

for the ground-state energy are plotted as a function of the strength λ of the Yukawa potential. For $|\lambda| \gtrsim 2.0 \text{ F}^{-1}$, the ground-state values of Osborn are lower than even the lower bounds of our calculation. This indicates clearly that Osborn's results are not accurate and the collapse is a spurious phenomenon. For $\lambda = -1.0$ and -1.4 F^{-1} , however, our upper bounds are only about 1% different from the results of Osborn's calculation, indicating that his approximations are reasonably valid when the potential depth is relatively small.

From Fig. 2, we see that the values of $\sqrt{-E_U}$ lie very nearly on a straight line, while for $\sqrt{-E_L}$, there is a deviation from a straight line at large values of $|\lambda|$. This deviation is a consequence of the fact that we have used for E_1 in Eq. (2) the energy of the first excited state as reported by Osborn. This value of E_1 is very likely too low, since Osborn's first excited state also shows a collapse behavior for $|\lambda| \gtrsim 2 \text{ F}^{-1}$, which is probably spurious. In fact, if we assume a linear relationship between $\sqrt{-E_1}$ and λ , then for $\lambda = -3.0 \text{ F}^{-1}$, the value of E_1 would be -62.2 MeV instead of -156.0 MeV , which was used to obtain the lower bound in Table I. This new value of E_1 gives a slightly better lower bound of about -210 MeV , which does lie nearly on a straight line drawn through the values of $\sqrt{-E_L}$ for $|\lambda| \leq 2.0 \text{ F}^{-1}$.

IV. CONCLUSION

The purpose of this investigation is to compare the results obtained for a three-body system with a local

Yukawa potential using the Faddeev method and a variational technique. The result of this comparison shows that with the type of trial function used here, the variational method yields upper and lower bounds that are close to each other and bracket nicely the ground-state energy obtained by Ball and Wong. In particular, the value of the upper bound is nearly the same as Ball and Wong's result when the potential strength is not exceedingly small. On the other hand, our variational study does rule out definitely Osborn's claim of a ground-state collapse phenomenon, since his ground-state energy is even lower than the lower bound of our calculation.¹³

In concluding, it should be mentioned that the variational method, used in conjunction with the Monte Carlo technique, can be straightforwardly extended to four-body problems. This has already been done not only for purely attractive potentials,¹⁴ but also for potentials with a repulsive component.^{8,9} As is the case here, the variational bounds obtained in most of these four-body calculations are close to each other. This is satisfying, since it means that these bounds can be used to check the results of future calculations when an exact and numerically feasible method for the four-body system becomes available.

¹³ After this manuscript was completed, it was brought to our attention that very recently, Humberston *et al.* [J. W. Humberston, R. L. Hall, and T. A. Osborn, Phys. Letters **27B**, 195 (1968)], using a variational method, have reached the same conclusion regarding Osborn's results for large values of $|\lambda|$.

¹⁴ Y. C. Tang and R. C. Herndon, Nucl. Phys. **A93**, 692 (1967).

Study of $\text{He}^3 + \text{He}^4$ and $\text{H}^3 + \text{He}^4$ Systems with the Resonating-Group Method†

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The resonating-group method, which employs a completely antisymmetric wave function, has been used to consider the problem of elastic scattering of He^3 and H^3 by He^4 . The nucleon-nucleon potential used in the calculation is purely central, but fits the low-energy scattering data satisfactorily. The calculated phase shifts are compared with those which have been obtained from an analysis of experimental data, and rather good agreement is found. In particular, the calculated $l=3$ phase shift for both systems exhibits a resonance behavior which indicates the presence of a 2F level lying between the ${}^2F_{7/2}$ and ${}^2F_{5/2}$ levels found experimentally. Effective, local, l -dependent potentials between the He^4 cluster and the He^3 or H^3 cluster have also been derived. They are approximately energy-independent and contain features which have been found in phenomenological analyses of elastic scattering in light systems. Effects due to specific distortion of the He^3 or H^3 cluster have also been examined in an approximate fashion. Here it is found that the effects on the 2F excitation energy and on the nonresonant phase shifts are only of minor importance.

I. INTRODUCTION

THE present work is an extension of a previous investigation made by Tang, Schmid, and Wildermuth¹ (hereafter referred to as TSW), which was

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concerned with a study of the scattering of He^3 by He^4 , and which employed the resonating-group method in the one-channel approximation. This extension is con-

¹ Y. C. Tang, E. Schmid, and K. Wildermuth, Phys. Rev. **131**, 2631 (1963).

sidered desirable because extensive experimental data²⁻⁵ on the elastic scattering of He⁴ by both He³ and H³ have now become available, and these data have been carefully analyzed to yield phase-shift solutions for a number of partial waves. Thus a detailed comparison between theory and experiment should provide a rather critical test of the accuracy of the simplified version of the resonating-group method used in the present calculation.

In a study of the simpler case of He⁴+He⁴ scattering, Okai and Park⁶ have found that the values of the phase shifts obtained with the resonating-group method agree quite well with those determined experimentally. In the present calculation we also find a large measure of agreement with experiment. The fact that such agreement is obtained for both of these cases strengthens one's confidence that phase shifts calculated with the resonating-group method can be used as starting values for analyses of elastic-scattering data for more complicated systems, such as He³+H³. Such a use of the calculated phase shifts will help to alleviate ambiguities inherent in any attempt to extract phase shifts from a limited amount of experimental information.⁷ For example, in his analysis of *p*+He³ scattering, Tombrello⁸ has used this procedure to obtain a phase-shift set which correctly predicts the existence of resonance levels in the nucleus Li⁴.

In this study we shall also make a careful examination of the behavior of the scattering wave functions. The purpose here is to find effective, local potentials between the He⁴ and the He³ or H³ clusters which will yield the same scattering phase shifts as does the resonating-group calculation. The result is that a number of such potentials can indeed be found, and these have the properties of being angular-momentum-dependent and only moderately energy-dependent. They also contain the important features discovered by a number of authors^{5,9,10} from phenomenological, local-potential analyses of elastic-scattering data for systems involving light nuclei.

The effective potentials obtained here should be very useful in studies of more complicated systems, where a

straightforward application of the resonating-group method would be impractical because of mathematical complexity and a need for lengthy computing time. For instance, in an analysis¹¹ of the Λ -hypernuclear system ΛC^{13} , an effective α - α potential was employed, a procedure which reduced this complicated 13-body problem to a more amenable four-body problem.

The present calculation is made under the assumption that distortion of the clusters is properly accounted for by the use of a totally antisymmetrized wave function. This should be a fairly valid approximation for scattering at most energies, since the requirements of the Pauli principle tend to keep the two clusters apart throughout the collision.¹² When bound and resonant states having predominantly He⁴+He³ (or H³) structure are considered, however, it may be necessary to allow more adequately for the distortion of the He³ or H³ cluster. In this calculation we briefly examine the influence of this extra distortion by using a procedure previously employed in studying the energy levels of light nuclei,¹³ namely, by giving a certain degree of freedom to the internal motion of the He³ or H³ cluster.¹⁴ As has been discussed before,¹³ this procedure is relatively crude; nevertheless, we believe that it gives a fair estimate of the effect of such distortion on the energies of the bound and resonant levels.

In Sec. II we give a brief discussion of the formulation of this problem. In Sec. III the results of this calculation are presented. Here the calculated phase shifts and differential cross sections are compared with those determined experimentally; a study of the behavior of the scattering function and the effective potentials is also made. Section IV is a short discussion of the effect of distortion of the He³ or H³ cluster over and above that already implicitly given by the Pauli exclusion principle. Finally, in Sec. V we summarize the results of the present investigation and discuss several improvements which can be made in future calculations.

