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Hindrance Factors for Beta Decays of Heavy Nuclei*

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The schematic model proposed by Brown and Bolsterli, and applied to the giant dipole resonance, is extended to the β and γ transitions of heavy nuclei. Special emphasis is placed on the extent to which forbidden β decays of heavy nuclei are hindered because of possible giant resonances corresponding to those found in the electromagnetic transitions. A new method of calculating hindered forbidden nuclear matrix elements is proposed on the basis of the schematic model. It is shown that the hindrance factor due to the giant resonance effect is roughly 4 for the first-forbidden β transitions. Systematics of unique first-forbidden transitions are examined from this viewpoint.

I. INTRODUCTION

IT is customary[1] to classify β transitions into allowed, first-forbidden, second-forbidden, etc., according to the spin and parity changes between initial and final nuclear states. The *ft* values of β transitions have proved to be useful in determining forbiddenness of the transitions and, accordingly, spins and parities of relevant nuclear states. However, discrimination in *ft* values between different forbiddennesses is sometimes obscured by the fact that a number of allowed transitions have *ft* values comparable to those of first forbidden transitions and similarly for higher forbidden ones. The purpose of this paper is to discuss the origin of such hindrance (retardation) phenomena and also to point out a characteristic difference between allowed and forbidden transitions. In this section the present status of the hindrance phenomena is briefly reviewed.

A. Allowed Transitions

It has long been known that the "normal allowed" β transitions are much hindered in comparison with the so-called "super-allowed" transitions, for the latter of which nuclear matrix elements have the order of magnitude predicted by the single-particle shell model. Several ways to explain such hindrance phenomena have been tried.

1. "Core-Overlap" Effect

The oldest idea is that hindrance arises from small "core overlap" between the parent and daughter nuclei.[2,3] If this effect is due to a difference of deformation between the initial and final cores, it is expected to be most clearly manifested by β decays in the transition regions. Though the existence of such an effect[4] has been indicated by the study of transition regions, it seems to be improbable[5] that the origin of the hindrance phenomena can always be attributed to lack of core overlap.

2. Pairing Correlation Effects

Several years ago the pairing model was applied to the study of β-decay systematics, and it was shown[6] that for a number of normal allowed transitions the isotope dependence of *ft* values can be well reproduced if the coupling constant is phenomenologically renormalized for each type of transition (i.e., $g_{9/2} \leftrightarrow g_{7/2}$, etc.). The renormalized coupling constant was found to have about the same magnitude for both spherical and deformed nuclei. The most recent study[7] of deformed nuclei showed that the experimental transition rates are typically 20 times lower than predicted by the pure Nilsson model, and eight times lower than predicted by the Nilsson model with pairing corrections.

3. Gamow-Teller Giant Resonance Effects

In 1961 isobaric analog states were experimentally discovered[8] in the study of (p,n) reactions. The state $T_-|i\rangle$, isobaric to the initial state $|i\rangle$, was shown to be a well-defined state with a narrow width having the order

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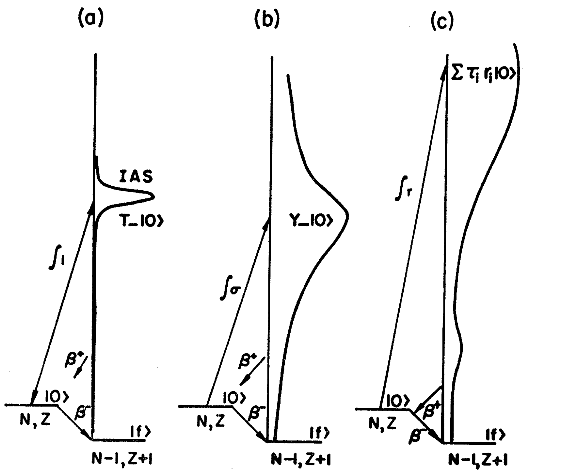

FIG. 1. β transitions on the collective states of (a) Fermi type $\int 1$, (b) Gamow-Teller type $\int \boldsymbol{\sigma}$, and (c) first forbidden $\int \mathbf{r}$. Two peaks in (c), $\int \mathbf{r}$, correspond to the collective states $|\mathrm{coll.}_{>}T_0-1(T_0-1)\rangle$ and $|\mathrm{coll.}_{<}T_0-1(T_0-1)\rangle$ (see Sec. III).

of 100 keV or less for medium and heavy nuclei. This fact explains[9] why Fermi transition matrix elements are generally so small for heavy nuclei. The discovery of isobaric analog states led to the conjecture[9–11] that the Gamow-Teller transition strength might also be concentrated in the several MeV energy region near the isobaric analog resonance, to which β transitions are energetically forbidden. This idea, the possible existence of Gamow-Teller giant resonance effects, seem to be in agreement with a variety of experimental evidence.[12–17] (See Fig. 1.)

It should be mentioned here that the isobaric analog state can be regarded as the state[18] in which $\bar{n}p$ (neutron-hole, proton) states with the spin (parity) $J^\pi = 0^+$ are coherently superposed; the Gamow-Teller giant resonance corresponds to the $\bar{n}p$ states with $J^\pi = 1^+$. The long life of the 0^+ state follows from isospin symmetry, and the life of the 1^+ state is closely related to the validity of supermultiplet symmetry.[19] To discuss the origin of the hindrance factors requires taking into account the existence of such collective modes, namely, the inclusion of many higher configurations in the conventional configuration mixing treatments. Each of the contributions is known to be small, but the total effect becomes important when they contribute coherently.[9,20] However, if the existence of giant resonance effects can be assumed, a much simpler treatment is possible.[21] It can be shown that even if the supermultiplet symmetry is significantly broken in actual heavy nuclei, the $1^+\bar{n}p$ collective state plays an important role in hindering Gamow-Teller β transitions.

B. Forbidden Transitions

It is already known[3,22,23] that phenomenological renormalized coupling constants are necessarily introduced also in the cases of first-forbidden transitions. Since many higher configurations must be taken into account in calculating the transition matrix elements,[22,23] it will be of interest to see whether giant resonance effects exist in forbidden β transitions. In contrast with the cases of allowed transitions for which the $0^+\bar{n}p$ states have no corresponding partners in the $\bar{p}p$ or $\bar{n}n$ states and similarly true for the main part of the 1^+ $\bar{n}p$ states, in the case of $J^\pi = 1^-$ the corresponding $\bar{p}p$ and $\bar{n}n$ states are responsible for the $E1$ giant resonance.[24] Therefore, if the knowledge of electromagnetic transitions is fed in, a fairly reliable estimate should be obtained for the problem of how much hindrance in forbidden β decays is to be expected, owing to giant resonance effects.

As the first step in answering this question, extensions of the schematic model of Brown and Bolsterli[24] are proposed and discussed in Secs. II and III. The isospin formalism is used, since the important role of isospin[25] in the photonuclear effects of heavy nuclei has already been pointed out.[26] In Sec. IV, ways of carrying out more realistic calculations are surveyed and a new method is proposed. Numerical estimates are obtained in Sec. V.

II. SCHEMATIC MODELS

First, we reformulate the model proposed by Brown and Bolsterli[24] and then extend it to heavy nuclei.

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A. Light Nuclei

We consider the system in which all the unperturbed particle-hole states are degenerate with an energy ΔE_0 for the unperturbed Hamiltonian H_0, and the interaction H_I having a constant value of matrix elements G is switched on among the particle-hole states. The charge-independent model Hamiltonian[27] is given by using a creation (annihilation) operator $A^\dagger(\lambda\nu)(A(\lambda\nu))$ of a particle-hole pair $(\lambda\nu)$ as follows:

$$H=H_0+H_I\,, \tag{1}$$

with the unperturbed part H_0,

$$H_0=2\Delta E_0\sum_\nu \mathbf{A}^\dagger(\lambda\nu)\cdot\mathbf{A}(\lambda\nu)\,, \tag{2}$$

and the interaction Hamiltonian

$$H_I=2G\mathbf{A}^\dagger\cdot\mathbf{A} \tag{3a}$$

$$=2G(A_0{}^\dagger A_0+A_{+1}{}^\dagger A_{+1}+A_{-1}{}^\dagger A_{-1})\,, \tag{3b}$$

where the isovector $\mathbf{A}=(A_0,A_{+1},A_{-1})$ is defined by

$$\mathbf{A}=\sum_\nu \mathbf{A}(\lambda\nu)\,. \tag{4}$$

The suffix $\lambda(\nu)$ stands for an unoccupied (occupied) state in the ground state (see Fig. 2), and we assume that for each value of ν only one value of λ contributes, and vice versa. Therefore the sums on a single ν in Eqs. (2) and (4) represent the sums over all possible pairs $(\lambda\nu)$. The components of $\mathbf{A}(\lambda\nu)$ can be written in terms of the creation (annihilation) operators $a^\dagger(a)$ for a photon and $b^\dagger(b)$ for a neutron as follows:

$$A_0(\lambda\nu)=\tfrac{1}{2}(b_\nu{}^\dagger b_\lambda-a_\nu{}^\dagger a_\lambda)\,, \tag{5a}$$

$$A_{-1}(\lambda\nu)=\tfrac{1}{2}\sqrt{2}[T_-,A_0(\lambda\nu)]=\tfrac{1}{2}\sqrt{2}a_\nu{}^\dagger b_\lambda\,, \tag{5b}$$

$$A_{+1}(\lambda\nu)=-\tfrac{1}{2}\sqrt{2}[T_+,A_0(\lambda\nu)]=-\tfrac{1}{2}\sqrt{2}b_\nu{}^\dagger a_\lambda\,. \tag{5c}$$

The model Hamiltonian in (1) has the following properties:

$$H|0\rangle=H_0|0\rangle=0 \tag{6}$$

for the wave function $|0\rangle$ in the ground state $(N=Z)$,

$$H_0\mathbf{A}^\dagger(\lambda\nu)|0\rangle=\Delta E_0\mathbf{A}^\dagger(\lambda\nu)|0\rangle \tag{7}$$

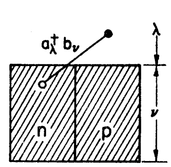

FIG. 2. Schematic picture of particle-hole excitations in a light nucleus $(N-Z)$.

