$^{31}P(d,\alpha)^{29}Si$ Reaction from 7.3 to 12.0 MeV*

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Cross sections for the reaction $^{31}P(d,\alpha)^{29}Si$ have been measured for laboratory deuteron energies from 7.300 to 12.020 MeV. The data have been analyzed in terms of Ericson fluctuation theory, from which the coherence width in ^{33}S has been obtained for the whole range of data and for the data divided into halves. The fraction of the reaction proceeding via a direct mechanism has also been determined by Ericson-type analysis. To obtain this fraction, it was necessary to take into account the particle decay of the compound nucleus through 3040 channels in order to calculate realistically the fluctuation damping coefficient.

I. INTRODUCTION

WHEN nuclei are sufficiently excited that levels of the compound nucleus are widely overlapping, interference effects among the overlapping levels will cause fluctuations in the cross sections as a function of incident energy. Ericson[1] and Brink and Stephen[2] have shown that a description of these fluctuations can be formulated in terms of three parameters. The first is the average width (coherence width) Γ of the compound-nucleus states while the second is the direct-reaction fraction y of the observed cross section. The third is the effective number N_{eff} of M sets of angular momentum projections that damp the fluctuations as a result of the multiplicity of incoherent emission channels.

The coherence width can be determined by means of a generalized correlation function defined by

$$R_{ab}(\epsilon)=\left\langle 0.5\left(\frac{\sigma_a(E)}{\langle\sigma_a\rangle}-1\right)\left(\frac{\sigma_b(E+\epsilon)}{\langle\sigma_b\rangle}-1\right)+0.5\left(\frac{\sigma_a(E)}{\langle\sigma_a\rangle}-1\right)\left(\frac{\sigma_b(E-\epsilon)}{\langle\sigma_b\rangle}-1\right)\right\rangle, \quad (1)$$

where σ_a and σ_b are the differential cross sections for α groups a and b at a given angle and where $a=b$ for auto-correlation and $a\neq b$ for cross correlation. The brackets indicate energy averaging. By specifying the statistical nature of the reaction amplitude one can directly determine Γ from

$$R_{a=b}(\epsilon)=R_{a=b}(0)/[1+(\epsilon)^2/(\Gamma)^2]. \quad (2)$$

The fraction of the cross section resulting from a direct mechanism defined by

$$y=\sigma_{\text{DI}}/(\sigma_{\text{DI}}+\langle\sigma_{\text{CN}}\rangle) \quad (3)$$

is determined from

$$R_{a=b}(0)=(1-y^2)/N_{\text{eff}}. \quad (4)$$

Here σ_{DI} is the direct-reaction cross section and $\langle\sigma_{\text{CN}}\rangle$ is the average compound nucleus contribution to the cross section. The calculation of y is dependent upon an independent determination of N_{eff} which will be discussed in detail below.

A number of investigators[3–6] have determined the average level width or coherence width for an assortment of medium weight nuclei. In addition, in most cases an attempt has been made to determine y. In all cases, the values of y obtained have been less than 0.50. Hence the associated errors make their measurements nearly indeterminate. It was the intention of the present work to make a reasonably accurate determination of both the coherence width and the fraction of direct reaction for the $^{31}P(d,\alpha)^{29}Si$ reaction covering a region of bombarding energy from 7.300 to 12.020 MeV in the laboratory. This corresponds to an excitation energy in ^{33}S from approximately 22.1 to 26.5 MeV, which suggests that for the (d,α) reaction the direct component might dominate and hence make the measurement of its fractional contribution meaningful.

II. EXPERIMENTAL

The deuteron beam for this experiment was provided by The University of Texas EN tandem accelerator. The range of deuteron energy covered was from 7.300 through 12.020 MeV in the laboratory in steps of approximately 12 keV. The beam was focused onto a target located at the center of a 20-in.-diam scattering chamber by a quadrupole lens system. Self-supporting phosphorus targets were prepared by the evaporation of natural red phosphorus, following the technique described by Hooten.[7] The target used in the present work had a thickness of 147 μg/cm^2 as determined from low-energy elastic-scattering measurements. Six surface barrier solid-state detectors were used at laboratory angles of 170°, 150°, 130°, 110°, 90°, and 40°. The selection of the first five angles allowed a reasonable angular distribution over at least one quadrant. The detector at 40° provided an indication of any forward peaking in the reaction cross section, which might be expected if a significant portion of the reaction mechanism were direct. All six of the detectors had a depletion depth of

* Work supported in part by the U. S. Atomic Energy Commission.
[1] T. Ericson, Ann. Phys. (N. Y.) **23**, 390 (1963).
[2] D. M. Brink and R. O. Stephen, Phys. Letters **5**, 77 (1963).
[3] B. W. Allardyce, P. J. Dallimore, I. Hall, N. W. Tanner, A. Richter, P. von Brentano, and T. Mayer-Kuckuk, Nucl. Phys. **85**, 193 (1965).
[4] M. L. Halbert, F. E. Durham, and A. Van Der Woude, Phys. Rev. **162**, 899 (1967).
[5] G. G. Seaman, R. B. Leachman, and G. Dearnaley, Phys. Rev. **153**, 1194 (1967).
[6] G. Dearnaley, W. R. Gibbs, R. B. Leachman, and P. C. Rogers, Phys. Rev. **139**, B1170 (1965).
[7] B. W. Hooten, Nucl. Instr. Methods **27**, 338 (1964).

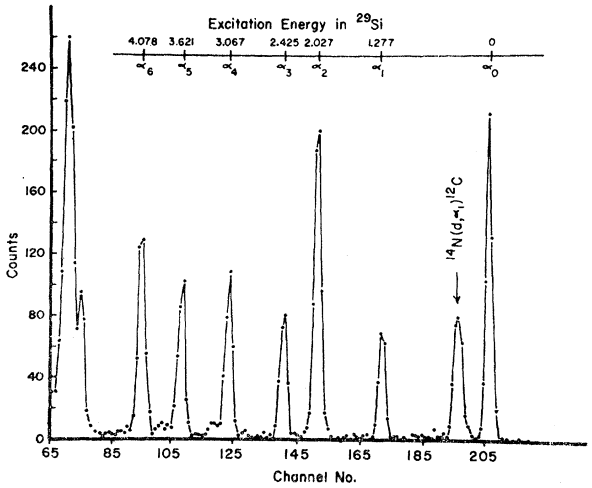

FIG. 1. A typical ^{31}P$(d,\alpha)^{29}$Si pulse-height spectrum showing the seven α groups resolved.