II. FORMULATION

The formulation of this problem follows closely that of TSW. For completeness, however, a brief description of the calculation will be given here. Basically, the method uses a variational technique in which is employed a cluster-type scattering wave function and a seven-particle Hamiltonian operator of the form

$$H = -\frac{\hbar^2}{2M} \sum_{i=1}^7 \nabla_i^2 + \sum_{i>j=1}^7 V_{ij}, \quad (1)$$

where M is the nucleon mass and V_{ij} is the potential

² R. J. Spiger and T. A. Tombrello, Phys. Rev. **163**, 964 (1967); R. J. Spiger, Ph.D. thesis, California Institute of Technology, 1967 (unpublished).

³ J. S. Vincent, E. T. Boschitz, and R. E. Warner, Bull. Am. Phys. Soc. **12**, 17 (1967). We wish to thank Professor B. F. Bayman for giving us a copy of this contributed paper.

⁴ M. Ivanovich, P. G. Young, and G. G. Ohlsen, Nucl. Phys. **A110**, 441 (1968).

⁵ F. Dunnill, T. J. Gray, H. T. Fortune, and N. R. Fletcher, Nucl. Phys. **A93**, 201 (1967).

⁶ S. Okai and S. C. Park, Phys. Rev. **145**, 787 (1966).

⁷ In the case of *d*+ α scattering, the phase-shift set obtained by L. S. Senhouse, Jr., and T. A. Tombrello, Nucl. Phys. **57**, 624 (1964), using mostly differential cross-section data, was indeed later found to be incorrect by L. C. McIntyre and W. Haeberli, Nucl. Phys. **A91**, 382 (1967), and by D. R. Thompson and Y. C. Tang, Phys. Letters **26B**, 194 (1968).

⁸ T. A. Tombrello, Phys. Rev. **138**, B40 (1965).

⁹ S. Ali and A. R. Bodmer, Nucl. Phys. **80**, 99 (1966).

¹⁰ J. L. Gammel and R. M. Thaler, Phys. Rev. **109**, 2041 (1958).

¹¹ R. C. Herndon and Y. C. Tang, Phys. Rev. **149**, 735 (1966).

¹² P. G. Burke and F. A. Haas, Proc. Roy. Soc. (London) **A252**, 177 (1959); H. S. W. Massey and B. L. Moiseiwitsch, *ibid.* **A205**, 483 (1951).

¹³ Y. C. Tang, K. Wildermuth, and L. D. Pearlstein, Phys. Rev. **123**, 548 (1961).

¹⁴ Such a precaution need not be applied to the α cluster because of its low compressibility (see Ref. 13).

energy of the nucleon pair (i, j). This two-nucleon potential is assumed to be purely central with exchange terms and has the form

$$V_{ij} = -V_0 \exp(-\kappa r_{ij}^2) (w + mP_{ij}^s + bP_{ij}^s - hP_{ij}^t) + \epsilon_{ij} e^2 / r_{ij}, \quad (2)$$

where the quantities P_{ij} are the space, spin, and isospin exchange operators, respectively, and $\epsilon_{ij} = 1$ if both i and j represent protons ($\epsilon_{ij} = 0$, otherwise). The exchange constants are taken to satisfy the normalization condition

$$w + m + b + h = 1, \quad (3)$$

and the ratio x of the s -wave singlet-to-triplet potential is given by

$$x = w + m - b - h. \quad (4)$$

For the parameters V_0 , κ , and x , we shall use the same values as did TSW for their potential I. These parameters are given by

$$V_0 = 72.98 \text{ MeV}, \quad \kappa = 0.46 \text{ F}^{-2}, \quad x = 0.63. \quad (5)$$

With these values the potential V_{ij} gives a reasonable fit to the low-energy, nucleon-nucleon scattering data. Thus in this formulation, with the exception of the exact nature of the exchange mixture, the Hamiltonian operator is completely specified by Eqs. (1), (2), and (5).

In the one-channel approximation, the scattering state will be represented by a completely antisymmetrized, seven-particle function Ψ of the form

$$\Psi = \mathcal{A}[\varphi_3 \varphi_4 F(\mathbf{r}) \xi(\sigma, \tau)], \quad (6)$$

where \mathcal{A} is an antisymmetrization operator, ξ is a spin-isospin function, and the vector $\mathbf{r} = \mathbf{R}_4 - \mathbf{R}_3$ connects the c.m. positions of the mass-3 and mass-4 clusters. The functions φ_3 and φ_4 are the internal spatial functions of the two clusters and are taken to have the following Gaussian forms:

$$\begin{aligned} \varphi_3 &= \exp(-\frac{1}{2}\bar{\alpha} \sum_{i=5}^7 (\mathbf{r}_i - \mathbf{R}_3)^2), \\ \varphi_4 &= \exp(-\frac{1}{2}\alpha \sum_{i=1}^4 (\mathbf{r}_i - \mathbf{R}_4)^2). \end{aligned} \quad (7)$$

In the above equations, the values of the width parameters $\bar{\alpha}$ and α are not the same as those used in TSW, but are chosen by using information from the most recent electron-scattering measurements.^{15,16} The rms radii a_3 and a_4 of the nucleon distribution in the clusters are related to the width parameters by the equations

$$a_3 = (1/\bar{\alpha})^{1/2}, \quad a_4 = (9/8\alpha)^{1/2}. \quad (8)$$

¹⁵ H. Collard, R. Hofstadter, E. B. Hughes, A. Johansson, M. R. Yearian, R. B. Day, and R. T. Wagner, Phys. Rev. **138**, B57 (1965).

¹⁶ R. F. Frosch, J. S. McCarthy, R. E. Rand, and M. R. Yearian, Phys. Rev. **160**, 874 (1967).

We use standard methods^{17,18} to extract these rms radii from the measured^{15,16} charge form factors. Thus we are led to choose

$$\bar{\alpha} = 0.362 \text{ F}^{-2}, \quad \alpha = 0.514 \text{ F}^{-2}, \quad (9)$$

which yield $a_3 = 1.66 \text{ F}$ and $a_4 = 1.48 \text{ F}$.

The function $F(\mathbf{r})$ which appears in Eq. (6) describes the relative motion of the two clusters and is obtained from the variational principle

$$\delta \int \Psi^* (H - E') \Psi d\tau = 0, \quad (10)$$

where E' is the total energy of the system. From this equation one derives an integrodifferential equation for $F(\mathbf{r})$, which shows that the interaction between the clusters is nonlocal. By use of the expansion

$$F(\mathbf{r}) = \sum_l r^{-1} f_l(r) P_l(\cos\theta) \quad (11)$$

and the integrodifferential equation for $F(\mathbf{r})$, one finds the following equation satisfied by the radial scattering function $f_l(r)$:

$$\left[\frac{\hbar^2}{2\mu} \left(\frac{d^2}{dr^2} - \frac{l(l+1)}{r^2} \right) + E - V_c(r) - V_l^*(r) \right] f_l(r) = 0. \quad (12)$$

Here μ is the reduced mass of the two-cluster system, E is the incident energy in the c.m. system, $V_c(r)$ is the Coulomb interaction energy of the two clusters, and $V_l^*(r)$ is the effective potential, which is given by

$$V_l^*(r) = V_D(r) + \frac{1}{f_l(r)} \int k_l(r, r') f_l(r') dr'. \quad (13)$$

Expressions for $V_D(r)$, $k_l(r, r')$, and $V_c(r)$ are given in TSW. In the determination of the Coulomb interaction, the antisymmetrization operator \mathcal{A} which appears in Eq. (6) is set equal to unity.¹⁹ The introduction of the effective potential $V_l^*(r)$ at this point does not aid in the actual solution of Eq. (12) for the scattering function $f_l(r)$, because $V_l^*(r)$ itself contains $f_l(r)$. Rather, Eqs. (12) and (13) are combined and solved, under the usual boundary conditions, for $f_l(r)$ and for either phase shifts δ_l or bound-state eigenenergies E , whichever are desired. The introduction of $V_l^*(r)$ simply makes transparent the fact that, once $f_l(r)$ has been obtained, Eq. (13) can be used to calculate a local potential which will yield the same scattering phase shifts as the actual nonlocal interaction. It should be noted that our $V_l^*(r)$ and δ_l are real because reaction channels are neglected, and that only the orbital angular momentum l enters the problem because a purely central nucleon-nucleon potential has been used.

