^{3}He and neutrons. Both the shallow and deep combinations of ^{3}He and α optical potential wells yield good fits to the experimental angular distributions provided that this criterion was satisfied. Spectroscopic factors extracted from both combinations agreed with each other within the extraction uncertainties of 10–20%.

The levels of ^{88}Y are very similar to those of ^{89}Zr [1] where the $1g_{9/2}$, $2p_{1/2}$, and $2p_{3/2}$ neutron hole states make up the low-lying states (Fig. 7). In ^{88}Y these hole states couple to the $2p_{1/2}$ proton state and each splits into two levels. The total $1g_{9/2}$ neutron strength is observed in the ground and 0.25-MeV states (4^- and 5^-, respectively) in ^{88}Y. In ^{85}Kr, on the other hand, the spectroscopic factor of the g.s. ($\frac{9}{2}^+$) represents only 60% of the full $1g_{9/2}$ neutron strength. Possibly the states in ^{85}Kr at 1.89- and 2.03-MeV excitation represent some of the remaining $\frac{9}{2}^+$ hole strength. The splitting of $\frac{9}{2}^+$ neutron hole states has also been observed in ^{87}Sr by Bassani.[2]

In summary, spectroscopic information gathered from (^{3}He,α) reaction studies on $N=50$ nuclei has given evidence that the neutron shell is essentially closed in all the five nuclei studied. The sequence of single neutron hole states, except the $1g_{9/2}$ hole state, remains the same as the proton number increases from 36 to 42. The $\frac{9}{2}^+$ neutron hole strength was found to be distributed over two or more states including the g.s. in ^{87}Sr and ^{85}Kr (where $Z\leq 38$) while for $Z>38$ (^{88}Y, ^{89}Zr, and ^{91}Mo) the total strength seems to be in the g.s. only (or g.s. and the first excited state because of the unpaired $2p_{1/2}$ proton in ^{88}Y). According to the single-particle shell model, at $Z=38$ the $2p_{3/2}$ and $1f_{5/2}$ proton subshells are closed while both $2p_{1/2}$ and $1g_{9/2}$ subshells are open. The splitting of $\frac{9}{2}^+$ neutron hole states is probably related to the number of open proton subshells in the target. Therefore, an investigation of the proton configurations in the $N=50$ nuclei would be very interesting.

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Dipole States in Sr^{88}

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A calculation of the dipole states in Sr^{88} is presented. In addition to the conventional one-particle–one-hole configurations, the calculation includes a set of two-particle–two-hole excitations, with the additional particle-hole pair coupled to angular momentum zero and isospin zero or one. This not only increases the number of degrees of freedom but also allows for the appearance of states with definite isospin in the final results. The dipole spectrum separates now into two groups of states with definite isospin. One group corresponds to isospin T equal to that of the ground state. It contains the giant dipole resonance of the nucleus and most of the ground-state dipole strength. The states in the second group are analog states with isospin $T+1$. They are fewer in number and lie systematically at higher energies relative to the members of the first group. They are also found to exhibit typical dipole features such as the presence of a coherent state at the high-energy side of the spectrum. The positions and the radiative strengths of the calculated levels are discussed.

I. INTRODUCTION

THE starting point for most theoretical considerations regarding the structure of nuclear dipole states is related to the observation that the electric dipole emission or absorption of a photon by a nucleus involves the transition of a single nucleon between orbitals of different parity. Thus the absorption of a photon causes the promotion of a nucleon from an occupied to an unoccupied level, i.e., to the formation of a particle-hole pair. The various possible pairs interact with each other, and this interaction is known[1] to be

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§ Supported in part by the U. S. Atomic Energy Commission.

[1] See, e.g., G. E. Brown, *Unified Theory of Nuclear Models and Forces* (North-Holland Publishing Co., Amsterdam, 1967).

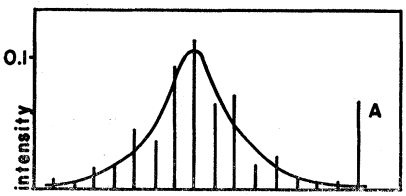

FIG. 1. Schematic representation of the fragmentation of a particle-hole excitation into neighboring $2p$-$2h$ states. The possible appearance of an analog state (A) is shown.

responsible for such familiar effects as the concentration of the dipole strength in the giant dipole resonance region. The particle-hole states, however, are not in general stationary states of the nuclear system. They are usually surrounded by a large number of complex multiparticle excitations with which they interact. A fragmentation of the elementary particle-hole modes is a result of this coupling and manifests itself (at least in a poor-resolution experiment) as a spreading of the original mode over a finite energy region. The particle-hole components, however, still dominate the spectrum, i.e., they determine the main features of the response of the nucleus to an external (dipole) perturbation and are therefore the appropriate doorways[2] to a photon-initiated reaction.

The study of the detailed fragmentation of the doorway states[3] is, in general, a very complicated problem whose gross features can be summarized at best by an estimate of the over-all spreading of the elementary modes. In some cases, however, it is found appropriate to isolate a limited set of configurations[4] (e.g., two-particle–two-hole excitations) and study in detail the effect of their participation. The selection is guided by the expected simplicity and systematic features of the resulting spectrum and by the predominant strength of the interactions included in such a calculation. The studies of the influence of low-lying vibrations on the dipole spectrum provide an interesting example of this approach.[5]

In this work the participation of a different class of two-particle–two-hole excitations is considered. The selection is basically motivated by the observation that in nuclei with $N>Z$ the traditional one-particle–one-hole ($1p$-$1h$) states are not always characterized by a definite isospin.[6] This is indicated by the fact that the isospin vector does not commute with the quasiparticle number operator (defined as the operator counting the number of particles plus the number of holes), and is also familiar from experience with isobaric analog states. The existence of analog states, on the other hand, suggests that the isospin quantum number is relevant even in heavy nuclei, i.e., that the dipole spectrum will tend to separate into classes of states with distinct isospin symmetries.[7,8] It is then clear that a calculation will fail to reproduce this trend unless it takes into account the appropriate set of two-particle–two-hole ($2p$-$2h$) excitations. In this paper such components are explicitly included, and a study of dipole states with definite isospin characteristics is presented. The appropriate configuration space is defined in Sec. II, followed by a discussion of the unperturbed energies of the individual excitations in Sec. III. The interactions between the various configurations are studied in Sec. IV. Sections V and VI contain the results and a summary of this work.

II. CONFIGURATION SPACE

There is some freedom in the choice of representation for the basis states to be selected, depending on the relative priority of the residual interactions that one chooses to emphasize. One possibility is to exhibit from the beginning the analog or nonanalog (lower T or normal) character of the representation and let the residual interactions subsequently introduce the dipole correlations. Another approach is to start from the conventional $1p$-$1h$ states, in which case the isospin symmetry only appears as a result of specific two-body collisions leading to $2p$-$2h$ excitations. The final results

[2] H. Feshbach, A. K. Kerman, and R. H. Lemmer, Ann. Phys. (N. Y.) **41**, 230 (1967).