300 μ at maximum bias voltage. The detectors used at the four most backward angles had active areas of 50 mm^2 whereas the two used at 90° and 40° had active areas of 25 mm^2. The detector holders were constructed in such a way that only particles incident from the direction of the target were detected. The solid angles for the 40° and 90° detectors were approximately 7×10^{-4} sr while the solid angles for the four most backward angle detectors were about 13×10^{-4} sr. The scattering chamber and its accessories are identical to those described by Foster.[8]

The detectors were biased at less than maximum bias voltages so that only the α groups of interest and very-low-energy protons, of the order of 3 MeV, were stopped. The α groups leading to the ground and first six excited states in ^{29}Si were resolved and are shown in the typical spectrum in Fig. 1. The experimental resolution was about 50 keV. The detector pulses were analyzed by two 400-channel pulse-height analyzers, both of which were controlled by a current integrator which monitored the charge collected in a Faraday cup located behind the scattering chamber. The spectra were transferred from the 400-channel analyzers to an on-line PDP-7 computer and printed out on a line printer as well as punched out on paper tape. A peak summing program APHRODITE[9] was used to sum about 50% of the peaks of interest. The excitation functions measured at 170° and 40° are shown in Fig. 2.

III. ANALYSIS

The bombardment of ^{31}P by deuterons having laboratory energies ranging from 7.300 to 12.020 MeV can lead to excitation of ^{33}S in the energy region from 22.078 to 26.459 MeV. In this region of excitation for a medium weight nucleus such as sulfur, the levels overlap so that the condition $\Gamma\gg D$ is satisfied as required for application of the Ericson theory. The various decay

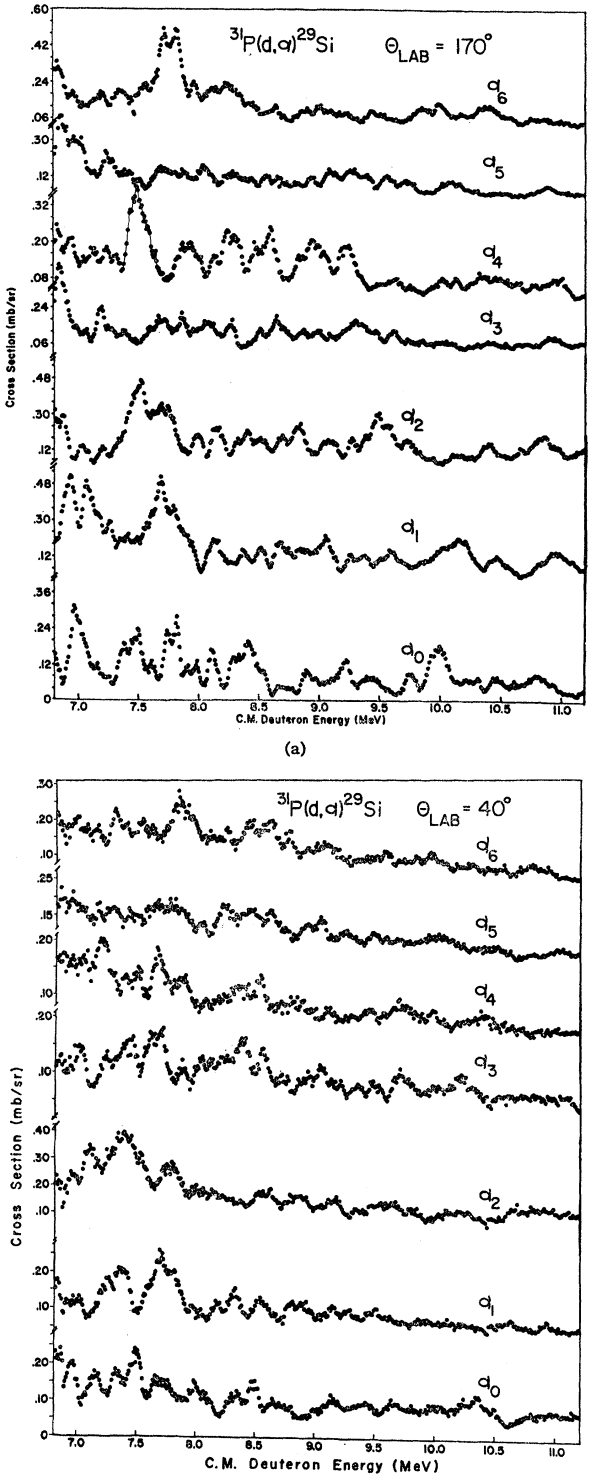

FIG. 2. (a) 170° excitation functions for the seven α groups observed. (b) 40° excitation functions for all seven α groups.

[8] J. L. Foster, Ph.D. thesis, The University of Texas, 1967 (unpublished).

[9] C. F. Moore and B. M. Foreman (private communication).

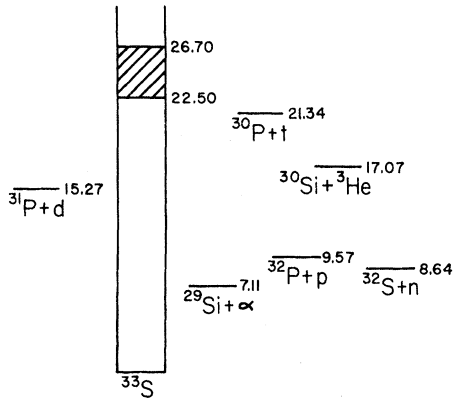

FIG. 3. Level scheme showing the open decay modes for the excited sulfur nucleus.

channels available to the highly excited sulfur, of which the α channel was the only one observed experimentally, are shown in Fig. 3.

