for the lower line and $r_N = 1.642$ F for the upper line. The suggested anomaly between ^{118}Sn and ^{120}Sn corresponds to a change in r_N of about 2% and a change in central-neutron density of about 6% and implies a restructuring of the interior of the nucleus in this region of mass number. However, a definite conclusion as to the existance of this anomaly cannot be made without further measurements.[9]

C. Imaginary Potential

An interesting feature of the optical-model analysis for the tin-isotope data was the large variation of the parameters of the imaginary potential within the contour of the r_M-a_M grid. To investigate this effect in detail, the ^{120}Sn analysis was used. Figure 7 shows the imaginary potential at three points, (a) the point of lowest χ^2, (b) the point of best χ^2 achieved with a_M constrained to be 0.65 F, and (c) the point of best χ^2 achieved with a_M constrained to be 0.70 F.

Figure 7 shows that the tails of these three potentials are very similar. However, the potential shapes within the nuclear volume very markedly. These three points have values of χ^2 varying from 7.9 to 10.3. This large imaginary potential parameter variation for small χ^2 variation was a consistent feature of the tin-isotope-data analysis. Thus it appears that an optical-model analysis cannot hope to determine accurately the shape of the imaginary potential throughout the entire nuclear volume.

[9] Subsequent to the preparation and submission of this paper, a reanalysis of pion-scattering data has been made by Auerbach *et al.*, Phys. Rev. Letters **21**, 162 (1968). This indicates no difference between the nuclear neutron and proton distributions in ^{208}Pb. This result is in disagreement with the work of Ref. 1, which forms the basis of the present analysis. The source of this discrepancy is being investigated.

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$g_{7/2}$-$h_{11/2}$ Anomaly in Stripping Studies near $A = 110$†

B. L. Cohen, R. A. Moyer, J. B. Moorhead, L. H. Goldman, and R. C. Diehl
University of Pittsburgh, Pittsburgh, Pennsylvania 15213
(Received 5 April 1968)

Judging from summed spectroscopic factors (ΣS) in (d,p) reactions at 12 MeV, the $g_{7/2}$ state seems to be almost empty while the $h_{11/2}$ state seems to be almost full in the isotopes of Pd, Cd, and In. This is the reverse of the results in the isotopes of Sn, and is sharply contrary to expectations from nuclear-structure theory, since the $g_{7/2}$ state lies much lower in energy than the $h_{11/2}$. This anomaly was investigated with 17-MeV-deuteron-induced (d,p) and (d,t) reactions. In the former, where the behavior is much less sensitive to the vagaries of the distorted-wave Born approximation than at 12 MeV, the same general results are obtained, although the effect is not as strong. In the (d,t) studies, there is very little anomaly in ΣS, but there are some suspicious aspects of this result. $S(d,p)$ for exciting various $7/2^+$ nuclear levels was found to have very different ratios at 12 and 17 MeV, and $S(d,p)/S(d,t)$ varies much more strongly than usual among these levels. It is concluded that the anomaly arises from a breakdown of the basic stripping theory.

THE $g_{7/2}$ single-particle state is one of the lowest-energy states in the 50–82 neutron shell, and the $h_{11/2}$ is one of the highest. One therefore expects that, for nuclei in which the shell is about half filled, the $g_{7/2}$ state will be mostly full, and the $h_{11/2}$ state will be mostly empty. Since the summed spectroscopic factor, $\sum S$, in a (d,p) stripping reaction on an even-even target is a direct measure of the "emptiness" of a single-particle state, one expects $\sum S \simeq 1$ for $h_{11/2}$ and $\sum S \simeq 0$ for $g_{7/2}$ in this region. This behavior was indeed found in a study of the isotopes of tin[1,2] with (d,p) and (d,t) reactions induced by 15-MeV deuterons, as shown in Table I. The results in Table I are in good agreement with predictions from nuclear structure theory,[1] and are indicative of what is expected for other nuclei with the same number of neutrons.

However, recent studies at this laboratory[3–5] with (d,p) reactions induced by 12-MeV deuterons on isotopes of Pd, Cd, and In have given very different results. These are shown in Table II.

