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High-Energy Scattering at Moderately Large Angles*

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The applicability of the eikonal approximation to high-energy nucleon-nucleus and hadron-hadron collisions is limited to small angles of scattering. However, experimental results have in some cases been interpreted by applying this theory beyond its angular range of validity. In the present paper, an expression for the scattering amplitude is obtained that has much of the simplicity of the eikonal approximation, but a much greater angular range of application. It provides a natural explanation for the disagreement between the Brookhaven and Virginia data on high-energy proton-He^4 elastic scattering in the neighborhood of the first minimum.

I. INTRODUCTION

SEVERAL recent papers have discussed the theory of high-energy nucleon-nucleus[1,2] and hadron-hadron[2,3] collisions from the point of view of the Glauber[4] multiple scattering formalism, which is based on the eikonal approximation. However, it has been known for more than a decade[5] that the eikonal approximation is expected to be valid only when the angle of scattering θ in the center-of-momentum (c.m.) system is small in comparison with the angle $\theta_c=(kR)^{-1/2}$, where $\hbar k$ is the momentum of each of the colliding objects in the c.m. system, and R is a typical linear dimension of these objects. Ross[6] has recently shown through numerical examples that this expectation is confirmed.

The results obtained in the papers cited in Refs. 1–3 have been applied to the interpretation of experiments in which the angular range is such that θ is not always small in comparison with θ_c. It is therefore desirable that the Glauber formalism be extended to larger angles. We shall refer in this paper to an angle as being *dynamically* small or large according to its smallness or largeness in comparison with θ_c. In contrast, an angle will be said to be *geometrically* small if it is small in comparison with 1 rad. Since $kR\gg 1$ in the situations considered in these papers, an interesting new range of angles is opened to theoretical investigation if θ is only required to be geometrically small, not dynamically small. It is the purpose of the present paper to effect this extension of the theory in a rather simple way.

II. DYNAMICALLY LARGE AND SMALL ANGLES

We consider the collision of two objects in the c.m. system, where the objects may be nuclei, nucleons, or other lumps of hadronic matter. Only spatial coordinates are considered, and all others such as spin or isospin are ignored. We assume that the relative motion of the centers of mass of the two objects can be described in the stationary case by a wave function $\psi(\mathbf{r})$, where $\mathbf{r}$ is the relative coordinate, such that in the absence of interaction

$$(\nabla^2+k^2)\psi=0. \tag{1}$$

Within the range of interaction, we assume that Eq. (1) is modified by replacement of k by $k+\kappa(\mathbf{r})$, and that ψ then describes elastic scattering. It is always assumed[4,5] that $|\kappa(\mathbf{r})|\ll k$, and that $\kappa(\mathbf{r})$ is so slowly varying that it changes by a small fraction of itself in a wavelength $2\pi/k$. The effect of inelastic processes on the elastic scattering can be taken into account approximately by making $\kappa(\mathbf{r})$ complex, and unitarity then requires that $\mathrm{Im}\kappa(\mathbf{r})\geqq 0$. $\kappa(\mathbf{r})$ evidently depends on the structure of the colliding objects; it may depend on k, and may also be expressible in terms of the properties of the component parts of the objects. The spatial extent of $\kappa(\mathbf{r})$ is found by taking the Lorentz contraction into account in the relativistic case.

The elastic scattering amplitude has been calculated in Sec. II of Ref. 5, where $U(\mathbf{r})$ can be replaced by $-2k\kappa(\mathbf{r})$. The results for dynamically large angles are[7]

$$f_l(\mathbf{k}_f,\mathbf{k}_0)=(k/2\pi)\int\kappa(\mathbf{r})\exp i[\mathbf{q}\cdot\mathbf{r}+\delta_0(\mathbf{r})+\delta_f(\mathbf{r})]d^3r,$$
$$\delta_0(\mathbf{r})=\int_0^\infty\kappa(\mathbf{r}-\hat{k}_0s)ds,\quad \delta_f(\mathbf{r})=\int_0^\infty\kappa(\mathbf{r}+\hat{k}_fs)ds. \tag{2}$$

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[1] W. Czyż and L. Leśniak, Phys. Letters **24B**, 227 (1967); **25B**, 319 (1967); R. H. Bassel and C. Wilkin, Phys. Rev. Letters **18**, 871 (1967); J. Formánek and J. S. Trefil, Nucl. Phys. **B3**, 155 (1967); W. Czyż and L. C. Maximon, Phys. Letters (to be published); T. T. Chou, Phys. Rev. **168**, 1594 (1968).