[27] Note that only the particle-hole states $A^\dagger(\lambda\nu)|0\rangle$ with $T=1$ are taken into account in (2) and (3). We may include the contributions of the states $A_0'^\dagger(\lambda\nu)|0\rangle$ with $T=0$, for which $A_0'(\lambda\nu)=\frac{1}{2}(b_\lambda{}^\dagger b_\nu+a_\lambda{}^\dagger a_\nu)$, into (2) and (3). Then we obtain another type of collective state, $A_0'^\dagger|0\rangle=\sum_\nu A_0'^\dagger(\lambda\nu)|0\rangle$, which exhausts the sum rule for the isoscalar transition operator $M_0'=A_0'^\dagger+A_0'$. An example of the isoscalar collective state $\mathbf{R}|0\rangle=(1/A)\sum^A\mathbf{x}_i|0\rangle$ is well known to be a spurious state because the c.m. of a nucleus s at rest.

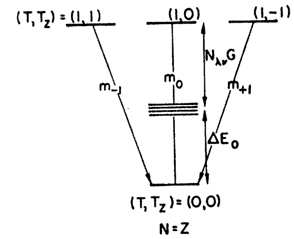

FIG. 3. Schematic diagram of β and f transitions and the associated collective states in light nuclei.

for the particle-hole states $\mathbf{A}^\dagger(\lambda\nu)|0\rangle(T=1)$,

$$[H_0,T_\pm]=[H_I,T_\pm]=0\,, \tag{8}$$

$$[H_I,\mathbf{A}^\dagger]|0\rangle=N_{\lambda\nu}G\mathbf{A}^\dagger|0\rangle\,, \tag{9}$$

where $N_{\lambda\nu}$ represents the number of degenerate unperturbed states $A_q(\lambda\nu)$ for $q=0,\ \pm1$. From (9) we obtain

$$H\mathbf{A}^\dagger|0\rangle=(\Delta E_0+N_{\lambda\nu}G)\mathbf{A}^\dagger|0\rangle\,. \tag{10}$$

The eigenstate, $A_q{}^\dagger|0\rangle=\sum_\nu A_q{}^\dagger(\lambda\nu)|0\rangle$, is referred to as a collective state; if G is positive, the energy of the collective state is higher than the original one. It can also be shown that all the noncollective states, after H_I is switched on, remain at the original energy ΔE_0 and have the form

$$\left(\frac{2N_{\lambda\nu}}{N_{\lambda\nu}-1}\right)^{1/2}\left(A_q{}^\dagger(\lambda_0,\nu_0)-\frac{1}{N_{\lambda\nu}}A_q{}^\dagger\right)|0\rangle\,. \tag{11}$$

Now let us introduce an idealized isovector transition operator as

$$\mathbf{m}=\mathbf{A}^\dagger+\mathbf{A}\,, \tag{12}$$

of which the qth components represent the β interaction for $q=\pm1$ and the electromagnetic interaction for $q=0$, except for the coupling constants. It can be proved that the transition between $|0\rangle$ and $\mathbf{A}^\dagger|0\rangle$ exhausts the sum rule of the transition strength due to the operator $\mathbf{m}$, and all the other transitions from $|0\rangle$ are forbidden, as seen from (11). The relationship between β and γ processes is obvious as shown in Fig. 3.

B. Heavy Nuclei $(N>Z)$

We extend the above argument to a heavy nucleus $(N>Z)$. It must be assumed[25,26] that total isospin is an approximately good quantum number also in the pertinent states of heavy nuclei.

As in Eq. (1) let us write the total Hamiltonian as

$$H=H_0+H_I\,, \tag{13}$$

where the unperturbed Hamiltonian H_0 and the interaction Hamiltonian H_I are assumed to satisfy the relations

$$H_I|0\rangle=H_0|0\rangle=0\,, \tag{14}$$

in which $|0\rangle$ satisfies $T_z|0\rangle=T_0|0\rangle$;

$$[T_\pm,H_I]=0 \tag{15}$$

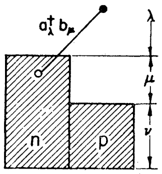

FIG. 4. Schematic picture of particle-hole excitation in heavy nucleus ($N\neq Z$).

and

$$[H_0,T_\pm]=\mp\Delta_c T_\pm. \tag{16}$$

The Δ_c in Eq. (16) represents the single-particle Coulomb displacement energy, which can be directly measured by the existence of an isobaric analog state. From Eqs. (15) and (16) it is clear that every eigenstate of H in (13) has a definite isospin T. In other words, the isospin projection operator[28] $P_T^{(T_z)}$ satisfies

$$[P_T^{(T_z)},H]=0, \tag{17}$$

where $P_T^{(T_z)}$ is the projection operator projecting the state with a definite isospin T out of an isospin mixture with fixed T_z.

A model Hamiltonian H satisfying the conditions (14)-(16) is given by

$$H_0=\sum_{\delta,T,T_z}\Delta E_0(\delta'\delta;T(T_z))|\delta'\delta;T(T_z)\rangle \times\langle\delta'\delta;T(T_z)| \tag{18}$$

and

$$H_I=\sum_{\delta,\epsilon,T}G_T(\delta'\delta,\epsilon'\epsilon)|\delta'\delta;T(T_z)\rangle\langle\epsilon'\epsilon;T(T_z)|, \tag{19}$$

where $|\delta'\delta;T(T_z)\rangle$ stands for a particle-hole state as in Sec. II A. Assuming $T_z=T_0=\frac{1}{2}(N-Z)$ for $|0\rangle$, we can denote a normalized unperturbed eigenstate[29] with total isospin T and T_z

$$|\delta'\delta;T(T_0-1)\rangle=[N_T(\delta'\delta)]^{-1/2}P_T^{(T_0-1)}a_{\delta'}^\dagger b_\delta|0\rangle. \tag{20}$$

The normalization coefficient $N_T(\delta'\delta)$ depends on the labels λ, μ, ν associated with the pair ($\delta'\delta$), where the labels for states λ, μ, and ν are shown in Fig. 4. The explicit forms are given by Eqs. (A7) and (A8). For each value of δ only one value of δ' is assumed to contribute, and vice versa. The numbers of pairs ($\delta'\delta$) are denoted as $N_{\lambda\mu}$, $N_{\mu\nu}$, and so on. The states $|\delta'\delta;T(T_z)\rangle$ for $T_z=T_0$ and T_0+1 can be obtained by multiplying T_+ and T_+^2 by $|\delta'\delta;T(T_0-1)\rangle$ in Eq. (20), respectively.

If we assume that all the unperturbed states are degenerate, $\Delta E_0(\delta'\delta;T(T_z))\equiv\Delta E_0(T,T_z)$, the problem can be solved in exactly the same way as in Sec. II A. The collective states $|\text{coll.}T(T_0-1)\rangle$ for negatron decays of $|0\rangle$ ($T_z=T_0$) are given by[30]

$$|\text{coll.}T(T_0-1)\rangle=\sum_\delta|(\delta'\delta);T(T_0-1)\rangle, \tag{21}$$

in which ($\delta'\delta$) represents (λν) for $T=T_0+1$; (λν), (λ,μ), and (μν) for $T=T_0$; (λν), (λμ), (μν), and (μ′μ) for $T=T_0-1$. The collective energies are written

$$E_{\text{coll.}}(T,T_0-1)=\Delta E_0(T,T_0-1)+\Delta_I{}^\beta(T), \tag{22}$$

with

$$\Delta_I{}^\beta(T)=N(T)G_T, \tag{23a}$$

where

$$N(T_0+1)=N_{\lambda\nu}, \tag{23b}$$

$$N(T_0)=N_{\lambda\nu}+N_{\lambda\mu}+N_{\mu\nu}\equiv N_\beta', \tag{23c}$$

and

$$N(T_0-1)\equiv N_{\lambda\nu}+N_{\lambda\mu}+N_{\mu\nu}+N_{\mu'\mu}\equiv N_\beta. \tag{23d}$$

We can easily derive the following relations between the collective states responsible for β and γ transitions of $|0\rangle(T_z=T_0)$:

$$T_+|\text{coll.}T(T_0-1)\rangle\propto|\text{coll.}T(T_0)\rangle \tag{24}$$

and

$$E_{\text{coll.}}(T,T_0)=\Delta E_0(T,T_0)+\Delta_I{}^\gamma(T) =E_{\text{coll.}}(T,T_0-1)-\Delta_c \tag{25}$$

for $T=T_0$ or T_0+1 (see Appendix B).

