Scattering of 24.5-MeV Protons from ^{89}Y

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Scattering of 24.5-MeV protons from ^{89}Y has been measured. Angular distributions were obtained for the ground state and the first four excited states as well as for several unresolved multiplets. The inelastic scattering data have been analyzed, using the collective model and a microscopic model with and without core-polarization effects.

I. INTRODUCTION

THE ground state and the first excited state of ^{89}Y are well understood in terms of the shell model. Analyses of data on transfer reactions[1,2] leading to ^{89}Y and of data on inelastic scattering of electrons,[3] protons,[2,4] and α particles[5] from ^{89}Y have given a clear picture of the nature of the second and third excited states. The situation for higher-lying states is not as promising. Some of the confusion concerning these states has been cleared up recently by high-resolution studies[6] of the level scheme of ^{89}Y. Several of the higher-lying states which had previously been thought to be single are now known to be closely spaced doublets or triplets.

In the present experiment the main aim was to study the microscopic model of inelastic proton scattering. Inelastic scattering from the first excited state of ^{89}Y is a good test for this theory, since the initial and final states are well represented by simple shell-model wave functions. Such an approach has been employed in interpreting data on the scattering of 61-MeV protons.[7] In this analysis, the inclusion of core-polarization effects was found to improve the theoretical situation by providing a better fit to the data with a more realistic effective interaction between target and projectile nucleons. A comparison of the results of the present experiment to those obtained at 61 MeV will give information on the energy dependence of the effective interaction. The effective interaction is expected to be energy-dependent, and the dependence of the spin- and isospin-independent part has been observed at proton bombarding energies between 23 and 52 MeV.[8] One can also obtain an upper limit on the spin-flip part of the effective interaction by setting the spin-independent part equal to zero. Inelastic scattering to the second excited state is a good case for this, since it has a strong monopole spin-flip amplitude. Scattering of protons from the first excited state of ^{89}Y is expected[9] to be a good test of an approximation made in the present microscopic scattering theory in which the space-exchange process is neglected.

A second aim of the experiment is the study of the higher-lying levels of ^{89}Y, using both collective and microscopic scattering theory. The picture of the second and third excited states that emerges is in good agreement with previous analyses of transfer reactions and inelastic scattering. The present experiment, however, sheds little light on states above the third excited state.

Figure 1 is a diagram of energy levels in ^{89}Y below 3 MeV. The ground state has been described as a pure single-particle state with a $2p_{1/2}$ proton outside a closed ^{88}Sr core. The first excited state at 0.908 MeV is obtained by promoting this proton to the $1g_{9/2}$ shell. The electromagnetic transition rate for the first-excited-state to ground-state transition is in good agreement with single-particle estimates with small configuration mixing.[10] Attempts have been made to describe the $\frac{3}{2}^-$, $\frac{5}{2}^-$ doublets at 1.507 and 1.745 MeV as a $2p_{1/2}$ proton coupled to the ^{88}Sr 2^+ first excited state.[11] The (α,α') and (e,e') data of Alster *et al.*[5] and of Peterson and Alster[3] have shown this to be a rather poor description. The shell-model description of these states would be obtained by promoting a $2p_{3/2}$ and a $1f_{5/2}$ particle into the $2p_{1/2}$ shell. The ^{90}Zr$(d,$He$^3)^{89}$Y results of Preedom *et al.*[1] are in agreement with this model, since they show

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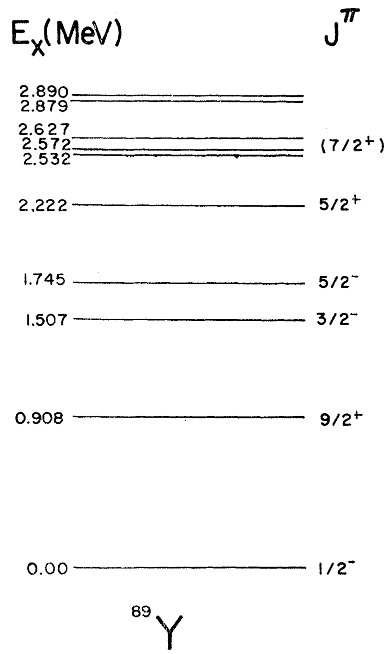

FIG. 1. Energy levels of ^{89}Y below 3 MeV. The excitation energies are taken from Ref. 6.

strong transitions to these two states from the ^{90}Zr ground state. It is difficult to make any qualitative statement concerning the states above 1.745 MeV. There are too many states to be described by the coupling of a $2p_{1/2}$ particle to the ^{88}Sr 3^- state at 1.84 MeV or $2p_{1/2}$ proton hole to the ^{90}Zr 3^- state at 2.75 MeV. The sorting out of these states awaits higher-resolution data than are presently available.

II. EXPERIMENTAL METHOD

This experiment was performed using the Michigan State University sector-focused cyclotron. An H^- beam was stripped inside the cyclotron and then focused by quadrupoles and bent 15° into a 36-in.-diam scattering chamber. The energy of the beam was determined to be 24.5 ± 0.2 MeV by a kinematic method using scattering from hydrogen and the first excited state of ^{12}C.[12] A ΔE-E silicon-detector telescope was used. The ΔE counter was a 1-mm surface-barrier detector, whereas the E counter was an Li-drifted 3-mm detector. The E counter was cooled to dry-ice temperature but was still a substantial contributor to the over-all energy resolution of 100 keV.

The target was a 1.75-mg/cm^2 self-supporting foil on loan from Oak Ridge National Laboratory. Elastic scattering from small carbon and oxygen contaminants was strong enough to obscure some of the data at many forward angles.