[2] See, also, the papers by R. J. Glauber in *High Energy Physics and Nuclear Structure*, edited by G. Alexander (North-Holland Publishing Co., Amsterdam, 1967), p. 310; W. Czyż and L. Leśniak, Ref. 1, p. 339; T. T. Chou and C. N. Yang, Phys. Rev. Letters **20**, 1213 (1968).

[3] L. Durand and R. Lipes, Phys. Rev. Letters **20**, 637 (1968); T. T. Chou and C. N. Yang, Ref. 2; Phys. Rev. **170**, 1591 (1968); **175**, 1832 (1968).

[4] R. J. Glauber, in *Lectures in Theoretical Physics*, edited by W. E. Brittin and L. G. Dunham (Interscience Publishers, Inc., New York, 1959), Vol. I, p. 315.

[5] L. I. Schiff, Phys. Rev. **103**, 443 (1956).

[6] D. K. Ross, Phys. Rev. **173**, 1965 (1968).

[7] For a numerical test of the accuracy of Eq. (2), see J. J. Tiemann, Phys. Rev. **109**, 183 (1958).

Here, $\hbar\mathbf{k}_0$ and $\hbar\mathbf{k}_f$ are the initial and final momentum vectors, $\mathbf{q}=\mathbf{k}_0-\mathbf{k}_f$, and $\hat{k}$ is a unit vector parallel to $\mathbf{k}$.

The amplitude for dynamically small angles is that obtained in the eikonal approximation. This can be derived in several ways; it is convenient for what follows to make use of the treatment of Ref. 5, and write it at first in one of the forms

$$f_{s1}(\mathbf{k}_f,\mathbf{k}_0)=(k/2\pi)\int\kappa(\mathbf{r})\exp i[\mathbf{q}\cdot\mathbf{r}+\delta_0(\mathbf{r})]d^3r,\quad(3)$$

$$f_{s2}(\mathbf{k}_f,\mathbf{k}_0)=(k/2\pi)\int\kappa(\mathbf{r})\exp i[\mathbf{q}\cdot\mathbf{r}+\delta_f(\mathbf{r})]d^3r.\quad(4)$$

Equations (3) and (4) are not identical, but become so when θ is so small that the component of $\mathbf{q}$ along $\hat{k}_0$ or $\hat{k}_f$ can be neglected. Now $\mathbf{q}\cdot\hat{k}_0=-\mathbf{q}\cdot\hat{k}_f=k(1-\cos\theta)\cong\frac{1}{2}k\theta^2$, and this is negligible when $(\mathbf{q}\cdot\hat{k}_0)R\ll 1$, where R is a typical (Lorentz-contracted) linear dimension of the colliding objects. This condition is evidently satisfied when $\theta\ll\theta_c$, that is, when the angle is dynamically small. We now choose the z axis to lie along $\mathbf{k}_0$, and denote by $\mathbf{b}$ the two-dimensional impact parameter vector (x,y). Then Eqs. (3) and (4) become

$$f_{s1}(\mathbf{k}_f,\mathbf{k}_0)\cong(k/2\pi)\int\int\kappa(\mathbf{b},z)\times\exp i\left(\mathbf{q}\cdot\mathbf{b}+\int_{-\infty}^{z}\kappa(\mathbf{b},z')dz'\right)d^2b\,dz,\quad(5)$$

$$f_{s2}(\mathbf{k}_f,\mathbf{k}_0)\cong(k/2\pi)\int\int\kappa(\mathbf{b},z)\times\exp i\left(\mathbf{q}\cdot\mathbf{b}+\int_{z}^{\infty}\kappa(\mathbf{b},z')dz'\right)d^2b\,dz.\quad(6)$$