Before applying Ericson fluctuation analysis, its applicability must be tested. Qualitatively, this can be done by comparing the excitation functions leading to the various final states in ^{29}Si, such as those shown in Fig. 2, and noting no apparent correlation between the various dips and rises. A more quantitative method involves the calculation of the cross correlation function defined by Eq. (1). The particular form of the correlation function employed permits an equal weighting of correlations between a given point and points measured at lower excitation energies as well as greater. One hundred twenty-six cross correlations between the different α groups at the six angles were carried out showing very little correlation. The correlations at each of the six angles should be relatively independent since the angular separation of the detectors is at least 20°, which is larger than a crude approximation of the coherence angle, given by $\theta \approx 1/kR \approx 16°$.[10]

A. Coherence Width Determination

Initially a straightforward correlation analysis was applied using for $\langle\sigma\rangle$ in the calculations a simple arithmetic mean. The resulting autocorrelation functions were not Lorentzian in shape but instead resembled the unmodulated correlation functions of Allardyce *et al.*[3] The values of Γ extracted from such functions were erratic, and varied between 70 keV for α_3 at 170° and 880 keV for α_5 at 150°. The average value of Γ was 278 keV.

The main reason for this behavior is the decrease in the average cross section as a function of energy. This decreasing cross section is an example of nonstationary or modulation effects as discussed by Hall[11] and by Allardyce *et al.*[3] Their method for accounting for the modulations was approximated. Although the modulated widths were more consistent than the unmodulated ones they still lacked over-all consistency and varied between 55 keV for α_0 at 170° and 150 keV for α_3 at 150°. The average value of Γ was 89 keV.

There is another method for circumventing the effects of this modulation. Rather than use the simple arithmetic mean for the average cross section in the correlation function, one can use a moving average. Program DOORWAY[12] is written to allow for such a choice for $\bar{\sigma}$. When this option in the program is exercised, the $\bar{\sigma}$ becomes $\bar{\sigma}(E)$, which represents a smooth variation of $\bar{\sigma}$ around which the cross section may be fluctuating. The quantities $\bar{\sigma}(E)$ may be calculated in two ways, as follows:

(a) The cross section $\bar{\sigma}(E)$ may be smeared with a rectangular resolution function of a prescribed width, preferably many times larger than the coherence width.

(b) $\bar{\sigma}(E)$ may be calculated by fitting the cross section over a prescribed region with a quadratic in the energy E. The second method was chosen for computational simplicity.

The extraction of widths from correlation functions using moving energy averages has been discussed by Gadioli *et al.*[13] Their method assumed that it is possible to write the cross section as the sum of a smoothly varying average and a fluctuating component about this average. If one lets σ_Δ be the cross section averaged over a small interval Δ, the problem is to discover an interval δ for which σ_δ reproduces the smoothly varying energy average. The method for determining δ is outlined in Ref. 14. The widths were extracted from the correlation functions using the moving energy average and a value of $\delta = 1049$ keV.

There are three specific corrections which can and should be made to these widths. These are corrections due to the interval between data points, finite sample size, and counting statistics, respectively.

The effect of the first correction, finite energy resolution ΔE, on Γ and $R(0)$ has been studied by Gibbs[14] and by Van Der Woude.[15] The latter shows that

$$R_e(0) = R(0)[1-(4/3!)(\Delta E/2\Gamma)^2+O((\Delta E/2\Gamma)^4)]$$

and

$$\Gamma_e/\Gamma = +(\Delta E/2\Gamma)^2+O((\Delta E/2\Gamma)^4), \qquad (5)$$

where the quantities subscripted by e have been obtained with finite energy resolution and the unsubscripted quantities are the desired values. The energy interval between data points in the present experiment was about 11 keV and the average coherence width is subsequently shown to be 69 keV for the entire energy

[10] D. M. Brink, R. O. Stephen, and N. W. Tanner, Nucl. Phys. **54**, 577 (1964).
[11] I. Hall, Phys. Letters **10**, 199 (1964).
[12] P. P. Singh (private communication).
[13] E. Gadioli, I. Iori, and A. Marini, Nuovo Cimento **39**, 996 (1965).
[14] W. R. Gibbs, Los Alamos Scientific Laboratory Report No. LA 3266, 1965 (unpublished).
[15] A. Van Der Woude, Nucl. Phys. **80**, 14 (1966).

TABLE I. Coherence widths in keV obtained from the whole range of data from autocorrelation analysis using a moving energy average.

	Γ for $E_d=7.300\rightarrow 12.020$ MeV					
	170°	150°	130°	110°	90°	40°
α_0	66±6.5	80±5.9	63±3.6	52±0.5	72±2.2	67±1.8
α_1	82±5.5	82±2.8	77±3.1	···	67±1.1	47±1.9
α_2	···	60±1.0	61±1.2	72±2.3	70±0.6	75±1.2
α_3	58±2.2	84±3.4	79±2.5	60±1.2	84±1.4	62±1.2
α_4	66±3.9	57±1.8	66±1.5	68±1.4	65±1.3	66±0.9
α_5	69±2.4	70±1.5	80±2.1	65±1.4	72±0.9	77±1.2
α_6	···	64±1.3	66±1.5	86±2.9	85±1.0	64±0.9

range. Therefore,

$$(\Delta E/2\Gamma)^2=63.5\times10^{-4}, \tag{6}$$

and hence the corrections to Γ and $R(0)$ are less than 1% and have been taken to be negligible.

Finite sample size and counting statistics corrections have been explicitly worked out by Gibbs.[14] The method employed followed the outline of Ref. 14 with only slight variations. The corrected widths for the entire range of data measured for the seven α groups at the six angles are shown in Table I. The uncertainties shown have been calculated as in Ref. 14. The data were halved and a similar extraction of the coherence widths carried out. The average widths for the entire range of data and for the first and second halves are 69±11, 61±12, and 77±13 keV, respectively, indicating that the average widths increase with excitation energy. The corresponding sample sizes are 25, 14, and 13.