[3] The fact that these states are also coupled to the continuum of scattered particles is not explicitly mentioned in this discussion but should also be kept in mind.

[4] The term "hallway states" has been used for excitations of this general type by R. A. Ferrell and W. M. MacDonald, Phys. Rev. Letters **16**, 187 (1966). It should be remembered, however, that the terminology often depends on the particular channel under consideration. Some of the $2p$-$2h$ configurations included in this work, although not excited by γ-ray absorption, are still directly coupled even to the elastic proton channel.

[5] J. LeTourneux, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. **34**, No. 11 (1965); A. K. Kerman and Ho Kim Quang, Phys. Rev. **135**, B883 (1964); C. B. Duke, F. B. Malik, and F. W. K. Firk, *ibid.* **157**, 879 (1967); N. Yudin, in Proceedings of the International Conference on Electromagnetic Interactions, Dubna, 1967 (unpublished); M. Danos and W. Greiner, Phys. Rev. **134**, B284 (1964); a number of additional references are given by W. Greiner, in *Recent Progress in Nuclear Physics with Tandems* (Heidelberg, 1966) and in several subsequent publications.

[6] This was emphasized already in the original considerations of M. H. Macfarlane and J. F. French, Rev. Mod. Phys. **32**, 567 (1960); Nucl. Phys. **26**, 168 (1961) and of A. M. Lane and J. M. Soper, Phys. Rev. Letters **7**, 420 (1961); Phys. Letters **1**, 28 (1962).

[7] The relevance of isospin for dipole states in heavy nuclei has been discussed by H. Morinaga, Z. Physik **188**, 182 (1965); S. Fallieros, B. Goulard, and R. H. Venter, Phys. Letters **19**, 398 (1965); M. H. Macfarlane, in *Isobaric Spin in Nuclear Physics*, edited by J. D. Fox and D. Robson (Academic Press Inc., New York, 1966); V. V. Balashov and E. L. Yadrowsky, Phys. Letters, **25B**, 67 (1967); C. J. Veje and D. F. Petersen, *ibid.* **24B**, 449 (1967); F. Lewis, Bull. Am. Phys. Soc. **12**, 906 (1967); B. Goulard and S. Fallieros, Can. J. Phys. **45**, 3221 (1967).

[8] Experimental (p,γ) reaction studies of $E1$ decays of analog states have been reported by J. L. Black and N. W. Tanner, Phys. Letters **11**, 135 (1964); J. Black, M. M. Islam, G. A. Jones, G. C. Morrison, and B. B. Taylor in *Isobaric Spin in Nuclear Physics*, edited by J. D. Fox and D. Robson (Academic Press Inc., New York, 1966), p. 863; S. M. Shafroth and G. J. F. Legge, *ibid.*, p. 219; Nucl. Phys. **A107**, 181 (1968); P. Axel, D. M. Drake, S. Whetstone and S. S. Hanna, Phys. Rev. Letters **19**, 1348 (1967); M. Hasinoff, H. M. Kuan, S. S. Hanna, and G. A. Fisher, Bull. Am. Phys. Soc. **6**, 893 (1968); F. Rauch, Bartol Research Foundation Report (unpublished); H. Ejiri, P. Richard, S. Ferguson, R. Heffner, and D. Perry, University of Washington Report (unpublished).

are of course independent of the choice of representation. The first approach is more convenient for the study of analog states, but we will mainly emphasize the second alternative. This reduces the problem to a special case of the study of $2p$-$2h$ effects and makes possible the extension of the configuration space beyond what is demanded by isospin requirements alone.

Our problem is then the study of the solutions of the Schrödinger equation

$$H|E\rangle=E|E\rangle, \tag{1}$$

where E is the excitation energy of the eigenstates $|E\rangle$ which contain both $1p$-$1h$ and other components. They are restricted in this work by the proper angular momentum and parity requirements, e.g., $J^\pi=1^-$ for an even-even nucleus. It is convenient to separate the total Hamiltonian H into three parts:

$$H=H_{ph}+\bar{H}+H', \tag{2}$$

where H_{ph} is limited to the $1p$-$1h$ space, $\bar{H}$ acts on all the remaining excitations, and H' is the interaction between the two spaces. This can be accomplished, e.g., with the help of the projection operators

$$\Lambda=\sum_{ph}|ph\rangle\langle ph|, \quad \bar{\Lambda}=1-\Lambda,$$

in which case

$$H_{ph}=\Lambda H\Lambda, \quad \bar{H}=\bar{\Lambda}H\bar{\Lambda}$$

and

$$H'=\Lambda H\bar{\Lambda}+\bar{\Lambda}H\Lambda.$$

The Schrödinger equation is then rewritten in the form

$$E\langle ph|E\rangle=\sum_{(ph)'}\langle ph|H_{ph}+H'[1/(E-\bar{H})]H'|(ph)'\rangle\langle(ph)'|E\rangle. \tag{3}$$

The fragmentation of the doorway states is indicated by the second term in the interaction matrix element, which has a great number of poles on the real-energy axis. The dipole matrix elements depend only on the particle-hole amplitudes $\langle ph|E\rangle$ and can be written in the form

$$\langle 0|Q_1|E\rangle=\sum_{ph}\langle 0|Q_1|ph\rangle\langle ph|E\rangle, \tag{4}$$

where $Q_1{}^\mu=\sum_{i=1}^{A}e_i r_i Y_1{}^\mu(i)$ is the dipole operator and the effective charges e_i are N/A and $-Z/A$ for protons and neutrons, respectively. An oversimplified illustration of a possible solution of Eq. (3) is given in Fig. 1. The fragmented dipole strength of a single p-h configuration (with strength about five times that of the central line) is represented by the vertical lines. An exceptionally large matrix element of H' can lead to the appearance of a state with particularly large strength. This is shown on the right side of Fig. 1 and illustrates the appearance of an analog state. It should be noted that the dipole strength of this state is still quite small relative to the total strength of all the other components. Not shown is the additional fragmentation of the analog into its neighboring (lower T) states which can be caused by the residual Coulomb interaction or by the common coupling of these states to the continuum of scattered proton waves.[9] These effects are neglected in the present work.

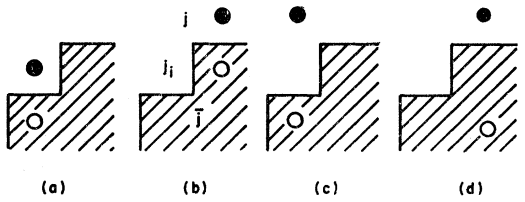

FIG. 2. The four possible types of $1p$-$1h$ configurations contributing to a dipole excitation. In (a) and (b) the proton or the neutron hole appears in the neutron-excess subshells and the excitation has definite isospin T. The configurations (c) and (d) can be analyzed into isoscalar and isovector components. The latter participate also in states with isospin $T+1$.