¹⁷ L. I. Schiff, Phys. Rev. **133**, B802 (1964).

¹⁸ K. Okamoto and C. Lucas, Nucl. Phys. **B2**, 347 (1967).

¹⁹ For a discussion of the validity of this approximation see Ref. 6.

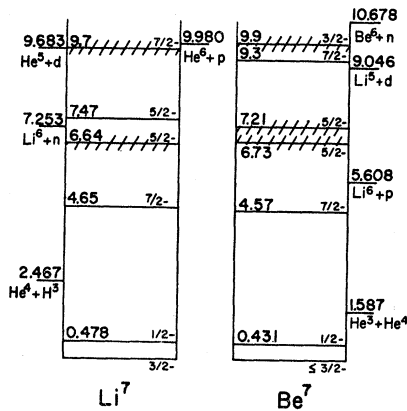


FIG. 1. Energy-level diagram for low-lying levels of Li^7 and Be^7 . The information is taken from Ref. 2.

The dependence of $f_l(r)$ on the exchange constants w , m , b , and h occurs only through the combination

$$A = 4w - m + 2b - 2h, \quad (14)$$

which serves to define A . This quantity is proportional to the strength of the spin-isospin-independent part of the nucleon-nucleon potential V_{ij} , and is the only adjustable parameter occurring in the present calculation. As is commonly done, we introduce a related parameter y in the following manner. We write the nucleon-nucleon potential V as a linear combination of a Serber potential V_S and a Rosenfeld potential V_R ; thus,

$$V = yV_S + (1-y)V_R. \quad (15)$$

Here V_S and V_R are given by Eq. (2) and

$$\text{Serber: } w = m, b = h, \quad (16)$$

$$\text{Rosenfeld: } m = 2b, h = 2w.$$

Under the assumption that both V_S and V_R have the same singlet-to-triplet ratio x , A and y are uniquely related by the expression

$$A = \frac{3}{4}(1+x)y. \quad (17)$$

This relation between A and y is useful because, from nucleon-nucleon scattering analyses, one expects y to be close to unity. In our calculation, however, we shall treat A as an adjustable parameter. We choose to do this because, as was mentioned in Sec. I, the effect of cluster distortion other than that given implicitly by the Pauli principle may not be insignificant. For example, it is known in the related problem of $\text{He}^4 + \text{He}^4$ elastic scattering²⁰ that this extra distortion effect gives rise to a weakly attractive potential between the clusters, and, therefore, it is felt that an adjustment in A may crudely compensate for the omission of such a polarization potential in our calculation.

Following TSW we adjust A (or, equivalently, y) so that our calculation yields the correct energy for the

²⁰ A. Herzenberg and A. S. Roberts, Nucl. Phys. 3, 314 (1957).

TABLE I. Calculated phase shifts, in degrees, for the $\text{He}^3 + \text{He}^4$ system. The parameters used are those of Eqs. (9) and (18). The c.m. energy \bar{E} is in MeV.

E	$l=0$	1	2	3	4	5	6
0.5	-1.51	-0.67	-0.01	0	0	0	0
1.0	-9.21	-5.03	-0.15	0.03	0	0	0
1.7	-22.31	-14.04	-0.83	0.32	-0.01	0	0
2.5	-35.35	-24.26	-2.25	1.84	-0.04	0	0
3.0	-42.24	-30.08	-3.32	4.42	-0.08	0.01	0
3.5	-48.31	-35.44	-4.44	10.35	-0.15	0.03	0
4.0	-53.73	-40.39	-5.53	27.16	-0.25	0.06	0
4.5	-58.60	-44.99	-6.56	83.63	-0.37	0.11	-0.01
5.0	-63.04	-49.29	-7.49	132.76	-0.52	0.17	-0.01
5.5	-67.13	-53.32	-8.30	147.13	-0.69	0.27	-0.02
6.0	-70.94	-57.12	-9.01	152.77	-0.88	0.40	-0.03
6.5	-74.25	-60.72	-9.62	155.62	-1.07	0.57	-0.05
7.0	-77.90	-64.14	-10.13	157.30	-1.26	0.78	-0.07
8.0	-84.20	-70.49	-10.93	159.13	-1.60	1.34	-0.13
10.0	-95.40	-81.68	-12.10	160.64	-1.91	3.07	-0.33
12.0	-105.21	-91.33	-13.23	161.20	-1.42	5.70	-0.59
14.0	-113.94	-99.85	-14.61	161.35	0.03	9.24	-0.86
16.0	-121.78	-107.50	-16.24	161.20	2.50	13.70	-1.04
18.0	-128.88	-114.47	-18.07	160.78	5.93	19.02	-1.07
20.0	-135.35	-120.87	-20.02	160.12	10.27	25.08	-0.91
22.0	-141.27	-126.79	-22.03	159.25	15.36	31.62	-0.54

bound 2P state (actually, the spin-orbit-split ground and first excited states—see Fig. 1) of Li^7 and Be^7 . Because the nucleon-nucleon potential of Eq. (2) can produce no spin-orbit splitting, we average the experimental energies of the $^2P_{3/2}$ ground state and $^2P_{1/2}$ first excited state, weighted according to the expectation value of $L \cdot S$. This procedure gives an average experimental 2P binding energy of 1.44 MeV for Be^7 and 2.31 MeV for Li^7 . A calculation of these binding energies as a function of A for both Be^7 and Li^7 shows that a value for A of

$$A = 1.258 \quad (18)$$

fits the averaged binding energy of both nuclei to within 60 keV. From Eqs. (17), (18), and (5), we then find that $y = 1.03$. It is satisfying that this value of y is

TABLE II. Calculated phase shifts, in degrees, for the $\text{H}^3 + \text{He}^4$ system. The parameters used are those of Eqs. (9) and (18). The c.m. energy \bar{E} is in MeV.

E	$l=0$	1	2	3	4	5	6
0.5	-9.45	-2.86	-0.05	0	0	0	0
1.0	-23.06	-10.23	-0.37	0.09	0	0	0
1.7	-37.49	-20.95	-1.36	0.81	-0.01	0	0
2.5	-49.46	-31.68	-2.91	4.38	-0.06	0.01	0
3.0	-55.40	-37.57	-3.91	11.88	-0.12	0.02	0
3.5	-60.54	-42.94	-4.84	40.73	-0.20	0.05	0
4.0	-65.08	-47.88	-5.64	120.55	-0.31	0.09	-0.01
4.5	-69.17	-52.45	-6.30	146.71	-0.45	0.15	-0.01
5.0	-72.93	-56.71	-6.81	154.05	-0.61	0.24	-0.02
5.5	-76.43	-60.71	-7.18	157.18	-0.77	0.36	-0.03
6.0	-79.73	-64.47	-7.43	158.81	-0.94	0.52	-0.04
6.5	-82.86	-68.03	-7.57	159.77	-1.10	0.72	-0.06
7.0	-85.85	-71.41	-7.64	160.38	-1.25	0.97	-0.09
8.0	-91.50	-77.69	-7.65	161.07	-1.45	1.61	-0.16
10.0	-101.75	-88.72	-7.67	161.54	-1.28	3.55	-0.36
12.0	-110.87	-98.22	-8.20	161.53	-0.05	6.39	-0.61
14.0	-119.06	-106.60	-9.35	161.26	2.38	10.16	-0.83
16.0	-126.48	-114.12	-10.96	160.76	6.04	14.84	-0.94
18.0	-133.26	-120.96	-12.90	160.05	10.84	20.38	-0.86
20.0	-139.47	-127.24	-15.02	159.15	16.63	26.61	-0.57
22.0	-145.20	-133.04	-17.24	158.08	23.12	33.23	-0.05

so close to unity. This result indicates that the present one-channel calculation may be reasonably good. The value of A found here differs from that found in TSW because the cluster sizes used here are not the same as in TSW. It should be emphasized that, once A is fixed in this manner, there are no adjustable parameters in the calculation, and the phase shifts, differential cross sections, and effective potentials may be computed in a straightforward fashion.