The z integrations are easily carried out, and Eqs. (5) and (6) both lead to the usual eikonal expression

$$f_s(\mathbf{k}_f,\mathbf{k}_0)=(ik/2\pi)\int e^{i\mathbf{q}\cdot\mathbf{b}}\left[1-\exp\left(i\int_{-\infty}^{\infty}\kappa(\mathbf{b},z)dz\right)\right]d^2b.\quad(7)$$

The physical picture underlying Eqs. (2)–(4) is as follows: Since $\kappa(\mathbf{r})$ is small and slowly varying, its three-dimensional Fourier transform with respect to $\mathbf{q}$ is small for large q. Then the principal effect of $\kappa(\mathbf{r})$ on the incident wave is to act as a complex refractive index which shifts its phase and decreases its amplitude. This may be thought of as the resultant of a very large number of very small angle scatterings that occur as the colliding objects move through each other. Each of these scatterings produces a spherical wave which can interfere with the original plane wave over an angular region such that the path difference between sphere and plane is of the order of a wavelength or less; at a distance ρ from the scattering point, this angle is of order $(k\rho)^{-1/2}$. Thus associated with each scattering is a paraboloidal volume with vertex at the scattering point and axis of opening along the direction of incidence. So long as one wishes to calculate only the scattering at dynamically small angles, that is, well within the paraboloid, the usual T-matrix formalism can be used with the modified incident wave $\exp i[\mathbf{k}_0\cdot\mathbf{r}+\delta_0(\mathbf{r})]$ or the modified outgoing wave $\exp i[\mathbf{k}_f\cdot\mathbf{r}-\delta_f(\mathbf{r})]$. These lead immediately to Eqs. (3) and (4), respectively.

When the scattering angle is dynamically large, there must be at least one large-angle scattering; in the case considered here there will be only one since such scatterings are very improbable. This means that the modified incident wave consists of two parts:[5] the phase-shifted incident wave $\exp i[\mathbf{k}_0\cdot\mathbf{r}+\delta_0(\mathbf{r})]$, and a wave already scattered through the angle θ. Both of these give contributions of the same order to the T-matrix element, since the first still requires a large-angle scattering and the second does not. It is shown in Sec. IV of Ref. 5 that the two contributions add to give Eq. (2); qualitatively, δ_0 and δ_f are both expected to appear since their paraboloids are distinct from each other when $\theta\gg\theta_c$.

III. MODERATELY LARGE ANGLES

An expression for the scattering amplitude is available that has the same accuracy at all angles that Eqs. (2) and (7) have at dynamically large and small angles.[8] It is rather complicated, and will not be used here since our primary objective is simplicity. Instead, we follow Ross[6] and interpolate between (2) and either (3) or (4):

$$f_1(\mathbf{k}_f,\mathbf{k}_0)=(k/2\pi)\int\kappa(\mathbf{r})\times\exp i[\mathbf{q}\cdot\mathbf{r}+\delta_0(\mathbf{r})+\gamma(\theta)\delta_f(\mathbf{r})]d^3r,\quad(8)$$

$$f_2(\mathbf{k}_f,\mathbf{k}_0)=(k/2\pi)\int\kappa(\mathbf{r})\times\exp i[\mathbf{q}\cdot\mathbf{r}+\gamma(\theta)\delta_0(\mathbf{r})+\delta_f(\mathbf{r})]d^3r.\quad(9)$$

[8] D. S. Saxon and L. I. Schiff, Nuovo Cimento 6, 614 (1957), Eq. (25).

Here, $\gamma(\theta)$ is a smooth, not necessarily real, function of the scattering angle that is zero for $\theta \ll \theta_c$ and unity for $\theta \gg \theta_c$. Neither (8) nor (9) satisfies the reciprocity[9] relation $f(\mathbf{k}_f,\mathbf{k}_0)=f(-\mathbf{k}_0,-\mathbf{k}_f)$, but their average does and can be used for interpolation between dynamically small and large angles.

