Symmetric-Core Collective Model for Odd-Odd Nuclei with Applications in the 2s-1d Shell

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A model for odd-odd nuclei is described in which the odd neutron and proton are coupled to a symmetric, rotating core. The Hamiltonian consists of four parts: H_R, the Hamiltonian for a rotating core of fixed shape; H_p and H_n, the Hamiltonians for a proton and a neutron moving in a symmetric oscillator potential with $\mathbf{l}\cdot\mathbf{s}$ and $\mathbf{l}^2$ terms; and V_{pn}, the residual neutron-proton interaction. This latter was taken to have a Gaussian radial dependence with a Serber exchange mixture with parameters picked to reproduce the low-energy singlet and triplet scattering lengths. The energy eigenvalues were obtained by an exact diagonalization of the total Hamiltonian using a core-particle basis to which a truncation procedure is applied to account for the filling of the shells. The model has been applied to ^{28}Al, ^{30}P, ^{32}P, and ^{36}Cl, determining the core-strength parameter P, the deformation parameter β, and the well-depth parameter λ by fitting to the known energy-level sequence. The state functions so obtained were used to calculate static magnetic dipole and electric quadrupole moments, and for ^{30}P the mixing ratio δ^2 for two 2^+ to ground transitions. The fit to the measured energy levels is quite successful; however, only in ^{28}Al and ^{30}P is the residual neutron-proton interaction needed. The values obtained for the static moments are rather poor. The results are compared, where possible, to a shell-model calculation.

I. INTRODUCTION

SOME time ago Chi and Davidson[1] showed in a detailed investigation that an asymmetric core collective model can explain quite successfully the low-lying level structure of odd-A nuclei in the $2s$-$1d$ shell. This model was a logical extension of the asymmetric core collective model of even-even nuclei first discussed in detail by Davydov and Filippov[2] which can be applied with some success to the even-even $2s$-$1d$ nuclei ^{24}Mg, ^{28}Si, and ^{32}S. Application of this model to odd-odd nuclei in this shell was immediately suggested; however, the then available experimental information was not sufficient to warrant such an investigation because of the computational difficulties involved. Currently, even though these nuclei are most in need of experimental study, there are several with a sufficient number of known levels to permit fixing the model parameters. Furthermore, since the appearance of the odd-A paper[1] this phenomenological collective model has been given a great deal of theoretical support by the microscopic calculations of Bar-Touv and Kelson.[3,4] In their first paper on even-even nuclei they solved the Hartree-Fock problem in this shell without imposing the condition of axial symmetry on the intrinsic states.[3] The results of the calculation showed that the $2s$-$1d$ shell contains two regions of significant asymmetry near ^{24}Mg and ^{32}S. In terms of the usual asymmetry parameter γ they found $\gamma(^{24}\text{Mg})\approx 25°$ and $\gamma(^{32}\text{S})\approx 32°$ which was quite consistent with what was found by Chi and Davidson,[1] $\gamma(^{25}\text{Mg})=28°$, $\gamma(^{31}\text{P})=31°$. In their second paper they extended their earlier calculation to the odd-A nuclei ^{25}Mg and ^{31}P and obtained results similar to the earlier phenomenological calculation.

As a result of these investigations, we felt that a calculation of the odd-odd nuclei would not only give a better understanding of the model, since logically it would contain explicitly the residual neutron-proton force, but also might provide structure predictions useful to further experimental investigation of these nuclei. The regions of significant asymmetry (operationally defined as $\gamma\gtrsim 20°$) being where they are, the most useful nuclei to investigate were thought to be ^{26}Al, ^{28}Al, ^{30}P, ^{32}P, ^{34}Cl, and ^{36}Cl. Of these only ^{34}Cl has an insufficient number of levels with assigned spins and parities to fit the model parameters.[5] The size of the basis, the desire to calculate the positions of levels up to $I=4$, and the nature of the diagonalization routines (which required double precision arithmetic) restricted the calculation to axially symmetric shapes. Even so, machine core size prevented us from including ^{26}Al (which has an $I=5$ ground state) in our investigation.

In Sec. II we discuss the model Hamiltonian and the basis that we have used to diagonalize it. In Sec. III we obtain the model parameters for the given odd-odd nuclei by fitting to their known level structure, and finally in Sec. IV we present certain conclusions concerning this model.

II. FORMULATION OF MODEL

The basic assumption of the model is that one has two particles outside of a deformed core. The core is assumed to be inert to vibrations and its only contribution to the total energy of the system is through

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[1] B. E. Chi and J. P. Davidson, Phys. Rev. **131**, 366 (1963).