We now proceed with the identification of the type of excitations included in the present work (i.e., the limitation of the space of $\bar{H}$) and the introduction of the appropriate notation. We will restrict ourselves to the case of an even-even nucleus with the number of neutrons exceeding the number of protons. The ground state will be represented by the ground configuration, i.e., ground-state correlations will not be taken into account. In order to emphasize the distinction between analog and normal excitations we also neglect the difference between individual proton and neutron wave functions. In this case the ground state $|0\rangle$ has a definite isospin $T=\frac{1}{2}(N-Z)$ and third component $T_3=-T$. The various types of possible particle-hole pairs with angular momentum and parity $J^\pi=1^-$ are shown in Fig. 2, where we emphasize the difference between configurations with the proton (neutron hole) in or above (below) the neutron excess shells. We denote by j the levels unoccupied by both protons and neutrons and by $\bar{j}$ the doubly occupied subshells. Finally, j_i represents the neutron-excess levels with i distinguishing between the individual subshells. Thus the symbol $|p_i\bar{p}\rangle$ corresponds to Fig. 2(a) with the quantum numbers j_i and $\bar{j}$ implicitly understood, while $|n\bar{n}_i\rangle$, $|p\bar{p}\rangle$, and $|n\bar{n}\rangle$ represent Figs. 2(b), 2(c), and 2(d), respectively.[10] It is noticed immediately that while the states $|p_i\bar{p}\rangle$ and $|n\bar{n}_i\rangle$ have a definite isospin equal to T, this is not true for the pairs $|p\bar{p}\rangle$ and $|n\bar{n}\rangle$, which contain both T and $T+1$ components. In order to construct states with definite isospin we have to involve also the analog of the ground state, which is defined by the relation

$$|a\rangle=(2T)^{-1/2}T_+|0\rangle \tag{5}$$

and can be expressed as a linear combination of con-

[9] See, e.g., D. Robson, Ann. Rev. Nucl. Sci. 16, 119 (1966).

[10] In the case of $n\bar{n}_i$ the bar refers to a neutron hole with quantum number j_i not $\bar{j}$.

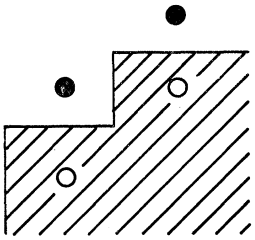

FIG. 3. A typical $2p$-$2h$ configuration is shown. The proton and the neutron hole have $J^\pi=0^+$, while the neutron and the proton hole are coupled to $J^\pi=1^-$.

figurations

$$|a\rangle=\sum_i g_i|[p_i\bar{n}_i]_0\rangle, \tag{6}$$

with

$$g_i=[(2j_i+1)/2T]^{1/2}.$$

The operator T_+ transforms neutrons into protons, and we represent by $[p_i\bar{n}_i]_0$ a proton–neutron-hole pair in the subshell i coupled to angular momentum zero. Clearly, the state $|a\rangle$ has also isospin T and third component $-T+1$. We can now in a straightforward manner replace the expressions $|p\bar{p}\rangle$ and $|n\bar{n}\rangle$ by linear combinations with definite isospin symmetry. We notice first that the configuration $|n\bar{p}\rangle$, which represents a neutron and a proton hole in the levels j and $\bar{j}$, respectively, has a unique isospin equal to $T+1$. Its analog, which we denote by $|A\rangle$, contains three components,

$$\begin{aligned}|A\rangle&=[2(T+1)]^{-1/2}T_+|n\bar{p}\rangle\\&=(T+1)^{-1/2}[2^{-1/2}(|p\bar{p}\rangle-|n\bar{n}\rangle)+T^{1/2}|n\bar{p};a\rangle],\end{aligned} \tag{7}$$

and has also isospin $T+1$. The expression $|n\bar{p};a\rangle$ represents a linear combination of $2p$-$2h$ excitations of the type shown in Fig. 3. We can now verify easily that the expressions

$$|\bar{A}\rangle=(T+1)^{-1/2}[-(\tfrac{1}{2}T)^{1/2}(|p\bar{p}\rangle-|n\bar{n}\rangle)+|n\bar{p};a\rangle]$$

and

$$2^{-1/2}(|p\bar{p}\rangle+|n\bar{n}\rangle), \tag{8}$$

have also definite isospin equal to T and represent normal, T-lower excitations of the nucleus. We have thus found that the introduction of excitations of the type $|n\bar{p};a\rangle$ is not only a necessary but also a sufficient condition for the construction of a basis with definite isospin. We prefer, however, to extend the basis a little further and include in this calculation all monopole excitation of the type $[p_i\bar{n}_i]_0$. In addition to the analog $|a\rangle$, we include therefore also all its T-lower counterparts represented by the expressions

$$|\bar{a}\rangle=\sum_i \bar{g}_i|[p_i\bar{n}_i]_0\rangle,$$

with

$$\sum_i g_i\bar{g}_i=0, \tag{9}$$

which clearly have isospin $T-1$. The $2p$-$2h$ excitations $|n\bar{p};\bar{a}\rangle$ are then states in the nucleus under consideration with $J^\pi=1^-$ and isospin T, and their participation will affect the properties of the T-lower dipole spectrum.

This essentially completes the definition of the unperturbed states whose interactions are studied in this work. We find it more convenient, however, to use in the following instead of $|a\rangle$ and $|\bar{a}\rangle$ the simpler configurations $|n\bar{p};[p_i\bar{n}_i]_0\rangle$, with the index i running through all the neutron-excess levels. Although our basis states do not have definite isospin, our final results will, because of the charge-independent nature of the nuclear interactions. The expressions introduced above will be, on the other hand, helpful for the understanding of the structure of the interaction matrix and the interpretation of the final results.

Before we conclude this section we should discuss the omission of excitations such as $|n\bar{p};[p_i\bar{n}_i]_2\rangle$, where the proton and the neutron hole are coupled to angular momentum two rather than zero. This is only motivated by simplicity and consistency with the fact that we did not take any dipole-quadrupole coupling into account. Since we want mainly to emphasize the isospin structure rather than all the detailed features of the dipole spectrum, we believe that this only clarifies the discussion and is justified.

III. UNPERTURBED ENERGIES

We will denote by $\epsilon_n(j)$, $\epsilon_p(j)$, and $\epsilon_p(j_i)$ the single-particle energies of neutrons in the levels j and of protons in levels j and j_i, while single-hole energies (i.e., the energies of the particles whose absence corresponds to the hole) of protons in state $\bar{j}$ and neutrons in $\bar{j}$ and j_i will be written as $\epsilon_p(\bar{j})$, $\epsilon_n(\bar{j})$, and $\epsilon_n(j_i)$, respectively.

