondary ionization on grain density measurements. Earlier experiments with significant statistics in this region have relied on secondary particles produced in interactions of calibration beam particles with emulsion nuclei. This requires not only identification and velocity determination based on scattering measurements but also that a rather large volume of emulsion be used for the measurements. In the experiment described below all measurements have been made on accelerator-produced particles of known mass and momentum (proton: 5, 8, 12, and 24 GeV/c; pion: 5 GeV/c) overlapping in a very small volume (one-half of 3.0 cm$\times$1.5 cm$\times$150 μ) of a single Ilford K-5 emulsion plate. A brief description of the experiment and the details of the measurements are given in Sec. II.

Comparison of our results (K-5 emulsion) with a recent experiment[14] in Ilford G-5 emulsion will determine the effects due to emulsion type. Herz and Stiller[12] have reported a consistent deviation in ratio of the blob density on the plateau to the value at minimum ionization for different types of emulsion—a 2% difference occurring between Ilford K-5 and G-5 emulsion. In Sec. III we show that no such difference is found between the results of this experiment and the G-5 data of Nicoletta *et al.*[14] Moreover, the results of both experiments are shown to be in agreement with the theoretical values of I and T_0; I can be calculated from an empirical formula given by Sternheimer[18] ($I=434$ eV) and T_0 should correspond to the energies of electrons whose range corresponds to the radius of a silver bromide grain ($T_0=2$–5 keV). Comparison of our data with previous experiments along with our conclusions is given in Sec. IV.

II. MEASUREMENTS

The nuclear emulsion used in this experiment was part of a small stack of Ilford K-5 emulsion with pellicle dimensions 10 cm$\times$10 cm$\times$600 μ exposed to four different primary beams of 5-, 8-, 12-, and 24-GeV/c protons and a beam of 5-GeV/c pions. The details of the exposure have appeared in the literature,[14] the essential point being that the beams enter the emulsion making angles with the edge of the pellicle of 75°, 60°, 50°, 40° for the 5-, 8-, 12-, and 24-GeV/c beams, respectively. The orientation of the overlapping beams is shown in Fig. 1. The 5-GeV/c beam is an approximately equal mixture of protons and pions with a total intensity double that of the other beams. All beams were flat in the plane of the emulsion with an average dip angle of 0.2°.

Blob counting was carried out using a Bausch and Lomb binocular microscope equipped with a built-in scattering (rotating) stage. The plate was placed under the objective so that the intersection point was at the center of rotation of the stage. The stage could be rotated until a particular beam was parallel to the x motion of the stage. Since the beam was flat, a length of the track could be followed by x motion alone. In order to pick up the other tracks parallel to the particular beam, the plate could be moved in the y direction up to 7 or 8 mm on either side of the center point. The x motion of the stage was limited to about 25 mm. For each of the four energy beams, a relatively small rectangular section of the plate of approximately 2 cm^2 was used. Tracks were followed under a magnification of about 1500$\times$, all tracks being followed for at least 1 cm of length, with the average length being about 2 cm. All counts were made by one individual in order to keep the counting criteria constant for all measurements. The criteria followed was that no blobs were counted whose center did not lie within the cylinder defined by the neighboring blobs along the particle's trajectory. Careful measurements on some typical tracks indicate that this corresponds to counting only blobs whose centers lie roughly within $\frac{1}{2}$ μ of the particle's trajectory as defined by the neighboring grains. Measurements of blob counts were recorded after each mm of track followed in order to isolate large observational error and to allow correction of the blob density for depth in the emulsion; a total of 4000 blobs was counted on a typical track. All tracks studied remained within the range of depth from 100 μ below the top to 20 μ above the bottom of the emulsion. Periodic

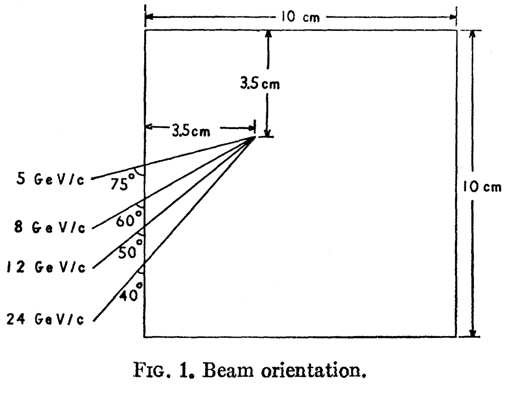

FIG. 1. Beam orientation.