[2] A. S. Davydov and G. F. Filippov, Nucl. Phys. **8**, 237 (1958).

[3] J. Bar-Touv and I. Kelson, Phys. Rev. **138**, B1035 (1965).

[4] J. Bar-Touv and I. Kelson, Phys. Rev. **142**, 599 (1966).

[5] P. M. Endt and C. Van der Leun, Nucl. Phys. **A105**, 1 (1967).

rotation. The total Hamiltonian is then

$$H=\frac{\hbar^2}{2}\sum_{k=1}^{3}\frac{L_k^2}{\mathcal{I}_k}+H_p(\mathbf{r}_p,\mathbf{l}_p,\mathbf{s}_p)+H_n(\mathbf{r}_n,\mathbf{l}_n,\mathbf{s}_n)+V_{pn}(\mathbf{r}_p,\mathbf{r}_n,\boldsymbol{\sigma}_p,\boldsymbol{\sigma}_n), \quad (1)$$

where L_k are the components of the core's angular momentum on the body fixed axis system; $\mathcal{I}_k$ are the corresponding moments of inertia about these axes; $H_p(\mathbf{r}_p,\mathbf{l}_p,\mathbf{s}_p)$ and $H_n(\mathbf{r}_n,\mathbf{l}_n,\mathbf{s}_n)$ are the Hamiltonians for the proton and neutron, respectively, and $V_{pn}(\mathbf{r}_p,\mathbf{r}_n,\boldsymbol{\sigma}_p,\boldsymbol{\sigma}_n)$ is the residual p-n interaction.

We now assume the core has an axis of symmetry which we shall choose as the 3 axis so that $\mathcal{I}_1=\mathcal{I}_2=\mathcal{I}_0\neq\mathcal{I}_3$. One also replaces the core's angular momentum by

$$\mathbf{L}=\mathbf{I}-\mathbf{j}_p-\mathbf{j}_n,$$

where $\mathbf{I}$ is the total angular momentum of the nucleus (a constant of the motion), and $\mathbf{j}_p$ and $\mathbf{j}_n$ are the angular momenta of the extra core proton and neutron. The condition of axial symmetry requires no rotation about this axis, and hence Eq. (1) becomes

$$H=\frac{\hbar^2}{2\mathcal{I}_0}\sum_{k=1}^{2}\{(I_k^2-2I_kj_{pk}-2I_kj_{nk}+j_{pk}^2+j_{nk}^2+2j_{pk}j_{nk})\}+H_p(\mathbf{r}_p,\mathbf{l}_p,\mathbf{s}_p)+H_n(\mathbf{r}_n,\mathbf{l}_n,\mathbf{s}_n)+V_{pn}(\mathbf{r}_p,\mathbf{r}_n,\boldsymbol{\sigma}_p,\boldsymbol{\sigma}_n). \quad (2)$$

Both $H_p(\mathbf{r}_p,\mathbf{l}_p,\mathbf{s}_p)$ and $H_n(\mathbf{r}_n,\mathbf{l}_n,\mathbf{s}_n)$ have been chosen as the Nilsson[6] Hamiltonian for a particle moving in a spheroidally deformed, harmonic oscillator well subject to an $\mathbf{l}\cdot\mathbf{s}$ interaction. This Hamiltonian is given by

$$H_i(\mathbf{r}_i,\mathbf{l}_i,\mathbf{s}_i)=p_i^2/2\mu_i+\tfrac{1}{2}(\mu_i\omega_0^2)[1-\beta Y_{20}(\theta_i,\varphi_i)]+C\mathbf{l}_i\cdot\mathbf{s}_i+D\mathbf{l}i^2, \quad (3)$$

where β is the deformation parameter introduced by Bohr,[7] μ_i is the reduced particle mass, and C and D are parameters introduced by Nilsson.[6]

The unsymmetrized basis is taken to be the product representation $D_{MK}{}^{I*}\chi_{j_p\Omega_p}\chi_{j_n\Omega_n}$, where $D_{MK}{}^{I*}$ are the $(2I+1)$-dimensional representation of the rotation group defined by Rose[8] and $\chi_{j_p\Omega_p}$ and $\chi_{j_n\Omega_n}$ are the particle solutions to the spherically symmetric potential in a coupled representation. If we now demand our solution to be symmetric under rotations through 180° about an axis perpendicular to the symmetry axis, the state function corresponding to Eq. (2) becomes