Clearly, proton and neutron energies for particles in corresponding levels are related by

$$\epsilon_p(j)=\epsilon_n(j)+\Delta_c-U(j), \tag{10}$$

and similarly for $\bar{j}$ and j_i. Here Δ_c represents the Coulomb displacement energy (including the neutron-proton mass difference) which, as the notation indicates, will be assumed to be independent of the quantum numbers of the particular level. This assumption obviously excludes isospin impurities, which are therefore ignored in the present work. The quantity $U(j)$ in Eq. (10) is the symmetry energy defined explicitly later, which results from the greater attraction on a proton (relative to a neutron) by the excess neutrons. Although its dependence on the quantum number of the level j is explicitly indicated, we will later on simplify the numerical work by assuming U to be a constant, independent of j. The unperturbed proton–proton-hole energy will be denoted by $\epsilon_p(j)-\epsilon_p(\bar{j})$, and will differ according to Eq. (10) from the energy of the corresponding $|n\bar{n}\rangle$ system, $\epsilon_n(j)-\epsilon_n(\bar{j})$, only by the difference in symmetry energy $U(j)-U(\bar{j})$. In a similar manner, the energies of particle-hole states $|p_i\bar{p}\rangle$ and $|n\bar{n}_i\rangle$ involving protons or neutron holes in one of the

neutron-excess levels are written as $\epsilon_p(j_i)-\epsilon_p(\tilde{j})$ and $\epsilon_n(j)-\epsilon_n(j_i)$, respectively, where again the subscript i specifies a particular subshell occupied only by neutrons. Finally, the unperturbed energy of the $2p$-$2h$ state $|n\bar{p};[p_i\bar{n}_i]_0\rangle$ is

$$\langle n\bar{p};[p_i\bar{n}_i]_0|H_{\rm sp}|n\bar{p};[p_i\bar{n}_i]_0\rangle = \epsilon_n(j)-\epsilon_p(\tilde{j})+\epsilon_p(j_i)-\epsilon_n(j_i), \quad (11)$$

where $H_{\rm sp}$ represents the part of the Hamiltonian containing only the particle-shell interactions. Because of Eq. (10), the right-hand side of Eq. (11) can be rewritten as

$$\epsilon_n(j)-\epsilon_n(\tilde{j})+U(\tilde{j})-U(j_i) = \epsilon_p(j)-\epsilon_p(\tilde{j})+U(j)-U(j_i). \quad (11')$$

Notice that if the approximation that U is independent of j is made, the unperturbed energies of the $|p\bar{p}\rangle$, $|n\bar{n}\rangle$, and $|n\bar{p}[p_i\bar{n}_i]_0\rangle$ systems, for a given set of j and $\tilde{j}$, all become equal to each other and depend only on the characteristics of the levels j and $\tilde{j}$, but not on the subshell index i. This will be true in the numerical application presented below. The set of single-particle and single-hole energies which were used in the case of Sr^{88} will be discussed in Sec. V. The fact that the unperturbed energies of the $2p$-$2h$ systems are at least approximately equal to those of the corresponding $1p$-$1h$ states can be considered by itself as an argument indicating the necessity of including them in a calculation, independent of the isospin considerations mentioned earlier. It should be remembered, however, that the importance of a given configuration depends on the size of its complete diagonal energy rather than its value in the absence of interactions.

IV. CONFIGURATION INTERACTIONS

Equation (2) was quite useful in indicating the general effect on the particle-hole configurations resulting from the possible participation of more complex multiparticle excitations. However, once we have specified the problem by defining the Hilbert space of the configurations that are considered relevant, we find it more convenient to proceed directly and construct the complete interaction matrix explicitly. The particle-hole interaction matrix elements are well known but we define them here in order to introduce an appropriate notation. The nuclear matrix elements, for example, between configurations of the type $|n\bar{n}_i\rangle$, will be written in the form $V_{nn}(jj_i;j'j_i')$ which, in turn, is a special case of an expression generally defined by

$$V_{\tau\tau'}(\alpha\beta;\gamma\delta) = \sum_{mm'}(\alpha\beta m-m|J0)(-1)^{\beta-m}(\gamma\delta m'-m'|J0)(-1)^{\delta-m'} \times(\phi_{\alpha m}{}^{\tau}(1)\phi_{\delta m'}{}^{\tau'}(2),\hat{V}_{12}\phi_{\beta m}{}^{\tau}(1)\phi_{\gamma m'}{}^{\tau'}(2)), \quad (12)$$

with $J=1$ for the dipole case. Obviously, the diagonal elements $V_{\tau\tau}$ are independent of τ, i.e., $V_{pp}=V_{nn}$ for the same α, β, γ, and δ.

The individual-particle wave functions $\phi_{\alpha m}{}^{\tau}$ represent a particle of charge τ, angular momentum α, and third component m. The two-body interaction is denoted by V_{12}, and the circumflex in Eq. (12) indicates that the exchange counterpart of the matrix element is also included; viz., $\hat{V}_{12}=V_{12}(1-P_{12})$, where P_{12} is the complete (space, spin, isospin) exchange operator. In a similar manner we write for the interaction between configurations of the $|p_i\bar{p}\rangle$ type the expression $V_{pp}(j_i\tilde{j};j_i'\tilde{j}')$ while the interaction between $|p_i\bar{p}\rangle$ and $|n\bar{n}_i\rangle$ will be given by $V_{pn}(j_i\tilde{j};j'j_i')$.

In dealing with excitations containing protons and neutrons in doubly unoccupied levels and proton and neutron holes in doubly occupied subshells we will find it more convenient to express these results by analyzing them into their appropriate isoscalar and isovector parts, i.e., we define

$$|s\rangle=2^{-1/2}(|p\bar{p}\rangle+|n\bar{n}\rangle) \quad (13a)$$

and

$$|v\rangle=2^{-1/2}(|p\bar{p}\rangle-|n\bar{n}\rangle). \quad (13b)$$

This is clearly useful not only because of the considerable difference in the radiative matrix elements of the isoscalar and isovector excitations but also because of their distinct role in the formation of wave functions with definite isospin symmetry. It was shown in Sec. III that the isoscalar configuration can only participate in states with total isospin T, whereas the isovector excitations contribute both to states with isospin T and $T+1$. Again the symbols $|s\rangle$ and $|v\rangle$ refer to excitations with one particle and one hole in the levels j and $\tilde{j}$, respectively, but now with symmetrized and antisymmetrized charge quantum numbers. Their unperturbed energies are

$$\begin{aligned}\langle s|H_{\rm sp}|s\rangle&=\langle v|H_{\rm sp}|v\rangle\\ &=\tfrac{1}{2}[\epsilon_p(j)-\epsilon_p(\tilde{j})+\epsilon_n(j)-\epsilon_n(\tilde{j})]\\ &=\epsilon_n(j)-\epsilon_n(\tilde{j})-\tfrac{1}{2}[U(j)-U(\tilde{j})]\\ &=\epsilon_p(j)-\epsilon_p(\tilde{j})+\tfrac{1}{2}[U(j)-U(\tilde{j})], \quad (14)\end{aligned}$$

with the U terms disappearing when the $U=$ constant approximation is made. $H_{\rm sp}$ is again the single-particle Hamiltonian introduced in Eq. (11).