Recently the method of counting maxima in the excitation functions in order to evaluate the coherence width has been used extensively. The method was originally proposed by Brink and Stephen[2] and further theoretical work regarding its application was carried out by Van Der Woude[15] and by Bizzeti and Maurenzig.[16] The number of maxima per unit energy interval is related to Γ by the relation

$$n=0.5b_N/\Gamma. \tag{7}$$

The factor b_N depends on the number of independent M sets and on the ratio $\Delta E/\Gamma$. In the present study, the points are sufficiently close together so that the effect of $\Delta E/\Gamma$ is negligible. Bizzeti and Maurenzig[16] have concluded that for

$$\Delta E/\Gamma=0, \quad b_N=1.10, \tag{8}$$

independent of N. In addition, they have explored in some detail the effects of "doubtful maxima." Essentially they have devised a method of guaranteeing uniformity in the selection of the maxima. Uniformity was achieved in the present study by estimating Ericson's variance of the difference $\sigma(E)-\sigma(E+\epsilon)$ given by

$$2R(0)\langle\sigma\rangle^2/[1+(\Gamma/\Delta E)^2]$$

[16] P. G. Bizzeti and P. R. Maurenzig, Nuovo Cimento 47, 29 (1967).

TABLE II. Coherence widths in keV for the first 2 MeV of the data from correlation analysis and from peak counting.

	Correlation width (keV)					
	170°		150°		130°	
α group	Auto-correlation	Peak counting	Auto-correlation	Peak counting	Auto-correlation	Peak counting
α_0	59±5.4	58±7.0	83±7.7	50±6.0	64±4.2	65±7.8
α_1	66±3.5	73±8.8	59±3.1	61±7.3	59±4.1	61±7.3
α_2	···	73±8.8	62±1.5	48±5.8	59±1.6	55±6.6
α_3	56±3.2	69±8.3	82±2.8	50±6.0	67±2.4	61±7.3
α_4	52±4.6	58±7.0	48±2.2	61±7.3	58±1.5	50±6.0
α_5	51±1.2	65±7.8	48±0.5	58±7.0	58±1.0	48±5.8
α_6	53±1.9	69±8.3	54±1.1	61±7.3	58±0.9	50±6.0

and then by defining a maximum to be a point larger than the points on either side of it by an amount larger than this variance. Unfortunately, the experimental variance was often of the same magnitude as the Ericson variance, and the situation was made difficult by the small value of $\Delta E/\Gamma$. These two difficulties coupled to lessen the effectiveness of the technique which otherwise is simple, speedy, and encompasses no bias due to the finite energy range of data. In addition, for a continuous curve of cross section versus energy, Van Der Woude[15] has shown that the statistical significance of the estimate of Γ is better for the peak counting method than for others.

In the present study the relatively small value of $\Delta E/\Gamma\approx0.16$ prompted the choice of $b_N=1.10$ for all values of N. Experimental cross-section uncertainty and the damping of the fluctuations because of N made it feasible to apply the peak counting technique only at angles of 170°, 150°, and 130°. Since the fluctuations damp out strongly in the last 2.7 MeV of the data, peaks were counted only in the first 2 MeV. The results for both peak counting and correlation analysis for the first 2 MeV of the data are shown in Table II. The standard deviations for the peak counting method shown were calculated using the results of Ref. 15. Even though the standard deviations associated with the peak counting results are less than those associated with the correlation analysis, less confidence is placed in the former than in the latter because of the problems already enumerated. In general there is fair agreement between the results for the two methods. The average value of Γ obtained at the three angles by both methods is 60±8 keV from correlation analysis and 59±6 keV from the method of peak counting.

B. Determination of y

In order to demonstrate that the fraction of direct reaction is sufficiently large to be measurable one can assume that it is not and seek a contradiction in the subsequent analysis. If $y=0$, the probability distribution of cross sections should be given by a γ distribution[17] with both parameters equal to N_{eff}. The probability distribution computed by DOORWAY using the

[17] E. Parzen, *Modern Probability and Its Applications* (John Wiley & Sons, Inc., New York, 1960).

moving energy averages for $\langle\sigma\rangle$ was fitted to this form and a best value for N was deduced by a χ^2 test. The probability fits obtained at 170°, the most backward angle for which data were taken, are shown in Fig. 4. Values for N were also deduced by using Gibb's biased value for $R(0)$,

$$R(0)=(n-1)/(1+nN), \tag{9}$$

and were shown to be in excellent agreement with the values obtained from the probability fits. The symbol n above denotes the sample size.

For a given set of spins, it is a simple matter to calculate the number of independent M sets at 0° and 180° under the assumption that the z axis is along the beam direction. It is reasonable to assume that the value of N would not vary greatly between 170° and 180°. A comparison of the values of N deduced from the probability fits at 170° and those calculated from angular-momentum considerations at 180° is shown in Fig. 4. Clearly, y cannot be ignored even at 170°.

It is clear from Eq. (4) that an accurate assessment of y is dependent on the knowledge of N_{eff}. The limiting number of M sets is given by

$$N=\tfrac{1}{2}[(2S+1)(2I+1)(2I'+1)(2S'+1)+1]$$

for all spins integer,

$$N=\tfrac{1}{2}[(2S+1)(2I+1)(2I'+1)(2S'+1)] \quad \text{otherwise,}$$

where S and I are the spins of the projectile and target, respectively, and the primed quantities refer to observed particle and residual nucleus spins. The value of N varies from 6 to 24 for the seven α groups resolved. As pointed out by Seaman *et al.*,[5] since the contribution to the cross section from each M set is usually different, the effective number of M sets N_{eff} is generally non-integral and less than the limiting value of N.

Bondorf and Leachman[18] have shown that a value for the effective number of M sets $N_{\text{eff}}^{\text{HF}}$ can be calculated from Hauser-Feshbach cross sections according to

$$N_{\text{eff}}^{\text{HF}}=\frac{[\sum_\mu \bar{\sigma}_\mu(\theta)]^2}{\sum_\mu[\bar{\sigma}_\mu(\theta)]^2}, \tag{10}$$

where μ refers to a given member of the set of spin projections $\{S_z,I_z,S_z',I_z'\}$ and it is assumed that each of the basic cross sections $\sigma_\mu(\theta)$ is independent. These cross sections can be calculated if the transmission coefficients in all open exit channels are known.