In order to discuss the correspondence of levels in neighboring nuclei, it is convenient to assume that

$$H_0=H_0{}^0+H_C+H_S, \tag{26}$$

where the Coulomb potential part including the neutron-proton mass difference is

$$H_C=(T_0-T_Z)\Delta_c, \tag{27}$$

and the T dependence of $E_0(\delta'\delta;T(T_z))$ is assumed to be given by the symmetry energy part[31]

$$H_S=(1/2T_0)\{T^2-T_0(T_0+1)\}\Delta_S', \tag{28}$$

so that the first term $H_0{}^0$ in Eq. (26) satisfies

$$[H_0{}^0,T_\pm]=0. \tag{29}$$

The constant terms in these equations are chosen in

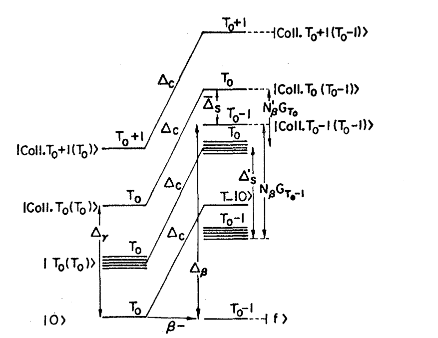

FIG. 5. Level scheme of β and γ transitions and the associated collective states in heavy nuclei.

[28] J. I. Fujita and K. Ikeda, Progr. Theoret. Phys. (Kyoto) 35, 622 (1966).

[29] It should also be mentioned that in this paper no Clebsch-Gordan coefficients appear explicitly, unlike the previous treatments (Refs. 9 and 10). However, it is an easy task to rewrite the discussion here to the case of eigenstates of angular momentum, because it can be done by a unitary transformation.

[30] The sum on δ in Eq. (21) is the one over-all possible pair of ($\delta'\delta$) as in Eq. (2).

[31] In Ref. 18 the symmetry energy part of the Hamiltonian H_S was treated as an idealized residual interaction producing isobaric analog states.

order to satisfy the relation

$$H_c|0\rangle = H_s|0\rangle = 0. \tag{30}$$

The relationship between Δ_S' and the observed symmetry energy is given in Appendix B.

1. Fermi and Gamow-Teller Cases

The schematic level scheme for the total Hamiltonian H is shown in Fig. 5. The quantity Δ_T^γ can be determined from the knowledge of electromagnetic transitions due to the transition operator Eq. (12), $m_0 = A_0^\dagger + A_0$. It is interesting to compare the above general forbidden cases with those of allowed transitions. For Fermi transitions, we have $(\delta'\delta) = (\mu\mu)$; for Gamow-Teller transitions predominant components are $(\delta'\delta) = (\mu'\mu)$. Schematic figures for allowed transitions are shown in Fig. 6.[32]

2. Electric-Dipole Case

The relationship between β and γ processes is clearly indicated in Eqs. (22) and (24) in the present model. For instance, let us examine the case of $\int r$. The peak energy of the famous $E1$ giant resonance state

$$P_{T_0}{}^{(T_0)} \sum_\delta \left(\frac{Z}{A} b_{\delta'}{}^\dagger b_\delta - \frac{N}{A} a_{\delta'}{}^\dagger a_\delta\right)|0\rangle \propto |\text{coll.}T_0(T_0)\rangle \tag{31}$$

is $\Delta E_0(T_0,T_0) + \Delta_I{}^\gamma(T_0)$ provided that particle-hole pair states $(\delta'\delta)$ with $J^\pi = 1^-$ are suitably chosen. The isobaric analog state of $|\text{coll.}T_0(T_0)\rangle$ is expressed as

$$(\tfrac{1}{2}T_0^{-1})T_- P_{T_0}{}^{(T_0)} \sum_\delta \left(\frac{Z}{A} b_{\delta'}{}^\dagger b_\delta - \frac{N}{A} a_{\delta'}{}^\dagger a_\delta\right)|0\rangle = P_{T_0}{}^{(T_0-1)} \sum_\delta a_{\delta'}{}^\dagger b_\delta |0\rangle, \tag{32}$$

which agrees with $|\text{coll.}T_0(T_0-1)\rangle$. On the other hand, the resonance related to the hindrance in β decays due to $\int \mathbf{r}$ is given by $|\text{coll.}T_0-1(T_0-1)\rangle$ since low-lying states of the nucleus have isospin $T = T_0 - 1$.

III. HINDRANCE FACTORS FOR β DECAYS

The solutions $|\text{coll.}T(T_z)\rangle$ in (21) are obtained on the assumption that all the unperturbed levels are degenerate and all the interaction matrix elements are a constant. It can be extended without difficulty to cases where the levels consist of two groups of degenerate levels. The hindrance factors F in these models can be calculated as follows. (Details are given in Appendix C.)

[32] The Fermi transition is a special case, where collective states with $T = T_0 - 1$ are spurious because of

$$P_{T_0-1}{}^{(T_0-1)} \sum_\mu a_\mu{}^\dagger b_\mu |0\rangle = P_{T_0-1}{}^{(T_0-1)} T_- |0\rangle = 0.$$

This is in contrast with the previous treatment in Ref. 18, where H_S in Eq. (28) is not taken out explicitly. In the Gamow-Teller case $Y_-|0\rangle$ includes small components with $T = T_0$, having analog excited states of the nucleus $T_z = T_0$.

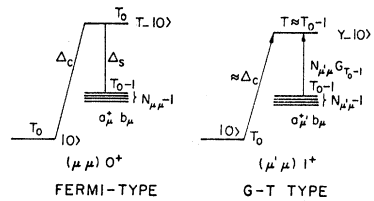

FIG. 6. Collective states corresponding to Fermi transition (left-hand side) and Gamow-Teller transition (right-hand side).

Suppose that we have

$$H_0|i'i; T_0-1(T_0-1)\rangle = (\Delta E_0 - \delta)|i'i; T_0-1(T_0-1)\rangle \tag{33a}$$

for $i = 1, 2, \cdots, n_1$ and

$$H_0|j'j; T_0-1(T_0-1)\rangle = \Delta E_0 |j'j; T_0-1(T_0-1)\rangle \tag{33b}$$

for $j = n_1+1,\ n_1+2,\ \cdots,\ n_1+n_2(=N)$, where ΔE_0 represents the quantity $\Delta E_0|\delta'\delta; T(T_z)\rangle$ appearing in (18). In this model we obtain two types of collective states $|\text{coll.}_{>(<)}; T_0-1(T_0-1)\rangle$ with the energy eigenvalues

$$E_{><} = \Delta E_0 + \tfrac{1}{2}\{-\delta + NG \pm[(\delta + (n_2-n_1)G)^2 + 4n_1n_2G^2]^{1/2}\}, \tag{34}$$

respectively, for which we have the relations

$$\epsilon \equiv E_< - \Delta E_0 + \delta = \frac{Gn_1\delta}{E_> + \delta - \Delta E_0} = -E_> + \Delta E_0 + GN. \tag{35}$$

The hindrance factor, which agrees with the transition matrix element itself in our model, is given by

$$F^{-1/2} = \langle f|m|i\rangle, \tag{36a}$$

which becomes, corresponding to the above two collective states,

$$F_>{}^{-1/2} = (\sqrt{n_2})\left(1 + \frac{n_1}{n_2}X\right)\left(1 + \frac{n_1}{n_2}X^2\right)^{-1/2} \tag{36b}$$

and

$$F_<{}^{-1/2} = (\sqrt{n_1})(1-Y)\left(1 + \frac{n_1}{n_2}Y^2\right)^{-1/2}, \tag{36c}$$

where

$$X = \frac{GN - \epsilon}{GN + \delta - \epsilon}, \quad Y = \frac{n_2G}{n_2G + \delta - \epsilon},$$

respectively. It is shown that the sum rule is exhausted by these two collective states:

$$\langle 0|m^\dagger m|0\rangle = n_1 + n_2 = F_>{}^{-1} + F_<{}^{-1}. \tag{37}$$

As an extreme case of $NG \ll \delta$, we have $X \approx Y \approx 0$, $F_>{}^{-1} \approx n_2$ and $F_<{}^{-1} \approx n_1$, while in case of $NG \gg \delta$ we have $X \approx Y \approx 1$ and $F_>{}^{-1} \approx N$, $F_<{}^{-1} \approx 0$. In this model, the β transitions from $|0\rangle$ to the other noncollective final states having energy eigenvalues $\Delta E_0-\delta$ or ΔE_0 are completely forbidden;

$$F_i^{-1/2}=F_j^{-1/2}=0 \tag{38}$$

for $i=1, 2, \cdots, n_1-1$ and $j=n_1+1, n_1+2, \cdots, n_1+n_2-1$.

In actual nuclei, δ is of the same order of $NG\approx n_2G$ as shown in Sec. V, whereas $n_2\gg n_1$. Therefore, the transition to the lower collective state is given by

$$F_<^{-1/2}\approx(\sqrt{n_1})(1-Y)\approx(\sqrt{n_1})\cdot\frac{\delta}{NG+\delta}, \tag{39}$$

$$\frac{F_<^{-1}}{n_1}\approx\left(\frac{\delta}{NG+\delta}\right)^2. \tag{40}$$

The hindrance factor $[\delta/(NG+\delta)]^2$ is attributed to the giant resonance $(E_>,F_>)$ effect.