We should note at this point that the off-diagonal matrix element $\langle s|H|v\rangle$ is not zero. Although the particle-hole interaction between isoscalar and isovector excitations vanishes, the complete matrix element does not, because of the difference between the single-proton and single-neutron energies, i.e.,

$$\begin{aligned}\langle s|H|v\rangle&=\langle s|H_{\rm sp}|v\rangle,\\ &=\tfrac{1}{2}[\epsilon_p(j)-\epsilon_p(\tilde{j})-\epsilon_n(j)+\epsilon_n(\tilde{j})],\\ &=\tfrac{1}{2}[U(\tilde{j})-U(j)],\end{aligned}$$

which vanishes only if $U(\tilde{j})=U(j)$.

TABLE I. Interaction matrix elements between $1p$-$1h$ configurations.

	$\|s'\rangle$	$\|v'\rangle$	$\|p_i'\bar{p}'\rangle$	$\|n'\bar{n}_i'\rangle$
$\|s\rangle$	$V_s(j\bar{j};j'\bar{j}')$	0	$2^{-1/2}V_s(j\bar{j};j_i'j')$	$2^{-1/2}V_s(j\bar{j};j'j_i')$
$\|v\rangle$	0	$V_v(j\bar{j};j'\bar{j}')$	$2^{-1/2}V_v(j\bar{j};j_i'j')$	$2^{-1/2}V_v(j\bar{j};j'j_i')$
$\|p_i\bar{p}'\rangle$	$2^{-1/2}V_s(j\bar{j};j_i'j')$	$2^{-1/2}V_v(j\bar{j};j_i'j')$	$\frac{1}{2}[V_s(j_i\bar{j};j'\bar{j}')+V_v(j_i\bar{j};j_i'j')]$	$\frac{1}{2}[V_s(j_ij;j'j_i')-V_v(j_i\bar{j};j'j_i')]$
$\|n\bar{n}_i\rangle$	$2^{-1/2}V_s(jj;j'j_i')$	$2^{-1/2}V_v(j\bar{j};j_i'j')$	$\frac{1}{2}[V_s(j_ij;j'j_i')-V_v(j_i\bar{j};j'j_i')]$	$\frac{1}{2}[V_s(jj_i;j'j_i')+V_v(jj_i;j'j_i')]$

In order to write down the interaction matrix elements of these configurations, we find it convenient to introduce the notation

$$V_s(\alpha\beta;\gamma\delta)=V_{nn}(\alpha\beta;\gamma\delta)+V_{np}(\alpha\beta;\gamma\delta)$$

and

$$V_v(\alpha\beta;\gamma\delta)=V_{nn}(\alpha\beta;\gamma\delta)-V_{np}(\alpha\beta;\gamma\delta) \quad (15)$$

for the isoscalar and isovector interaction, respectively. In Table I we summarize the matrix elements for all the $1p$-$1h$ states.

We finally have to identify the interaction of the $2p$-$2h$ configurations with each other and with the $1p$-$1h$ excitations. A typical interaction, shown diagrammatically in Fig. 4, indicates the importance in this case of monopole charge-exchange forces. Such interactions will be written in the following in terms of the basic matrix elements

$$M_{\alpha\beta}=\frac{1}{(2\alpha+1)(2\beta+1)}\times\sum_{mm'}(\phi_{\alpha m}{}^n(1)\phi_{\beta m'}{}^p(2),\hat{V}_{12}\phi_{\alpha m}{}^p(1)\phi_{\beta m'}{}^n(2)). \quad (16)$$

Note that the symmetry terms, discussed earlier, are closely related to these quantities. Indeed, the quantity U, which is explicitly defined by

$$U(\alpha)=\sum_i\sum_{m'}[(\phi_{\alpha m}{}^p(1)\phi_{j_im'}{}^n(2),\hat{V}_{12}\phi_{\alpha m}{}^p(1)\phi_{j_im}{}^n(2))-(\phi_{\alpha m}{}^n(1)\phi_{j_im'}{}^n(2),\hat{V}_{12}\phi_{\alpha m}{}^n(1)\phi_{j_im}{}^n(2))], \quad (17)$$

can easily be reexpressed in the form

$$U(\alpha)=\sum_i(2j_i+1)M_{\alpha j_i}, \quad (18)$$

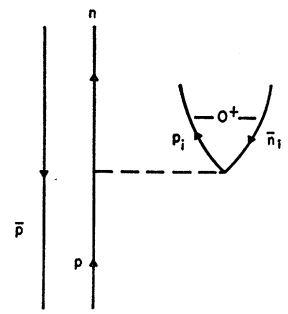

FIG. 4. Interaction diagram for the coupling between $1p$-$1h$ and $2p$-$2h$ excitations. This graph describes the particular transition from the configuration $p\bar{p}$ of Fig. 2(c) to $n\bar{p}[p_i\bar{n}_i]_0$ of Fig. 3.

with the summation over i in Eqs. (17) and (18) including all the excess neutrons. The interaction matrix element between the isovector and isoscalar modes and the $2p$-$2h$ excitations can now be written down. The results

$$\langle n'\bar{p}';[p_i\bar{n}_i]_0|H|s\rangle=\delta_{jj'}\delta_{\bar{j}\bar{j}'}(2j_i+1)^{1/2}2^{-1/2}(M_{jj_i}-M_{\bar{j}j_i}), \quad (19a)$$

$$\langle n'\bar{p}';[p_i\bar{n}_i]_0|H|v\rangle=\delta_{jj'}\delta_{\bar{j}\bar{j}'}(2j_i+1)^{1/2}2^{-1/2}(M_{jj_i}+M_{\bar{j}j_i}), \quad (19b)$$

follow immediately and are quite familiar from well-known analog-state considerations. The vanishing of the interaction with the isoscalar mode in the case where the M's are independent of the quantum numbers j should again be noted. The interaction between $2p$-$2h$ states is then given by

$$\langle n'\bar{p}';[p_i'\bar{n}_i']_0|H-H_{\rm sp}|n\bar{p};[p_i\bar{n}_i]_0\rangle=\delta_{j_ij_i'}V_v(j\bar{j};j'\bar{j}')+\delta_{jj'}\delta_{\bar{j}\bar{j}'}(2j_i'+1)^{1/2}M_{j_ij_i'}(2j_i+1)^{1/2}-(M_{jj_i}+M_{\bar{j}j_i})\delta_{jj'}\delta_{\bar{j}\bar{j}'}\delta_{j_ij_i'}, \quad (20)$$

with the first term representing the normal particle-hole interaction between the neutron-particle–proton-hole components, and the second resulting from the corresponding (charge exchange) interaction of the proton–neutron-hole monopole pairs. The third term simply indicates the modification of the diagonal energies of the two particles and the two holes resulting from their coexistence. We note finally that the two remaining interaction matrix elements are simply related to those of Table I. The relations

$$\langle n'\bar{p}';[p_i'\bar{n}_i']_{0^+}|H|p_i\bar{p}\rangle=-\delta_{j_ij_i'}[2/(2j_i+1)]^{1/2}\langle v'|H|p_i\bar{p}\rangle \quad (21a)$$

and

$$\langle n'\bar{p}';[p_i'\bar{n}_i']_{0^+}|H|n\bar{n}_i\rangle=\delta_{j_ij_i'}[2/(2j_i+1)]^{1/2}\langle v'|H|n\bar{n}_i\rangle \quad (21b)$$

can easily be established.