We now approximate to Eqs. (8) and (9) under the assumption that the scattering angle is geometrically small, i.e., $\theta \ll 1$. This means that δ_0 and δ_f are calculated with the approximation that the $\hat{k}_0$ that appears in δ_0 and the $\hat{k}_f$ that appears in δ_f are each perpendicular to $\mathbf{q}$, instead of departing from perpendicularity by the angle $\frac{1}{2}\theta$. In order to estimate the error involved in this approximation, it is convenient to rotate the earlier coordinates by $\frac{1}{2}\theta$ so that the new x axis lies along $\mathbf{q}$, and $\hat{k}_0$ and $\hat{k}_f$ lie in the xz plane. We can then expand the integrands of δ_0 and δ_f about the z axis; for example,

$$\begin{aligned}\delta_0(\mathbf{r}) &= \int_0^\infty \kappa(x-s\sin\tfrac{1}{2}\theta,\, y,\, z-s\cos\tfrac{1}{2}\theta)ds \\ &= \int_0^\infty \kappa(x,y,z-s)ds \\ &\quad -\tfrac{1}{2}\theta\int_0^\infty s(\partial/\partial x)\kappa(x,y,z-s)ds+O(\theta^2). \qquad (10)\end{aligned}$$

The order of magnitude of the term proportional to θ is $\theta\kappa R$, so that the contributions of this term to the phase can be neglected if $\theta \ll 1/\kappa R$. Since in typical cases κR is not large in comparison with unity, the neglect of all except the first term on the right side of Eq. (10) is justified if θ is geometrically small.

With this approximation, Eqs. (8) and (9) become

$$f_{m1}(\mathbf{k}_f,\mathbf{k}_0)=(k/2\pi)\int\int \kappa(\mathbf{b},z)\exp i\left(\mathbf{q}\cdot\mathbf{b}+\int_{-\infty}^{z}\kappa(\mathbf{b},z')dz'+\gamma(\theta)\int_z^\infty \kappa(\mathbf{b},z')dz'\right)d^2b\,dz, \qquad (11)$$

$$f_{m2}(\mathbf{k}_f,\mathbf{k}_0)=(k/2\pi)\int\int \kappa(\mathbf{b},z)\exp i\left(\mathbf{q}\cdot\mathbf{b}+\gamma(\theta)\int_{-\infty}^{z}\kappa(\mathbf{b},z')dz'+\int_z^\infty \kappa(\mathbf{b},z')dz'\right)d^2b\,dz. \qquad (12)$$

As with Eqs. (5) and (6), the z integrations are easily carried out, and (11) and (12) both lead to

$$\begin{aligned} f_m(\mathbf{k}_f,\mathbf{k}_0) &= \frac{ik}{2\pi[1-\gamma(\theta)]}\int e^{i\mathbf{q}\cdot\mathbf{b}}\left(e^{i\gamma(\theta)\chi(\mathbf{b})}-e^{i\chi(\mathbf{b})}\right)d^2b, \\ \chi(\mathbf{b}) &= \int_{-\infty}^{\infty}\kappa(\mathbf{b},z)dz. \end{aligned} \qquad (13)$$

Equation (13) is our approximation for the scattering amplitude at moderately large angles, i.e., angles that are geometrically small although they may be dynamically small, intermediate, or large. It reduces to the eikonal expression (7) when $\gamma(\theta)=0$ or $\theta \ll \theta_c$, and to

$$(k/2\pi)\int e^{i\mathbf{q}\cdot\mathbf{b}}\chi(\mathbf{b})e^{i\chi(\mathbf{b})}d^2b \qquad (14)$$

when $\gamma(\theta)=1$ or $\theta \gg \theta_c$. It is interesting to note that Eq. (14) is essentially an expression that has been conjectured as a description of pion-nucleon charge-exchange scattering.[10]

The neglect of the θ-dependent terms in the expansion (10) is considered from a different point of view in the Appendix. This consideration is important in reconciling an apparent conflict with one of Ross's numerical examples.[6]