Particular attention was given to establishing an accurate normalization of the data, which were taken relative to a monitor counter at 90°. First, a normalization was obtained by weighing the target and assuming it to be uniform. The integrated charge and the various geometric factors were then used to calculate the cross section. As a check in this result a method of normalization using a thick target was devised. This method, which will be described in more detail,[13] eliminates the effect of target nonuniformity. The spectrum from a 0.001-in.-thick Y foil was obtained at a back angle (101°). The shape of the elastic-scattered peak was used to determine the target thickness and uniformity over the actual beam spot. The thick-target spectrum obtained is shown in Fig. 2. Also shown is a calculated fit to the shape assuming that the target had several regions of different thickness within the beam spot. The calculated fit gives both the total number of counts and the average target thickness. The two methods of obtaining the normalization agreed to within 2%. The over-all error due to normalization is less than 6% and is essentially due to uncertainties in geometric factors such as beam position and collimator dimensions. The errors shown on the angular-distribution figures are the sum of statistical errors and those due to uncertainties in background subtraction. The difficulty in obtaining accurate backgrounds is illustrated by a typical spectrum shown in Fig. 3. The first excited state at 0.908 MeV is very weak and rides on the tail of the elastic peak. Another problem in background subtraction is illustrated by the broadening of the 1.745-MeV state. This broadening is due to a peak near 1.78 MeV which has been shown to be inelastic scattering of the elastically scattered protons within the detector volume.[14] We were able to subtract these events because they are proportional to the number of elastic events detected. Nonetheless, the errors in the cross section to the 1.745-MeV state become large at forward angles.

III. ANALYSIS

A. Elastic Scattering

The elastic scattering was analyzed in terms of an optical-model potential of the usual form,

$$U(r) = -VF(x) + 4W_D i \frac{d}{dx'} F(x') + \left(\frac{\hbar}{m_\pi c}\right)^2 V_s \frac{1}{r}\frac{d}{dr} F(x_s) \boldsymbol{\sigma}\cdot\mathbf{l} + U_c(r),$$

where $F(y) = (e^y + 1)^{-1}$,

$$x = (r - r_0 A^{1/3})/a,$$
$$x' = (r - r_0' A^{1/3})/a',$$
$$x_s = (r - r_s A^{1/3})/a_s,$$

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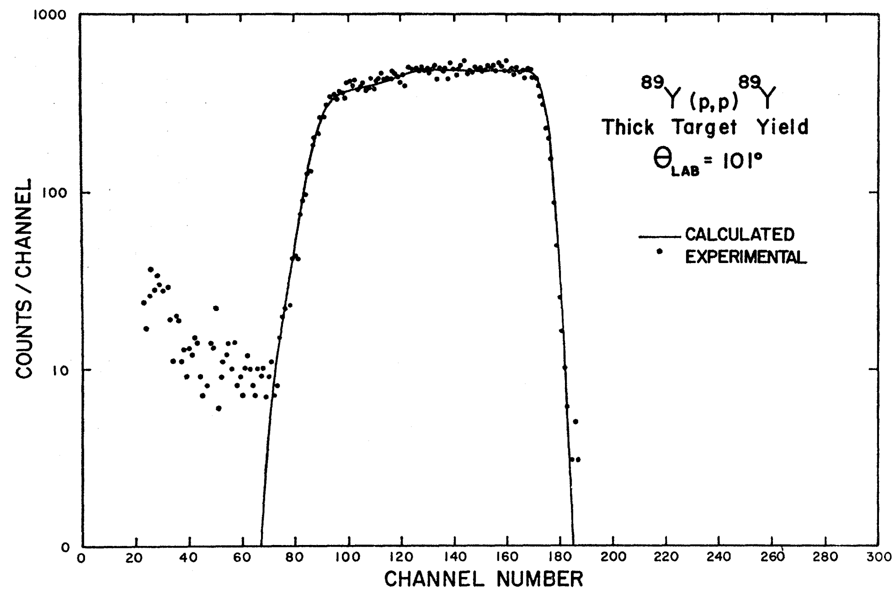

Fig. 2. Elastically scattering protons from a thick ^{89}Y target at 101° in the laboratory. The calculated curve was obtained by assuming the beam spot included several regions of different target thickness.

and $U_c(r)$ is the Coulomb potential for a uniformly charged sphere of radius $1.31A^{1/3}$ F. The spin-orbit strength V_s was constrained to be real. The parameter values were obtained by minimizing the quantity χ^2 by an automatic search routine:

$$\chi^2 = N^{-1} \sum_{i=1} \{[\sigma_{\text{theor}}(\theta_i) - \sigma_{\text{expt}}(\theta_i)]/\Delta\sigma_{\text{expt}}(\theta_i)\}^2,$$

where $\sigma_{\text{theor}}(\theta_i)$ and $\sigma_{\text{expt}}(\theta_i)$ are the theoretical and measured differential cross sections at angle θ_i, and $\Delta\sigma_{\text{expt}}(\theta_i)$ is the error assigned to σ_{expt} at the angle θ_i.

The fit to the experimental differential cross section divided by the Rutherford differential cross section is shown in Fig. 4. The parameters of the optical model given on the figure were used for the distorted-wave analysis of the inelastic scattering.