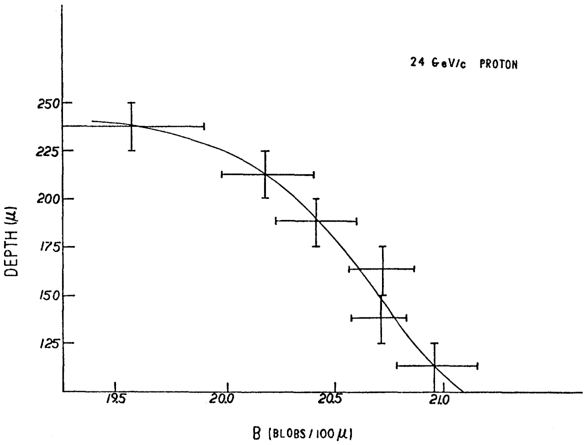

FIG. 2. Track depth versus blob density for a typical beam (24 GeV/c). Horizontal error bars represent statistical error. Vertical error bars represent combined depth range. The best fit is indicated by the solid curve.

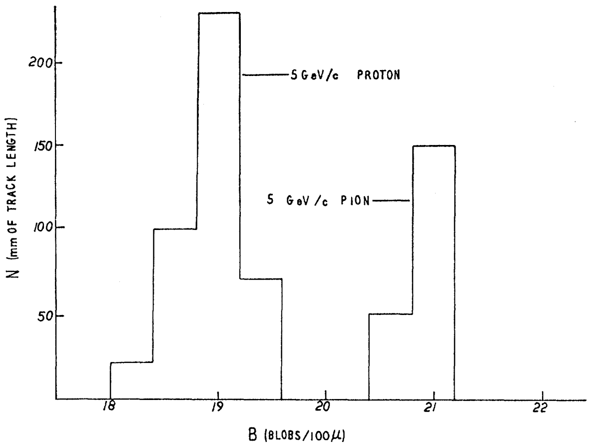

FIG. 4. Histogram of the number of tracks in mm of track length versus the blob density.

measurements of the depth of the track with respect to the bottom and top of the emulsion were made. By extrapolating between depth measurements, each mm of blob counts could be assigned to a distinct interval of depth in the emulsion.

The variation of blob density with depth for a typical beam (24 GeV/c) is shown in Fig. 2. Each datum point represents the number of blobs per 100 μ in a 25-μ interval of depth. The error bars for all 25-μ intervals represent statistical error $[(100/\sqrt{N})\%]$. The plots of blob density versus depth for all other beams are in essential agreement with that for the 24-GeV/c beam and are, therefore, not shown. A more detailed determination of the blob density–depth distribution was obtained as follows: The blob density for each 10-μ interval was normalized to the "best-fit curve" value for that beam at 150 μ and these results were statistically averaged over the four beams, giving us the distribution shown in Fig. 3. The error bars represent the statistical error corresponding to the total number of blobs counted at that depth. The shape of this distribution agrees within the statistics with those of the individual beams and provides us with an excellent profile of blob density versus depth. No evidence was seen for variation of blob density with position of the beam tracks in the 2 cm^2 used for measurements.

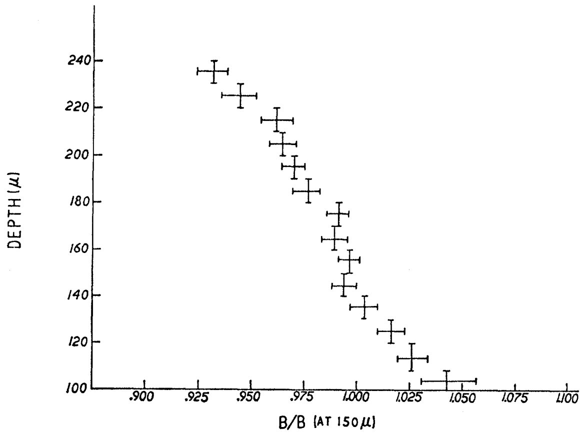

FIG. 3. Track depth versus blob density normalized to 150-μ depth. All beams were combined to obtain these results. The error bars represent statistical error.