A discussion of the main features of the interaction matrix elements seems appropriate at this point. In addition to the conventional particle-shell and particle-hole interactions we have considered matrix elements involving some $2p$-$2h$ configurations. It is interesting to note that the results of Eqs. (19)–(21) did not require the introduction of any matrix element other than the

quantities M and V which were already introduced in connection with the $1p$-$1h$ problem. It is easy to see that this is related to the monopole character of the configurations $[p_i\bar{n}_i]_0$. This excitation can be obtained by application of the appropriate component of the raising operator T_+ on the ground state of the system. Similarly, we can obtain $|n\bar{p}\rangle$ from $|p\bar{p}\rangle$ or $|n\bar{n}\rangle$ by simple transformations of the charge indices of the proton or the neutron hole. It is clear, therefore, that Eqs. (19)–(21) can be related to the expressions for the $1p$-$1h$ interactions by elementary transformations in isospin space.

V. DIPOLE SPECTRUM

A quantitative study of the dipole states in Sr^{88} will now be presented as an application of the formalism developed in the previous sections. The set of single-particle and single-hole energies used in this calculation is shown in Table II. They were obtained partly from experimental data and partly from simple optical-model estimates. No attempt to fit the data accurately with angular-momentum-dependent potentials was made, but rather, a single "compromise" potential was adopted. The symmetry energies appearing in Eq. (10) were found not to depend very critically on the quantum numbers of each particular level. Therefore, the approximation $U=\text{const}$, which also simplifies considerably the expressions obtained in the previous sections, was adopted. Actually, the approximation was made directly on the matrix element $M_{\alpha\beta}$ of Eq. (16), which from now on will be written simply as M independent of α and β. The symmetry energy is then, according to Eq. (18), a sum over the individual contributions from the various neutron-excess levels (weighted by the number of particles in each level) and is given by

$$U=2TM=6\ \text{MeV}. \tag{22}$$

The numerical value 6 MeV was obtained from data on the $g_{9/2}$ proton and neutron energies. Consistency with Eq. (22) then required the redefinition of some of the individual-particle energies in order to satisfy this relation exactly. Although the numbers in Table II are therefore not the most accurate values one can possibly obtain[11] we believe that they are quite satisfactory for the purposes of this work since they do represent the general trends and satisfy the consistency requirements of this calculation.

We now have to evaluate the remaining unknowns, i.e., the matrix elements of Eq. (12). As mentioned earlier, these contain direct and exchange parts both of which will be treated approximately. The familiar multipole expansion of the spatial part of the two-body interaction,

$$V(r_{12})=\sum_{\lambda\mu} v_\lambda(r_1;r_2)\frac{4\pi}{2\lambda+1}Y_\lambda^{*\mu}(1)Y_\lambda^{\mu}(2), \tag{23}$$

together with the coupling coefficients in Eq. (12) remind us that only terms with $\lambda=1$ enter in the direct part. The approximation is now made that the matrix elements of these terms are proportional to the corresponding dipole matrix elements of the interacting configurations. Hence we assume that the direct part of V is given by

$$[V(\alpha\beta\gamma\delta)]_{\text{dir}}=fd_{\alpha\beta}d_{\gamma\delta}, \tag{24}$$

where

$$d_{\alpha\beta}=(\alpha\|rY_1\|\beta),$$

and f is a constant characterizing the over-all strength of the nuclear force. Although f can be considered as an arbitrary parameter, its value was not obtained by fitting the final results with any experimentally known level but rather by comparison with calculated nuclear two-body matrix elements. The value $f=0.08$ MeV F^{-2} was adopted and was found to represent reasonably well the trends of these matrix elements.

The approximation of Eq. (24) leads to the so-called schematic model[1] for the direct parts. Its main effect is expected to be an exaggeration of the strength of the giant dipole resonance relative to that of the lower-lying states. The spin-dependent components of the nuclear force have also been neglected. They are important for magnetic excitations or for the study of spin-flip dipole states but should not affect critically the electric dipole properties of the spectrum that we are considering. Finally, although the coefficient of the spin- and isospin-independent interaction was considered in parts of this calculation the final results presented here were obtained with this coefficient also assumed to be equal to zero. This was done in order to decrease the number of parameters and to indicate specifically the effects of the $\tau_1\cdot\tau_2$ interaction. In turn, this assumption implies that the isoscalar excitations become uncoupled both from each other and from all the other configurations, i.e.,

TABLE II. Unperturbed energies and dipole matrix elements (F) of the $1p$-$1h$ configurations. The first seven states appear both in proton and neutron excitations. The next four are possible only for neutrons and the last three only for protons.

Particle	Hole	Unperturbed energy (MeV)	Dipole matrix element
$2d_{5/2}$	$2p_{3/2}$	8.1	3.0
$2d_{5/2}$	$1f_{5/2}$	8.5	0.4
$3s_{1/2}$	$2p_{3/2}$	10.0	1.7
$1g_{7/2}$	$1f_{5/2}$	11.1	4.1
$2d_{3/2}$	$2p_{3/2}$	11.3	1.0
$2d_{3/2}$	$1f_{5/2}$	11.7	1.6
$2d_{5/2}$	$1f_{7/2}$	13.1	1.9
$3s_{1/2}$	$2p_{1/2}$	8.1	1.2
$1h_{11/2}$	$1g_{9/2}$	8.7	5.7
$2d_{3/2}$	$2p_{1/2}$	9.4	2.2
$2f_{7/2}$	$1g_{9/2}$	12.5	2.2
$1g_{9/2}$	$1f_{7/2}$	8.3	4.6
$2p_{1/2}$	$2s_{1/2}$	10.5	1.3
$2p_{1/2}$	$1d_{3/2}$	11.8	1.2

[11] M. M. Stautberg, J. J. Kraushaar, and B. W. Ridley, Phys. Rev. **157**, 977 (1967).