III. RESULTS AND DISCUSSION

A. Phase Shifts

The phase shifts up to $l=6$ for both He³+He⁴ and H³+He⁴ elastic scattering at c.m. energies 0.5–22.0 MeV have been calculated and are listed in Tables I and II. A comparison up to $l=4$ of these calculated phase shifts with the real part of experimentally determined phase shifts is shown in Figs. 2 and 3. The experimental phase shifts in Fig. 2 are those of Spiger and Tombrello,² while those in Fig. 3 include some phase shifts determined by Ivanovich, Young, and Ohlsen,⁴ as well.

The agreement between theory and experiment for $l=0, 1,$ and 2 below 7-MeV c.m. energy is good. Evidently the effect of reaction channels on the real part of the phase shifts is small up to 7 MeV, even though reactions become possible at as low an energy as 4.0 MeV (proton emission) for the He³+He⁴ system and 4.8 MeV (neutron emission) for the H³+He⁴ system. The calculated $l=3$ phase shift shows a resonance behavior

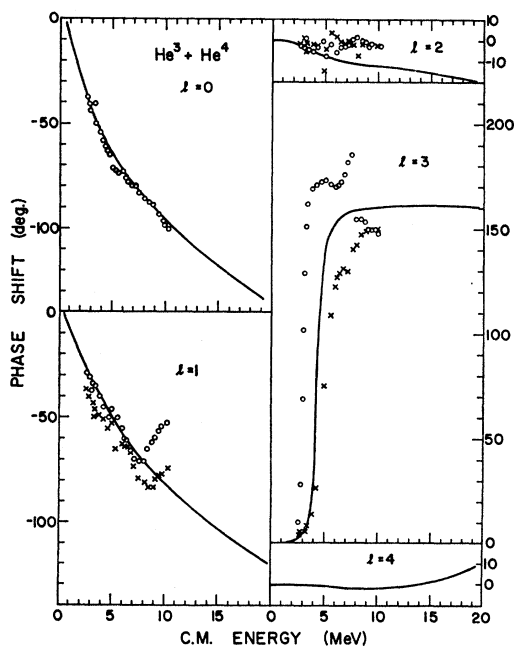


FIG. 2. Comparison of the calculated phase shifts for He³+He⁴ scattering with the experimental phase shifts of Ref. 2. The crosses represent the experimental values for $j=l-\frac{1}{2}$, and the circles represent those for $j=l+\frac{1}{2}$.

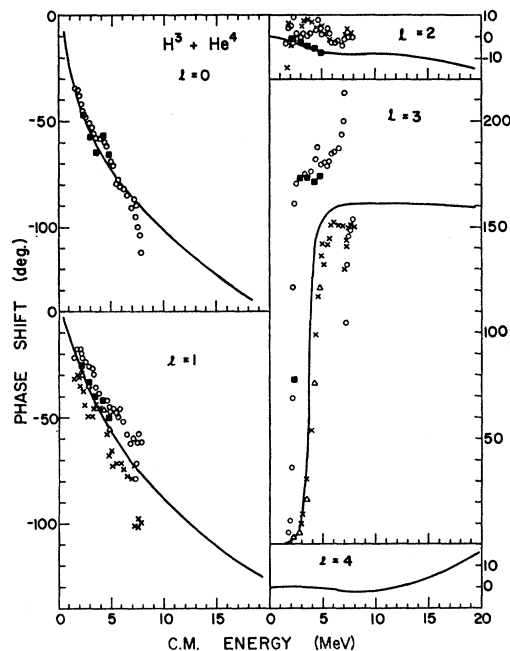


FIG. 3. Comparison of the calculated phase shifts for H³+He⁴ scattering with the experimental phase shifts of Refs. 2 and 4. The crosses and circles represent the experimental values of Ref. 2 for $j=l-\frac{1}{2}$ and $j=l+\frac{1}{2}$, respectively, and the triangles and squares represent those of Ref. 4 for $j=l-\frac{1}{2}$ and $j=l+\frac{1}{2}$, respectively.

very similar to that of the experimental points. In fact, it goes through resonance at an energy which lies between the energies at which the experimental ${}^2F_{7/2}$ and ${}^2F_{5/2}$ phase shifts go through resonance. This is just the type of behavior that one would expect from a calculation which omits spin-orbit effects, but which is otherwise correct. We can estimate from the experimental data the energy at which the calculated resonance should occur. To do this we average the experimental 2F energies, weighted according to the expectation values of $L \cdot S$, and obtain average c.m. resonance energies of 3.91 MeV for Be⁷ and 3.03 MeV for Li⁷, whereas the calculation gives 4.5 and 3.7 MeV, respectively, for these two resonances.

We believe that this discrepancy of about 0.6 MeV can be understood by invoking recently determined² properties of low-lying states in the mass-7 system. In the following discussion we will consider, for definiteness, the nucleus Be⁷. The 2P splitting is 0.431 MeV, and, on the basis of the $L \cdot S$ weighting procedure, the 2F splitting should be $7/3$ of this, or 1.01 MeV. However, the actual splitting is 2.16 MeV, and, furthermore, this large discrepancy exists even when theoretical calculations of the 2F splitting are made¹³ using a more accurate procedure than simple $L \cdot S$ weighting. Let us assume tentatively that the part of the 2F splitting due only to spin-orbit forces is the theoretically expected value of 1 MeV and see where this assumption leads. Since the resonating-group calculation gives the energy of an unsplit 2F level as 6.1-MeV excitation in Be⁷, we

would expect, on the basis of a total splitting of 1 MeV, that the ${}^2F_{5/2}$ level would occur at 6.7 MeV, and that the ${}^2F_{7/2}$ level would occur at 5.7 MeV. An inspection of Fig. 1 shows that the expected position of the ${}^2F_{5/2}$ level agrees with that found experimentally, whereas the expected position of the ${}^2F_{7/2}$ level is 1.1 MeV higher than that found experimentally. The agreement found for the ${}^2F_{5/2}$ level position may be fortuitous; however, to proceed with our argument we will assume that the agreement is significant. Under this assumption, explanation for the further lowering in energy of the ${}^2F_{7/2}$ level must be sought in effects other than those produced by spin-orbit interactions alone. We argue, in fact, that the inclusion in the calculation of an additional channel having an explicit $p+Li^{6*}$ structure could explain the occurrence of the ${}^2F_{7/2}$ level at 4.6 MeV rather than at 5.7 MeV. This expectation is based on the fact² that the $\frac{7}{2}^-$ level at 9.3 MeV has both a large reduced width for breakup into a proton and Li^6 in its first excited state (Li^{6*}) and a significant reduced width for α -particle emission. The implication then is that the ${}^2F_{7/2}$ level at 4.6 MeV also contains a significant