A straightforward interpretation of these results would be that the $g_{7/2}$ state is mostly *empty* in the isotopes of Pd, Cd, and In, while the $h_{11/2}$ state is

† Supported by the National Science Foundation.
[1] B. L. Cohen and R. E. Price, Phys. Rev. **121**, 1441 (1961).
[2] E. J. Schneid, A. Prakash, and B. L. Cohen, Phys. Rev. **156**, 1316 (1967).
[3] B. L. Cohen, J. B. Moorhead, and R. A. Moyer, Phys. Rev. **161**, 1257 (1967).
[4] J. B. Moorhead, B. L. Cohen, and R. A. Moyer, Phys. Rev. **165**, 1287 (1968).
[5] L. H. Goldman and J. Kremenek (private communication).

TABLE I. $\sum S(d,p)$ with 15-MeV deuterons on isotopes of tin.[a]

	Sn^{112}	Sn^{114}	Sn^{116}	Sn^{118}
$g_{7/2}$	0.31	0.16	0.13	0.16
$h_{11/2}$	0.88	0.83	0.85	0.70

[a] Reference 2.

mostly *full*; we refer to the sharp difference between these results and those from Sn as reinforced by their agreement with nuclear structure theory as "the $g_{7/2}$-$h_{11/2}$ anomaly." This anomaly is so strongly contrary to expectations from nuclear-structure theory that it is important to look for experimental explanations. Some of the possibilities considered and their rebuttal are as follows:

(1) *Some nuclear states were incorrectly assigned as* $g_{7/2}$. The $l=4$ angular distributions are quite characteristic and not easily confused, and it would take several errors of this type—at least one in each isotope—to explain the discrepancy in $\sum S$ for $g_{7/2}$. However, the clinching argument came from the $In^{115}(d,p)$ ground-state transition. That state is known to be 1^+ which requires $g_{7/2}$ stripping (to couple with the $g_{9/2}$ odd proton); the value in Table II is derived from the cross section for this transition alone, so there is no chance for an error of the type under discussion.

(2) *The $h_{11/2}$ state is fragmented and many components were missed.* Only a single nuclear state was assigned as $h_{11/2}$ in each of the Pd and Cd isotopes studied, and in all cases it is a previously known and assigned (from isomerism), low-energy state. At some angles, these were among the most strongly excited levels, so it seems very unlikely that other $h_{11/2}$ levels, even if an order of magnitude less strongly excited, would be missed unless they lie within ~ 7 keV of another more strongly excited level. Since the average level spacing is about 30 keV, the probability for this is about 23%. If another state were nearly as strongly excited as the known $h_{11/2}$ state, it would be detected from the angular distribution even if it were degenerate in energy with another level of the type normally encountered.

These arguments were given strong support by the studies of the $In^{115}(d,p)$ reaction. Since the target angular momentum is $\frac{9}{2}$, the $h_{11/2}$ state is expected to be split into 10 nuclear levels with $I=1$ to 10. Essentially all of these levels were found in the experiment.

(3) *There was something wrong with the DWBA calculations used in the analysis.* The DWBA analysis used to determine the results in Table II was also used for analyzing the $s_{1/2}$, $d_{5/2}$, and $d_{3/2}$ states in the same experiments, and those results seemed plausible. But the strongest argument against this explanation was derived from the Sn^{116} measurement listed as the last items in Table II; it agrees substantially with the 15-MeV result. It seemed very difficult to understand how there could be a large difference in the DWBA calculations between neighboring nuclei. The large difference in the measured cross sections between Sn and the others seemed like too physical a phenomenon to be explained away by such an artificial device.

TABLE II. $\sum S(d,p)$ with 12-MeV deuterons (13-MeV for Cd^{112}).

	Pd^{106}	Pd^{108}	Cd^{112}	Cd^{114}	In^{115} [a]	Sn^{116}	Sn^{116} [b]
$g_{7/2}$	0.86	0.76	0.74	0.52	0.67	0.18	0.16
$h_{11/2}$	0.29	0.28	0.40	0.33	0.62	0.69	0.76

[a] The values for the odd proton target, In^{115}, are $(2j+1)^{-1} \Sigma S'$ (cf. Ref. 4).
[b] The analysis for Sn^{116} was done with and without cutoffs in the DWBA integration. The first values given (no cutoff) are obtained on the same basis as other entries in this table; the second (with cutoffs) were obtained the same way as in Table I.

TABLE III. σ_{DW} (μb/sr) at the peak of the angular distribution for (d,p) reactions induced by 12-MeV deuterons. $Q=4.0$ MeV.