The measured blob density for each track was corrected to its equivalent value at 150 μ from the top of the emulsion using the distribution shown in Fig. 3. As an example of the results of this correction, we show in Fig. 4 the histogram for number of tracks versus blob density for the mixed 5-GeV/c beam. The separation between protons and pions is quite distinct and the spectra are sharply peaked, as would be expected from the statistics involved in each measurement (~4000 blobs). The histograms for the remaining beams show the same characteristics and are, therefore, not shown.

The average blob density (corresponding to 150 μ) is given for each beam in Table I, normalized to the average blob density of the 5-GeV/c proton beam. The values of blob density were converted to grain density by the method of O'Ceallaigh.[21] The relative grain

TABLE I. Blob and grain densities for all beams.

Beam momentum (GeV/c)	γ	Blob density relative to 5-GeV/c proton	Grain density relative to 5-GeV/c proton
5 (proton)	5.4	1.000	1.000
8 (proton)	8.6	1.039	1.044
12 (proton)	12.9	1.059	1.067
24 (proton)	25.7	1.085	1.096
5 (pion)	35.8	1.100	1.114

---

[21] C. O'Ceallaigh, CERN Report No. B.S. 11, 1 54 (unpublished).

TABLE II. Determination of errors.

Beam momentum (GeV/c)	σ_{B^*} [calculated]	σ_{B^*} [observed]	σ_{B^*}' [calculated]	σ_{B^*}' [observed]
5 (proton)	0.019	0.014	0.0040	0.0029
8 (proton)	0.019	0.014	0.0037	0.0033
12 (proton)	0.019	0.017	0.0039	0.0035
24 (proton)	0.019	0.036	0.0041	0.0079
5 (pion)	0.019	0.0045	0.0057	0.0014

densities are given in Table I along with the corresponding values of γ. The average value of blob density measured and the corresponding value of the grain density at $\gamma=5.4$ are 18.95 and 21.07, respectively.

Two alternative methods were used to calculate the blob densities (normalized to the 5-GeV/c proton). In the first method, the average blob densities for each beam in each 10-μ interval of depth were calculated. After normalization to the 5-GeV/c proton blob density at the corresponding depth, a weighted average of this ratio was computed for each beam and multiplied by the average blob density of the 5-GeV/c proton beam. The resulting blob densities were converted to grain densities by O'Ceallaigh's method[21] and normalized to the 5-GeV/c proton values. The resulting relative blob and grain densities are in agreement (within statistical error) with the values given in Table I. The second method involved using the blob density at 150 μ from the best-fit curves representing blob density versus depth distributions, such as that shown in Fig. 2. The relative blob and grain densities obtained are again in agreement, within statistics, with the results in Table I.

III. DETERMINATION OF ERROR

The variance of the relative blob (or grain) density B^* from the mean value of the relative blob density $\bar{B}^*$ for each energy beam can be obtained from

$$\sigma_{B^*}^2=\sum_{i=1}^{n}\frac{(B^*_i-\bar{B}^*)^2}{n}\,, \tag{1}$$

where n is the total number of tracks in the beam since most of the tracks were of the same length (2 cm). It is also important to calculate the variance due to imperfect counting procedure by counting a number of tracks twice at different times but by the same observer. This observer variance, denoted σ_{obs}, should be related to the variance due to statistical error, σ_s, and the variance in grain density measurements by

$$\sigma_{B^*}^2=\sigma_s^2+\sigma_{\text{obs}}^2\,. \tag{2}$$

In Table II we list the values of σ_{B^*} as measured and as calculated from Eq. (2) for each beam. The statistical variance was obtained from the standard expression