$$\Psi_M{}^I=\sum_{j_p,\Omega_p,j_n,\Omega_n}\frac{A_M{}^I(j_p,\Omega_p,j_n,\Omega_n)}{\sqrt{2}}\{D_{MK}{}^{I*}\chi_{j_p,\Omega_p}\chi_{j_n,\Omega_n}+(-1)^{I-j_p-j_n}D_{M-K}{}^{I*}\chi_{j_p,-\Omega_p}\chi_{j_n,-\Omega_n}\}, \quad (4)$$

with

$$K=|\Omega_p+\Omega_n|\leq I,$$
$$K=0,\quad \Omega_p \text{ or } \Omega_n>0.$$

The quantities K, Ω_p, and Ω_n are the projections of the total angular momentum, and the angular momenta of the proton and neutron on the axis of symmetry. Except for ^{28}Al the $d_{5/2}$ shell is truncated from the basic states $\chi_{j\Omega}$, and thus considered to be part of the core.

The remaining term to be discussed is the residual p-n interaction. For initial quick scans of the parameter space we used

$$V_{pn}(\mathbf{r}_p,\mathbf{r}_n,\boldsymbol{\sigma}_p,\boldsymbol{\sigma}_n)=\lambda_0\delta(r_p-r_n)(1-\alpha+\alpha\boldsymbol{\sigma}_p\cdot\boldsymbol{\sigma}_n), \quad (5)$$

which being of zero range has the nice analytic property that the derived level order is independent of the harmonic-oscillator parameter $\hbar\omega_0$. The parameter α was fixed by the zero-range Rosenfeld mixture.[9] Since only two nuclei were found to require the use of this interaction the final fitting was done using the interaction potential used by Newby[10] in his study of the odd-even shift in rare-earth nuclei. This potential has a Gaussian radial dependence with a Serber exchange mixture. It has the advantage of reproducing the low-energy singlet and triplet scattering lengths.

The entire Hamiltonian matrix was diagonalized for each nucleus. The parameter set is given by

$$P=\hbar^2/2\mathcal{I}_0\kappa\hbar\omega_0,$$
$$B=\beta/\kappa,$$
$$\lambda=\lambda_0/\kappa\hbar\omega_0.$$

Here P is the core strength parameter which is, apart from a factor of $\frac{4}{3}$, identical to that used before[1] and κ is the parameter introduced by Nilsson,[2] $\kappa=-C/2\hbar\omega_0$. The quantity $\kappa\hbar\omega_0$ is simply a scale factor and does not affect the level order. Wherever possible the results obtained were compared with the shell-model results of Glaudemans, *et al.*[11] In the latter calculation, all nuclei beyond ^{28}Si in the $2s$-$1d$ shell have been fit with the same set of parameters. The basic physical notion that one has of the collective model leads us to believe that the associated parameters should vary from nucleus to nucleus.

III. APPLICATION OF MODEL TO SOME NUCLEI IN $2s$-$1d$ SHELL

A. Energy Levels

We have applied the model discussed above to several nuclei in the latter half of the $N=2$ oscillator shell. Here the odd-nucleon numbers, in the notation of Ref. 1, have the range $9\leq\zeta_i\leq19$. In this subsection we shall discuss the energy eigenvalues and the eigenfunctions

[6] S. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. **29**, No. 16 (1955).
[7] A. Bohr, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. **26**, No. 14 (1952).
[8] M. E. Rose, *Elementary Theory of Angular Momentum* (John Wiley & Sons, Inc., New York, 1957).
[9] L. Rosenfeld, *Nuclear Forces* (North-Holland Publishing Co., Amsterdam, 1948).
[10] N. D. Newby, Jr., Phys. Rev. **125**, 2063 (1962).
[11] P. W. M. Glaudemans, G. Wiechers, and P. J. Brussaard, Nucl. Phys. **56**, 529 (1964); **56**, 548 (1964).

while in the next subsection we shall be concerned with the static electric quadrupole and magnetic dipole moments as well as the $M1$ and $E2$ transition probabilities calculated from these eigenfunctions.

As stated before, we have restricted ourselves to the latter half of the shell simply because of memory size in the available computer. This also restricted the calculation to cores with axial symmetry. Within this set we have fit the model to those odd-odd nuclei with at least three low-lying, positive-parity levels whose spins were well known. In Figs. 1–3 we show a comparison of the experimental[5] and calculated energy-level structures of these nuclei. In these figures we also compare, where possible, our calculation with the shell-model calculation of Glaudemans *et al.*[11] In Table I we list the values of the parameters for each nucleus fitted. Here we have taken $\kappa=0.05$, the usually accepted value.

We consider in turn the details of each of the fitted nuclei in the shell.