Deuteron optical model parameter	$g_{7/2}$ no cutoff	$g_{7/2}$ cutoff	$h_{11/2}$ no cutoff	$h_{11/2}$ cutoff	$s_{1/2}$ no cutoff	$s_{1/2}$ cutoff
"Compromise"	59	54	54	58	2400	2000
Perey *A*—Pd	70	57	63	49	2500	2100
Perey *A*—Cd	69	64	83	56	3300	2300
Perey *A*—Sn	89	16	71	11	490	630
Perey *B*—Pd	64	63	52	55	3000	2300
Perey *B*—Cd	66	64	64	56	3100	2700
Perey *B*—Sn	47	24	26	19	1280	890
BBC	63	55	47	48	3000	2000

On the other hand, there is room for large differences within the framework of currently accepted DWBA practice. Moreover, the deuteron elastic scattering angular distributions in this region do change rapidly from element to element, and in particular, they are quite different for Sn than for Pd and Cd.[6] As a result, there are differences in DWBA results among them as shown in Table III. The listings there are σ_{DW}, defined by the expression for the differential cross section $d\sigma/d\Omega$ as

$$\frac{d\sigma}{d\Omega}(d,p)=1.5(2j+1)\sigma_{DW}S.$$

Listings are given with and without a lower cutoff in the DWBA integration, the cutoffs being taken at 7.7 F. The "compromise" parameters are those suggested by Perey[7] and by Winner and Drisko[8] as a set that varies smoothly with A and gives reasonably good fits to the data for all targets. These are the ones used in the analyses leading to Tables I and II, with the cutoff in Table I and without the cutoff in Table II. The Perey *A* and Perey *B* are different sets given in Ref. 7 as fits to elastic scattering from individual elements. The Barnes, Bockelman, and Comfort (BBC) potential was suggested in Ref. 9 as being especially good for (d,p) analysis in this mass region but with lower bombarding energy.

[6] R. K. Jolly, E. K. Lin, and B. L. Cohen, Phys. Rev. **130**, 2391 (1963); G. Mairle and U. Schmidt-Rohr, Max-Planck-Institut für Kernphysik Report No. 19651 V113 (unpublished).
[7] C. M. Perey and F. G. Perey, Phys. Rev. **132**, 755 (1963).
[8] D. R. Winner and R. M. Drisko, Nucl. Data (to be published).
[9] J. R. Comfort, C. K. Bockelman, and P. D. Barnes, Phys. Rev. **157**, 1965 (1967).

TABLE IV. σ_{DW} (μb/sr) at the peak of the angular distribution for (d,p) reactions induced by 17-MeV deuterons, $Q=4.0$ MeV.

Deuteron optical model parameter	$g_{7/2}$ no cutoff	$g_{7/2}$ cutoff	$h_{11/2}$ no cutoff	$h_{11/2}$ cutoff	$s_{1/2}$ no cutoff	$s_{1/2}$ cutoff
Compromise	120	110	130	92	1900	1130
Perey *A*—Pd	140	66	170	60	750	700
Perey *A*—Sn	120	57	150	50	700	570
Perey *B*—Pd	85	68	105	60	1200	700
Perey *B*—Sn	70	57	82	48	750	590

We see from Table III that there can indeed be large differences in σ_{DW} for Pd and Cd on the one hand and Sn on the other. However, there is a very strong correlation between the values for $g_{7/2}$ and $h_{11/2}$ in all cases, so none of the sets listed can change the $g_{7/2}$ and $h_{11/2}$ results in opposite directions as is required to eliminate the anomaly. Moreover, use of either Perey potential would destroy the agreement for the $s_{1/2}$ states, and presumably also for the $d_{5/2}$ and $d_{3/2}$ states.

On the other hand, Table III does indicate that experiments at 12 MeV are very sensitive to the choice of optical-model parameters in the analysis. This may be due to the fact that this is a region where Coulomb effects can be very important. Upon exploration it was found that the situation is much more favorable at 17 MeV, which is currently the maximum available beam energy at this laboratory. The 17-MeV equivalent of Table III is Table IV. The deuteron optical-model parameters were taken from an analysis[10] of 15-MeV elastic deuteron scattering; no data or analyses are available at 17 MeV.

First we may note that the cross sections for $g_{7/2}$ and $h_{11/2}$ are considerably higher in Table IV than in Table III. When Coulomb effects are unimportant, cross sections decrease with increasing bombarding energy, so this difference indicates that Coulomb effects are rather important at 12 MeV; this may account for the great sensitivity there.