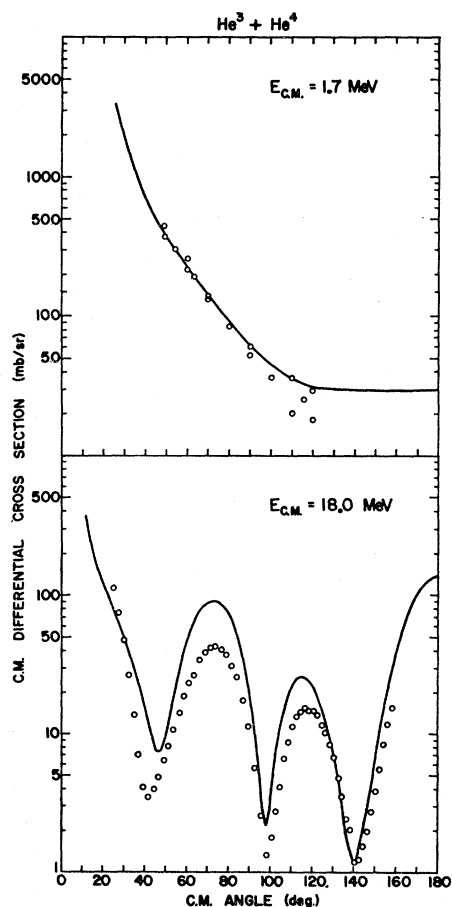


FIG. 4. Comparison of the calculated differential cross section for He^3+He^4 scattering with experimental data at 1.7 and 18.0 MeV. The data at 1.7 MeV are those of Ref. 21, and the data at 18.0 MeV are those of Ref. 3.

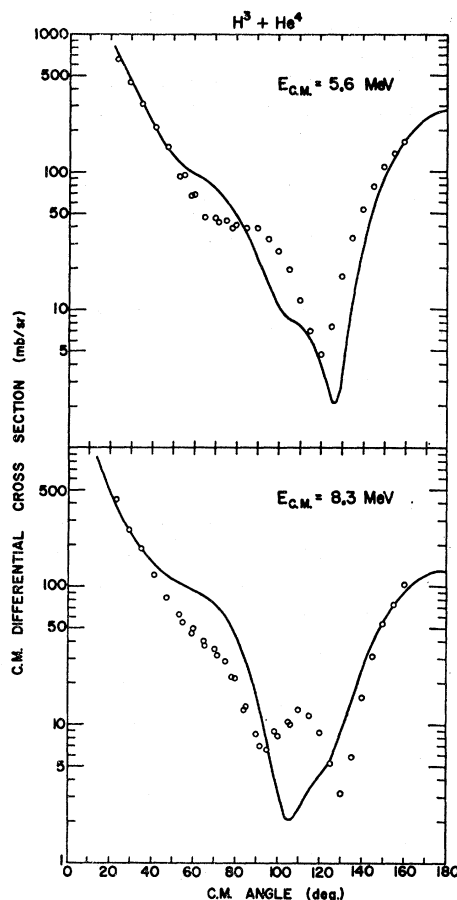


FIG. 5. Comparison of the calculated differential cross section for H^3+He^4 scattering with experimental data at 5.6 and 8.3 MeV. The data are those of Ref. 22.

amount of $p+Li^{6*}$ structure, and that, therefore, its coupling to the 9.3-MeV level of the same J^π but different L (caused, for example, by spin-orbit interactions) might be sufficient to account for its observed further lowering in energy. The position of the 6.7-MeV, ${}^2F_{5/2}$ level would not be expected to have been appreciably affected by the presence of channels other than that included in the present calculation. This is because the nearby $\frac{5}{2}^-$ level at 7.2 MeV has an α -particle reduced width which is less than 1% of that for the ${}^2F_{5/2}$ level, and hence the coupling between these two levels should be small.

B. Cross Sections

In Figs. 4 and 5 the calculated c.m. differential cross sections for He^3+He^4 and H^3+He^4 elastic scattering are compared with those from experimental measurements at several illustrative energies. As indicated by the phase-shift calculation (Sec. III A and Figs. 2 and 3), one expects good agreement between calculated and experimental differential cross sections at low energies where reactions are nonexistent or of weak intensity,

and where one is still below the influence of the ${}^2F_{7/2}$ resonance. Such agreement is shown in Fig. 4 for the 1.7-MeV data of Miller and Phillips.²¹ In this figure is also shown a comparison with the 18-MeV data of Vincent, Boschitz, and Warner.³ At this latter energy it is probably a good assumption that the energy splitting of the 2F states has a moderately small effect on the scattering cross section, and that reaction channel effects should be the principal cause of differences between the calculated and experimental differential cross sections. Figure 4 shows that, indeed, the most significant difference between theory and experiment is the noticeably larger magnitude of the calculated cross section, whereas the calculated positions of the maxima and minima agree well with the experimental positions. It is to be expected that the omission of reaction channels in the calculation would cause such a difference in the magnitude of the cross section; however, the effect of reaction channels on the positions of the maxima and minima in the present type of fully antisymmetrized calculation is not obvious. It is known, however, that when a standard optical model, which employs a local imaginary potential to describe reaction effects, is used to analyze elastic-scattering data, the positions of the maxima and minima are rather insensitive to the strength of this imaginary potential.

Figure 5 shows a comparison with the present calculation of the data of Flynn, Leland, and Rosen²² for H³+He⁴ elastic scattering at 5.6 and 8.3 MeV. Here the calculation reproduces the experimental shape only moderately well at 5.6 MeV and rather poorly at 8.3 MeV. It is believed that the major difference between calculation and experiment at 8.3 MeV is due to the proximity in energy of a broad resonance in the neutron-emitting channel caused by the 9.7-MeV level in Li⁷, which has significant widths for both neutron and α -particle emission. In fact, Wildermuth and McClure,²³ in their comprehensive discussion of cluster calculations, have pointed out that resonances in individual reaction channels can have a marked influence on the elastic-scattering cross section. At 5.6 MeV it is possible that the 9.7-MeV level has some effect on the experimental cross section, even though these data occur 0.5 MeV farther from the 9.7-MeV level than do the 8.3-MeV data. The influence of the 7.47-MeV level, however, can be ignored here, since it has a very small α -particle width.² In addition, the 5.6-MeV data may show some small effect of the spin-orbit splitting of the $l=3$ phase shifts (see Fig. 3), which effect is not included in our calculation.

²¹ P. D. Miller and G. C. Phillips, Phys. Rev. **112**, 2048 (1958).

²² E. R. Flynn, W. T. Leland, and L. Rosen (private communication).

²³ Karl Wildermuth and Walter McClure, in *Springer Tracts in Modern Physics*, edited by G. Hohler (Springer-Verlag, Berlin, 1966), Vol. 41.

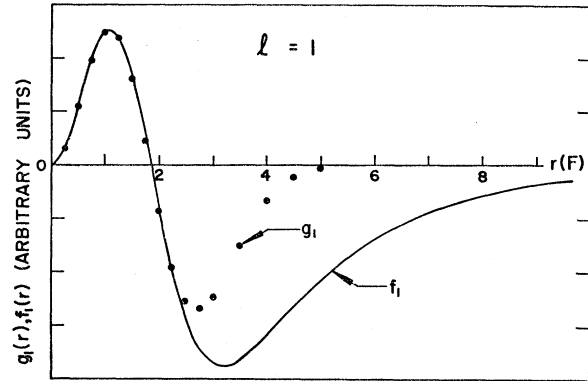


FIG. 6. The radial bound-state function for the 2P ground state of the He³+He⁴ system. The curve represents the function f_1 as calculated from Eq. (12), while the solid dots represent the function g_1 as calculated from Eq. (19).