As a special case for $n_1=1$, the hindrance factor in Eq. (39) becomes

$$F_<^{-1/2}=\delta/(NG+\delta) \tag{41}$$

for $n_2\gg 1$. If δ goes to zero, $F_<^{-1/2}$ also tends to zero as expected from the completely degenerate case in Sec. II B.

It should be remarked here that if the transition strength given by Eq. (39) is equally distributed among n_1 levels, it agrees with the one given by (41).

In Sec. IV it is shown that more realistic treatment also leads to the expression quite similar to Eq. (41).

IV. MORE REALISTIC METHOD

A. Hindrance Factor in Commutator Method

A method using the commutator of the nuclear Hamiltonian and the transition operator has been developed earlier,[9,33] and applied to the Fermi[34] and Gamow-Teller β transitions of spherical[14] and deformed[16] nuclei. In this section the physical meaning of this method is reexamined on the basis of above arguments on the schematic models.

First, let us briefly recapitulate an outline of the method.[33] We start from the identity

$$\langle f|m|0\rangle=\frac{\langle f|[H,m]-m\Delta|0\rangle}{E_f-E_0-\Delta}, \tag{42}$$

which is valid for any value of Δ except $\Delta=E_f-E_0$. If we insert the model wave functions $|0\rangle_0$ and $|f\rangle_0$ in place of $|0\rangle$ and $|f\rangle$, generally the two sides of Eq. (42) are not equal. If we choose the value of Δ to be

$$\Delta=\frac{\langle 0|m^\dagger[H,m]|0\rangle}{\langle 0|m^\dagger m|0\rangle}, \tag{43}$$

then we can expect that a better estimate for a hindered matrix element is obtained from the right-hand side of Eq. (42), for various reasons. (a) The right-hand side corresponds to a sort of perturbation approach starting from a collective model.[14,35,36] (b) It is the deviation from the random-phase approximation (RPA) that gives the m a finite value, as clearly seen from the numerator in Eq. (42); and the effective transition operator $[H,m]-m\Delta$ generally has no sharp selection rules,[9] unlike m itself.

Now, let us examine the relationship between this method and the schematic model, especially its prediction, Eq. (41). The latter model is quite simple but explains the essential feature of hindrance phenomena due to the effect of collective states. Suppose that H in Eq. (42) is given by Eq. (13) and the true wave functions in Eq. (42), $|0\rangle$ and $|f\rangle$, are replaced by zeroth-order model wave functions $|0\rangle_0$ as schematically shown in Fig. 5 and $|f\rangle_0=|\delta_0'\delta_0; T_0-1(T_0-1)\rangle$ in Eq. (33a). Then the left-hand side of Eq. (42) becomes

$${}_0\langle f|m|0\rangle_0={}_0\langle f|\sum_\delta a_{\delta'}{}^\dagger b_\delta|0\rangle_0=\sqrt{N_T}(\delta_0'\delta_0), \tag{44}$$

where $N_T(\delta_0'\delta_0)$ given by Eq. (A11) is close to 1 when $T_0\gg 1$. On the other hand it can be proved that the right-hand side of Eq. (42) becomes

$$\delta/(NG+\delta), \tag{45}$$

agreeing with Eq. (41).

We can thus conclude here that the left-hand side of Eq. (42) gives a value of $O(1)$, the value of the nuclear matrix element for superallowed transitions, whereas the same model wave functions give the appropriate hindrance factor, Eq. (41).

B. More Realistic Method for Forbidden Transitions

In Sec. III we obtained two different estimates on the hindrance factor for the ground-ground β transition, $F^{-1/2}=0$ in Eq. (38) and $F^{-1/2}=\delta/(NG+\delta)$ in Eq. (41). The preceding argument suggests a means for evaluating hindrance factors more realistically as improvements on the schematic model. [The mathematical meaning of (42) is discussed in Ref. 36.]

The basic idea is to insert H of (13) into the numerator of (42) and replace E_f, E_0, and Δ in the denominator of (42) by phenomenological values. Inserting (13) into (42) leads to

$$\langle f|m|0\rangle$$
$$=\frac{\langle f|[H_0,m]-m\Delta_0|0\rangle+\langle f|[H_I,m]-m\Delta_I|0\rangle}{E_f-E_0-\Delta}, \tag{46}$$

[33] J. I. Fujita and K. Ikeda, Progr. Theoret. Phys. (Kyoto) **36**, 288 (1966).

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where

$$\Delta_0=\frac{\langle 0|m^\dagger[H_0,m]|0\rangle}{\langle 0|m^\dagger m|0\rangle}, \tag{47a}$$

$$\Delta_I=\frac{\langle 0|m^\dagger[H_I,m]|0\rangle}{\langle 0|m^\dagger m|0\rangle}, \tag{47b}$$

and

$$\Delta=\Delta_0+\Delta_I. \tag{47c}$$

The Δ_0 defined by (47a) can be rewritten as

$$\Delta_0=\Delta^0+\Delta_S+\Delta_C, \tag{48}$$

according to Eq. (26), in which

$$\Delta_C=\frac{\langle 0|m^\dagger[H_C,m]|0\rangle}{\langle 0|m^\dagger m|0\rangle}, \tag{49a}$$

$$\Delta^0=\frac{\langle 0|m^\dagger[H_0{}^0,m]|0\rangle}{\langle 0|m^\dagger m|0\rangle}, \tag{49b}$$

and

$$\Delta_C=\frac{\langle 0|m^\dagger[H_S,m]|0\rangle}{\langle 0|m^\dagger m|0\rangle}. \tag{49c}$$

It is assumed that all wave functions in the numerator of (46) are replaced by the model wave functions, $|0\rangle_0$ or $|f\rangle_0$.

Now instead of calculating Δ by Eqs. (49) the energy shift Δ in the denominator of Eq. (46) can be expressed in terms of the peak energy Δ_γ of a giant resonance in the corresponding γ process; making use of the relations

$$m_\gamma\propto[T_+,m] \tag{50a}$$

and

$$T_+|0\rangle=0, \tag{50b}$$

we obtain

$$\Delta_\gamma\equiv\frac{\langle 0|m_\gamma{}^\dagger[H,m_\gamma]|0\rangle}{\langle 0|m_\gamma{}^\dagger m_\gamma|0\rangle}\approx\frac{\langle 0|m_\gamma{}^\dagger P_{T_0}{}^{(T_0)}[H,m_\gamma]|0\rangle}{\langle 0|m_\gamma{}^\dagger P_{T_0}{}^{(T_0)}m_\gamma|0\rangle}=\Delta+\bar{\Delta}_S-\Delta_C, \tag{51a}$$

where

$$\bar{\Delta}_S=\frac{\langle 0|m^\dagger HP_{T_0}{}^{(T_0-1)}m|0\rangle}{\langle 0|m^\dagger P_{T_0}{}^{(T_0-1)}m|0\rangle}-\frac{\langle 0|m^\dagger HP_{T_0-1}{}^{(T_0-1)}m|0\rangle}{\langle 0|m^\dagger P_{T_0-1}{}^{(T_0-1)}m|0\rangle}. \tag{51b}$$

In order to compare our results with experimental data in Sec. V, several simplifying assumptions are made. (i) Validity of the RPA for H_I. Then the contribution of H_I to the numerator of (46) can be omitted. This approximation does not presume that the total effect of H_I is negligible but, on the contrary, its contribution to Δ_I is essentially important for our method as seen from the derivation of Eq. (45). (ii) The quantity $\bar{\Delta}_S$ is an experimentally measurable quantity as the energy difference between two peaks of giant resonances with total isospins T_0 and T_0-1, respectively, as seen from Eq. (51b), although such experimental data are not available at present. Therefore, $\bar{\Delta}_S$ is estimated on the basis of the schematic model described in Sec. II,

$$\bar{\Delta}_S=\Delta_S+(N_\beta'-1)G_{T_0}-(N_\beta-1)G_{T_0-1}, \tag{52}$$

for which N_β and N_β' are defined by Eqs. (23).

On the basis of these two assumptions, we obtain the approximate formula from Eq. (46).

$$\langle f|m|0\rangle={}_0\langle f|m_{\text{eff}}|0\rangle_0, \tag{53a}$$

where

$$m_{\text{eff}}\approx\frac{E_f{}^0-\langle E_f{}^0\rangle}{E_f-E_0-(\Delta_\gamma+\Delta_C-\bar{\Delta}_S)}m. \tag{53b}$$

In (53b), $E_f{}^0$ and $\langle E_f{}^0\rangle$ are defined to satisfy the relations

$$H_0|f\rangle_0=E_f{}^0|f\rangle, \tag{54a}$$

$$\langle E_f{}^0\rangle=\frac{{}_0\langle 0|m^\dagger H_0m|0\rangle_0}{{}_0\langle 0|m^\dagger m|0\rangle_0}. \tag{54b}$$

The choice of model wave functions is not unique.[37]

(a) The simplest one is to choose $|f\rangle_0$ as a particle-hole state of $|0\rangle_0$. Then the transition is hindered by the factor appearing in Eq. (53b), or essentially the hindrance factor given by Eq. (41) for the case $n_1=1$.