The general expression for the transmission coefficient T_{lj} as given by Feshbach[19] is

$$T_{lj}=1-e^{-4\eta_{lj}}, \tag{11}$$

where η_{lj} is the imaginary part of the potential scattering phase shift. As illustrated in Fig. 3, there are a

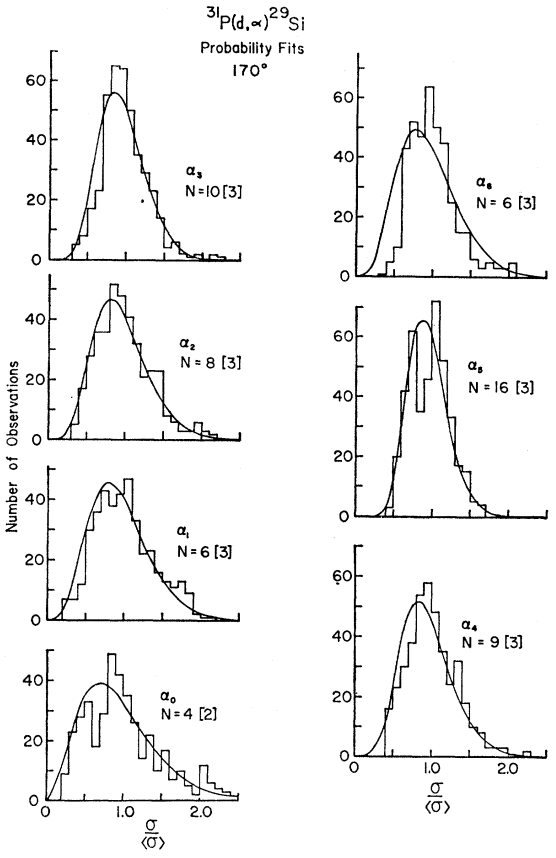

FIG. 4. Probability distribution fits for determining N_{eff} at 170° assuming no direct interaction. The value of N_{eff} obtained from the fits for each α group are indicated. The bracketed numbers are N_{eff} calculated from angular-momentum conservation at 180°.

number of open decay channels available to the excited ^{33}S nucleus. Thus it is necessary to calculate the transmission coefficients for all these channels which contribute significantly. The imaginary phase shifts η_{lj} were calculated using the optical model. Two different computer codes were used in order to account for all the decay channels and to check one code against the other. Code OPTIX[20] was used for the deuterons, alphas, and protons, code PEREY[21] was used for neutrons, protons, and deuterons. Because of the comparatively small number of open decay channels to the $^3He+^{30}Si$ system from the region of excitation covered in ^{33}S, the ^3He contribution was ignored. In addition, although the $t+^{30}P$ system is shown in Fig. 3, its contribution is completely negligible. The optical-model parameters were taken from Huizenga and Igo[22] for α particles, from Perey and Perey[23] for deuterons, and from Rosen[24] for neutrons and protons. The transmission coefficients ob-

[18] J. Bondorf and R. B. Leachman, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. **34**, No. 10 (1965).
[19] H. Feshbach, in *Nuclear Spectroscopy*, edited by F. Ajzenberg-Selove (Academic Press Inc., New York, 1960), Part B, p. 1041.
[20] W. J. Thompson and F. Gille, Florida State University Technical Report No. 9, 1965 (unpublished).
[21] F. G. Perey, Phys. Rev. **131**, 745 (1963).
[22] J. R. Huizenga and G. Igo, Nucl. Phys. **29**, 462 (1962).
[23] C. M. Perey and F. G. Perey, Phys. Rev. **152**, 923 (1966).
[24] L. Rosen, Ann. Phys. (N. Y.) **34**, 96 (1965).