1. $\zeta_1=13$, $\zeta_2=15$; ^{28}Al, ^{28}P

These nuclei consist of a $1d_{5/2}$ hole and a $2s_{1/2}$ particle. For ^{28}Al all of the $1d_{5/2}$ states were truncated from the neutron basis, whereas no particle states were truncated from the proton basis. The most obvious feature of the observed level structure is the pair of close doublets separated by almost 1 MeV. In order to obtain the correct spacing and sequence in the lowest doublet the extra core particles were taken to be rather weakly interacting. However, this interaction was absolutely necessary for when it was turned off the spin sequence was 2^+, 3^+ and no values of the other parameters could invert it. The spins of the second doublet are not known, but the parameter fit gives the order as 3^+, 4^+ with the fourth excited state as 1^+ as is observed. These

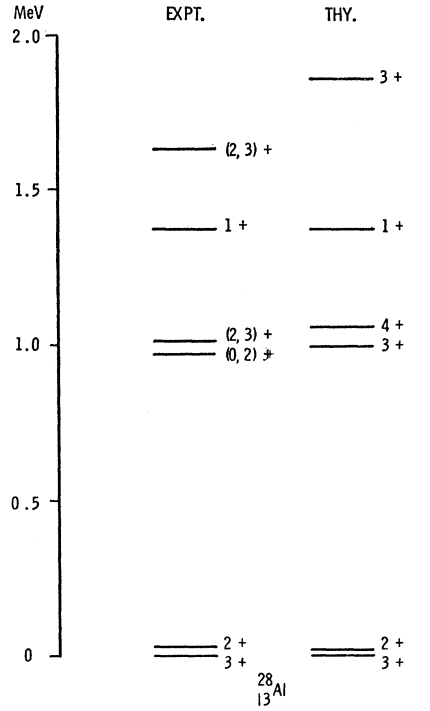

FIG. 1. A comparison of the experimental measured energy levels with the values given by the model for the odd-odd nucleus ^{28}Al. The model parameters are given in Table I.

results are shown in Fig. 1. Our previous experience[1] leads us to believe that the ^{28}P system will be fit with very similar parameter values.

2. $\zeta_1=\zeta_2=15$; ^{30}P

This self-conjugate nucleus is perhaps the most interesting of the ones studied and we shall consider the re-

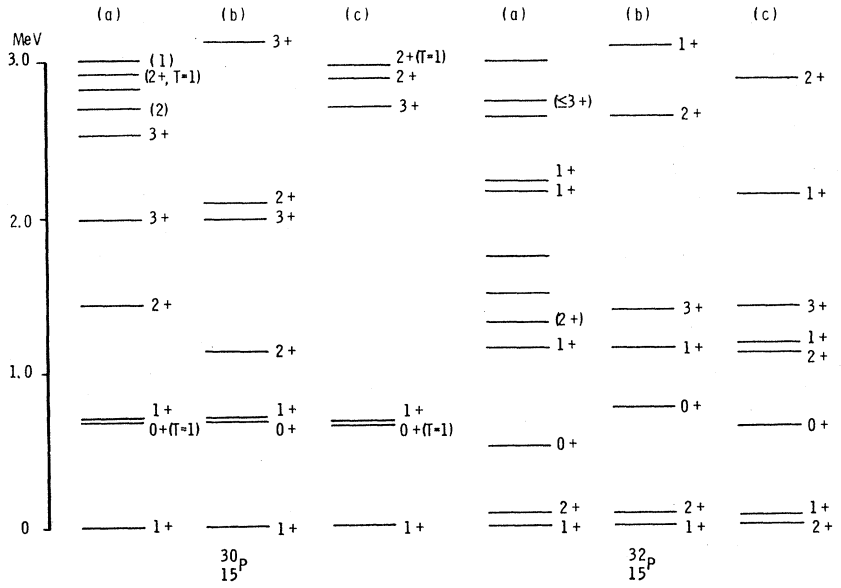

FIG. 2. A comparison of the experimental measured energy levels (a) with the values given by the model (b) and with the values given by the shell-model calculation of Ref. 11 (c) for the odd-odd nuclei ^{30}P and ^{32}P. The model parameters are given in Table I.