IV. CONCLUDING REMARKS

In the eikonal approximation, $e^{i\chi(\mathbf{b})}$ is simply the transmission factor or S matrix element $S(\mathbf{b})$ that corresponds to the impact parameter $\mathbf{b}$ or (with spherical symmetry) the orbital-angular-momentum quantum number $l=bk-\frac{1}{2}$. It is often assumed[1–3] that $S(\mathbf{b})$ can be written as a product of matrix elements $S_{\alpha\beta}$, each of which describes the scattering of a component part α of one of the colliding objects on a component part β of the other. The eikonal approximation expresses the elementary scattering amplitude as the two-dimensional Fourier transform of $a_{\alpha\beta}=1-S_{\alpha\beta}$. The total amplitude is then the Fourier transform of

$$a=1-S=1-\prod_{\alpha,\beta}S_{\alpha\beta}=1-\prod_{\alpha,\beta}(1-a_{\alpha\beta}), \qquad (15)$$

and the number of $a_{\alpha\beta}$'s multiplied together in any term on the right side of Eq. (15) cannot exceed the number of interacting pairs of component parts of the two objects. Physically, this limitation occurs because the over-all scattering angle is very small in the eikonal approximation.

When Eq. (13) is used, however, the total scattering amplitude is the Fourier transform of

$$(S^\gamma-S)/(1-\gamma)=a-\tfrac{1}{2}\gamma a^2-\tfrac{1}{6}\gamma(2-\gamma)a^3-\cdots, \qquad (16)$$

where a is given by Eq. (15) if the elementary scatterings can be described by the eikonal approximation. Thus, as would be expected physically, multiple scatterings of the component parts of arbitrarily high order occur when the over-all scattering angle is not dynamically small, that is, when $\gamma(\theta)>0$.

It should also be noted that, as has been done with the eikonal approximation,[1–3] the $S_{\alpha\beta}$ can be regarded as operators. Then matrix elements of (13) can be

[9] R. Glauber and V. Schomaker, Phys. Rev. **89**, 667 (1953).
[10] N. Byers and C. N. Yang, Phys. Rev. **142**, 976 (1966).

taken between initial and final states of the colliding objects that differ through rearrangement of the component parts of each. This corresponds to inelastic scattering in the nuclear case, and to the diffractive excitation process of Chou and Yang[3] in the hadronic case.

Finally, it is interesting to compare the two available sets of high-energy proton-He^4 scattering data in the context of Eq. (13). The Brookhaven group has published elastic cross sections at 1000 MeV,[11] and the Virginia group at 600 MeV,[12] with angular ranges such that values of invariant momentum transfer squared up to 0.7 $(\mathrm{BeV}/c)^2$ appear in both experiments. The nucleon-nucleon amplitude used in analyzing these data is of the form $kg(q^2)$. This means that $\kappa(\mathbf{r})$ is independent of k so that the earlier work[1,2] based on the eikonal expression (7) predicts that the differential cross section in the c.m. system is proportional to k^2. In actuality, for momentum transfer squared less than 0.5 $(\mathrm{BeV}/c)^2$, the cross section is proportional to k^2 everywhere except in the neighborhood of the minimum (see Fig. 1).

This behavior is to be expected when Eq. (13) is used. For dynamically small angles, $\gamma\cong 0$ in both sets of experiments, and for dynamically large angles, $\gamma\cong 1$ in both cases. Thus in each of these situations, γ is fixed in going from 600 to 1000 MeV, and the cross section is expected to scale in proportion to k^2, as indeed it does. In the dynamically intermediate region, however, where the angle might still be regarded as geometrically small, $\gamma(\theta)$ is changing rapidly, and not in the same way for the two sets of experiments since their θ_c values are somewhat different. Since the two minima occur in the neighborhood of the two values of θ_c (20° to 30° in the c.m. system), the two sets of data should not scale in proportion to k^2 near this first minimum. In fact they do not: The minimum cross section in the 600-MeV data is relatively higher and occurs at a larger momentum transfer than in the 1000-MeV data. This observation provides support for the superiority of (13) with respect to (7) at moderately large angles.