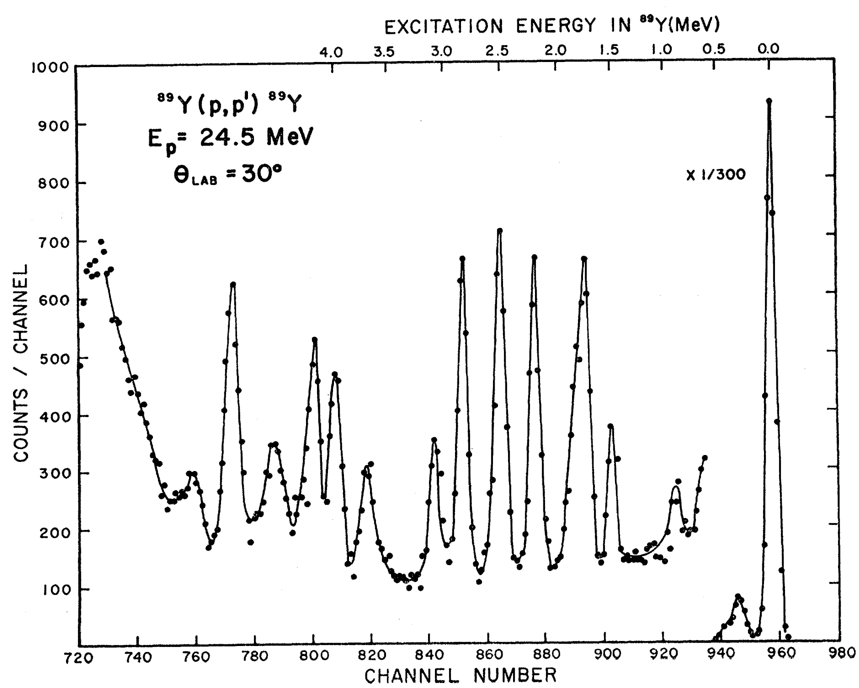

Fig. 3. Spectrum of protons scattered from ^{89}Y at 30° in the laboratory. The broad peak near channel 945 is due to contaminants in the target.

B. Inelastic Scattering

The theoretical predictions for the inelastic scattering were computed in the distorted-wave approximation.[15] All exchange effects between the projectile and target are ignored except insofar as they are included in the effective interactions or form factors. The data are analyzed using both the macroscopic collective model and the microscopic shell model. In the microscopic description, however, some core-polarization effects are included by means of a macroscopic model.[16]

1. Collective-Model Analysis

In this model the ground state of ^{89}Y is a single proton occupying a $2p_{1/2}$ state outside a closed ^{88}Sr core. The excited states are then the $2p_{1/2}$ proton coupled to the various collective excitations of the ^{88}Sr core. The cross section is proportional to β_L^2, where β_L is the deformation parameter and is the only unknown parameter in the theory.[17] The β_L may also be determined from the electromagnetic transition strength.

Another interesting consequence of the simple collective model is the sum rule relating transition probabilities. In particular, if the extra core particle has $j=I_i$, then one expects for every core excitation with $J_c>I_i$ of ^{88}Sr, $2I_i+1$ excited states of the core plus one-particle system. (In our case $2I_i+1=2$ and we expect a sequence of doublets if this model is correct.)

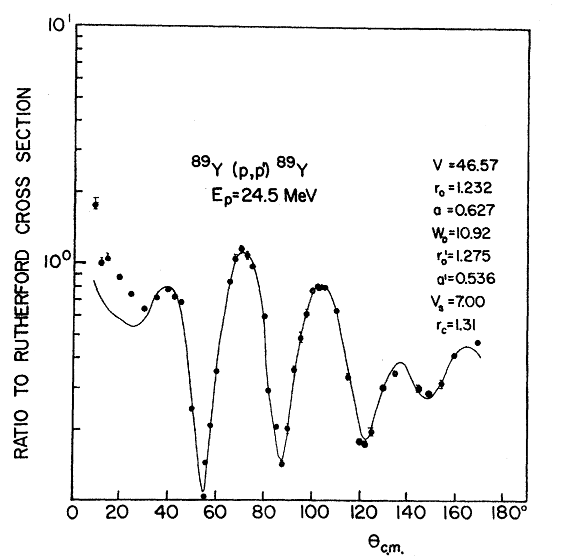

FIG. 4. Ratio of elastically scattered protons from ^{89}Y to the Rutherford cross section. The solid curve is an optical-model fit to the data points. The errors shown are statistical and do not include angular errors, which are important in the forward angles where the Rutherford cross section depends very strongly on angle.

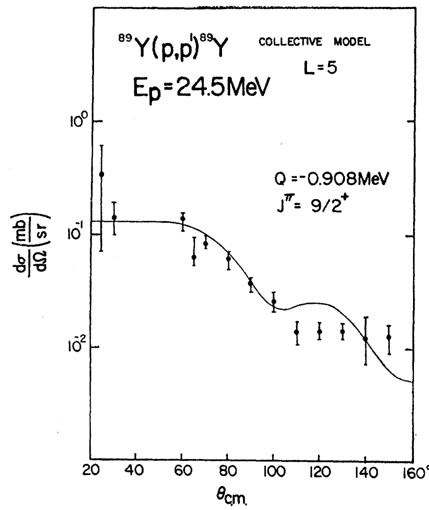

FIG. 5. Collective-model fit to the scattering of 24.5-MeV protons from the first excited state of ^{89}Y.

In this case the sum rule reads

$$\sum_{I_f} \sigma_{I_i\to I_f}(\theta)=\sigma_{0\to\lambda}(\theta)_{\text{core}}.$$

It is convenient to introduce a partial deformation parameter,

$$\beta_L(I_f)[(2I_f+1)/(2I_i+1)(2L+1)]^{1/2}\beta_L.$$

The sum rule then reads

$$\sum_{I_f} \beta_L^2(I_f)=\beta_L^2.$$

The 0.908-MeV state. Although the 0.908-MeV $\frac{9}{2}^+$ level in ^{89}Y is not believed to be a collective level, it is convenient to analyze it as such for comparison with other collective analyses as well as for comparison with a microscopic analysis. Figure 5 shows the experimental cross section along with the collective-model prediction. A β_5 of 0.06 is found in good agreement with Awaya's value of 0.07.[4] This value of β_5 is also in rough agreement with that obtained by Gray *et al.*[18] for the excitation of the 5^- state in ^{90}Zr. Such a value is more suggestive of a single-particle transition.