C. Radial Wave Functions and Effective Potentials

Figure 6 shows the radial function $f_1(r)$ for the $l=1$ bound state of Be⁷ as determined from Eq. (12). For comparison we have also shown the values of a function $g_1(r)$ given by

$$g_1(r) = Cr^2(1 - 24\alpha_{av}r^2/35) \exp(-6\alpha_{av}r^2/7), \quad (19)$$

with $\alpha_{av} = 0.44 \text{ F}^{-2}$, which is a weighted average of the values of α and $\bar{\alpha}$ in Eq. (9). With this expression for $g_1(r)$, the function

$$\Psi = \alpha [\varphi_3^{av} \varphi_4^{av} r^{-1} g_1(r) P_1(\cos\theta) \xi(\sigma, \tau)], \quad (20)$$

with φ_3^{av} and φ_4^{av} given by Eq. (7), except that both α and $\bar{\alpha}$ are replaced by α_{av} , is the usual shell-model wave function describing the lowest configuration $(1s)^4(1p)^3$ in an oscillator well of width parameter α_{av} . Further, to facilitate comparison, the constant C is chosen such that the functions f_1 and g_1 have the same value at $r=1.1 \text{ F}$, the position of the first maximum. From Fig. 6 it is seen that for $r \gtrsim 2.5 \text{ F}$, $g_1(r)$, which according to Eq. (19) has an improper asymptotic behavior, differs markedly from $f_1(r)$, whereas at smaller radii f_1 and g_1 are nearly identical. This indicates that in situations where relatively small radii would dominate a calculation, the employment of $g_1(r)$ would be a useful simplification, since it is easier to calculate with the function $g_1(r)$ than with the function $f_1(r)$. However, to obtain quantitative results, the value for the constant C in Eq. (19) would certainly have to be correctly determined.

The function $f_1(r)$ shown in Fig. 6 has a rather large peak at $r=1.1 \text{ F}$, a value much smaller than the sum of the rms radii of He³ and He⁴. This, however, cannot mean that the two clusters have a large probability to be close to each other, because such a situation would violate the Pauli exclusion principle. Rather, one must realize that the function $f_1(r)$ occurs under the antisymmetrization operator, and thus has no clear interpretation in terms of cluster separation when the

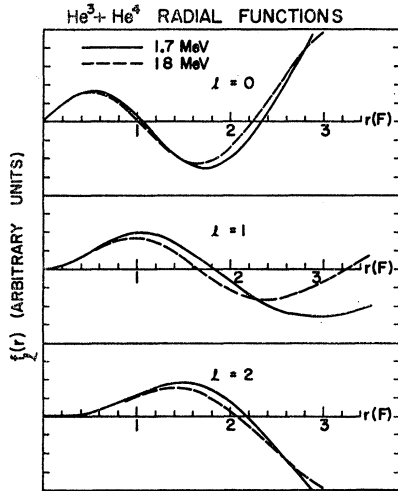


FIG. 7. The radial scattering function $f_l(r)$ of Eq. (12) for $l=0, 1, 2$. The curves shown are those at c.m. energies of 1.7 and 18.0 MeV for the $\text{He}^3 + \text{He}^4$ system.

clusters overlap strongly. In fact, to examine the probability for close packing of all seven nucleons within a sphere of radius R , one must calculate, with the properly normalized wave function Ψ , the value of the expression

$$P(R) = \int \Psi^* \prod_{i=1}^7 \theta(R - \tilde{r}_i) \Psi d\tau, \quad (21)$$

where \tilde{r}_i is the distance of the i th nucleon from the c.m. of Be^7 , and $\theta(t)$ is the unit step function given by

$$\begin{aligned} \theta(t) &= 1 \text{ for } t > 0 \\ &= 0 \text{ for } t \leq 0. \end{aligned} \quad (22)$$

The computation of $P(R)$ is quite difficult; however, we believe that it can be performed by using a Monte Carlo technique which has recently been devised for few-body problems.²⁴

The radial scattering functions $f_l(r)$ for $\text{He}^3 + \text{He}^4$ scattering with $l=0, 1$, and 2 at 1.7 and 18 MeV are shown in Fig. 7. Similar curves (not shown) are obtained for $\text{H}^3 + \text{He}^4$ scattering. Each of the functions in Fig. 7 has one of its zeros not far from $r = R_{\text{rms}}$, where R_{rms} is the rms radius of the folded-matter distribution and is given by

$$R_{\text{rms}} = (a_3^2 + a_4^2)^{1/2} = 2.25 \text{ F}. \quad (23)$$

The positions of these zeros are seen to be only weakly energy-dependent—a change of less than 0.2 F in these positions occurs when the energy is changed by almost 20 MeV. These positions, however, are more strongly l -dependent—a change of 0.6 F occurs when l is changed from 0 to 1.

The effective potentials $V_l^*(r)$, as defined in Eq. (13), are shown in Fig. 8 for $l=0$ at 1.7 and 18 MeV and for

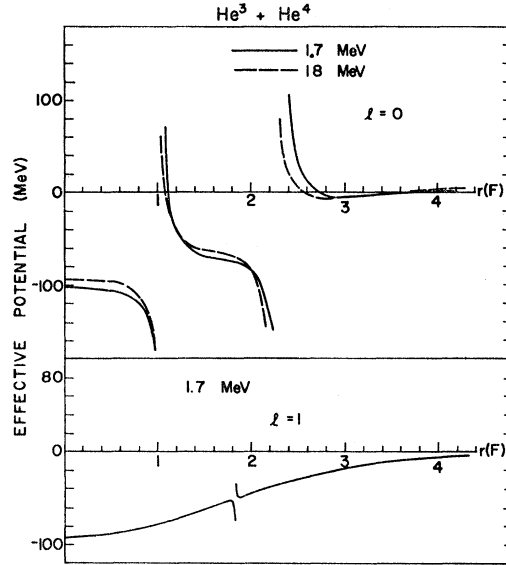


FIG. 8. The effective potential $V_l^*(r)$ of Eq. (13) for $l=0$ and 1 in the $\text{He}^3 + \text{He}^4$ system. The potential $V_0^*(r)$ is shown for c.m. energies of 1.7 and 18.0 MeV, and the potential $V_1^*(r)$ is shown for a c.m. energy of 1.7 MeV. Except for the positions of narrow singularities, the curve for $V_1^*(r)$ at 18.0 MeV is very close to the one shown at 1.7 MeV.

$l=1$ at 1.7 MeV. Because of the nonlocal nature of the interaction between the clusters, singularities occur in the effective potentials at values of r for which $f_l(r) = 0$. We should mention, however, that such singularities occur only because a one-channel approximation has been used in our calculation. If reaction channels had been included, then the function $f_l(r)$ occurring in Eq. (13) would be complex, and, in general, one would not expect the real and imaginary parts to vanish simultaneously. Therefore, the singularities in $V_l^*(r)$ would be absent; nevertheless, for small reaction probabilities, relatively rapid changes in $V_l^*(r)$ would still occur, and these changes would be increasingly damped for increasing reaction probabilities. Also, it should be noted that a singularity in the potential will have little effect on the scattering whenever the spatial extent of the singularity is small compared to the wavelength λ of relative motion of the two clusters. Thus, since large reaction cross sections will occur at relatively high energies, and large wavelength λ will occur at relatively low energies, in the region of surface interaction where the potential depth is small, one expects that in most cases²⁵ the potential $V_l^*(r)$ can be represented by a smoothed potential which closely resembles $V_l^*(r)$ but is free from narrow singularities. Such a potential is illustrated in Fig. 9 for $l=0, 1, 2$, and 3 at 8.3 MeV.

