^{6}Li$(p,p')^6$Li(3.56 MeV) Reaction from $E_p=24.3$ to 46.4 MeV and the Effective Interaction*

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The total cross section for the ^{6}Li$(p,p')^6$Li(3.56 MeV) reaction was measured by observing the isotropic 3.56-MeV de-excitation γ rays in a 20-cm^3 Ge(Li) detector and was found to decrease slowly with energy. In a microscopic model of the reaction, this cross section depends only on the spin-isospin–dependent part V_{11} of the effective interaction. At energies above 34 MeV, the data are well described by a potential, $V_{11}=Ve^{-\alpha r}/\alpha r$, where the range $1/\alpha$ is taken to be 1.0 F, and $V=12.1$ MeV. At 25 MeV, $V=11.1$ MeV fits the data. The impulse approximation predicts a cross section larger than the data for the energy range of this experiment, the ratio decreasing from 2.9 at 25 MeV to 1.4 at 45 MeV. Available evidence on the spin-isospin–dependent interaction is summarized. For proton energies between 23 and 52 MeV, the strength of V_{11} lies in the range 12.5 ± 2.5 MeV and does not appear to vary with energy.

I. INTRODUCTION

In a microscopic model[1,2] the cross section for the inelastic scattering of protons depends on a nuclear matrix element which contains both the wave functions of the target and the effective interaction V_{eff} between the projectile and the target nucleons. Thus, if one wishes to use inelastic scattering as a spectroscopic tool, he must have foreknowledge of V_{eff}. However, except at energies well above 100 MeV, where V_{eff} is essentially the free nucleon-nucleon interaction,[1,3] there is no simple way to determine V_{eff} from external information. For this reason, several authors have studied inelastic scattering in cases where the wave functions are expected to be particularly simple or particularly well known.[3–6]

Two approaches have been taken. In one of these V_{eff} is obtained from the known two-body force (impulse approximation)[1] and the calculation contains no adjustable parameters. Such studies have been aimed primarily at determining the region of validity of the impulse approximation. Fairly good agreement with experiment is obtained for energies greater than 45 MeV.[3,5]

The other approach is empirical in nature. Here V_{eff} is assumed to have a simple form, often Yukawan, and its parameters are fixed by comparison with experiment.[4,6] If such an approach is to be useful for spectroscopy, the parameters of the empirical interaction must not depend strongly on the target nucleus and the multipolarity of the transition.

Most of the transitions studied to the present time are dominated by the spin-independent part of V_{eff}. However, certain transitions in the light nuclei are sensitive primarily to the spin-dependent part of the interaction. The energy dependence of the ^{7}Li(p,n)-^{7}Be(431 keV) reaction has been studied from 23 to 52 MeV[4] and is consistent with a constant value of the spin-isospin–dependent interaction V_{11}. Measurements of the ^{14}C$(p,n)^{14}$N and ^{18}O$(p,n)^{18}$F reactions[7,8] near 14 MeV have also yielded values of V_{11}.

In this paper, we describe a measurement of the variation with energy of the total cross section for the ^{6}Li$(p,p')^6$Li(3.56 MeV) reaction between 24.3 and 46.4 MeV. Because of the quantum numbers of the states involved only V_{11} can contribute to this reaction, so the cross section can be analyzed to yield this quantity directly. In Secs. II and III, the experimental procedure and results are described. In Sec. IV we present a brief resume of the theory. The available data on V_{11} are summarized in Sec. V and are discussed in Secs. VI and VII.

II. EXPERIMENTAL PROCEDURE

The energy levels of ^{6}Li are shown in Fig. 1. The 3.56-MeV state has a spin and parity of 0^+, so the angular distribution of γ rays leading to the ground state is isotropic in the rest frame of the recoiling nucleus and approximately isotropic in the lab. In addition, this state is the highest-lying particle-stable state,[9] so it is not fed with appreciable probability by γ-ray transitions from above. For these reasons, a measurement of the intensity of the 3.56-MeV γ-ray at a single angle is a measure of the total cross section for the ^{6}Li$(p,p')^6$Li-(3.56 MeV) reaction.

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[1] A. K. Kerman, H. McManus, and R. M. Thaler, Ann. Phys. (N. Y.) 8, 551 (1959).

[2] V. A. Madsen, Nucl. Phys. 80, 177 (1966); G. R. Satchler, *ibid.* 77, 481 (1966); N. K. Glendenning and M. Veneroni, Phys. Rev. 144, 839 (1966).

[3] R. M. Haybron and H. McManus, Phys. Rev. 140, B638 (1965).

[4] P. J. Locard, S. M. Austin, and W. Benenson, Phys. Rev. Letters 19, 1141 (1967).

[5] H. McManus, F. Petrovich, and D. Slanina, Bull. Am. Phys. Soc. 12, 12 (1967); F. Petrovich, D. Slanina, and H. McManus, Michigan State University Report No. MSPT-103, 1967 (unpublished).

[6] G. R. Satchler, Nucl. Phys. A95, 1 (1967).