(b) As shown in Sec. V, the actual forbidden transitions are rather close to Eq. (38). If the transition strength is equally distributed among the n_1 states, the hindrance factor agrees with Eq. (41) as already remarked in Sec. III. However, it is conceivable that the group of n_1 levels by themselves constitute a sort of giant resonance, as shown in Fig. 1, because of the repulsive interacting among neutron holes and protons. In this case transitions should be hindered more than expected from Eqs. (53b) or (41). The latter effect remains to be investigated in future quantitative studies of individual hindrance factors for the forbidden transitions.

V. COMPARISON WITH EXPERIMENTS

First-forbidden β-transition rates are generally hindered to some extent compared with single-particle values. It is very interesting to discuss the general trends of the hindrance in terms of the present theory. The first forbidden β-transition has six transition operators of $\int\mathbf{r}$, $\int\boldsymbol{\sigma}\times\mathbf{r}$, $\int\boldsymbol{\sigma}\cdot\mathbf{r}$, $\int\gamma_5$, $\int\boldsymbol{\alpha}$, and $\int B_{ij}$, among which $\int\boldsymbol{\alpha}$ is related to $\int\mathbf{r}$ on the basis of conserved vector current theory.[22,38] The component $\int\mathbf{r}$ is related to the $E1$ γ radiation, whose giant resonance energy is known experimentally. In the present theory the hindrance factor F_g due to the giant resonance effect [Eq. (41), single-particle transition in closed shell

[37] As discussed in detail in Ref. 36, the quantity Δ is not exactly independent of the choice of the model space, but such an effect is neglected in this paper.

[38] J. I. Fujita, Phys. Letters **24B**, 123 (1967); Y. Fujii and J. I. Fujita, Phys. Rev. **140**, B239 (1965).

TABLE I. Hindrance factors of B_{ij} β transitions.

Region of mass No.	$\log_{10}F_p$(expt)	$\langle\log_{10}F_p(\text{expt})\rangle_{\text{av}}$[a]	$\log_{10}F$(theoret)[b]	Δ (MeV)	Main transition process
72–86	0.7 –1.2	1.06	0.61	24.2	$(1g_{9/2})_n \leftrightarrow (1f_{5/2})_p$
88–112	0.6 –1.4	0.81	0.38	21.9	$(2d_{5/2})_n \leftrightarrow (2p_{1/2})_p$
122–138	0.95–1.65	1.25	0.66	23.1	$(1h_{11/2})_n \leftrightarrow (1g_{7/2})_p$
200–204	0.85–1.15	0.99	0.56	24.6	$(3p_{3/2})_n \leftrightarrow (3s_{1/2})_p$

[a] Average values of $\log_{10}F_p$(expt).
[b] Rough estimates obtained for mean values of A and $N-Z$ in each region of mass number and transition process. We neglected the contribution of Q values (E_f-E_0) in Eq. (54), which gives small fluctuations.

region] is given from Eq. (53b):

$$F_g^{-1/2}\approx\frac{E_f{}^0-\langle E_f\rangle^0}{E_f-E_0-\Delta_\gamma-\Delta_C+\bar{\Delta}_S}\,. \tag{55}$$

In the above expression we neglected the second term of Eq. (46), which is considered to give small fluctuations. For medium and heavy nuclei, we know experimentally that $\Delta_\gamma\approx 36A^{-1/6}$ MeV,[39] $\Delta_C\approx 1.44ZA^{-1/3}-1.1-2.5m_ec^2$ MeV,[40] and $\Delta_S\approx 50(N-Z)A^{-1}$ MeV.[41] Assuming $G_{T_0}=G_{T_0-1}$ for simplicity, we get, from Eqs. (23) and (52),

$$\bar{\Delta}_S\approx\Delta_S-N_{\mu\mu'}G_{T_0}\approx\Delta_S-a(\Delta_\gamma-\hbar\omega)\,,$$

where

$$a=(N_{\mu\mu'})_\beta/(N_{\lambda\nu}+N_{\lambda\mu}+N_{\mu\nu})_{E1\gamma}\,. \tag{56}$$

The mean value of transition energy $\langle E_f\rangle^0$ in the numerator of Eq. (55) can be assumed to be $\langle E_f\rangle^0\approx\hbar\omega=41A^{-1/3}$ MeV since most of the possible transition processes with parity change are associated with one $\hbar\omega$ jump. On the other hand, the $E_f{}^0$ for the ground-state transitions have negative values since the Fermi surface of protons is lower than that of neutrons. The quantity a in Eq. (56) and $E_f{}^0$ can be obtained from the level schemes[42] of the simple j-j coupling shell model. The hindrance factor given by Eq. (55) shows no marked dependence on any quantities such as A, Z, and Δ_C, since $\bar{\Delta}_S$ cancels a considerable part of Δ_C and $\hbar\omega/\Delta_\gamma$ is proportional to $A^{-1/6}$. This fact agrees with experiment.[43] A numerical estimate for the transition ^{210}Bi $\rightarrow$ ^{210}Po gives a hindrance factor of 4. Generally, the uniform hindrance factor $F\approx 3\sim 4$ for the $\int\mathbf{r}$ component of $\Delta J=0, \pm 1$ β decay may be attributed to the effect of the giant resonance.

The experimental data on the component $\int\mathbf{r}$ are scanty because some additional measurement, such as circular polarizations, is necessary to extract $\int\mathbf{r}$ from the other components. Furthermore, most of the known $\int\mathbf{r}$ are hindered due to spin (j,K) selection rules. An experimental value $\int\mathbf{r}$ is available for the transition ^{140}La$(3^-)\rightarrow{}^{140}$Ce(2^+) in the closed shell region.[44] It is hindered by a factor of 3.8 compared with the estimate based on the pairing plus Q-Q force model.[6] This is close to the theoretical hindrance $F_g=3$ due to the giant resonance for this transition.

In view of the existence of many experimental data on $\int B_{ij}$ let us extend our argument to the case of the unique first-forbidden transition.[45] Although the $M2$ γ transition corresponding to the B_{ij} β transition has not been well investigated yet, we know that $M2$ transitions are generally hindered in nuclei with $A>30$.[46] The Δ_γ for B_{ij} whose collective state also consists of configurations with one $\hbar\omega$ jumped states may be conjectured to be not quite different from that for $\mathbf{r}$.[47] Assuming the $E1$ giant resonance energy for Δ_γ corresponding to the B_{ij}, we obtain theoretical hindrance factors for each mass region.

Figure 8 shows experimental transition probabilities and the hindrance factors of $\int B_{ij}$ for medium and heavy nuclei in spherical mass regions. The numerical values are presented in Appendix D. Hindrance factors F_S and F_p are obtained as

$$F_{S(p)}{}^{-1}=\left|\int B_{ij}\right|^2_{\text{expt}}\Big/\left|\int B_{ij}\right|^2_{S(p)}\,,$$

where $|\int B_{ij}|_S$ and $|\int B_{ij}|_p$ are the values based on the simple j-j coupling shell model[43] and the pairing model,[6] respectively. The $|\int B_{ij}|_S$ corresponds to the $|\int B_{ij}|_p$ obtained by assuming the associated V^2 and $U^2=1$. The F_S and F_p show no marked dependence on A. They are larger than 4 and distributed around the values $F_S\approx 40$ and $F_p\approx 10$. Table I summarizes the theoretical and experimental hindrance factors. The theoretical hindrance factor (giant resonance effect)

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[45] Most of the unique transitions are free from effects of cancellation among matrix elements and angular momentum (j,l,k) selection rules, in contrast with the case of nonunique transitions with ranks 1 and 0.
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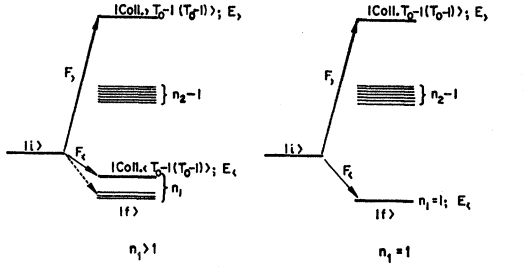

FIG. 7. Schematic level structures in the models in Sec. III for $n_1>1$ and $n_1=1$.

based on Eq. (55) is approximately $F_g\approx 4(\log_{10}F_g\approx 0.6)$. It is interesting to see that the transition of one particle outside a closed shell has hindrance factors close to the predictions in Eq. (55) (see Fig. 8).

The strength function of $\int B_{ij}$ for nonclosed shell nuclei is expected to be close to Eq. (38) (see Fig. 7), where transitions to the noncollective states are more hindered. For actual nuclei it is conceivable that the transition strength of the $|\text{coll.}_<T_0-1(T_0-1)\rangle$ is more or less distributed on the other low-lying states because of the coupling and the nondegeneracy of the unperturbed levels (n_1). The model of the equal strength distribution over the n_1 levels, which we mentioned before and for which hindrance factor is given by

$$F^{-1}=F_<^{-1}/n_1\approx F_g^{-1},$$

seems to give the lower limit of experimental ft values as shown in Fig. 8. The mean values of the hindrance factors F_p are larger than the $F_g\approx 4$ by a factor of 2.5, which can be attributed to the effect due to the mixing among nearby lying levels.