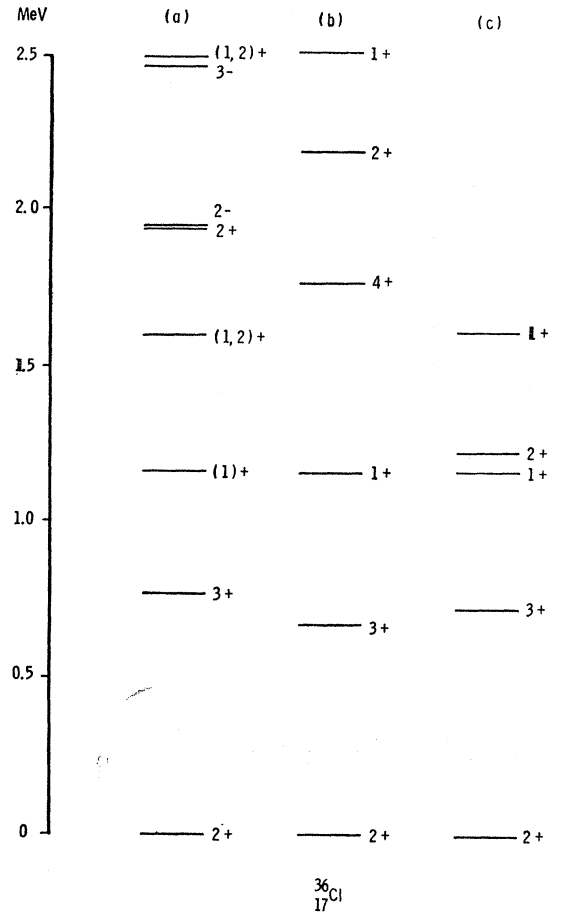

FIG. 3. A comparison of the experimental measured energy levels (a) with the values given by the model (b) and with values given by the shell-model calculation of Ref. 11 (c) for the odd-odd nucleus ^{36}Cl. The model parameters are given in Table I.

sults in somewhat greater detail. The ground and first excited state spins, 1^+ and 0^+, respectively, were the controlling factors in the choice of parameter values. In fact, only by including the residual p-n interaction were we able to obtain this proper order. Again, when it was turned off the order was reversed and no values of the other parameters could reinvert it. One can understand this by considering the simplified problem of each particle occupying a single deformed Nilsson orbital which in this case is the $\frac{1}{2}+[2,1,1]$ state for both neutron and proton. In the limit of noninteracting particles, the diagonal contributions to the Hamiltonian matrix are given by

$$E(I,K)=(\hbar^2/2\mathcal{I}_0)[I(I+1)-K^2+(-1)^{I+1}a_pa_n\delta_{K,0}] +\epsilon_p+\epsilon_n. \quad (6)$$

The first two terms in the brackets come from the rotational part of the Hamiltonian while the third term is due to the Coriolis force. The quantities ϵ_p and ϵ_n are the single-particle energies and a_p and a_n the corresponding decoupling parameters.[6] Since the two extra core particles are assumed to occupy the same Nilsson orbital we may write these as

$$a_p=a_n=a.$$

In this case the odd nucleons can couple to form $K=0$ and $K=1$ bands. From Eq. (6) it can be seen that the level sequence is $I(K)=0(0)$, $1(1)$, $1(0)$. One cannot account for the experimental level sequence by allowing K-band mixing between the two $I=1$ levels (although this is important), since this leads to the conclusion that $(a^2-1)^2$ be negative.

Clearly the residual p-n interaction will be important here. Indeed Newby[10] has shown that this interaction plays a central role in the odd-even shift in the $K=0$ bands in the rare-earth deformed nuclei. The correction to the total energy of Eq. (6) resulting from this interaction is given by

$$\Delta E(I,K)=\langle\Omega_p',\Omega_n'|V_{pn}|\Omega_p,\Omega_n\rangle +(-1)^{I+1}\langle\Omega_p',\Omega_n'|V_{pn}|-\Omega_p,-\Omega_n\rangle\delta_{K,0}, \quad (7)$$

where

$$\langle\Omega_p',\Omega_n'|V_{pn}|\Omega_p,\Omega_n\rangle = \sum_{j_p',j_n',j_p,j_n} C_{j_p'\Omega_p'}{}^*C_{j_n'\Omega_n'}{}^*C_{j_p\Omega_p}C_{j_n\Omega_n} \times\langle j_p'\Omega_p',j_n'\Omega_n'|V_{pn}|j_p\Omega_p,j_n\Omega_n\rangle$$

and the deformed orbital expansion coefficients $C_{j\Omega}$ are taken in the coupled representation.