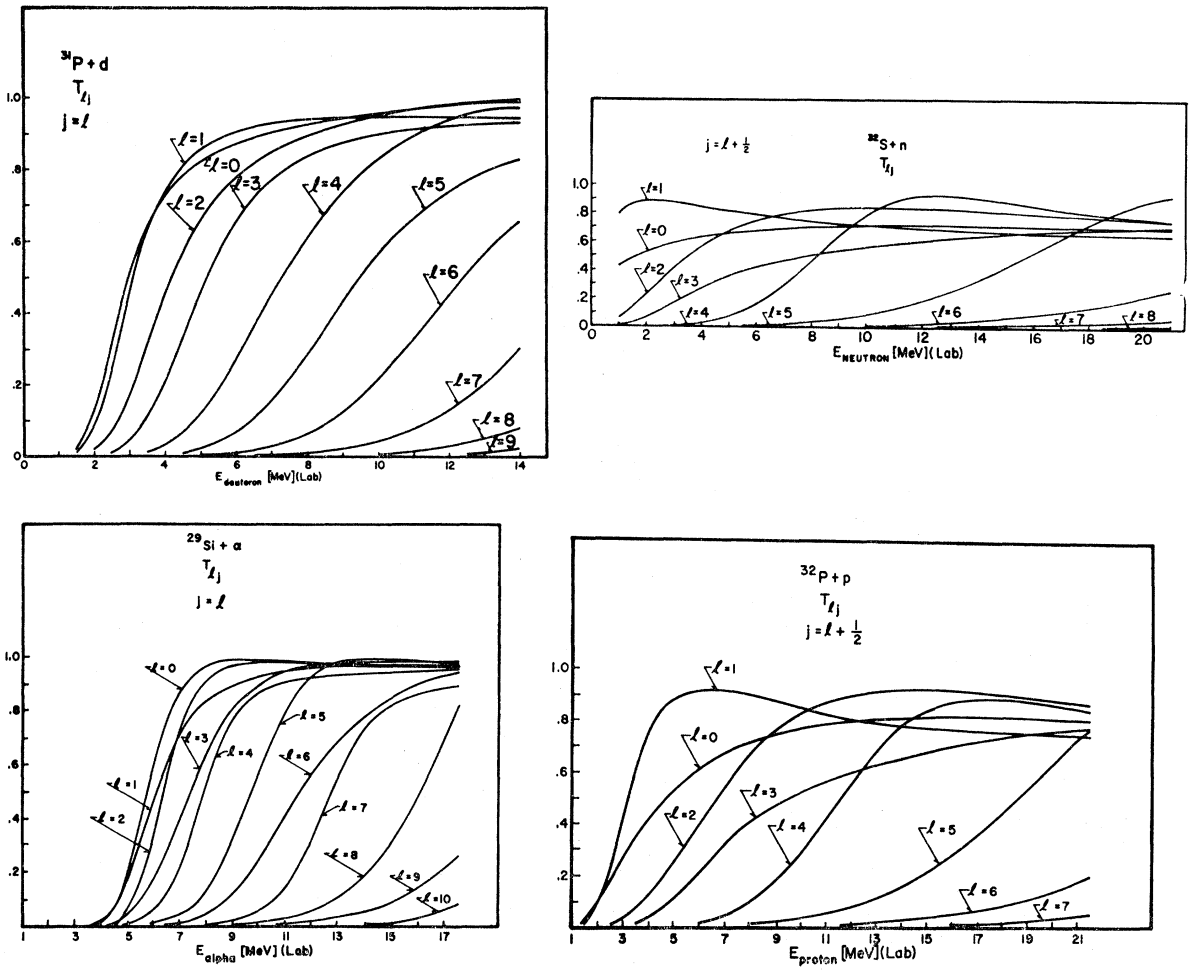

FIG. 5. Transmission coefficients for the open particle-decay channels which were considered.

tained are shown in Fig. 5. Since the deuteron transmission coefficients so obtained were only slightly dependent on the spin-orbit interaction, a single set of T_{lj} was used for all three j values.

For the purpose of calculating $N_{\text{eff}}{}^{\text{HF}}$, an incident laboratory energy of 9.5 MeV was chosen, this being approximately the midpoint of the energy range covered. This corresponds to an excitation energy of 24.195 MeV in ^{33}S. A decay channel was considered to be open so long as its associated transmission coefficient had a value of at least 0.03 for some choice of l. For the neutron, proton, and α-particle decay channels, levels were considered in the residual nuclei beyond the point where their individual energies, spin, and parities were known. The level-density formulas of Gilbert, Chen, and Cameron[25] were used to approximate the level densities in this region. No approximations were necessary for the deuteron channel. Eighteen deuteron channels, 420 α channels, 793 neutron channels, and 1809 proton channels were accounted for, giving a total of 3040 in all.

For those cases where the target and projectile are not both spin-0 particles, the number $N_{\text{eff}}{}^{\text{HF}}$ is usually an overestimate. Gibbs has shown that a more realistic value for N_{eff} defined as $N_{\text{eff}}{}^{J\pi}$ may be obtained.[26] The computer code SNUF[27] calculates both $N_{\text{eff}}{}^{\text{HF}}$ and $N_{\text{eff}}{}^{J\pi}$. The latter is obtained from $N_{\text{eff}}{}^{\text{HF}}$ as described in detail in Ref. 26. $N_{\text{eff}}{}^{J\pi}$ can, however, be approximated in terms of $N_{\text{eff}}{}^{\text{HF}}$ as shown by Gibbs,

$$N_{\text{eff}}{}^{J\pi}=N_{\text{eff}}{}^{\text{HF}}\Lambda/(\Lambda+N_{\text{eff}}{}^{\text{HF}}-1), \qquad (12)$$

where Λ represents the number of independent reaction amplitudes which are approximately equal to the number of contributing J, π values. The values of $N_{\text{eff}}{}^{J\pi}$, found from the approximation method and from the more realistic calculation in SNUF, were found to be almost identical. The resulting values for $N_{\text{eff}}{}^{J\pi}$ and $N_{\text{eff}}{}^{\text{HF}}$ for α_0, α_1, and α_2 as a function of angle are shown in Fig. 6.

Having made a realistic estimate of N_{eff}, one can deduce y from Eq. (4). One must take care, however,

[25] A. Gilbert, F. S. Chen, and A. G. W. Cameron, Can. J. Phys. 43, 1248 (1965).

[26] W. R. Gibbs, Phys. Rev. 153, 1206 (1967).

[27] W. R. Gibbs (private communication).

TABLE III. y and its variance for the entire range of data.

	170°		150°		130°		110°		90°		40°	
	Y	σ_R^2	Y	σ_R^2	Y	σ_R^2	Y	σ_R^2	Y	σ_R^2	Y	σ_R^2
α_0	0.59	0.050	0.69	0.030	0.76	0.021	0.93	0.001	0.87	0.008	0.88	0.005
α_1	0.73	0.023	0.86	0.008	0.84	0.010	0.52	0.065	0.92	0.003	0.83	0.010
α_2	...	...	0.92	0.002	0.93	0.003	0.88	0.006	0.96	0.001	0.94	0.002
α_3	0.86	0.008	0.84	0.010	0.88	0.007	0.90	0.003	0.92	0.003	0.91	0.003
α_4	0.78	0.018	0.87	0.007	0.91	0.003	0.90	0.004	0.90	0.004	0.93	0.002
α_5	0.88	0.007	0.90	0.003	0.88	0.004	0.90	0.003	0.95	0.002	0.93	0.002
α_6	...	...	0.92	0.003	0.93	0.003	0.88	0.007	0.94	0.002	0.95	0.002

that the correct value for $R(0)$ is used in obtaining y. $R(0)$ must be corrected for the effect of counting statistics, and for finite sample size effects, both of which have been discussed by Gibbs.[14] A good approximation to y is given to be

$$y=\left(1-\frac{4n^2NR_0(0)}{(n-1)(4n-4+N)}\right)^{1/2}, \tag{13}$$

where $R_0(0)$ is the value of the correlation function corrected for the effects of counting statistics. Values of y were deduced from the above relationship.

The values of y calculated from the entire range of data and their respective variances are given in Table III. The value of y for all groups at all six angles is about 0.87 with an rms error of approximately 10% or less. Over all there seems to be very little variation in y as a function of angle or α group. Values of y were also obtained for the data split into halves, and they are shown in Table IV. These results seem to indicate an increase in y of the order of 5% for the upper half of the energy range.

C. Test of the $2I+1$ Rule

The spin of the sixth excited state in ^{29}Si has been given a tentative assignment[28] of $\frac{3}{2}$ and definite assignments of $\frac{5}{2}$ and $\frac{7}{2}$ have only recently been made for the fourth and fifth excited states, respectively.[29] One possible method for determining the spins of the states in the residual nucleus is by the application of the $2I+1$ rule which has been previously explored by a number of investigators.[30–34] According to the $2I+1$ rule, the integrated cross section in a specific nuclear reaction for the excitation of a final state of spin I is proportional to $2I+1$. MacDonald[32] has outlined the most favorable conditions for applications of the rule, all of which are reasonably well satisfied in the present case except that the direct-reaction component is very large. It has been shown by Hinds *et al.*[35] that the effects of the direct reactions can be minimized by using data from only the back quadrant. It is therefore

TABLE IV. y obtained from the data in halves. (A) y for $E_d=7.300\rightarrow 9.660$ MeV. (B) y for $E_d=9.660\rightarrow 12.020$ MeV.