But the really great improvement of Table IV over Table III lies in the fact that there are no great differences between Pd and Sn, and that the ratio between the two is very nearly the same for any choice of potential, with or without cutoffs. It was therefore decided to carry out a series of measurements with 17-MeV deuterons.

The protons were detected with photographic plates in the focal plane of a magnetic spectrometer. In each set of runs, measurements were made at two or three angles near the peaks in the angular distributions for $l=4$ and $l=5$ transitions. At least two, and in many cases three, separate sets of runs were made with each target.

The product of incident beam times target thickness was measured by counting elastically scattered deuterons with NaI(Tl) scintillation detectors mounted at 38° on each side of the beam. This dual detector arrangement eliminates errors due to shifts in the angle of the incident beam. Elastic deuteron cross sections at these angles were determined with targets of sufficient thickness to make direct thickness measurements feasible, and were checked by using these targets to measure elastic deuteron scattering at 11.8 MeV where they are known from other work[6] and at 7 MeV where they can be assumed to be the Rutherford cross section.

[10] C. M. Perey and F. G. Perey, Phys. Rev. **134**, B353 (1964).

TABLE V. $\sum S(d,p)$ with 17-MeV deuterons.

	Pd^{106}	Pd^{108}	Cd^{112}	Cd^{114}	In^{115} [a]	Sn^{114}	Sn^{116}
$g_{7/2}$	0.73	0.53	0.31	0.34	1.10	···	0.22
$h_{11/2}$	0.38	0.34	0.47	0.71	···	0.64	0.71

[a] The value for the odd proton targets In^{115} is $(2j+1)^{-1}\sum S'$ (cf. Ref. 4).

In Sn^{116} and Pd^{108}, (d,p) cross sections at 17 MeV were made for all $l=4$ states. In Pd^{106}, data were obtained for the 0.311- and 0.364-MeV $g_{7/2}$ states; since these contribute 82% of the strength in Ref. 1, $\sum S$ for them was multiplied by 1/0.82. In each Cd isotope, only the principal $l=4$ peak was studied. Corrections needed there were 1/0.75 in Cd^{112} and 1/0.52 in Cd^{114}. The single known $h_{11/2}$ state was observed in all cases. The results are listed in Table V as obtained with "compromise" parameters and the cutoff. Conversions to other parameters may easily be made from Table IV.

In comparing Table V with Table II, we see that our anomaly has decreased somewhat, but has not by any means disappeared. Although the Cd results are much more like the Sn and theoretical expectations than they were in Table II, the Pd results still have much larger $\sum S$ for $g_{7/2}$ than for $h_{11/2}$, and $\sum S$ for $g_{7/2}$ in In^{115} is even larger than at 12 MeV.

If one is to believe that the anomaly is due to nuclear structure rather than to nuclear reaction phenomena, that is, that the $g_{7/2}$ actually *is* mostly empty and the $h_{11/2}$ actually *is* mostly full in the Pd isotopes, then this effect should also be in evidence in pickup reactions, such as (d,t). Measurements of (d,t) cross sections exciting the same states as those excited in the (d,p) studies were therefore undertaken. This, of course, requires the use of targets with A two mass units higher than the targets used in the (d,p) studies. Optical-model parameters for tritons were taken from Ref. 11, and since they are so uncertain, compromise parameters only were used for the deuterons with no cutoff. The results are listed in Table VI. The results for Sn^{116} and Sn^{118} are in good agreement with the 15-MeV measurements of Ref. 2.

We note from Table VI that there is little sign of our anomaly except in Cd^{116}. The $g_{7/2}$ result in Cd^{116} is very unreliable since only one state was observed and it was assumed that the ratio of its strength to that of the other $g_{7/2}$ states is the same (1.08) as in the 12-MeV

[11] J. C. Hafele, E. R. Flynn, and A. G. Blair, Phys. Rev. **155**, 1238 (1957).

TABLE VI. $[1/(2j+1)] \sum S(d,t)$ with 17-MeV deuterons.

	Pd^{108}	Pd^{110}	Cd^{114}	Cd^{116}	Sn^{116}	Sn^{118}
$g_{7/2}$	0.63	0.85	0.70	0.40	0.88	1.00
$h_{11/2}$	0.17	0.25	0.23	0.34	0.14	0.25

(d,p) work. This is well known to be a very unreliable assumption, and we shall soon see further evidence for its unreliability. On the other hand, there is no such difficulty with the $h_{11/2}$ state from Cd^{116}.