If we require the 0^+ and 1^+ members of the $K=0$ band to have an inverted order then the following inequality must hold:

$$\langle\Omega_p',\Omega_n'|V_{pn}|-\Omega_p,-\Omega_n\rangle<(-\hbar^2/2\mathcal{I}_0)(1+a^2). \quad (8)$$

One can get a rough idea of how sensitive the configuration $(\frac{1}{2}+[2,1,1])^2$ will be to a particle-particle interaction by determining the matrix elements in Eq. (7). These are shown in Table II evaluated for different values of the deformation parameter β. The form of the potential used was that of Newby[10] and the inertial parameter was determined from the first excited state of ^{28}Si. It is clear from this table that in general the fitting will be very sensitive to the residual interaction.

The parameters obtained from the fit are given in Table I and the results of this fit are shown in Fig. 2, where they are compared not only with experiment but with a recent shell-model calculation.[11] We have

TABLE I. The values of the model parameters: P, the core strength parameter, β the deformation parameter, λ_0 the residual p-n well-depth parameter, and $\hbar\omega_0$ the over-all scale parameter for odd-odd nuclei in the second half of the $2s$-$1d$ shell. The quantity β_0 is the inverse oscillator length defined by $\beta_0^2=m\omega_0/\hbar$.

Nucleus	ζ_p	ζ_n	P	β	$\lambda_0\beta_0^3/4\pi$	$\hbar\omega_0$ (MeV)
^{28}Al	13	15	0.12	−0.10	−2.54	32.0
^{30}P	15	15	0.19	0.15	−4.46	28.6
^{32}P	15	17	0.17	0.10	0	32.0
^{36}Cl	17	19	0.56	0.72	0	6.8

TABLE II. Matrix elements for the residual p-n interaction as a function of the deformation parameter $\delta=\frac{3}{2}(5/4\pi)^{1/2}\beta$. Both extra core nucleons are assumed to be in the $\frac{1}{2}+[2, 1, 1]$ Nilsson orbital. The level sequence so generated is shown in the last column. The form of the potential is identical to that of Ref. 10. These correspond to the self-conjugate nucleus ^{30}P. All matrix elements are in MeV.

δ	$\langle\frac{1}{2},\frac{1}{2}\|V_{pn}\|\frac{1}{2},\frac{1}{2}\rangle$	$\langle\frac{1}{2},-\frac{1}{2}\|V_{pn}\|\frac{1}{2},-\frac{1}{2}\rangle$	$\langle\frac{1}{2},-\frac{1}{2}\|V_{pn}\|-\frac{1}{2},\frac{1}{2}\rangle$	$\frac{\hbar^2}{2\mathcal{I}_0}(1+a^2)$	Level sequence $I(K)$
−0.2	−2.902	−2.194	−0.676	−0.722	1(1), 0(0), 1(0)
−0.1	−2.989	−2.550	−0.782	−0.876	1(1), 0(0), 1(0)
0.1	−2.880	−2.481	−0.739	−0.485	1(0), 1(1), 0(0)
0.2	−2.798	−2.090	−0.641	−0.302	1(0), 1(1), 0(0)

truncated the $1d_{5/2}$ states from both the proton and neutron bases which assumes an inert ^{28}Si core. The shell-model calculation of Ref. 11 also assumes this.

One final comment is that one should expect that near ^{28}Si the value of $(\kappa\hbar\omega_0)P$ should correspond roughly to the inertial parameter determined from the first excited state in ^{28}Si. For ^{28}Si this parameter is 295 keV, whereas for ^{30}P it is 272 keV.

3. $\zeta_1=15$, $\zeta_2=17$; ^{32}P, ^{32}Cl

For this system, the residual p-n interaction was not needed to yield the correct level order for the ground and first excited state doublet. Including this interaction has little effect on the first few levels and its effects can be absorbed into the scale factor. The $1d_{5/2}$ states were truncated from the proton bases and the $(1d_{5/2}, 2s_{1/2})$ states from the neutron basis. The latter restriction must be viewed with skepticism since (d,p) reactions on ^{31}P seem to indicate $2s_{1/2}$-state mixtures available to the captured neutron.[12] A possible remedy for such situations will be discussed in the conclusions. The parameters are given in Table I and the results are displayed in Fig. 2, where they are compared with experiment and a recent shell model calculation.[11] It should also be mentioned that for a somewhat different set of parameters the level sequence (1^+, 2^+, 0^+, 1^+, 2^+, 3^+) can be obtained; however, the second 1^+ level does not fit quite as well. For ^{32}Cl only the ground-state spin is known ($I=2^+$) and apparently it has no near neighbor to form a close doublet.[5]

4. $\zeta_1=17$, $\zeta_2=19$; ^{36}Cl, ^{36}K

For this system we have assumed the level sequence to be 2+, 3+, 1+, $\cdots$, which order does not require the residual p-n interaction. In ^{36}K only the ground-state spin is known.[5] The parameters are shown in Table I and the fit compared with experiment and a recent shell-model calculation[11] in Fig. 3. Thus, near the end of the shell one should not expect the model to work well, which is born out by the unrealistic parameters β and $\hbar\omega_0$ in Table I. A similar situation was found earlier[1] in ^{35}Cl and this fit has been included for completeness.