	170°		150°		130°		110°		90°		40°	
	Y	σ_R^2	Y	σ_R^2	Y	σ_R^2	Y	σ_R^2	Y	σ_R^2	Y	σ_R^2
						(A)						
α_0	0.61	0.051	0.63	0.050	0.73	0.030	0.93	0.002	0.83	0.012	0.85	0.010
α_1	0.79	0.018	0.87	0.018	0.72	0.029	...	...	0.92	0.003	0.77	0.021
α_2	...	...	0.91	0.004	0.89	0.005	0.88	0.007	0.91	0.003	0.94	0.002
α_3	0.78	0.020	0.87	0.008	0.87	0.009	0.90	0.005	0.91	0.003	0.83	0.012
α_4	0.66	0.046	0.82	0.014	0.89	0.005	0.90	0.004	0.88	0.006	0.93	0.003
α_5	0.91	0.004	0.95	0.001	0.93	0.002	0.95	0.001	0.92	0.003	0.94	0.002
α_6	...	...	0.92	0.003	0.93	0.002	0.89	0.003	0.91	0.004	0.93	0.003
						(B)						
α_0	0.66	0.052	0.82	0.018	0.68	0.048	0.98	0.049	0.92	0.004	0.96	0.002
α_1	0.76	0.031	0.80	0.022	0.86	0.013	0.68	0.037	0.93	0.003	0.98	0.001
α_2	0.82	0.019	0.96	0.002	0.97	0.001	0.90	0.007	0.98	0.001	0.95	0.002
α_3	0.93	0.003	0.93	0.010	0.94	0.002	0.94	0.002	0.94	0.002	0.92	0.004
α_4	0.86	0.012	0.83	0.017	0.92	0.003	0.90	0.007	0.93	0.003	0.92	0.005
α_5	0.85	0.014	0.85	0.014	0.84	0.015	0.93	0.003	0.96	0.002	0.96	0.002
α_6	0.89	0.007	0.88	0.008	0.96	0.002	0.93	0.003	0.97	0.001	0.94	0.002

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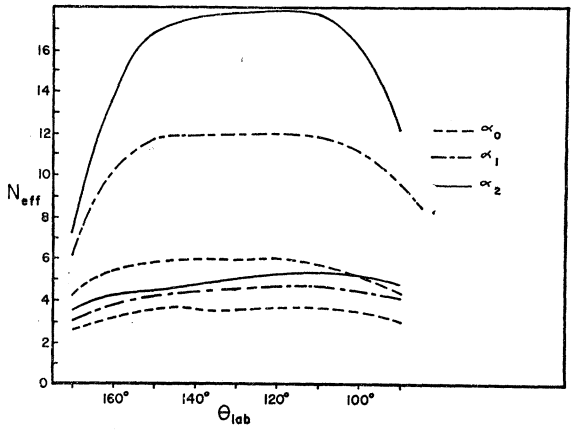

FIG. 6. Typical $N_{\text{eff}}^{\text{HF}}$ and $N_{\text{eff}}^{J\pi}$ curves as a function of angle. The lower of the two values in each case is $N_{\text{eff}}^{J\pi}$.

possible that, with a large amount of data, the cross sections integrated over angles $\geq 90°$ might still approximate a $2I+1$ proportionality.

In the present analysis the data were integrated from 90° to 180° at every twelfth energy giving a total of 33 cases. The choice of every twelfth energy insured comparison between points separated in energy by an amount greater than the coherence width of the fluctuations. The integrated results for each α group were then averaged to minimize the effect of local fluctuation and the relative intensities versus $2I+1$ are displayed in Fig. 7. The results are clearly in agreement with accepted spin values of $\frac{1}{2}$, $\frac{3}{2}$, $\frac{5}{2}$, and $\frac{3}{2}$ for the ground and first three excited states, respectively, of ^{29}Si but would indicate spin values of $\frac{3}{2}$, $\frac{5}{2}$, and $\frac{5}{2}$ instead of $\frac{5}{2}$, $\frac{7}{2}$, and $\frac{3}{2}$ for the spins of the fourth, fifth, and sixth excited states, respectively. In view of the large direct interaction cross section, however, it is surprising that the $2I+1$ rule fits so well even for the lowest four states of ^{29}Si.

IV. DISCUSSION AND SUMMARY

The use of a moving energy average in the autocorrelation analysis has been found satisfactory for determining coherence widths in the region of overlapping levels. The consistency of the measured coherence widths obtained from correlation analysis and from the peak counting technique lends additional credence to the results. A trend of increasing width as a function of excitation energy was indicated by the values of 61 and 77 keV for the first and second halves of the data, respectively.

The failure of the probability fits at 170° to predict values of N in agreement with those obtained from angular-momentum considerations at 180° showed clearly that y could not be ignored even at 170°, and hence that theoretical values for N_{eff} were necessary. Gibb's code provides two choices for N_{eff}, namely, $N_{\text{eff}}^{\text{HF}}$ and $N_{\text{eff}}^{J\pi}$. In a recent paper, Seaman *et al.*[5] comment that the number of (S_z,I_z,S_z',I_z') sets will be small compared to the number of (j,l,J,j',l') sets when

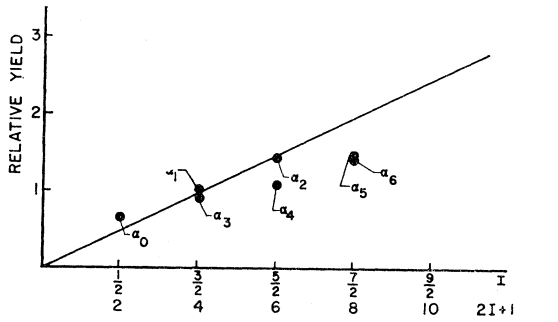

FIG. 7. Relative intensity of the various ^{31}P$(d,\alpha)^{29}$Si α groups as a function of $2I+1$, when I is the spin of the residual state in ^{29}Si, with cross-section averages performed over 33 energies.