$$\sigma_s=1/\sqrt{N}\,, \tag{3}$$

where N is the number of blobs counted in an average track. σ_s and σ_{obs} were the same for all beams and had the values 0.016 and 0.011, respectively. To estimate the error in $\bar{B}^*$ (or $\bar{G}^*$), we divide σ_{B^*} by the square root of the number of tracks used to determine $\bar{B}^*$ for that beam. The measured and calculated values of this quantity are listed in Table II under σ_{B^*}'. Besides eliminating any gross errors in the total counts, the technique of recording blob counts after each mm of track counted and correcting for depth seems to give a significant reduction in σ_{B^*} over merely confining

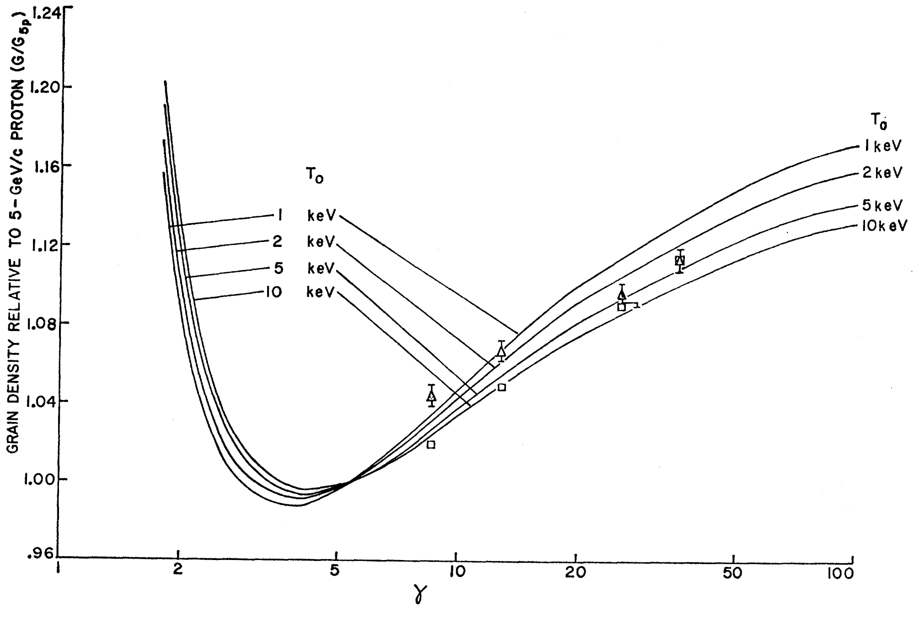

FIG. 5. Relative grain density versus $\gamma=(1-\beta^2)^{-1/2}$ for a mean ionization potential $I=434$ eV and cutoff energy $T_0=1$, 2, 5, and 10 keV. Square data points are from Nicoletta *et al.* (Ref. 14). Triangular data points are the result of this investigation. The error bars represent statistical error and are approximately the same for both experiments.

TABLE III. Comparison of results.

			Quoted			$I=434$ eV
Author	Type of emulsion		I (eV)	T_0 (keV)	$G_{\mathrm{plateau}}/G_{\mathrm{min}}$	T_0 (keV)
Pickup *et al.*[a]	Ilford	G-5	216	10	1.10	1000
Shapiro *et al.*[b]		G-5	554	<5	1.12	100
Michaelis *et al.*[c]		G-5	...	...	1.087	>1000
Morrish[d]		G-5	...	...	1.16	10
Fleming *et al.*[e]		G-5	...	...	1.14	20
Stiller *et al.*[f]		G-5	376	2–5	1.143	20
Alexander *et al.*[g]		G-5	530	29	1.133	50
Jongejans[h]		G-5	501	100	1.128	50
Patrick *et al.*[i]		G-5 and K-5	442	2	1.18[j]	2[j]
					1.12[k]	100
Stiller[l]	Ilford	G-5	...	...	1.07[m]	>1000
		K-5	...	...	1.09[m]	>1000
		L-4	...	...	1.13[m]	50
	NIKFI	BR	...	...	1.16[m]	10
	Kodak	NTB-4	...	...	1.12[m]	100
Buskirk *et al.*[n]	Ilford	K-5	...	...	1.15[m]	10–20
Herz *et al.*[o]	Ilford	K-5	...	...	1.119[m]	100
		G-5	...	...	1.085[m]	>1000
		L-4	...	...	1.090[m]	>1000
	Kodak	NRB-4	...	...	1.105[m]	500
	Gevaert	715	...	...	1.062[m]	>1000
	NIKFI	BR	...	...	1.092[m]	>1000
		BM	...	...	1.111[m]	300
Anzon *et al.*[p]	NIKFI	R	...	...	1.108[m]	...
	Ilford	G-5	...	...	1.103[m]	...
Nicoletta *et al.*[q]	Ilford	G-5	434	5	1.114[r]	5
This work	Ilford	K-5	434	5	1.114[r]	5