The 1.51- and 1.74-MeV states. The 1.51-MeV level has been assigned a $J^\pi=\frac{3}{2}^-$ and the 1.74-MeV level a $J^\pi=\frac{5}{2}^-$.[3-5] Such a doublet suggests the collective model, since these are just the values of J^π expected to arise from the coupling of a $p_{1/2}$ particle to the 2^+ excitation of the core. However, the $B(E2)$ values[3] to these levels are about one single-particle unit, which suggests a less collective description of these states. The results of the collective model, including Coulomb excitation, are shown in Fig. 6. The angular distributions are in reasonable agreement with experiment and, as can be seen

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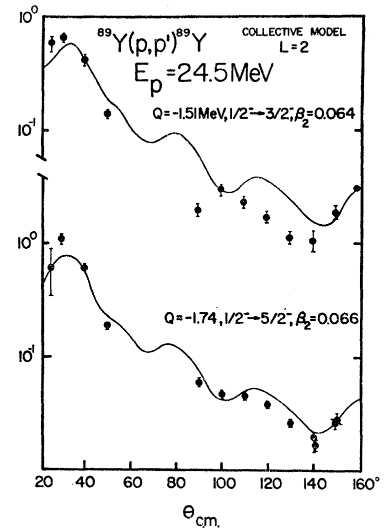

FIG. 6. Collective-model fits to inelastic scattering from the second and third excited states of ^{89}Y.

from Table I, the cross sections are approximately proportional to $2I_f+1$ as predicted by the simple collective model. The values obtained are consistent with those extracted from the corresponding electromagnetic measurements. The inelastic scattering to the $\frac{5}{2}^-$ level does seem to be slightly enhanced compared to the corresponding electromagnetic transition. As in the case of the electromagnetic transitions,[3] we find here that the transition strengths to this doublet exhausts only about 30% of the sum rule if we take $\beta_2=0.114$ in ^{88}Sr.[3] The value of β_2 found for these two states indicates that they might be more reasonably described by the shell model.

Higher-lying states. Angular distributions were obtained for the $\frac{5}{2}^+$ 2.21-MeV state as well as for multiplets at 2.52 and 2.86 MeV. One can analyze the data under the assumption that only one level was excited at each energy. This could easily be the case if there is just one level of strong collective character near 2.52 and 2.86 MeV. Previous to the high-resolution work[6] which resolved these levels into multiplets, the 2.52-MeV state was thought to be $\frac{7}{2}^+$ and the 2.86-MeV state either $\frac{7}{2}^+$ or $\frac{5}{2}^+$.

TABLE I. Deformation parameters. The electromagnetic β_L's are taken from the work of Peterson and Alster (Ref. 3). The β_L's from (p,p') on ^{88}Sr are from the work of Stautberg *et al.* (Ref. 2).

E_x (MeV)	J^π	Nucleus	L	Electromagnetic $\beta_L(I_f)$	From (p,p') $\beta_L(I_f)$	$\beta_L{}^2(I_f)$	β_L
0.908	$\frac{9}{2}^+$	^{89}Y	5		0.0408	0.00166	0.0604
1.507	$\frac{3}{2}^-$	^{89}Y	2	0.0378±0.0183	0.0404	0.00163	0.0639
1.745	$\frac{5}{2}^-$	^{89}Y	2	0.0413±0.0060	0.0512	0.00261	0.0660
1.84	2^+	^{88}Sr	2	0.111 ±0.003	0.13	0.017	0.13
2.222	$\frac{5}{2}^+$	^{89}Y	3	0.116 ±0.005	0.103	0.0106	0.157
2.532	$\frac{7}{2}^+$	^{89}Y	3	0.123 ±0.005	0.115	0.0132	0.152
2.86	$(\frac{7}{2}^+)$	^{89}Y	3	0.114 ±0.005	0.975	0.0095	0.129
2.74	3^-	^{88}Sr	3	0.180 ±0.003	0.200	0.040	0.200

As can be seen from Fig. 7, scattering from these three states has angular distributions that are well described by an angular-momentum transfer of $L=3$ when the cross sections are calculated using the collective model including Coulomb excitation. In this simple model one would expect only a doublet, however, with J^π of $\frac{5}{2}^+$ and $\frac{7}{2}^+$. Furthermore, all three are very strongly excited, yielding electromagnetic transitions about 10 times the single-particle value.[3] In this experiment, as can be seen in Table I, the sum rule is exhausted only if all three of the octupole excitations are included. Although the 2.21- and 2.52-MeV levels fail to exhaust the sum rule, their excitation strengths are in the ratio predicted by the simple collective model.

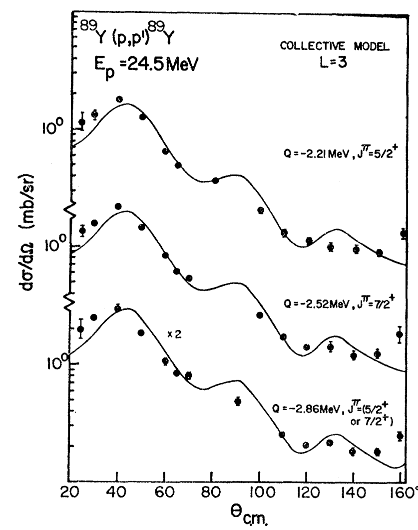

FIG. 7. Collective-model fits to inelastic scattering from states in ^{89}Y near 2.21, 2.52, and 2.86 MeV.