In Fig. 9 we compare the general trend of the hindrance of the first-forbidden $\int B_{ij}$ with that of the Gamow-Teller $\int \boldsymbol{\sigma}$. The values F_p are plotted against the energy difference between $T_-|0\rangle$ and $|0\rangle$, namely, $\bar{\Delta}=\Delta_C\pm Q_{\beta\mp}\pm m_ec^2$ (the upper and lower signs refer to the β^- and β^+ decays with $Q_{\beta\mp}$). The F_p for Gamow-Teller transitions increase with the increase of $\bar{\Delta}$ as a function of $\bar{\Delta}^2$, as is consistent with the arguments in Refs. 9 and 13. However, the F_p for $\int B_{ij}$ remains constant, as expected from the preceding argument. This characteristic difference arises from the fact that the hindrance for an allowed transition is essentially the phenomenon within one harmonic-oscillator shell, but for a forbidden transition it is the one between two harmonic oscillator shells.

In the present analysis based on Eq. (55) we assumed that the deviation from the RPA is less important for H_I than for H_0, at least for the purpose of the studying of the systematic behavior of ft values. The contribution due to H_I in Eq. (55) should be examined in the next step.

VI. CONCLUSION

Our work started from the study of collective levels which absorb a large portion of the sum rule limit for the transition strength. The sum rule can be written as

$$\sum_f |\langle f|m|0\rangle|^2=\langle 0|m^\dagger m|0\rangle=|\langle C|m|0\rangle|^2, \quad (57)$$

where

$$|C\rangle=m|0\rangle/(\langle 0|m^\dagger m|0\rangle)^{-1/2}. \quad (58)$$

If $|C\rangle$ belongs to the complete set $|f\rangle$, the sum rule is exhausted by one transition, $|0\rangle\rightleftarrows|C\rangle$. Then even if $|C\rangle$ is not an exact eigenstate and has a finite width,

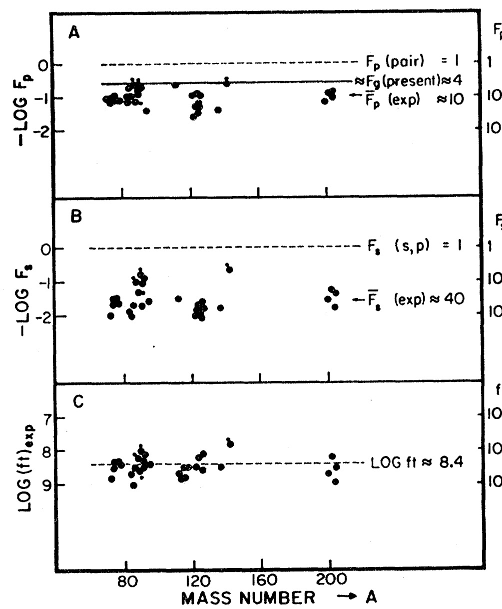

FIG. 8. Experimental $\log ft$ values (C), hindrance factors F_s based on the shell model (B), and those F_p based on the pairing model (A) for the unique first-forbidden transitions. The values of the ground-state ground-state transitions of $2^-\rightarrow 0^+$ and $J_i\rightarrow J_f=J_i\pm 2$ in the spherical mass region are plotted. Points accompanying satellites indicate transitions of one nucleon outside a closed shell.

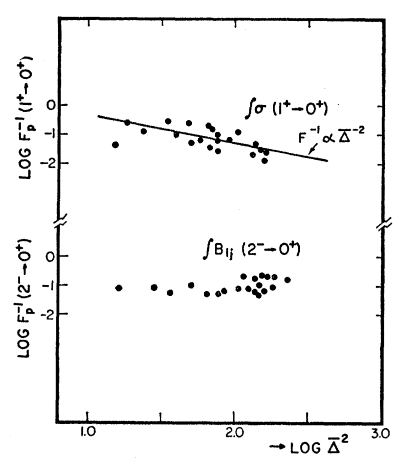

FIG. 9. Hindrance factors E_p of unique first-forbidden $2^-\rightarrow 0^+$ and Gamow-Teller $1^+\rightarrow 0^+$ transitions versus $\log\bar{\Delta}^2$, where $\bar{\Delta}=\Delta_c\pm Q_{\beta\mp}\pm m_ec^2$. The upper and lower signs correspond to negatron and positron decays with Q_{β^-} and Q_{β^+}.

existence of such a virtual state can cause the hindrance phenomena.

In Secs. II and III hindrance phenomena were studied on the basis of schematic models, which are simple extensions of the one proposed by Brown and Bolsterli.[24] It was suggested that knowledge of the $E1$ giant resonance can be applied to the calculation of hindrance factors for the β-decay matrix element $\int\mathbf{r}$. As discussed in Sec. II, the collective motions which have no analogs in the electromagnetic transitions play important roles for allowed β transitions, whereas for the forbidden β transitions the responsible collective modes are closely related to the familiar ones known in the study of γ-ray processes. The hindrance factor due to the giant resonance effect is roughly estimated to be $F\approx 4$ for the first-forbidden β decays.

Since little is known of $\int\mathbf{r}$ in β decays, we examined in Sec. V also the empirical systematics of $\int B_{ij}$. A characteristic difference between the hindrance expected for allowed and forbidden β transitions was noted and discussed in terms of the semiempirical systematics.

In Sec. IV a new method of calculation was proposed for the treatment of nuclear matrix elements for forbidden β decays, which makes use of knowledge of the corresponding electromagnetic transitions. The physical meaning of this method is clearly seen from the schematic models in Sec. II. The merit of this method is that we can easily estimate the core-polarization effect[10] leading to a giant resonance.

Although whole arguments in this paper are semiquantitative, it will be worthwhile to search, by future precise experiments using nuclear reactions, for the possible existence of the giant resonances corresponding to forbidden β transitions.

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APPENDIX A: EIGENSTATES OF ISOSPIN

Explicit expressions of the projection operators $P_T^{(T_z)}$ are given in Ref. 28. The first few terms are written as

$$P_{T_0-1}{}^{(T_0-1)}=1-\frac{1}{2T_0}T_-T_+-\frac{1}{4T_0(2T_0+1)}T_-{}^2T_+{}^2\cdots, \quad \text{(A1a)}$$

$$P_{T_0}{}^{(T_0-1)}=\frac{1}{2T_0}T_-T_+-\frac{1}{2T_0(2T_0+2)}T_-{}^2T_+{}^2\cdots, \quad \text{(A1b)}$$

and

$$P_{T_0+1}{}^{(T_0-1)}=\frac{1}{2(2T_0+1)(2T_0+2)}T_-{}^2T_+{}^2\cdots, \quad \text{(A1c)}$$

in which

$$T_z|0\rangle=T_0|0\rangle=\tfrac{1}{2}(N-Z)|0\rangle, \quad \text{(A2a)}$$

and

$$T_\pm=T_x\pm iT_y. \quad \text{(A2b)}$$

The components of an isopin operator are expressed in the second quantized form:

$$T_z=\tfrac{1}{2}\sum_\delta (b_\delta{}^\dagger b_\delta-a_\delta{}^\dagger a_\delta), \quad \text{(A3a)}$$

$$T_+=\sum_\delta b_\delta{}^\dagger a_\delta, \quad \text{(A3b)}$$

and

$$T_-=\sum_\delta a_\delta{}^\dagger b_\delta. \quad \text{(A3c)}$$

In the above expressions $a^\dagger(a)$ and $b^\dagger(b)$ are the creation (annihilation) operators for a proton and a neutron.

By straightforward but tedious calculations assuming $T_+|0\rangle=0$ we obtain

$$P_{T_0+1}{}^{(T_0-1)}a_\lambda{}^\dagger b_\nu|0\rangle=-\frac{1}{(T_0+1)(2T_0+1)}\{a_\lambda{}^\dagger b_\nu+(b_\lambda{}^\dagger b_\nu-a_\lambda{}^\dagger a_\nu)T_-\}|0\rangle+\frac{1}{2(T_0-1)(2T_0+1)}b_\lambda{}^\dagger a_\nu T_-{}^2|0\rangle, \quad \text{(A4a)}$$

$$P_{T_0}{}^{(T_0-1)}a_\lambda{}^\dagger b_\nu|0\rangle=\frac{T_0+2}{T_0(T_0+1)}a_\lambda{}^\dagger b_\nu|0\rangle+\frac{T_0+3}{2T_0(T_0+1)}(b_\lambda{}^\dagger b_\nu-a_\lambda{}^\dagger a_\nu)|0\rangle-\frac{1}{2T_0(T_0+1)}b_\lambda{}^\dagger a_\nu T_-{}^2|0\rangle, \quad \text{(A4b)}$$

$$P_{T_0-1}{}^{(T_0-1)}a_\lambda{}^\dagger b_\nu|0\rangle=\frac{2T_0{}^2-T_0-2}{T_0(2T_0+1)}a_\lambda{}^\dagger b_\nu|0\rangle-\frac{2T_0+3}{2T_0(2T_0+1)}(b_\lambda{}^\dagger b_\nu-a_\lambda{}^\dagger a_\nu)T_-|0\rangle+\frac{1}{2T_0(2T_0+1)}b_\lambda{}^\dagger a_\nu T_-{}^2|0\rangle, \quad \text{(A4c)}$$

$$P_{T_0}{}^{(T_0-1)}a_\lambda{}^\dagger b_\mu|0\rangle=\frac{1}{2T_0}(a_\lambda{}^\dagger b_\mu+b_\lambda{}^\dagger b_\mu T_-)|0\rangle, \quad \text{(A5a)}$$

$$P_{T_0-1}{}^{(T_0-1)}a_\lambda{}^\dagger b_\mu|0\rangle=\frac{2T_0-1}{2T_0}a_\lambda{}^\dagger b_\mu|0\rangle-\frac{1}{2T_0}b_\lambda{}^\dagger b_\mu T_-|0\rangle, \tag{A5b}$$

$$P_{T_0}{}^{(T_0-1)}a_\mu{}^\dagger b_\nu|0\rangle=\frac{1}{2T_0}(a_\mu{}^\dagger b_\nu-a_\mu{}^\dagger a_\nu T_-)|0\rangle, \tag{A6a}$$

$$P_{T_0-1}{}^{(T_0-1)}a_\mu{}^\dagger b_\nu|0\rangle=\frac{2T_0-1}{2T_0}a_\mu{}^\dagger b_\nu|0\rangle+\frac{1}{2T_0}a_\mu{}^\dagger a_\nu T_-|0\rangle. \tag{A6b}$$