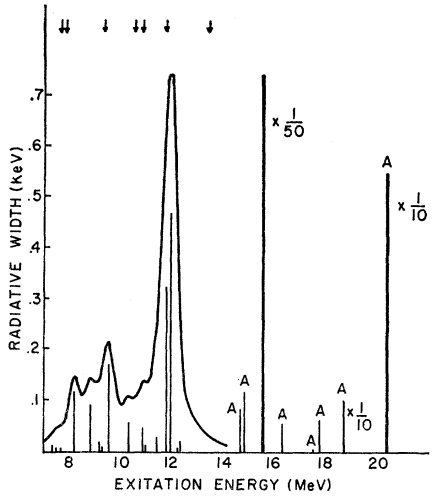

FIG. 5. The dipole spectrum of Sr^{88}. The vertical lines correspond to the radiative width of each state shown. The analog states (A) are seen on the right side of the spectrum. The giant dipole resonance appears at ~16 MeV. The continuous curve represents an incoherent sum of the widths of the remaining states distributed around their positions with Breit-Wigner functions and a spreading width of 0.5 MeV. A number of states were too weak to be shown individually but their contribution was included in the curve. The arrows indicate the positions of the analog states shifted down by $(1+T^{-1})U=7$ MeV.

$V_s=0$. Since the particle-hole interaction produces negative energy shifts in the isoscalar case, we should expect the excitation energies of these states to decrease when $V_s\neq 0$. We should remember also that the spurious state remains in this case isolated in this group of states and does not affect the remaining parts of the spectrum.

We can now discuss the exchange parts of the matrix elements of Eq. (12). These, in general, contain many terms of the expansion given by Eq. (23); i.e., many multipole components of the nuclear force are involved. We have made the approximation of neglecting all but the monopole components (whenever they are present) which will be shown to be closely related to the matrix elements M of Eqs. (6) and (22). This approximation is justified by the relatively small magnitude of the neglected terms compared with their monopole counterparts or with the direct dipole terms discussed earlier. We believe, however, that it is probably also more consistent not to include these terms as we can illustrate by considering, for example, the quadrupole interactions. Our calculation has ignored quadrupole couplings to begin with, by considering, e.g., only averages over fragmented single-particle states (the fragmentation is often due to the coupling of the particle with a quadrupole vibration) and also by ignoring the interaction between dipole and quadrupole excitations.[5] In these cases the quadrupole forces enter in the direct terms of the interaction matrix elements. It would therefore seem inconsistent to allow for the presence of such multiple components in exchange terms when their direct effects have already been disregarded. The monopole forces, on the other hand, have been included systematically in this work. It is clear, for example, that the direct part of the matrix element M in Eq. (16) contains contributions only from the $\lambda=0$ multipole component. The important role that this quantity plays in this work indicates that all the contributions of the monopole force should be consistently included. Not only is this confirmed by an evaluation of the appropriate matrix elements but it can also be shown that some qualitative features of the final spectrum are somewhat distorted if these monopole exchange terms are omitted. Since the exchange part of M is, in general, negligible compared to the direct part, we can easily see that the monopole exchange term of the matrix elements V of Eq. (12) is directly proportional to M. The coefficients of proportionality can be easily evaluated, and this makes it possible to include exchange in V.

Before we discuss the results of this calculation, we should specify in more detail the configurations that were included. Harmonic-oscillator wave functions were used and almost all the excitations that are connected to the ground state by a nonvanishing dipole matrix element were included. Only the configurations $1g_{7/2}1f_{7/2}^{-1}$, and $1h_{9/2}1g_{9/2}^{-1}$ were excluded for reasons of practical convenience. Their energies are considerably higher than those of any of the other levels and their presence is not expected to change significantly the properties of our spectrum. We notice in addition that these are precisely

TABLE III. Excitation energies and radiative widths Γ_γ for the normal dipole states (a) and the analog states (b) in Sr^{88}.

(a)		(b)	
$E_{exc.}$ (MeV)	Γ_γ (eV)	$E_{exc.}$ (MeV)	Γ_γ (eV)
15.56	37×10^3	20.46	6×10^3
13.69	1	18.82	1×10^3
12.40	23	17.90	65
12.32	9	17.59	4
12.00	470	16.40	58
11.87	320	14.86	120
11.48	29	14.75	86
11.00	11		
10.93	50		
10.91	1		
10.61	3		
10.60	4		
10.59	1		
10.52	1		
10.40	60		
9.60	180		
9.42	2		
9.30	7		
9.27	21		
8.87	93		
8.24	120		
8.00	~0		
7.80	~0		
7.80	~0		
7.79	~0		
7.67	3		
7.48	6		
7.40	12		

the states that will play an important role in the spin-flip excitations of the nucleus. As was mentioned earlier in connection with the omission of the spin-spin interactions, we do not intend to study these excitations here in any detail. Independent estimates indicate that the states with strong spin-flip components will appear about 2–3 MeV higher than the normal giant dipole states.

We now present the main features of the results of this calculation. The secular equation of the configurations discussed in Sec. II was solved with the interactions described in Sec. III and the approximations mentioned in the preceding paragraphs. The positions and the dipole strengths of the resulting states were obtained. In Fig. 5 we summarize the main features of the spectrum. The vertical lines represent the radiative widths. The excitation energies are shown in the abscissa. The levels labeled A are analog states with isospin $T+1$, which, as expected from the previous discussions, appear automatically in the results. It is clear that the number of analogs is considerably smaller than that of the normal (isospin T) states and that their dipole strengths are relatively small. They do exhibit, on the other hand, all the characteristics of a dipole spectrum; e.g., the dipole strength within the analog group is clearly concentrated at a relatively high energy. The continuous curve in Fig. 5 has been drawn rather arbitrarily in order to give an over-all impression of the distribution of the dipole strength among the weaker states. It represents a spreading of the individual states with Breit-Wigner functions added incoherently and with a spreading width of 0.5 MeV. The giant dipole resonance appearing at $\sim$16 MeV will obviously dominate the spectrum but the presence of the weaker states is also clear. It may be of some interest to notice the residual fluctuations in the continuous curve. These are clearly not associated with a local increase of the level density but rather with a concentration of strength in some regions. We can attempt to correlate the positions of the peaks in this curve with the excitation energies of the analog states. Well-known arguments suggest the presence of a T-lower counterpart for each analog state at an energy $(1+T^{-1})U$ (7 MeV in our case) below the analog. This simple correspondence tends to be destroyed by residual effects connected with the fact that there are usually many more isospin T states than there are analogs. The arrows in Fig. 5 show the energies of the analogs shifted down by 7 MeV and indicate the extent to which the peaks in the radiative width have any connection with these energies. Although the number of peaks compares reasonably with the number of analogs, no complete correspondence between the excitation energies is indicated. In particular, the giant dipole state is about 2 MeV closer to the corresponding coherent analog than the simple rule would indicate. This is due to the fact that the energy shift of the T-lower coherent mode is larger than the corresponding

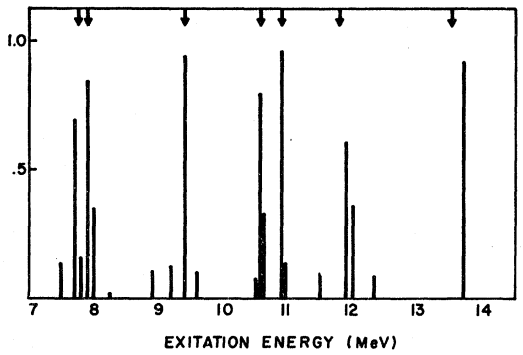

FIG. 6. Distribution of the $2p$-$2h$ strength among the normal ($T=6$) dipole states. Levels with very small strength are not shown. The arrows indicate the positions of the analog states shifted down by $(1+T^{-1})U=7$ MeV.

shift in its $T+1$ counterpart. In turn this difference is related to the larger number of configurations that contribute to the T-lower spectrum.