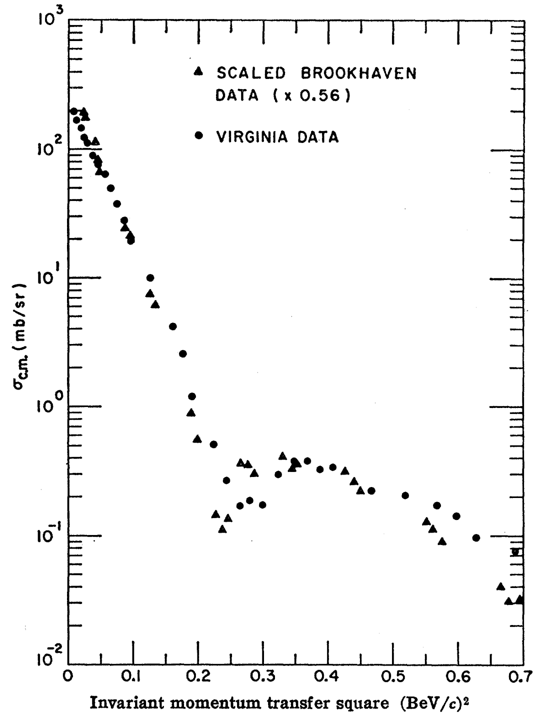

FIG. 1. Elastic proton-He^4 differential cross sections in the c.m. system from Brookhaven at 1000 MeV laboratory energy (Ref. 11) and from Virginia at 600 MeV (Ref. 12). The Brookhaven data are scaled down by the square of the ratio of the proton momenta in the c.m. system: $(0.878/1.170)^2=0.56$.

APPENDIX

There appears to be no reason why the expansion in Eq. (10), and the subsequent neglect of the θ-dependent terms, cannot be applied to Eq. (3). If this were done, it would be equivalent to the neglect of the component of $\mathbf{q}$ parallel to $\hat{k}_0$ in the exponent of the integrand, which, as shown in Sec. II, leads to the eikonal expression (7). However, Ross showed, by means of a numerical example in Sec. II of his paper,[6] that neglect of this longitudinal component of $\mathbf{q}$ seriously overestimates the amplitude even for quite moderate scattering angles. Another numerical result, quoted in Sec. IV of his paper, shows that at $\theta=90°$, inclusion of the δ_f term in Eq. (2) increases the amplitude by a factor 30 as compared with its omission. It is the purpose of this Appendix to show that these results make plausible the expansion procedure of Eq. (10) when applied to (8) with γ appreciably different from zero, but not when applied to (3).

We write Eq. (8) in the form

$$f_1(\mathbf{k}_f,\mathbf{k}_0)=(k/2\pi)\int e^{i\mathbf{q}\cdot\mathbf{r}}F_1(\mathbf{r})F_2(\mathbf{r})d^3r\,,$$

$$F_1(\mathbf{r})=\kappa(\mathbf{r})e^{i\delta_0(\mathbf{r})}=(2\pi)^{-3/2}\int\phi_1(\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{r}}d^3k\,,\quad(17)$$

$$F_2(\mathbf{r})=e^{i\gamma(\theta)\delta_f(\mathbf{r})}=(2\pi)^{-3/2}\int\phi_2(\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{r}}d^3k\,.$$

It follows at once that

$$f_1(\mathbf{k}_f,\mathbf{k}_0)=(k/2\pi)\int\phi_1(\mathbf{k})\phi_2(\mathbf{q}-\mathbf{k})d^3k\,.\quad(18)$$

If we suppose for simplicity that $\kappa(\mathbf{r})$ is spherically symmetric, then $\phi_1(\mathbf{q})$ has axial symmetry about the vector $\hat{k}_0$ through the origin of $\mathbf{k}$ space, and falls off

[11] H. Palevsky, J. L. Friedes, R. J. Sutter, G. W. Bennett, G. J. Igo, W. D. Simpson, G. C. Phillips, D. M. Corley, N. S. Wall, R. L. Stearns, and B. Gottschalk, Phys. Rev. Letters **18**, 1200 (1967).

[12] E. T. Boschitz, W. K. Roberts, J. S. Vincent, K. Gotow, P. C. Gugelot, C. F. Perdrisat, and L. W. Swenson, Phys. Rev. Letters **20**, 1116 (1968).

anisotropically with a scale factor of order $1/R$ as $\mathbf{k}$ moves away from the origin. $\phi_2(\mathbf{k})$ has a similar behavior with $\hat{k}_f$ substituted for $\hat{k}_0$, but in addition has a δ-function singularity at the origin and a $1/k$ singularity at the origin along the $\pm\hat{k}_f$ directions. The latter singularity and the ϕ_1-like behavior both decrease as $\gamma \to 0$, and only the δ function remains when $\gamma = 0$.