The suffixes λ, μ, and ν are given in Fig. 4. If we note that

$$a_\lambda{}^\dagger b_\nu|0\rangle,\quad \tfrac{1}{2}T_0{}^{-1/2}(b_\lambda{}^\dagger b_\nu-a_\mu{}^\dagger a_\nu)T_-|0\rangle,$$
$$\tfrac{1}{2}\{2T_0(T_0-1)\}^{-1/2}b_\lambda{}^\dagger a_\nu T_-{}^2|0\rangle,\quad a_\lambda{}^\dagger a_\mu|0\rangle,$$
$$(2T_0-1)^{-1/2}b_\lambda{}^\dagger b_\mu T_-|0\rangle,\quad a_\mu{}^\dagger b_\nu|0\rangle,$$

and $(2T_0-1)^{-1/2}a_\mu{}^\dagger b_\nu T_-|0\rangle$ are normalized states in Eqs. (A4)-(A6), then we can easily normalize the states.

The normalized states are expressed as

$$|\lambda\nu;T=T_0+1,T_z=T_0-1\rangle=[(2T_0+1)(T_0+1)]^{1/2}\times P_{T_0+1}{}^{(T_0-1)}a_\lambda{}^\dagger b_\nu|0\rangle, \tag{A7a}$$

$$|\lambda\nu;T=T_0,T_z=T_0-1\rangle=\frac{(\sqrt{T_0}+1)T_0}{\sqrt{T_0{}^2+8T_0+4}}\times P_{T_0}{}^{(T_0-1)}a_\lambda{}^\dagger b_\nu|0\rangle, \tag{A7b}$$

$$|\lambda\nu;T=T_0-1,T_z=T_0-1\rangle=\frac{T_0(2T_0+1)}{\sqrt{4T_0{}^4+7T_0{}^2+12T_0+4}}\times P_{T_0-1}{}^{(T_0-1)}a_\lambda{}^\dagger b_\nu|0\rangle, \tag{A7c}$$

$$|\lambda\mu;T=T_0,T_z=T_0-1\rangle=(\sqrt{2T_0})P_{T_0}{}^{(T_0-1)}\times a_\lambda{}^\dagger b_\mu|0\rangle, \tag{A8a}$$

$$|\lambda\mu;T=T_0-1,T_z=T_0-1\rangle=\left(\frac{2T_0}{2T_0-1}\right)^{1/2}P_{T_0-1}{}^{(T_0-1)}\times a_\lambda{}^\dagger b_\mu|0\rangle, \tag{A8b}$$

$$|\mu\nu;T=T_0,T_z=T_0-1\rangle=(\sqrt{2T_0})P_{T_0}{}^{(T_0-1)}\times a_\mu{}^\dagger b_\nu|0\rangle, \tag{A9a}$$

$$|\mu\nu;T=T_0-1,T_z=T_0-1\rangle=\left(\frac{2T_0}{2T_0-1}\right)^{1/2}P_{T_0-1}{}^{(T_0-1)}\times a_\mu{}^\dagger b_\nu|0\rangle. \tag{A9b}$$

Note that orthogonality conditions are clearly satisfied;

$$\langle\delta'\delta;T(T_z=T_0-1)|\epsilon'\epsilon;T(T_z=T_0-1)\rangle=\delta_{\delta'\epsilon'}\delta_{\delta\epsilon}, \tag{A10}$$

and, if $T_0\gg1$, we have

$$P_{T_0-1}{}^{(T_0-1)}a_{\delta'}{}^\dagger a_\delta|0\rangle\approx a_{\delta'}{}^\dagger a_\delta|0\rangle. \tag{A11}$$

APPENDIX B

Equation (24) is proved as follows:

$$\begin{aligned}T_+|\text{coll.}T(T_z=T_0-1)\rangle&=\frac{1}{\sqrt{N_T}}T_+P_T{}^{(T_0-1)}\sum_\delta a_{\delta'}{}^\dagger b_\delta|0\rangle\\&=\frac{1}{\sqrt{N_T}}P_T{}^{(T_0)}\sum_\delta[T_+,a_{\delta'}{}^\dagger b_\delta]|0\rangle\\&=\frac{1}{\sqrt{N_T}}P_T{}^{(T_0)}\sum_\delta(b_{\delta'}{}^\dagger b_\delta-a_{\delta'}{}^\dagger a_\delta)|0\rangle\end{aligned} \tag{B1}$$

for $T=T_0$ and T_0+1 as collective states of $|0\rangle$ with $T_z=T_0$, which agrees with Eq. (24).

Also the following relations are useful:

$$\begin{aligned}\Delta_I{}^\gamma(T)&\equiv\langle0|\sum_\delta(b_\delta{}^\dagger b_{\delta'}-a_\delta{}^\dagger a_{\delta'})H_IP_T{}^{(T_0)}\sum_\delta(b_{\delta'}{}^\dagger b_\delta-a_{\delta'}{}^\dagger a_\delta)|0\rangle/\langle0|\sum_\delta(b_\delta{}^\dagger b_{\delta'}-a_\delta{}^\dagger a_{\delta'})P_T{}^{(T_0)}\sum_\delta(b_{\delta'}{}^\dagger b_\delta-a_{\delta'}{}^\dagger a_\delta)|0\rangle\\&=\langle0|\sum_\delta b_\delta{}^\dagger a_{\delta'}H_IT_-T_+P_T{}^{(T_0-1)}\sum_\delta a_{\delta'}{}^\dagger b_\delta|0\rangle/\langle0|\sum_\delta b_\delta{}^\dagger a_{\delta'}T_-T_+P_T{}^{(T_0-1)}\sum_\delta a_{\delta'}{}^\dagger b_\delta|0\rangle\\&=\langle0|\sum_\delta b_\delta{}^\dagger a_{\delta'}H_IP_T{}^{(T_0-1)}\sum_\delta a_{\delta'}{}^\dagger b_\delta|0\rangle/\langle0|\sum_\delta b_\delta{}^\dagger a_{\delta'}P_T{}^{(T_0-1)}\sum_\delta a_{\delta'}{}^\dagger b_\delta|0\rangle\equiv\Delta_I{}^\beta(T)\end{aligned} \tag{B2}$$

for $T=T_0$ or T_0+1.

Note that the symmetry energy Δ_S to be compared with experiments is defined by

$$\begin{aligned}\Delta_S&\equiv\frac{\langle0|b_\delta{}^\dagger a_{\delta'}HP_{T_0}{}^{(T_0-1)}a_{\delta'}{}^\dagger b_\delta|0\rangle}{\langle0|b_\delta{}^\dagger a_{\delta'}P_{T_0}{}^{(T_0-1)}a_{\delta'}{}^\dagger b_\delta|0\rangle}-\frac{\langle0|b_\delta{}^\dagger a_{\delta'}HP_{T_0-1}{}^{(T_0-1)}a_{\delta'}{}^\dagger b_\delta|0\rangle}{\langle0|b_\delta{}^\dagger a_{\delta'}P_{T_0-1}{}^{(T_0-1)}a_{\delta'}{}^\dagger b_\delta|0\rangle}\\&=\Delta_S{}'+G_{T_0}(\delta'\delta;\delta'\delta)-G_{T_0-1}(\delta'\delta;\delta'\delta),\end{aligned} \tag{B3}$$

as seen from Eqs. (19) and (28). If $G_{T_0}=G_{T_0-1}$ for every $(\delta'\delta)$, we get $\Delta_S=\Delta_S'$. However, if $G_{T_0}\neq G_{T_0-1}$, large effects arise on the $\Delta_I{}^{\beta}(T)$ in Eq. (B2).

APPENDIX C

In this Appendix the equations given by Eq. (33) are solved.