2. *Shell-Model Analysis*

In this description the zero-order ground state of ^{89}Y is taken to be a single proton occupying a $2p_{1/2}$ state outside a closed core. The excited states are then described either by exciting this particle to a higher-lying shell-model orbit or by promoting one of the core particles to one of the unoccupied levels lying above the closed shell. By zero-order is meant the simple individual particle state without the inclusion of core polarization. The single-particle states are taken as eigenstates of a Woods-Saxon potential. The relevant parameters are given in Table II. In this case the form factor is given by the nuclear matrix element of some effective operator V. Following Johnson *et al.*,[19] we write $V=\sum_i v_{ip}$, where v_{ip} is an effective two-body

[19] M. B. Johnson, L. W. Owen, and G. R. Satchler, Phys. Rev. **142**, 748 (1966).

interaction between the projectile p and the ith target nucleon. Also,

$$v_{ip} = -(V_0 + V_1 \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_p) g(r_{ip}),$$

where

$$V_S = V_{S\alpha} + V_{S\beta}(\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_p), \quad S = 0 \text{ or } 1.$$

This yields a form factor $F_{LSJ}(r)$ as follows:

$$F_{LSJ}(r_p) = [M_L \delta(S,0) + N_{LJ} \delta(S,1)] I_L(r_p),$$

where these quantities are defined by Johnson *et al.*[19] The first term corresponds to a spin transfer of 0; the second term to a spin transfer of 1. $\mathbf{L}$ is the orbital angular-momentum transfer and $\mathbf{J} = \mathbf{L} + \mathbf{S}$ is the total angular-momentum transfer. It has recently been shown that if one includes core polarization, one should use,

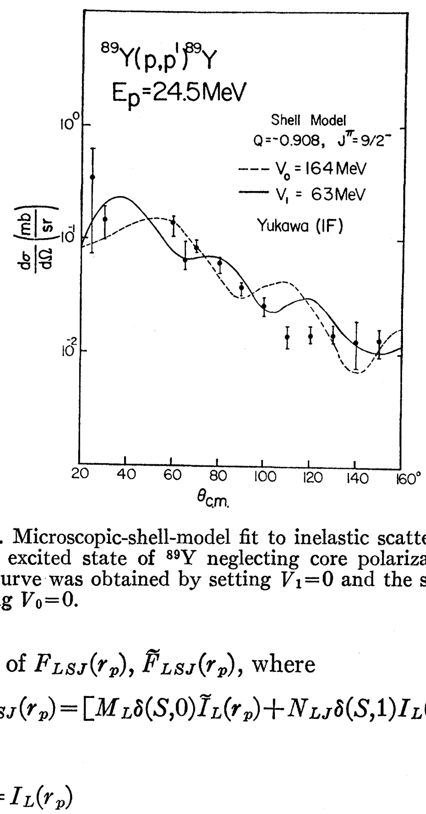

FIG. 8. Microscopic-shell-model fit to inelastic scattering from the first excited state of ^{89}Y neglecting core polarization. The dashed curve was obtained by setting $V_1=0$ and the solid curve by setting $V_0=0$.

instead of $F_{LSJ}(r_p)$, $\tilde{F}_{LSJ}(r_p)$, where

$$\tilde{F}_{LSJ}(r_p) = [M_L \delta(S,0) \tilde{I}_L(r_p) + N_{LJ} \delta(S,1) I_L(r_p)]$$

and

$$\tilde{I}_L(r_p) = I_L(r_p) + (4\pi V_0)^{-1} Y_L(Q) \langle n_2 L_2 j_2 | k_v | n_1 L_1 j_1 \rangle k_p(r_p),$$

and $k_p(r_p)$ is the usual collective form factor. The other quantities are defined elsewhere.[16] There is, of course, an isospin dependence of $F_{LSJ}(r)$, but since we shall only be concerned with the excitation of protons by protons, we only use the combination $V_S = V_{S\alpha} + V_{S\beta}$.[18] Throughout the analysis the form of $g(r)$ will be taken to be that of Yukawa potential with a range of 1.0 F.

With each form factor $F_{LSJ}(r)$ there will be associated one amplitude and a labeling triad (LSJ). Since the amplitudes belonging to different values of J are rigorously incoherent and those corresponding to

TABLE II. Shell-model parameters.

Potential parameters for bound protons Orbit	$2p_{1/2}$	$2p_{3/2}$	$1f_{5/2}$	$1g_{9/2}$
Binding energy (MeV)	7.88	9.65	9.46	5.68
Well depth (MeV)	61.0	61.0	61.0	61.9
Nuclear radius $= r_0 A^{1/3}$,			$r_0 = 1.2$ F	
Coulomb radius $= r_C A^{1/3}$,			$r_C = 1.25$ F	
Diffuseness $= a$,			$a = 0.7$ F	
Spin-orbit parameter $= \lambda$,			$\lambda = 25$	

different values of S are approximately so,[4] we may write

$$\frac{d\sigma}{d\theta} = \text{const} \times \frac{2J_f+1}{2J_i+1} \sum_{LSJ} \sigma_{LSJ}(\theta),$$

where $\sigma_{LSJ}(\theta)$ is the partial cross section corresponding to the form factor $\tilde{F}_{LSJ}(r)$.

Three of the states to which the collective model was applied in Sec. III B 1 exhibit deformation parameters more suggestive of shell-model transitions than of collective ones. These are the $\frac{9}{2}^+$ state at 0.908 MeV, the $\frac{3}{2}^-$ state at 1.51 MeV, and the $\frac{5}{2}^-$ state at 1.74 MeV.