The above consideration of effective potentials also leads to the conclusion that the standard optical model, which employs smoothly varying local potentials, should attain its greatest validity at energies for which the scattering system of interest has relatively large

²⁴ R. C. Herndon and Y. C. Tang, *Methods in Computational Physics* (Academic Press Inc., New York, 1966), Vol. 6, p. 153.

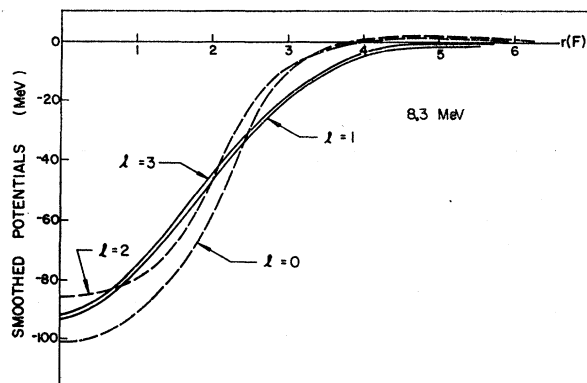


FIG. 9. Smoothed effective potentials as discussed in Sec. III C. These are shown at 8.3 MeV in the He³+He⁴ system for $l=0, 1, 2,$ and $3.$

reaction probabilities. Indeed, this is the usual domain of application of the optical model.

From a semiquantitative viewpoint, the smoothed potentials of Fig. 9 are quite similar to the real parts of optical potentials found by other authors^{3,5} in their phenomenological analyses of He³+He⁴ scattering data. For instance, our calculated central potential depth is around 90 MeV,²⁶ which is about equal to the value obtained by Dunnill, Gray, Fortune, and Fletcher.⁵ On the other hand, it is to be noted from Fig. 9 that our calculated potential has one interesting feature which is not contained in the usual optical potential. This is the odd-even effect, wherein the potentials in the odd-orbital-angular-momentum states are quite different from those in the even-orbital-angular-momentum states, and we see that this effect is particularly prominent in the important surface region. The lack of such a feature in the optical potential of Dunnill *et al.*⁵ might explain why their fits to the experimental data are rather poor. In the optical-model analysis of Vincent *et al.*³ at 18 MeV, a good fit to the experimental differential cross section was obtained; however, to accomplish this they had to use an imaginary potential with an unrealistically long range. It is our belief that, had they included the odd-even effect in their analysis, a fit of similar quality could have been obtained with, however, a more reasonable imaginary potential.

The odd-even effect found here has also been noted by Gammel and Thaler¹⁰ in their phenomenological analysis of p +He⁴ scattering. There it was found that to obtain good quality fits to experimental data, such a feature had to be contained in the potential. In particular these authors found that in the region 2–6 F the odd- l potentials are deeper than the even- l potentials,

²⁵ When the reaction cross section is small, the averaging effect of λ may not be sufficient to smooth the rapidly varying part of V_i^* which occurs at small values of $r.$

²⁶ Note that the central depth of 90 MeV is about equal to $\mu V/M,$ where V is the nucleon optical-potential depth of about 50 MeV.

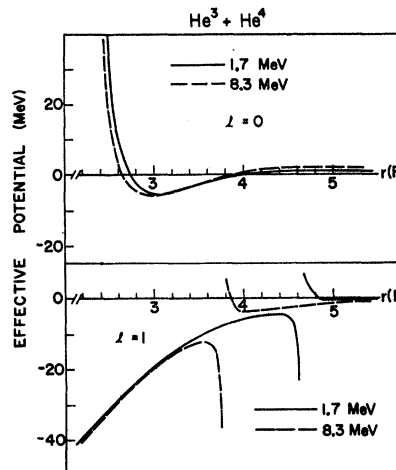


FIG. 10. The effective potential $\tilde{V}_l(r)$ of Eq. (24) for $l=0$ and 1 in the He³+He⁴ system. The curves shown are those at c.m. energies of 1.7 and 8.3 MeV.

a property which is also present in the smoothed potentials of our calculation.²⁷

Next we discuss the construction of another effective, local, l -dependent potential $\tilde{V}_l(r),$ which yields the same phase shifts as those obtained with our resonating-group calculation. This construction is motivated by the fact that in our antisymmetrized calculation the function $f_l(r),$ and consequently $V_i^*(r),$ has no clear interpretation in terms of cluster separation in the region $r \lesssim R_{rms},$ where the clusters overlap strongly. Thus, in the construction of an effective potential it is appropriate to de-emphasize this region. For $\tilde{V}_l(r)$ we shall therefore take a hard-core potential of the form

$$\begin{aligned} \tilde{V}_l(r) &= \infty & \text{for } r < r_{l0} \\ &= V_i^*(r) & \text{for } r \geq r_{l0}, \end{aligned} \quad (24)$$

where r_{l0} is the value of r at which the zero near $r = R_{rms}$ of $f_l(r)$ occurs.²⁸ For example, at 1.7 MeV the values of r_{l0} are equal to 2.33, 1.83, and 2.20 F for $l=0, 1,$ and $2,$ respectively. For $l \geq 3$ we set r_{l0} equal to zero, because, for values of E of interest here, the only zero of $f_l(r)$ occurring inside R_{rms} is at the origin. In addition, it should be mentioned that the centrifugal barrier becomes increasingly important as the orbital angular momentum l increases. This prevents the associated

²⁷ Note that this is not a general property of the odd-even effect. In $d+\alpha$ scattering, for example, the even- l potentials will be deeper than the odd- l potentials.

²⁸ It is appropriate to mention here that this particular way of constructing the hard-core potential has also been proposed by Okai and Park (Ref. 6) in their paper on α - α scattering. We should further point out that the features of the local potentials found in phenomenological analyses by other authors [Ref. 9; and O. Endo, I. Shimodaya, and J. Hiura, Progr. Theoret. Phys. (Kyoto) 31, 157 (1964)] of α - α scattering data are entirely consistent with those of our constructed potential. From the point of view of our present discussion, the use of a hard core in the phenomenological approach is merely an artifact which avoids the construction of a potential inside the region of $r \lesssim R_{rms}.$

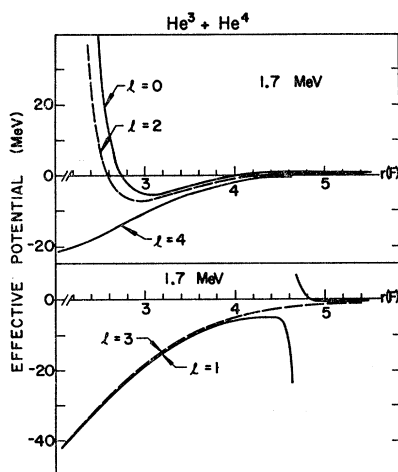


FIG. 11. The effective potential $\tilde{V}_i(r)$ of Eq. (24) for $l=0, 1, 2, 3,$ and 4 in the He^3+He^4 system at a c.m. energy of 1.7 MeV.

potential at small values of r from having much effect on the scattering.

The effective potentials $\tilde{V}_i(r)$ in the range $2-6$ F for various values of l and E are shown in Figs. 10 and 11. Figure 10 shows the energy dependence of \tilde{V}_0 and \tilde{V}_1 and indicates that this dependence is only moderate. The potential \tilde{V}_1 has a singularity near $r=4$ F, but, as has already been discussed, this singularity will have little influence on the scattering cross section. Figure 11 shows the potentials at a fixed energy of 1.7 MeV for $l=0$ to 4 . Here the most interesting point is that the shapes of the calculated even- l potentials are very similar to those obtained by Ali and Bodmer⁹ in their analysis of α - α scattering data using a phenomenological, l -dependent, energy-independent potential. For example, our calculated $l=4$ potential has no repulsive core, and indeed Ali and Bodmer have found that no repulsive core is needed in their $l=4$ potential.