[12] W. C. Parkinson, Phys. Rev. **110**, 485 (1958).

B. Magnetic Dipole and Electric Quadrupole Matrix Elements

The previously described fitting procedure not only determines the eigenvalues ϵ, but the components $S_{\epsilon\nu}$ of the eigenvectors which in turn may be used to calculate transition probabilities between states and the static moments for the various states.

We have calculated the ground-state magnetic dipole and electric quadrupole moments as well as the $M1$-$E2$ mixing ratio, defined as[13]

$$\delta^2(I\to I-1)=\frac{T(E2:I\to I-1)}{T(M1:I\to I-1)}. \tag{9}$$

These calculations, and indeed the results, are much the same as the earlier odd-A calculation[1] except that two nucleons are now involved. As usual we make use of the definition for the transition probability per unit time for emission of a photon of energy $h\nu$ and multipolarity λ;

$$T(\lambda)=\frac{8\pi(\lambda+1)}{\lambda\hbar[(2\lambda+1)!!]^2}\left(\frac{h\nu}{hc}\right)^{2\lambda+1}B(\lambda), \tag{10}$$

where $B(\lambda)$ is the usual reduced transition matrix element.[13] For $M1$ transitions we find

$$B(M1)=\frac{3}{4\pi}[\mu_N\sum_{k=1}^{2}\sum_{\gamma\gamma'}S_{f\gamma'}S_{i\gamma}\langle j'\|G_k\|j\rangle M_{\gamma'\gamma}{}^{G_k}]^2, \tag{11}$$

where

$$\langle j'\|G\|j\rangle=(g_s-g_l)_k\langle j'\|s_k\|j\rangle+(g_l-g_c)_k\langle j'\|j_k\|j\rangle, \tag{12}$$

$$\begin{aligned}M_{\gamma'\gamma}{}^{G_k}=&C(I1I';K\Delta K)C(j_k1j_k';\Omega_k,\Delta\Omega_k)\delta_{\Delta K,\Delta\Omega_k}\delta_{\Omega_p,-\Omega_p}\\&+(-1)^{I-j_p-j_n}C(I1I';-K,\Sigma K)\\&\times C(j_k'1j_k;-\Omega_n,\Sigma\Omega_n)\delta_{\Omega_p',-\Omega_p}\delta_{\Sigma K,\Sigma\Omega_k}.\end{aligned} \tag{13}$$

For E2 transitions,

$$\begin{aligned}B(E2)=[e\sum_{\gamma\gamma'}S_{f\gamma'}S_{i\gamma}(a_cA_{\gamma'\gamma}&\\&+a_p\langle j_p'\|\rho^2Y_2\|j_p'\rangle M_{\gamma'\gamma}{}^p)]^2,\end{aligned} \tag{14}$$

where

$$\begin{aligned}M_{\gamma'\gamma}{}^p=&C(I2I';K,\Delta K)C(j_p2j_p';\Omega_p,\Delta\Omega_p)\delta_{\Delta K,\Delta\Omega_p}\\&+(-1)^{I-j_p-j_n}C(I2I';-K,\Sigma K)\\&\times C(j_p2j_p';-\Omega_p,\Sigma\Omega_p)\delta_{\Sigma\Omega_p,\Sigma K},\end{aligned} \tag{15}$$

[13] J. P. Davidson, Rev. Mod. Phys. **37**, 105 (1965).

TABLE III. Comparison of the calculated values with experimental values of the ground-state magnetic dipole and electric quadrupole moments. For ^{32}P and ^{36}Cl values of P and β which best fit the calculated and measured values are also given.