Let us assume the form of solutions, which correspond to the collective states $|\text{coll.}_{>(<)};\, T_0-1(T_0-1)\rangle$, as

$$C\sum_i |i'i;\, T_0-1(T_0-1)\rangle + D\sum_j |j'j;\, T_0-1(T_0-1)\rangle. \quad \text{(C1)}$$

From (13), (18), (19), (C1), and (33) we obtain the relations

$$\frac{C}{E-\Delta E_0}=\frac{D}{E-\Delta E_0+\delta}, \quad \text{(C2)}$$

and the dispersion relation

$$\frac{n_1}{E-\Delta E_0+\delta}+\frac{n_2}{E-\Delta E_0}=\frac{1}{G} \quad \text{(C3)}$$

for $G=G_{T_0-1}$, and the normalization conditions

$$n_1C^2+n_2D^2=1. \quad \text{(C4)}$$

Solving (C3), we obtain the energy eigenvalues of the collective states, $E_>$ and $E_<$, given by Eq. (34). In order to obtain the results (36) and (37), the following relations are useful:

$$-2\Delta E_0+E_>+E_<=NG-\delta, \quad \text{(C5a)}$$

$$(E_>-\Delta E_0)(E_<-\Delta E_0)=-n_2G\delta, \quad \text{(C5b)}$$

and

$$-\Delta E_0+E_>=-\epsilon+NG. \quad \text{(C5c)}$$

APPENDIX D

TABLE II. Log *ft* values of the unique first-forbidden transitions and the hindrance factors of non-well-deformed nuclei in medium and heavy mass regions.

Parent	Daughter	Decay mode		$Q_{\beta^-}(Q_{ce})$	$\log_{10}ft$[a]	$\log_{10}F_s$[b]	$\log_{10}F_p$[c]	$j_n \leftrightarrow j_p$[d]
${}_{33}As_{39}{}^{72}$	${}_{32}Ge_{40}{}^{72}$	$2^- \to 0^+$	+	4.3	8.8	2.0	1.05	$1g_{9/2} \leftrightarrow 1f_{5/2}$
${}_{33}As_{41}{}^{74}$	${}_{32}Ge_{42}{}^{74}$	$2^- \to 0^+$	+	2.5	8.5	1.7	1.05	$1g_{9/2} \leftrightarrow 1f_{5/2}$
${}_{33}As_{41}{}^{74}$	${}_{34}Se_{40}{}^{74}$	$2^- \to 0^+$	−	1.4	8.3	1.5	1.15	$1g_{9/2} \leftrightarrow 1f_{5/2}$
${}_{33}As_{43}{}^{76}$	${}_{34}Se_{42}{}^{76}$	$2^- \to 0^+$	−	3.0	8.3	1.5	1.10	$1g_{9/2} \leftrightarrow 1f_{5/2}$
${}_{33}As_{45}{}^{78}$	${}_{34}Se_{44}{}^{78}$	$2^- \to 0^+$	−	4.3	8.4	1.6	1.10	$1g_{9/2} \leftrightarrow 1f_{5/2}$
${}_{37}Rb_{47}{}^{84}$	${}_{36}Kr_{48}{}^{84}$	$2^- \to 0^+$	+	2.7	8.7	1.9	1.20	$1g_{9/2} \leftrightarrow 1f_{5/2}$
${}_{37}Rb_{49}{}^{86}$	${}_{38}Se_{48}{}^{86}$	$2^- \to 0^+$	−	1.8	8.5	1.7	0.95	$1g_{9/2} \leftrightarrow 1f_{5/2}$
${}_{37}Rb_{51}{}^{88}$	${}_{38}Sr_{50}{}^{88}$	$2^- \to 0^+$	−	5.2	8.2	1.0	0.60	$2d_{5/2} \leftrightarrow 2p_{1/2}$
${}_{39}Y_{51}{}^{90}$	${}_{40}Zr_{50}{}^{90}$	$2^- \to 0^+$	−	2.3	8.0	0.8	0.65	$2d_{5/2} \leftrightarrow 2p_{1/2}$
${}_{39}Y_{53}{}^{92}$	${}_{40}Z_{52}{}^{92}$	$2^- \to 0^+$	−	3.6	8.1	0.9	0.70	$2d_{5/2} \leftrightarrow 2p_{1/2}$
${}_{47}Ag_{65}{}^{112}$	${}_{48}Cd_{64}{}^{112}$	$2^- \to 0^+$	−	4.0	8.7	1.5	0.60	$2d_{5/2} \leftrightarrow 2p_{1/2}$
${}_{51}Sb_{71}{}^{122}$	${}_{52}Te_{70}{}^{122}$	$2^- \to 0^+$	−	2.0	8.5	2.0	0.95	$1h_{11/2} \leftrightarrow 1g_{7/2}$
${}_{53}I_{71}{}^{124}$	${}_{52}Te_{72}{}^{124}$	$2^- \to 0^+$	+	3.2	8.2	1.7	1.30	$1h_{11/2} \leftrightarrow 1g_{7/2}$
${}_{53}I_{73}{}^{126}$	${}_{52}Te_{74}{}^{126}$	$2^- \to 0^+$	+	2.2	8.1	1.6	1.25	$1h_{11/2} \leftrightarrow 1g_{7/2}$
${}_{53}I_{73}{}^{126}$	${}_{54}Xe_{72}{}^{126}$	$2^- \to 0^+$	−	1.2	8.6	2.1	1.30	$1h_{11/2} \leftrightarrow 1g_{7/2}$
${}_{59}Pr_{83}{}^{142}$	${}_{60}Nd_{82}{}^{142}$	$2^- \to 0^+$	−	2.2	7.8	0.65	0.60	$2f_{7/2} \leftrightarrow 2d_{5/2}$
${}_{81}Tl_{119}{}^{200}$	${}_{80}Hg_{120}{}^{200}$	$2^- \to 0^+$	+	2.5	8.7	1.55	1.15	$3p_{3/2} \leftrightarrow 3s_{1/2}$
${}_{81}Tl_{121}{}^{202}$	${}_{80}Hg_{122}{}^{202}$	$2^- \to 0^+$	+	1.2	8.4	1.25	0.90	$3p_{3/2} \leftrightarrow 3s_{1/2}$
${}_{81}Tl_{123}{}^{204}$	${}_{80}Hg_{124}{}^{204}$	$2^- \to 0^+$	+	0.3	8.5	1.35	1.05	$3p_{3/2} \leftrightarrow 3s_{1/2}$
${}_{81}Tl_{123}{}^{204}$	${}_{82}Pb_{122}{}^{204}$	$2^- \to 0^+$	−	0.8	9.0	1.85	0.85	$3p_{3/2} \leftrightarrow 3s_{1/2}$
${}_{36}Kr_{49}{}^{85}$	${}_{37}Rb_{48}{}^{85}$	$\frac{9}{2}^+ \to \frac{5}{2}^-$	−	0.7	9.1	2.0	0.70	$1g_{9/2} \leftrightarrow 1f_{5/2}$
${}_{38}Sr_{51}{}^{89}$	${}_{39}Y_{50}{}^{89}$	$\frac{5}{2}^+ \to \frac{1}{2}^-$	−	1.5	8.6	1.3	1.15	$2d_{5/2} \leftrightarrow 2p_{1/2}$
${}_{38}Sr_{53}{}^{91}$	${}_{39}Y_{52}{}^{91}$	$\frac{5}{2}^+ \to \frac{1}{2}^-$	−	2.7	8.3	1.0	0.75	$2d_{5/2} \leftrightarrow 2p_{1/2}$
${}_{39}Y_{52}{}^{91}$	${}_{40}Zr_{51}{}^{91}$	$\frac{1}{2}^- \to \frac{5}{2}^+$	−	1.5	8.5	1.7	0.90	$2d_{5/2} \leftrightarrow 2p_{1/2}$
${}_{43}Tc_{52}{}^{95m}$	${}_{42}Mo_{53}{}^{95}$	$\frac{1}{2}^- \to \frac{5}{2}^+$	+	0.7	8.4	1.6	1.40	$2d_{5/2} \leftrightarrow 2p_{1/2}$
${}_{50}Sn_{73}{}^{123}$	${}_{51}Sb_{72}{}^{123}$	$\frac{11}{2}^- \to \frac{7}{2}^+$	−	1.4	8.8	1.9	1.65	$1h_{11/2} \leftrightarrow 1g_{7/2}$
${}_{50}Sn_{75}{}^{125}$	${}_{51}Sb_{74}{}^{125}$	$\frac{11}{2}^- \to \frac{7}{2}^+$	−	2.3	8.8	1.9	1.50	$1h_{11/2} \leftrightarrow 1g_{7/2}$
${}_{51}Sb_{74}{}^{125}$	${}_{52}Te_{73}{}^{125}$	$\frac{7}{2}^+ \to \frac{11}{2}^-$	−	0.8	8.5	1.8	0.90	$1h_{11/2} \leftrightarrow 1g_{7/2}$
${}_{51}Sb_{76}{}^{127}$	${}_{52}Te_{75}{}^{127}$	$\frac{7}{2}^+ \to \frac{11}{2}^-$	−	1.6	8.5	1.8	0.95	$1h_{11/2} \leftrightarrow 1g_{7/2}$
${}_{55}Cs_{82}{}^{137}$	${}_{56}Ba_{81}{}^{137}$	$\frac{7}{2}^+ \to \frac{11}{2}^-$	−	0.5	8.5	1.8	1.40	$1h_{11/2} \leftrightarrow 1g_{7/2}$

[a] Experimental log *ft* values corrected for shape factor (Ref. 42).
[b] Logarithmic hindrance factors obtained on the basis of the simple *j-j* coupling shell model (Ref. 43) in which use was made of the radial part of the wave function obtained from the harmonic-oscillator potential.
[c] Logarithmic hindrance factors calculated in terms of the pairing model. The U^2 and V^2 factors obtained in Ref. 6 were used.
[d] Main components of shell-model states assumed for transition process.