To summarize, we have found a number of effective, local potentials which can yield the phase shifts of the resonating-group calculation. These potentials have the properties of being l -dependent and only moderately energy-dependent. Furthermore, they possess the feature of an odd-even effect, which has been previously noted in the optical-model analysis of $p+\text{He}^4$ scattering,¹⁰ but has not otherwise been widely used in analyses of scattering data with phenomenological potentials.

IV. SPECIFIC DISTORTION EFFECT

As mentioned in Sec. I, the effect of distortion of the He^3 or H^3 cluster over that already implicitly given by the Pauli principle might be expected to be important when bound and resonant states are considered. In the present section we report on a brief study of such an effect in the He^3+He^4 system. With our assumption of Eq. (6) for the total wave function, the mode of dis-

TABLE III. Parameters for the 2P bound state in the He^3+He^4 system. The values of $\bar{\alpha}$ and E_T are optimum for the given value of A .

A	$\bar{\alpha}$ (F ⁻²)	E_T (MeV)
1.15	0.444	-37.13
1.20	0.456	-37.51
1.25	0.470	-37.92

tortion for the He^3 cluster which can be most easily investigated is a radial compression. For the bound 2P state the optimum value of $\bar{\alpha}$, for a given value of A or y , is found as follows. One evaluates the total energy E_T of the system given by²⁹

$$E_T(\bar{\alpha}) = E(\bar{\alpha}) + E_\alpha + E_h(\bar{\alpha}), \quad (25)$$

where E_α is the experimental energy of the α particle (-28.30 MeV), and E_h is the energy of the He^3 cluster. The normal procedure would then be to use for E_h the expression

$$E_h = \frac{3\hbar^2\bar{\alpha}}{2M} - 3(w+m)V_0 \left(\frac{\bar{\alpha}}{\bar{\alpha}+2\kappa} \right)^{3/2} + e^2 \left(\frac{2\bar{\alpha}}{\pi} \right)^{1/2}, \quad (26)$$

which is the expectation value of the He^3 Hamiltonian with the Gaussian function of Eq. (7).¹ A variation of the parameter $\bar{\alpha}$ until E_T attains its minimum value would then yield the optimum values of $\bar{\alpha}$ and E_T as a function of A or y . Unfortunately this straightforward procedure will lead to an inaccurate estimate of the importance of the distortion effect. This is because a nonsaturating nucleon-nucleon potential is used in our calculation, and hence the value of $\bar{\alpha}$ which minimizes E_h of Eq. (26) corresponds to a value of the rms radius for He^3 which is much smaller than the experimental value. Thus, in order to obtain a more realistic estimate of the radial distortion effect, we shall adopt a method which has been used in previous cluster-model calculations.¹³ This method consists in replacing E_h of Eq. (26) with the relation

$$E_h = -7.72 + \frac{1}{8}K(1 - 0.362/\bar{\alpha})^2 \text{ MeV}, \quad (27)$$

which involves a quantity K , the rigidity of the He^3 cluster. For the estimation of K we use the result of a calculation³⁰ on the ground-state properties of He^3 with a potential having a hard core of radius 0.45 F. This yields

$$K = 80 \text{ MeV}, \quad (28)$$

which should be a reasonably good estimate since the energy, rms radius, and charge form factor of He^3 calculated with this particular potential agree very well with the corresponding experimental values. The result

²⁹ The quantity E_T here is not the same as the quantity E' of Eq. (10). The expectation value of the α -particle Hamiltonian with the Gaussian function of Eq. (7) is used in the expression for E' , while the experimental value of the α -particle energy is used in the expression for E_T .

³⁰ Y. C. Tang and R. C. Herndon, Phys. Letters **18**, 42 (1965).

of the variation of $\bar{\alpha}$ in Eqs. (25), (27), and (28) for three values of A is given in Table III. From this table and the fact that the experimental value of E_T is -37.46 MeV we find, with the radial distortion effect taken into account, that the quantities A , $\bar{\alpha}$, and a_3 take on the following values:

$$A = 1.194, \bar{\alpha} = 0.454 \text{ F}^{-2}, a_3 = 1.48 \text{ F}. \quad (29)$$

The changes from the values of 1.258, 0.362 F^{-2} , and 1.66 F, respectively, for the calculation without distortion are not large. Consequently the indication is that the He³ cluster is sufficiently rigid so that the amount of radial distortion is small in the ground state of Be⁷.

With $A = 1.194$ we have also considered the 2F resonant state with the effect of distortion of the He³ cluster taken into account. It is found that the optimum value of $\bar{\alpha}$ is 0.391 F^{-2} and that the 2F excitation energy is 6.5 MeV. This latter value is only 0.4 MeV larger than the value when distortion is neglected, thus indicating that the distortion effect is also not important for the 2F resonant state in Be⁷.

The value of A is changed when distortion of the He³ cluster is considered. We have therefore calculated the nonresonant phase shifts with $A = 1.194$ and $\bar{\alpha} = 0.362 \text{ F}^{-2}$ and have compared them with those of the no-distortion calculation. The comparison shows that the differences are quite small. We conclude, therefore, that no significant modifications of the results presented in Sec. III are made necessary by the existence of the specific distortion effect.

V. CONCLUSIONS

This investigation shows that, for incident energies at which reaction cross sections are relatively small, the resonating-group method in the one-channel approximation can be used quite successfully to explain the elastic scattering of He³ or H³ by He⁴. With a near-Serber mixture for the nucleon-nucleon potential, it is found that, when the c.m. energy is less than about 7 MeV, the phase shifts calculated with this method agree very well with those determined from phenomenological phase-shift analyses of experimental data.

In the energy region where there exist individual levels of the compound system which have significant amounts of both He³+He⁴ (or H³+He⁴) and other cluster structures, our one-channel approximation leads, as expected, to less favorable comparison with experiment. Here we have found, for example, that the angular dependence of the differential cross section is not reproduced satisfactorily by the calculation. To

remedy this, it appears that one would have to include other channels. For instance, if the n -Li^{6*} channel had been included in the calculation of H³+He⁴ scattering at 8.3 MeV, the resultant agreement with experiment might have been greatly improved.

At energies where there are a large number of open reaction channels, such as at the 18-MeV comparison of Fig. 4, we note that the major difference between theory and experiment is the larger magnitude of the calculated cross section, whereas the calculated positions of the maxima and minima agree well with the experimental positions. This suggests that, at such energies, the simplest way to improve the calculation would be to include a phenomenological imaginary potential in the formulation. The more desirable approach of explicitly performing a many-channel calculation is presently impractical.

For simplicity, spin-orbit interactions have not been included in our calculation. This is not a serious defect, but does prevent us from making a detailed comparison between calculated and experimental differential cross sections in energy regions where resonant levels of He³+He⁴ or H³+He⁴ structure exist. Thus, in these particular energy regions, it would be desirable to include a spin-orbit component in the nucleon-nucleon potential. As has been shown in corresponding bound-state calculations,¹³ this inclusion will make the computation more involved but not prohibitive.

In addition, we have used our calculated radial scattering functions to find a number of effective local potentials between the He⁴ and the He³ or H³ clusters. These potentials allow one to obtain some insight into the scattering calculation by means of replacing the complicated antisymmetrized problem by a conceptually simpler unantisymmetrized problem akin to that of the standard optical model. Moreover, such potentials will be useful in making approximate calculations on the properties of heavier nuclei for which a description as a system of clusters is appropriate. We have also noted that, besides their l dependence, these effective potentials possess an odd-even feature. This latter feature has previously been observed in a phenomenological analysis of p +He⁴ scattering data, but has not been used in usual optical-model analyses of nucleon-nucleus scattering.

ACKNOWLEDGMENTS

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