The 0.908-MeV state. The zero-order shell-model description of this state is taken to be that of a single proton outside a closed ^{88}Sr core occupying the $1g_{9/2}$ single-particle state. The participating triads (LSJ) for this transition are (505), (515), (514), and (314). The (514) component is negligible because of its small nuclear matrix element (N_{LJ}). Table III gives the values of the nuclear matrix elements used in the shell-model description. Because small angular-momentum transfers are favored, the $S=1$ term is dominated by the (314) triad. One can get an upper limit to both V_0 and V_1 by setting V_1 and V_0, respectively, to zero and ignoring any core polarization. The resulting angular distributions are shown in Fig. 8. The upper limits obtained are 164 MeV for V_0 and 63 MeV for V_1. If one still ignores core polarization but takes $V_0 = 100$ MeV, which was recently[16] found to be a reasonable value, then an upper limit to V_1 is about 50 MeV. Figure 8 shows that the angular distribution is rather poorly reproduced by either the $S=0$ or $S=1$ term alone. Although the angular distribution can be improved by adjusting the ratio $|V_1/V_0|$, this has not been done here, since the effects of core polarization are not known for this transition. To get a rough estimate of the core

TABLE III. The nuclear matrix elements M_L and N_{LJ} relevant to the shell-model analysis in the text.

Transition					
$\frac{1}{2}^- \to \frac{9}{2}^+$	$p_{1/2} \to g_{9/2}$	M_5	N_{34}	N_{54}	N_{55}
		-0.28209	-0.37613	-0.04205	0.30902
$\frac{1}{2}^- \to \frac{3}{2}^-$	$p_{1/2}^{-1} \to p_{3/2}^{-1}$	M_2	N_{01}	N_{21}	N_{22}
		-0.28209	-0.32573	-0.11516	0.34549
$\frac{1}{2}^- \to \frac{5}{2}^-$	$p_{1/2}^{-1} \to f_{5/2}^{-1}$	M_2	N_{22}	N_{23}	N_{43}
		-0.28209	-0.23033	-0.06156	0.42649

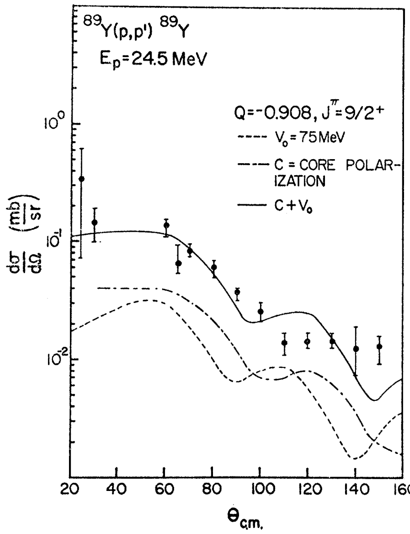

FIG. 9. Microscopic-shell-model fit to inelastic scattering from the first excited state of ^{89}Y including core polarization. Also shown are the individual contributions from the core and the single-particle transitions.

participation, however, the coupling parameter $Y_L(Q)$ [16] was taken to be the same as that found in the electromagnetic excitation of the 2.31-MeV 5^- level in ^{90}Zr. With this estimate of the strength of the core participation, a V_0 of about 75 MeV is required with $V_1=0$. The fit to the experimental cross section including core polarization is shown in Fig. 9. It is similar to that found when the collective model alone is used. This value of V_0 is intermediate between the value ($\sim$100 MeV) found at 19 MeV [16] for the excitation of the 5^- level in ^{90}Zr and the value (50 MeV) found at 62 MeV [7] for the excitation of this same level in ^{89}Y using the ^{90}Zr core parameters. This energy dependence is very similar to that found from analysis of ^{7}Li$(p,p')^7$Li.[8]

The 1.51- and 1.74-MeV states. Excitation of these states is considered to be via the excitation of a proton in the $2p_{3/2}$ or $1f_{5/2}$ state to the $2p_{1/2}$ state. For example,

$$|\tfrac{3}{2}^-\rangle = |2p_{3/2}{}^{-1}; 2p_{1/2}{}^2\rangle |^{88}\mathrm{Sr}\rangle,$$
$$|\tfrac{5}{2}^-\rangle = |1f_{5/2}{}^{-1}; 2p_{1/2}{}^2\rangle |^{88}\mathrm{Sr}\rangle,$$
$$|\tfrac{1}{2}^-, GS\rangle = |2p_{1/2}\rangle |^{88}\mathrm{Sr}\rangle.$$

This description is consistent with the small deformation parameters found for these states as well as the spectroscopic factors from a recent $(d,{}^3\mathrm{He})$ experiment by Preedom *et al.*[1] Actually the spectroscopic factor found for the pickup of a $1f_{5/2}$ proton indicates that there should perhaps be a $(f_{5/2}{}^{-1}g_{9/2}{}^2)$ component present in the $\frac{5}{2}^-$ state which is neglected in the present work. Coulomb excitation is included with the macroscopic form factors.

These two states are particularly interesting since excitation of the $\frac{3}{2}^-$ state introduces the (LSJ) triads (011), (211), (202), and (212), whereas excitation of the $\frac{5}{2}^-$ states involves the triads (212), (202), (213), and (413). Again the interference between different triads is neglected. Contributions from the (211) and (213) triads are negligible compared with the others because of small nuclear matrix elements. In the case of the excitation of the $\frac{3}{2}^-$ state, the nuclear matrix elements N_{01} and N_{22} are approximately equal but the calculation favors the lower L component. In the case of the $\frac{5}{2}^-$ state, however, the nuclear matrix element N_{43} is about equal to $2N_{22}$, so that, although small L transfers are favored, the triads (212) and (413) contribute about equally. The net result is that in the excitation of the $\frac{3}{2}^-$ state the $S=1$ part of the force gives a cross section that for $V_0=V_1$ is about 10 times as large as the $S=0$ cross section.[4] For the excitation of the $\frac{5}{2}^-$ state, however, the $S=1$ part of the force introduces an additional cross section that is approximately equal of that of the $S=0$ component with $V_0=V_1$. The relatively strong monopole $S=1$ term participating in the excitation of the $\frac{3}{2}^-$ state places an upper limit on V_1 of $\sim$36 MeV. An upper limit to V_0 obtained from this excitation is $\sim$120 MeV. For a V_0 of 100 MeV, a V_1 of 20 MeV is required. When core polarization is included, V_1 is reduced still further. When one compares the experimental angular distributions to these two states (Fig. 6), one realizes that the similarities rather than the differences in angular distributions are most apparent. As is shown in Fig. 10, any large component of $S=1$ in the case of the $\frac{3}{2}^-$ excitation will be hard to reconcile, since the (202) part of the cross section has a peak near 30°, whereas the monopole part does not. Having seen that the evidence points towards a rather weak $S=1$ term, we ignore it in studying the effects of core polarization.