[a] Reference 1.
[b] Reference 2.
[c] Reference 3.
[d] Reference 4.
[e] Reference 5.
[f] Reference 6.
[g] Reference 7.
[h] Reference 8.
[i] Reference 9.
[j] Corrected for secondary ionization.
[k] Our estimate for uncorrected grain density.
[l] Reference 10.
[m] Blob density ratios.
[n] Reference 11.
[o] Reference 12.
[p] Reference 13.
[q] Reference 14.
[r] $G_{5\text{-GeV}/c\text{ pion}}/G_{5\text{-GeV}/c\text{ proton}}$.

measurements to a 50-μ interval of depth.[14] Due to the close agreement between σ_{B^*} and σ_s ($\sim$0.016), we used statistical error in determining all error bars for each of the curves.

IV. RESULTS AND DISCUSSION

We are interested in determining the correct value of T_0 to be used in the Sternheimer formula.[15–19] The theoretical values of G^* (grain density normalized to the 5-GeV/c proton grain density) for different values of γ were calculated using $I=434$ eV, and the value of $\delta(\gamma)$ was calculated from the empirical formula given by Sternheimer.[22] The theoretical curves for G^* versus γ for different values of T_0 are plotted normalized to the 5-GeV/c proton ($\gamma=5.4$) and are shown in Fig. 5 along with the experimental data (represented by circles) and the experimental data from the previous experiment[14] on G-5 emulsion (represented by squares). The error bars on the data for this experiment take into account the finite (80 000) number of blobs in the normalization point ($\gamma=5.4$). The majority of the observed points for this experiment lie closest to the theoretical curve for $T_0=5$ keV, although values of T_0 from 2 to 10 keV are within the statistical error. This fit is obviously in agreement with the previous G-5 experiment.[14] One gets similar results for both experiments when the experimental data are normalized to the 8-, 12-, and 24-GeV/c proton, as well as the 5-GeV/c pion, grain densities. We therefore find no evidence of the dependence of ionization loss on type of emulsion, in disagreement with the results of Stiller *et al.*[10,12] Therefore, it is tempting to combine the data of the two experiments (G-5 and K-5) to attempt a more accurate determination of the value of T_0. The average relative grain densities are shown in Fig. 6 as open circles and are in agreement with a value of $T_0=5$ keV. Accepting the proposal of Messel and Ritson,[23] the maximum transferable energy T_0 in the restricted energy-loss calculation should be replaced by the energy of a δ ray with a range about equal to the grain size; for G-5 and K-5 emulsion this would be between 2–5 keV.

In Table III we have summarized the results of earlier experiments on the relativistic rise of grain density in various emulsions. As mentioned earlier, many previous experimenters best fit their data by finding the values of both the mean ionization potential I and T_0. While the shape of the relativistic increase is not very sensitive to the value of I chosen, the value of T_0 quoted does depend somewhat on the value of I used. In other experiments, only the ratio of the grain

---

[22] To obtain $\delta(\gamma)$ for $I=434$ eV we extrapolated between the results of Refs. 15 and 17 using the procedure outlined in Ref. 18.

[23] H. Messel and D. M. Ritson, Phil. Mag. **41**, 1129 (1950).