	$\langle\mu\rangle(\mu_N)$		$\langle Q\rangle(b)$				fitted$\langle\mu\rangle$, $\langle Q\rangle$	
Nucleus	expt	theoret	expt	theoret	P	β	$\langle\mu\rangle(\mu_N)$	$\langle Q\rangle(b)$
^{28}Al	...	9.584	...	−0.1159	...	...	...	...
^{30}P	...	1.865	...	0.0124	...	...	...	...
^{32}P	−0.2523	0.5732	...	−0.0245	0.55	0.03	−0.2523	−0.0013
^{36}Cl	1.284	2.714	−0.0168	0.5872	0.27	0.05	1.314	−0.0168

$$A_{\gamma'\gamma}=\delta_{j_p',j_p}\delta_{j_n',j_n}[\delta_{\Omega_p',\Omega_p}\delta_{\Omega_n',\Omega_n}C(I2I';K,\Delta K)\delta_{\Delta K,0}+(-1)^{I-j_p-j_n}\delta_{\Omega_n',-\Omega_n}\delta_{\Omega_p',-\Omega_p}\times C(I2I';-K,\Sigma K)\delta_{\Sigma K,0}] \quad (16)$$

and as before

$$a_c=3Z_cR_0^2\beta/4\pi,$$
$$a_p=\hbar/m\omega_0,$$

Z_c being the charge of the core. The $C(I_1I_2I;m_1m_2)$ are the usual Clebsch-Gordan coefficients defined in Ref. 8. From the above expressions δ follows immediately. The static moments are just

$$\langle\mu\rangle=g_cI+\left(\frac{I}{I+1}\right)^{1/2}\sum_{k=1}^{2}\sum_{\gamma,\gamma'}S_{\epsilon\gamma}S_{\epsilon\gamma'}\times\langle j_k'\|G_k\|j_k\rangle M_{\gamma'\gamma}{}^{G_k}, \quad (17)$$

$$\langle Q\rangle=\left(\frac{16\pi I(2I-1)}{(I+1)(2I+3)}\right)^{1/2}\sum_{\gamma\gamma'}S_{\epsilon\gamma}S_{\epsilon\gamma'}\times(a_ca_{\gamma\gamma'}+a_p\langle j_p'\|\rho^2Y_2\|j_p\rangle M_{\gamma'\gamma}{}^{p}). \quad (18)$$

In the above we have used the abbreviations $\Sigma K=K+K'$, $\Delta K=K'-K$, and $g_c=Z/A$.

The results of these calculations are shown in Table III and are quite similar to the odd-A case.[1] In Table III we also show the values obtained for P and β by fitting to the magnetic dipole and electric quadrupole moments for ^{32}P and ^{36}Cl. We have also calculated δ for the two $2+\rightarrow 1+$ ground-state transitions and find them several orders of magnitude smaller than the values obtained by Harris *et al.*[14] in the nucleus ^{30}P.

IV. CONCLUSION

We have seen that by exact calculation using a very simple nuclear model, which is in every respect a logical extension of one used to study odd-A nuclei in this shell, that the energy levels for odd-odd nuclei in the last half of the 2s-1d shell compare quite favorably with those observed. That is, the Hamiltonian of Ref. 1 is extended to the case of two extra core nucleons, and it is diagonalized exactly with respect to a similar core-particle basis. However, the fit is achieved at $\gamma=0^0$ (axial symmetry) here, whereas in Ref. 1 it was found that the best fit for the nuclei ^{29}Si, ^{29}P, ^{31}P, and ^{35}Cl was near $\gamma=30^0$. This does not mean that better fits may not be found at other values of γ, nor does it support a recent microscopic calculation which indicates that nuclei in this shell may be symmetric[15] since only the value $\gamma=0^0$ was used here. Only in two cases was the residual p-n interaction at all necessary to obtain the level sequence and then it was vital. The results for ^{36}Cl very near the end of the shell are similar to those obtained for ^{35}Cl earlier[1] and have been included only for completeness since such a collective model is doubtlessly not suitable so near the spherical nucleus ^{40}Ca. We note that not only are the model parameters not reasonable (P and β too large, $\hbar\omega_0$ too small) but that the sign of the quadrupole moment is wrong. However, the results for ^{28}Al, ^{30}P, and ^{32}P are all quite good with reasonable model parameters.

This calculation has the same deficiencies observed in the odd-A case for the electric quadrupole matrix elements which are too small by far to account for the strengths of the $E2$ transitions measured recently in ^{30}P. This would seem to be a characteristic of the model. In ^{32}P the fact that the magnetic moment has the wrong sign may indicate that the ground-state wave function should have a contribution from the $d_{5/2}$ orbitals.

We should also expect the model not to work too well in predicting spectroscopic factors for single- and two-nucleon transfer reactions because of the truncation of the shell-model states. Since in these models one usually takes the Pauli principle into account by removing filled states from the available basis, a method which does not remove complete shell-model states might produce more consistent results. In the present method one simply is not taking full advantage of the configuration mixing introduced by the collective potential. Possibly a more suitable procedure would be to choose a basis of particle states composed of deformed Nilsson orbitals. This would also remove ambiguities in truncation which arise in the present method and would make available to the basis components from the $d_{5/2}$ subshell.

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