The fact that the spectrum now contains an appreciable amount of $2p$-$2h$ components should be remembered. In Fig. 6 we show the intensity of $2p$-$2h$ components in various T-lower states. The arrows have the same meaning as in Fig. 5 and indicate in this case a very clear correlation with the peaks in the $2p$-$2h$ intensity distribution.

The presence of relatively weak analogs in the middle of the giant dipole resonance is worth noticing. States of this type have been observed[8] in a number of (p,γ) reaction experiments. It should also be remembered that direct neutron emission from analog states is not allowed (by isospin selection rules) as long as their corresponding parent states are bound. The highest lying analog (which can be considered as the giant dipole state of this group) corresponds, however, to an unbound parent and therefore emission of at least low-energy neutrons is possible in this case. This, as well as the modulation of the neutron emission probability caused by the presence of an analog state,[12] may be relevant in connection to the observation of high-energy peaks in some (γ,n) reaction experiments.[13]

In Table III(a) the excitation energies and the ground-state radiative widths of the various T-lower states are shown. The corresponding quantities for the analog states are given in Table III(b). The dipole strengths and the oscillator strengths will not be given in detail but we discuss here their main features. The giant dipole resonance and the remaining T-lower states contain, respectively, 82 and 10% of the total dipole strength. The configurations $g_{7/2}f_{7/2}^{-1}$ and $h_{9/2}g_{9/2}^{-1}$ which were not included in this work account for 1% of the total strength. Finally, the remaining 7% is associated with the analog states. This can be compared

[12] D. Robson, J. D. Fox, P. Richard, and C. F. Moore, Phys. Letters **18**, 86 (1965).

[13] B. L. Berman, J. T. Caldwell, R. R. Harvey, M. A. Kelly, R. L. Bramblett, and S. C. Fultz, Phys. Rev. **162**, 1098 (1967); and B. M. Spicer (private communication).

with an estimate provided by the expression[14]

$$|C_{T+1}|^2=\frac{1}{T+1}\left(1-\frac{3T}{2A^{2/3}}\right) \tag{25}$$

for the total ground-state strength of all the states with isospin $T+1$. For Sr^{88}, Eq. (25) gives $|C_{T+1}|^2=0.076$, which compares well with the present result and is about half of the value obtained from the geometrical estimate $|C_{T+1}|^2=(T+1)^{-1}$. Similar reduction factors have been discussed by Petersen and Veje[7] and by Macfarlane.[7]

The oscillator strength of the giant dipole state was found to exceed by a factor of 1.3 the classical limit of the well-known (energy-weighted) dipole sum rule. This is a familiar feature of most shell-model calculations which tend to overconcentrate the dipole strength in an energy region of <30 MeV. Experimentally, it is known[15] that only the classical sum is exhausted in this region. The use of harmonic-oscillator wave functions and the fact that ground-state correlations have not been taken into account are presumably responsible for this result. The sums of the oscillator strengths of the remaining T-lower states and of the analog states each contain about 15% of the classical limit with $\sim$10% already concentrated in the strongest state of the analog group. The detailed structure of the wave functions obtained in this calculation will not be presented here. We mention, however, that the configurations $2d_{5/2}1f_{7/2}^{-1}$ and $1g_{7/2}1f_{5/2}^{-1}$ are the predominant components on the two upper analog states. Their amplitudes are -0.74 and 0.54, respectively, for the 20.6-MeV state, while the corresponding amplitudes for the state at 18.5 MeV are 0.67 and -0.58.

VI. SUMMARY AND CONCLUSIONS

Some properties of dipole states in nuclei with $N>Z$ were studied in this work and were illustrated by a calculation of the dipole spectrum in Sr^{88}. In addition to the conventional $1p$-$1h$ excitations, a selected set of $2p$-$2h$ configurations was included. This led not only to an increase in the number of degrees of freedom but also to the appearance in the final spectrum of states with definite isospin. The spectrum contained both analog states with isospin $T+1$ and a considerably larger set of normal states with isospin T equal to that of the ground state of the nucleus. This set accounts for most of the ground-state dipole strength and includes the giant dipole resonance.

The importance of monopole charge-exchange interactions was emphasized in the calculation. The effects of the monopole and the dipole nuclear-force components were systematically included but quadrupole and higher multipolarities were neglected.

The analog states were seen to exhibit typical features of a dipole spectrum, e.g., a coherent mode at a relatively high energy. The existence of such a mode at $\sim$21 MeV in Zr^{90} was predicted earlier by similar considerations. This expectation is in agreement with the results of recent (p,γ) reaction experiments by Axel *et al.*[8] and by Hanna *et al.*[8] The observation of the coherent state in Sr^{88} will clearly be of interest.

The analog states are obviously related to their corresponding parent spectrum by a rotation in isospin space. The parent states are neutron–proton-hole excitations in Rb^{88} and can be formed, for example, in a μ^- capture reaction from Sr^{88}. The highest of these states is expected to lie above the threshold for neutron emission and its presence should affect the properties of the s-wave (or d-wave) strength function in this region. A spreading of this state, among the compound states around it, should also be expected. This spreading is not affected by a rotation in isospin space and will contribute to the total width of the corresponding analog, even in the absence of residual Coulomb interactions. When these effects are taken into account the analog will no longer be the isospin counterpart of a single state but rather the counterpart of a group of compound states. These compound states, on the other hand, are significantly influenced by the presence of the shell-model excitations whose properties were discussed in this work. The results presented in this paper characterize then the over-all (doorway) excitation and not the individual components which result from its residual fragmentation.

ACKNOWLEDGMENTS

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[14] B. Goulard and S. Fallieros (Ref. 7).

[15] S. C. Fultz, R. L. Bramblett, B. L. Berman, J. T. Caldwell, and M. A. Kelly, in *Proceedings of the International Conference on Nuclear Physics, Gatlinburg, Tennessee, 1966* (Academic Press Inc., New York, 1967), p. 397.