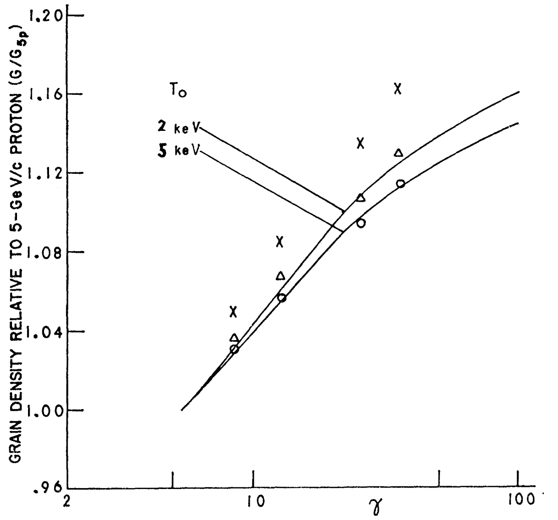

FIG. 6. Relative grain density versus $\gamma=(1-\beta^2)^{-1/2}$ for a mean ionization potential $I=434$ eV and cutoff energy $T_0=2$ and 5 keV. Combined data from Nicoletta *et al.* (Ref. 14) and this investigation are represented by circles. 10 and 25% corrections for secondary ionization at minimum are represented by triangles and X's, respectively. No error bars are shown, but combined statistics of both experiments correspond to approximately 0.3% error.

density at plateau to the grain density at minimum ionization is quoted. In Table III we present the quoted values of I and T_0 and/or the ratio $G_{\text{plateau}}/G_{\text{min}}$ given by the various authors, as well as our estimate of the value of T_0 (based on $I=434$ eV) which best fit their data in order to make comparison easier. It is important to note, however, that what is listed by many authors is the ratio of the blob density at plateau to the blob density at minimum ionization, thereby making our comparison even more difficult. We estimate that conversion to grain density should increase the ratio $G_{\text{plateau}}/G_{\text{min}}$ in Refs. 10–13 by at least 0.015, although an exact determination by us would be impossible. This could have a sizable effect in reducing the value of T_0 which best fits their data.

It is obvious from this experiment and the previous experiment of Nicoletta *et al.*[14] that there is no sizable contribution due to secondary ionization. However, after correction for secondary ionization corresponding to a 10% contribution at minimum ionization (represented as triangles in Fig. 6) we get excellent fit to the curve for $T_0=2$. For convenience, the data after a correction of 25% contribution due to secondary ionization at minimum ionization are also given (by X's). However, the range of a 5-keV electron, according to our crude calculations, is about 0.5 μ, which we estimated in Sec. II to be the radius of the effective cylinder which contains the centers of the blobs counted. The range corresponding to an electron of kinetic energy of about 2 keV is on the order of 0.2 μ. Therefore, we feel we are in agreement with Sternheimer's[18] formula for $I=434$ eV and a value of T_0 (5 keV) corresponding to the measurement technique used with no correction for secondary ionization. Use of a value of T_0 corresponding to the undeveloped grain radius (~ 0.2 μ, $T_0 \sim 2$ keV) would require a 10% correction for the secondary ionization predicted by Patrick and Barkas.[9] In any given experiment involving blob counting, therefore, it is necessary to perform the blob counting according to strict criteria which determine the maximum distance from the particle trajectory (as determined from the line connecting the centers of neighboring grains). The energy of an electron having this approximate range will then give a crude estimate of the T_0 to be used in Sternheimer's formula.[24] Merely requiring that the blob counting be consistent will not be sufficient unless some correction is made for secondary ionization.

We plan in future experiments to investigate the effect on the slope of the relativistic rise of changes in the magnitude of the grain density at minimum ionization.

ACKNOWLEDGMENTS

The authors are very grateful to Professor P. L. Jain for the loan of the emulsion plate and the microscope used in this experiment. We would also like to thank Dr. R. M. Sternheimer for helpful discussion.

---

[24] For example, including grains ionized by secondary electrons of range 1.2 μ should require $T_0=10$ keV in Sternheimer's formula.