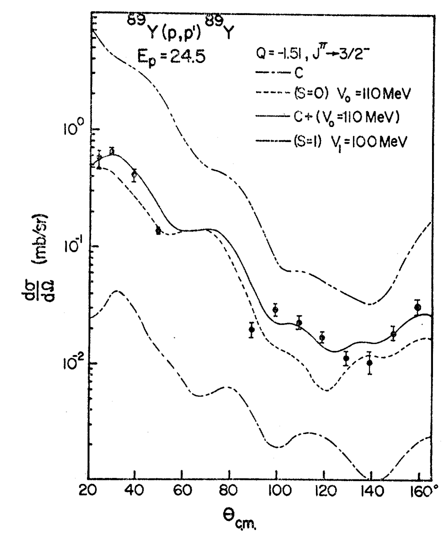

FIG. 10. Microscopic-model fits to scattering from the second excited state of ^{89}Y. The curve labeled C is the core-polarization contribution. The dashed curve labeled $S=0$ includes no core polarization or spin flip, whereas the solid curve includes core polarization. The curve labeled $S=1$ includes only spin flip.

The core-polarization parameters[16] are calculated from the measured electromagnetic transitions to these states as obtained from a recent electron scattering experiment.[3] These calculations yield

$$\tfrac{1}{2}^- \to \tfrac{3}{2}^-: \quad 1.08 \leq e_{\mathrm{eff}}/e \leq 1.68, \quad 1260 \leq C_2 \leq 10\,700 \text{ MeV},$$

$$\tfrac{1}{2}^- \to \tfrac{5}{2}^-: \quad 1.39 \leq e_{\mathrm{eff}}/e \leq 1.65, \quad 1150 \leq C_2 \leq 2550 \text{ MeV},$$

where C_2 is a measure of the core deformability.[16] Using the hydrodynamical value of $\hbar\omega_2$ ($\sim$15 MeV), the energy of the virtual core excitations, we find that the intensity of the admixed core state in the ground and excited states is on the order of 1%. The values obtained for the core parameter $Y_2(Q)$ associated with these excitations are given below in MeV^{-1}:

$$0.094 \times 10^{-3} < Y_2(1.51 \text{ MeV}) < 0.80 \times 10^{-3},$$

$$0.39 \times 10^{-3} < Y_2(1.74 \text{ MeV}) < 0.872 \times 10^{-3},$$

whereas in the excitation of the first 2^+ state in ^{90}Zr the Y_2(2.18 MeV) was found to be 1.61×10^{-3} MeV^{-1}. The approximate factor of 3 difference between these values and that found for ^{90}Zr seems significant. If the core excitation strength actually measured the participation of the ^{88}Sr core, then, ignoring the blocking effects due to the Pauli principle, one would expect the values of Y_2 for the three states to be approximately equal. Inclusion of the blocking effects due to the Pauli principle would tend to lower the value of Y_2 in ^{90}Zr relative to that in ^{89}Y. Since Y_2 is much larger in ^{90}Zr, we interpret the core-excitation process in a more general way as representing that part of Hilbert space that is omitted in the simple shell-model description. Figure 10 shows the inclusion of core excitation as described above applied to the excitation of the $\frac{3}{2}^-$ state. The calculation uses the mean value of the $B(E2)$'s and requires a V_0 of 110 MeV. However, because of the uncertainty in the value of the $B(E2)$, it can really only be said that $90 \lesssim V_0 \lesssim 120$ MeV. This is in agreement with other calculations[16] that include core polarization. The fit seems to be qualitatively better than either the direct or core-polarization terms alone. The range of values is an upper limit to V_0 because the $S=1$ part of the interaction has been neglected.

A similar analysis of the $\frac{5}{2}^-$ excitation was performed including core polarization. A discrepancy arises in this case, however, in that it is found that $165 \leq V_0 \leq 195$ MeV even when core polarization is included. This condition prevails because the $(2p_{1/2}, 1f_{5/2})$ form factor is smaller than the $(2p_{1/2}, 2p_{3/2})$ form factor by about a factor of $\sqrt{2}$. If the two form factors were approximately equal in strength, then a consistent value of V_0 could be found. In terms of the core participation, the coupling strength needs to be enhanced by about a factor of 2.4 in order to yield the correct magnitude for the cross section for a V_0 of 100 MeV. It is unlikely that the $S=1$ mechanism can account for the large experimental cross section of the $\frac{5}{2}^-$ state, since its inclusion would have a much larger effect on the $\frac{3}{2}^-$ cross section and lead to a much smaller value of V_0 for the $\frac{3}{2}^-$ excitation. Our phenomenological representation of the effective interaction may not represent the correct multipole behavior of the various parts of the force, although the inconsistency is, perhaps, more likely to be suggestive of a poor description of the $\frac{5}{2}^-$ state. An admixture of a $g_{9/2}{}^2$ configuration will improve the agreement of the state description with the available spectroscopic factor. Such an admixture would imply stronger core participation, since the $g_{9/2}{}^2$ component would not contribute to the excitation of this level. This might make the core parameters more compatible with those associated with ^{90}Zr. A similar admixture of a $g_{9/2}{}^2$ component in the $\frac{3}{2}^-$ state would also imply a stronger participation of the core in that transition and would also allow the upper limit on the magnitude of the $S=1$ term to be raised. Since a small admixture of the core might boost the cross section to agree with experiment, a description of the $\frac{5}{2}^-$ state was attempted which is just a generalization of the weak-coupling model, namely,

$$|\tfrac{5}{2}^-\rangle = a\,|1f_{5/2}{}^{-1}\rangle + b\,|(2p_{1/2} \times \hbar\omega_2)\tfrac{5}{2}^-\rangle.$$