If now we apply the Fourier transforms in Eq. (17) to (3), we see that f_{s1}, which is f_1 with $\gamma = 0$, is equal to $(k/2\pi)\phi_1(\mathbf{q})$. Then, the first of Ross's numerical examples quoted above shows that $\phi_1(\mathbf{q})$ falls off rapidly as $\mathbf{q}$ acquires a longitudinal component, that is, as it departs significantly from the plane through the origin perpendicular to $\hat{k}_0$. On the other hand, if we consider Eq. (8) from the same point of view when γ is appreciably different from zero, we see that there is a substantial overlap of ϕ_1 and ϕ_2 in the integrand of (18). This occurs when $\mathbf{k}$ is roughly equal to $\frac{1}{2}\mathbf{q}$ and in the plane through the origin perpendicular to $\hat{k}_0$, and $\mathbf{q}-\mathbf{k}$ is roughly equal to $\frac{1}{2}\mathbf{q}$ and in the plane through the origin perpendicular to $\hat{k}_f$. This overlap accounts for the second of Ross's examples.

The conclusion is that when $\gamma = 0$, inclusion of $\mathbf{q}\cdot\hat{k}_0$, the longitudinal component of $\mathbf{q}$, causes (3) to be considerably smaller than (7). But when γ is appreciably different from zero, inclusion of the $\gamma\delta_f$ term in (8) compensates for this decrease. Since $\gamma(\theta)$ and $\mathbf{q}\cdot\hat{k}_0$ increase together as θ increases from zero, it is then permissible to make use of the expansion (10) in going from (8) to (11).

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Nuclear-Matter Sizes in the Tin Isotopic Sequence*†

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Differential cross sections and polarizations have been measured for 39.6-MeV protons elastically scattered from ^{116}Sn, ^{118}Sn, ^{120}Sn, ^{122}Sn, and ^{124}Sn. These data were analyzed with the optical model of Greenlees, Pyle, and Tang to extract nuclear-matter rms radii. These radii are significantly greater than the corresponding proton radii, and indicate that the neutrons added between successive even isotopes are added to the surface of the neutron distribution, with the exception of the isotope pair ^{118}Sn-^{120}Sn, in which case an anomaly occurs which suggests a structural rearrangement.

I. INTRODUCTION

THE optical model has been used extensively in the analysis of proton elastic-scattering data in the energy region below 50 MeV[1,2] and has been remarkably successful in representing such data. However, ambiguities exist in the parametrization, making it impossible to quote a unique set of parameters even for the scattering from one isotope at one energy. It is, therefore, difficult to extract any physically significant information from such analyses.

Recently, Greenlees, Pyle, and Tang[3] have produced a reformulation of the model which derives the form factors of the real-central and spin-orbit parts of the potential from the nuclear-matter distribution and appropriate components of the nucleon-nucleon potential. It has proved possible in this formulation to extract, from proton elastic-scattering data, the nuclear matter rms radius to an accuracy of 2–3%, and the volume integral of the real-central potential to about 5%. Reference 3 analyzed a range of elements with A from 58 to 208 at incident proton energies of 14.5, 30, and 40 MeV. The nuclear-matter rms radii obtained were independent of energy for a given A and significantly greater than the corresponding nuclear proton rms radii obtained from electron scattering and μ-mesonic x-ray studies. The volume integrals of the real-central potential were simply related for various mass numbers at a given energy, but the results suggested a gradual decrease in these integrals with increasing incident proton energy.

For analyses using the model of Ref. 3 it is desirable to have both elastic differential cross-section and polarization data. The present experiment involved the measurement of such data for protons with an average incident energy of 39.6 MeV scattered from a range of tin isotopes. The primary motivation was the analysis of the data, using the model of Ref. 3, to study the variation of nuclear-matter rms radii within an isotopic sequence.

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[3] G. W. Greenlees, G. J. Pyle, and Y. C. Tang, Phys. Rev. **171**, 1115 (1968).