With the exception of Cd^{116}, the $g_{7/2}$ is apparently much more full in every case than the $h_{11/2}$. The general agreement between Pd and Sn isotopes here is strong evidence that our first and second experimental explanations of the (d,p) anomaly discussed above are not valid. If we were assigning nuclear states as $g_{7/2}$ incorrectly, this would make the agreement in Table VI rapidly worse, and if we were missing a large fraction of the $h_{11/2}$ strength, the values of ΣS for $h_{11/2}$ in Table VI should become too large.

One is tempted at this point to accept Table VI as correct and assign the anomaly in the Pd data of Table V as due to difficulties with DWBA. (Note that there is essentially no chance whatever that the experimental measurements that lead to Table VI are correct while those that lead to Table V are incorrect—almost every detail of the two experiments is the same, and the same nuclear states are studied.) However it should be pointed out that the chances for difficulties with DWBA in Table VI are greater than in Table V. Coulomb effects are much more serious in the former, and triton optical-model potentials are not nearly as well known as are proton potentials.

There is one further aspect of the data that is worthy of consideration; this concerns the detailed S values for the individual nuclear levels in the Pd isotopes. These are listed in Table VII. Note that their sums agree with the $\sum S$ in Tables V and VI except for the aforementioned factor of 1/0.82 that must be applied to the Pd^{107} results.

We see in Table VII that the rather acceptable value of $S(d,t)$ for Pd^{110} in Table VI is very heavily dependent on the contribution of the 0.427-MeV nuclear level. However, this level is a rather mysterious one. We see immediately from Table VII that its $S(d,p)/S(d,t)$ ratio is very much different from the others; it is smaller than the next smallest ratio by a factor of 2.5, whereas the other four cases agree within a factor of 2.0. Moreover, its angular distribution in Ref. 3 includes a sizeable rise in the forward direction such as is not seen in any of the other 11 $l=4$ angular distributions in Refs. 3 and 4. It does not seem very satisfactory for this to be the level that "saves the day" by causing our anomaly to disappear in the (d,t) data.

TABLE VII. S values for individual $7/2^+$ levels from $Pd(d,p)$ and $Pd(d,t)$ reactions at 17 MeV. The Sn^{117} results are shown for comparison.

Nucleus	Excitation energy	$S(d,p)$	$(2j+1)^{-1} \times S(d,t)$	$S(d,p)/S(d,t)$	$S(d,p)$ (12 MeV)	$S(d,p)$(12 MeV) / $S(d,p)$(17 MeV)
Pd-107	0.311	0.33	0.36	0.92	0.26	0.79
	0.364	0.28	0.15	1.86	0.45	1.61
Pd-109	0.245	0.22	0.19	1.16	0.44	2.00
	0.427	0.22	0.59	0.37	0.20	0.91
	0.644	0.09	0.08	1.13	0.12	1.33
Sn-117	0.72	0.22	1.00	0.22	0.18	1.22

In general, one cannot help but be disturbed by the strong variations in the $(d,p)/(d,t)$ ratios. This ratio is not expected to be exactly constant from state to state, but such strong variations among strongly excited states is unusual, and it is unexpected theoretically since states should be excited in proportion to the amount of the single quasiparticle state they contain. An even stronger aspect of Table VII is seen in the comparison between $S(d,p)$ as determined at 12 and at 17 MeV. We see that for the two principal states in each isotope, these ratios are very different. This difference cannot be blamed on DWBA as the Q values are quite close. It cannot be blamed on experimental error because it is very easy to recheck the ratios of two peaks in a photographic plate spectrum, and repeated checks confirmed the different cross-section ratios at the two energies for both isotopes. The only ready explanation is that our determinations of S are very strongly dependent on details of nuclear structure. Since the discrepancies in question are in the most strongly excited states and are of the order of a factor of 2, determinations of S by standard techniques must be considered uncertain by at least that amount.

We therefore explain the $g_{7/2}$-$h_{11/2}$ anomaly as a breakdown in the basic assumptions of stripping theory. How far this breakdown extends is not clear. It is apparently worse at 12 than at 17 MeV, but it is clearly still present at 17 MeV. It may be that it applies only to the $g_{7/2}$ and $h_{11/2}$ states in nonclosed shell nuclei; difficulties should appear first in high-l states due to poor angular-momentum matching. The results for these states do seem rather consistent among all the tin isotopes, and they are also consistent with theoretical expectations there. As yet there is no evidence for anomalous behavior of the s and d states in the Pd and Cd isotopes; this matter is under further investigation.