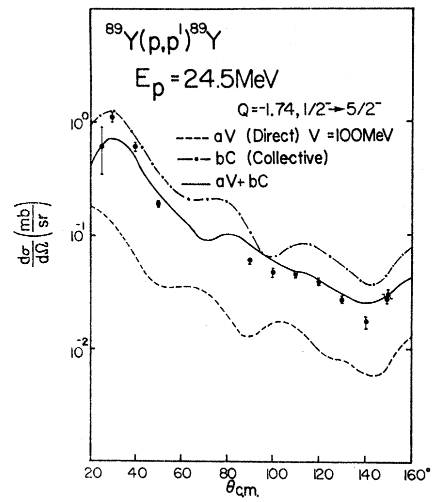

FIG. 11. Fit to the scattering of protons from the third excited state of ^{89}Y. A weak-coupling model is used with interference between direct and collective contributions.

The quantities a and b were taken to be real and were selected to yield the correct value of $B(E2)$ and approximately the correct cross section, V_0 being taken equal to 100 MeV. Such an analysis yields $a \simeq 0.87$ and $b \simeq -0.035\langle\hbar\omega_2\|\alpha_2^*\|0\rangle$. The fit to the data is then shown in Fig. 11. Also shown is the contribution due to the core and that due to the direct terms ($V_0 = 100$ MeV). Consistency between the cross section and $B(E2)$ is obtained for this state only by allowing the core and direct terms to interfere destructively. Now,

$$\langle\hbar\omega_\lambda\|\alpha_\lambda^*\|0\rangle = -(2\lambda+1)^{-1/2}\beta_\lambda,$$

where β_λ is the deformation parameter of the ^{88}Sr core in the presence of the extra core proton. Taking $\beta_2=0.114$ [1] (R_c is taken to be $1.25A^{1/2}$),

$$b \approx 0.64.$$

a^2+b^2 need not equal 1, since the two states are not necessarily orthogonal. This must certainly be regarded as a tentative and incomplete description of this state.

IV. CONCLUSION

The $\frac{9}{2}^+$ state at 0.908 MeV is characterized by a small ($\beta_5=0.06$) deformation parameter and seems to be reasonably well described in terms of a single-particle state when the coupling to the core is included. The quality of the fit is only fair, but neither the range of the force nor the mixture of $S=1$ was optimized.

Although the $\frac{3}{2}^-$ and $\frac{5}{2}^-$ states at 1.51 and 1.74 MeV are reasonably well described in terms of the collective model, a microscopic analysis in which the $\frac{3}{2}^-$ state was taken to be a hole in the $2p_{3/2}$ shell plus two $p_{1/2}$ particles outside the core was also found to be consistent with the data. The analysis of both of these states in terms of single-hole states is suggested by the small transition probabilities and by the large spectroscopic factors. The analysis of the $\frac{5}{2}^-$ state was not so straightforward, however, because the inelastic cross section seems to be enhanced relative to the electromagnetic transition. In this case, therefore, a V_0 much larger than that needed for the other states is required. Alternatively, the participation of the core is incompletely accounted for in terms of the corresponding electromagnetic transition. Other important uncertainties arise because we ignore knock-out terms and multiple excitation. In addition, the multipole dependence of the spin-dependent part of the force is unknown. Information obtained on the effective forces V_0 and V_1 are summarized in Tables IV and V. For a Yukawa force of range 1 F an upper limit to $|V_1|=|V_{1\alpha}+V_{1\beta}|$ of 36 MeV has been established. Since $V_{1\beta}$ has been shown to be approximately 11 MeV[20] at this energy, this puts an upper limit on $V_{1\alpha}$ of 47 MeV. Ignoring the difficulty with the $\frac{5}{2}^-$ state, it seems that a real Yukawa of range 1 F and a strength of about 75 to 100 MeV does represent at least the strength of the effective proton-proton interaction at 24.5 MeV when core polarization is included.

TABLE IV. Measured values of V_0 and V_1 assuming a Yukawa shape with range of 1 F.

State (MeV)	Core polarization	V_0 (MeV)	V_1 (MeV)
0.908	No	164	0
0.908	No	0	63
0.908	No	100	50
0.908	Yes	75	0
1.51	No	120	0
1.51	No	0	36
1.51	No	100	20
1.51	Yes	110	0

TABLE V. Information on V_0, the central part of the effective interaction, from ^{89}Y$(p,p')^{89}$Y* (0.908 MeV). The spin-flip part of the interaction V_1 is set equal to zero. A Yukawa shape with a range of 1 F is assumed.

E_p (MeV)	Core polarization	V_0 (MeV)	Reference
14.7	No	205	4, 17
18.9	No	205	2, 17
24.5	No	164	Present work
24.5	Yes	75	Present work
61.2	No	91	7
61.2	Yes	50	7

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[20] P. J. Locard, S. M. Austin, W. Benenson, and G. M. Crawley, Bull. Am. Phys. Soc. 12, 1178 (1967).