orbital angular momenta are large compared to nuclear and projectile spins. In the above, l is the orbital angular momentum, j is the channel spin defined by $\mathbf{l}+\mathbf{S}$, and J is the spin of the compound nucleus. It is further concluded that for such conditions $N_{\text{eff}}^{\text{HF}} \ll \Lambda$, where Λ is the number of independent reactions amplitudes, and hence the reaction amplitude for each M set is independent. In the present study, values of l through $l=9$ and values of J through $J=7$ were taken to be important and since these numbers are large compared to nuclear and projectile spins, then one would conclude that $N_{\text{eff}}^{\text{HF}} \ll \Lambda$. However, Table V, which shows the calculated values of $N_{\text{eff}}^{\text{HF}}$ and Λ as a function of angle, clearly demonstrates that $N_{\text{eff}}^{\text{HF}}$ is not $\ll\Lambda$. It was consequently necessary to use for N_{eff} the values of $N_{\text{eff}}^{J\pi}$ given by Gibb's more realistic calculation. There are still certain inherent inaccuracies in the values for $N_{\text{eff}}^{J\pi}$. The Gilbert and Cameron level densities were used to calculate the number of exit channels in the highly excited regions of the residual nuclei. There may easily be significant errors in these predicted values. This possibility was checked by removing from the code 1000 decay channels which had associated with them the smallest transmission coefficients. Values of $N_{\text{eff}}^{J\pi}$ were found to vary only of the order of 2%, which demonstrated that serious error is not being encountered from this source. In addition, there is some error introduced by ignoring the ^{3}He$+^{30}$P contribution to the open decay channels. Nevertheless, in the determination of y, no estimate of the error associated with $N_{\text{eff}}^{J\pi}$ is included since the magnitude of this error is unknown.

The other problem in the determination of y is to obtain meaningful and reasonably consistent values for $R(0)$. Values of both $R(0)$ and Γ obtained from autocorrelation employing an arithmetic mean for the average cross section were extremely erratic. Values of y deduced from these $R(0)$ would have ranged from 0.0 to 0.75 in a random fashion. Values of $R(0)$ obtained after modulation had been added were still erratic and somewhat sensitive to the functional form of the modulation. The values of $R(0)$ obtained using the moving energy average were relatively constant over a wide range of the averaging interval chosen and gave con-

TABLE V. Calculated value for $N_{\text{eff}}^{\text{HF}}$ and Λ, the number of independent reaction amplitudes, as a function of angle.

Lab angle	α_0		α_1		α_2		α_3		α_4		α_5		α_6	
	$N_{\text{eff}}^{\text{HF}}$	Λ	$N_{\text{eff}}^{\text{HF}}$	Λ	$N_{\text{eff}}^{\text{HF}}$	Λ	$N_{\text{eff}}^{\text{HF}}$	Λ	$N_{\text{eff}}^{\text{HF}}$	Λ	$N_{\text{eff}}^{\text{HF}}$	Λ	$N_{\text{eff}}^{\text{HF}}$	Λ
170°	4.23	5.21	6.18	4.96	7.28	5.82	6.06	4.88	7.04	5.91	6.96	5.94	6.41	6.47
150°	5.80	6.54	11.70	5.75	16.82	5.62	11.68	5.66	16.67	5.64	16.61	5.64	19.39	5.85
130°	5.95	6.96	11.86	6.71	17.77	6.57	11.86	6.61	17.76	6.57	17.75	6.57	23.61	6.11
110°	5.75	7.14	11.79	6.81	17.73	7.04	11.80	6.71	17.76	7.06	17.76	7.07	23.58	6.88
90°	4.32	7.15	8.54	6.85	12.13	7.13	8.49	6.75	12.01	7.14	11.98	7.15	15.13	7.09
40°	5.91	7.03	11.82	6.59	17.73	6.32	11.80	6.49	17.72	6.32	17.70	6.32	23.32	5.91

sistent results for y. The rigorous justification of the use of the moving average in determining $R(0)$ is difficult although some work has been done previously in this regard. Gadioli *et al.*[36] have investigated the errors involved when a nonstatistical effect, such as a large direct-reaction component, is present and when the average cross section is a constant with energy. They then attempted to extrapolate these results to encompass a varying average cross section. Unfortunately, their results do not include any predictions for the relative antocorrelation function of the type used in the present analysis. In addition, they stress that their results are only a first approximation.

Therefore, to approximate the error in y, one can use the tables in Ref. 14 from which an error estimate of $\pm 10\%$ is obtained for a mean value of y of 0.87. The calculated y values show rather good consistency as a function of angle and of α group, as is indicated in Table III.

It has been suggested by Gibbs[27] that perhaps the effect of the use of the moving average is to reduce the effective sample size by a factor of approximately $\frac{1}{4}$. The effect of the smaller size, as can be seen from Eq. (13), would be to make the predicted value for y too large. Calculation of a few sample cases indicates that the error introduced in this way is still no greater than about 20%. Thus it is believed that the average value for y is $0.87_{-20\%}^{+10\%}$. There was about a 3% variation above and below this value on the average for the data divided into halves, with the larger result being associated with the higher-energy data. The small variation in y is somewhat surprising in that the fluctuations appear to be strongly damped in the upper-half energy region as compared with the lower half.

In summary, as high excitation energies in the compound nucleus are reached, so many competing exit channels become available that account must be taken of the nonstationary effects, that is, a decreasing cross section. This problem cannot be overcome by taking data over only a small energy region since in general the coherence width increases as a function of excitation energy and the resulting sample size would be too small to allow a realistic evaluation of y. Therefore, further theoretical work similar to that carried out by Gadioli[36] would be useful to clearly define the bias associated with the moving average and to facilitate the accurate measurement of y. In addition, at these higher excitation energies, the accurate determination of $N_{\text{eff}}^{J\pi}$ becomes increasingly difficult because of the errors associated with the usual level-density formulas and the extremely large number of open particle-decay channels.

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