[7] C. Wong, J. D. Anderson, J. McClure, B. Pohl, V. A. Madsen, and F. Schmittroth, Phys. Rev. 160, 769 (1967).

[8] S. D. Bloom, J. D. Anderson, W. F. Hornyak, and C. Wong, Phys. Rev. Letters 15, 264 (1965).

[9] T. Lauritsen and F. Ajzenberg-Selove, Nucl. Phys. 78, 1 (1966).

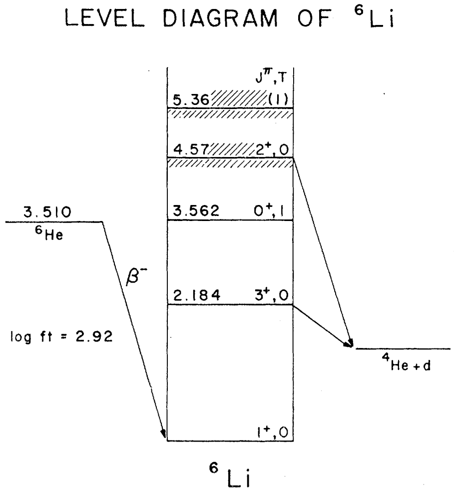

FIG. 1. Energy-level diagram of ^{6}Li adapted from Ref. 9.

ronous cyclotron bombarded a target rolled to a thickness of 1.0 mm from lithium metal enriched to 99.3% ^{6}Li.[10] The energy loss ΔE in this target ranged from 850 keV at 25 MeV to 500 keV at 46.4 MeV. The mean proton energies E_p were obtained by subtracting $\frac{1}{2}\Delta E$ from the proton energy determined from the beam transport system and are accurate to ± 0.1 MeV.

The de-excitation γ rays were detected in a 20-cm^3 Ge(Li) detector placed 12 cm from the target and at the largest convenient angle (155°) to the beam to minimize Doppler-shift effects and to reduce the background from the forward-peaked ^{6}Li(p,n) neutrons. The energy resolution of the detection system was 13 keV.

III. RESULTS

Figure 2 shows spectra taken at $E_p=24.3$ and 46.4 MeV. Spectra at other angles were similar, with roughly the same ratio of signal to background. For the particular detector geometry used, the 3.56-MeV double-escape peak contained most of the events, and the analysis was performed on this peak. The peak is substantially broadened by Doppler shifts. To facilitate an accurate background subtraction, the line shape at 24.3 MeV was computed[11] using an angular distribution previously measured [12] at 24.4 MeV. The result is shown in Fig. 3, where the calculated line shape is compared with a line shape obtained from the γ-ray spectrum by subtracting a linear background fitted to the regions labelled "Bg" shown in Fig. 2. The calculated and experimental line shapes agree quite closely except in a region near channel 510, where background γ rays (see below) are known to contribute. The same channels

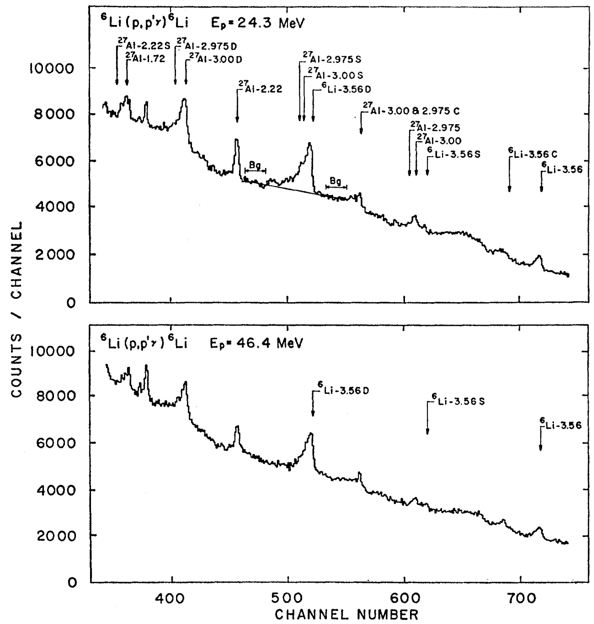

FIG. 2. γ-ray spectrum from ^{6}Li(p,p') at the highest and lowest energies of the experiment. In this figure C = Compton edge, S = single-escape peak, D = double-escape peak. Thus the peak labeled ^{6}Li-3.56D is the double-escape peak of the 3.56-MeV deexcitation γ ray from ^{6}Li. The labels are probable identifications of the source of the extraneous peaks in the spectrum. The regions marked Bg were least-squares-fitted with a linear background; the same channels were fitted at all energies.

fitted in the 24.3-MeV spectrum were fitted to determine backgrounds for all spectra in this experiment. As can be seen from the γ-ray spectrum taken at 46.4 MeV, the contribution from the tail of the line shape appears to be substantially less at the higher energies. This can be taken as evidence that the backward peak seen in the angular distribution at 24.4 MeV is less prominent at higher energies.

There are several sources of background which must be considered in the evaluation of these data. γ rays can be produced by interactions of scattered protons or neutrons in the aluminum beam pipe and neutron-induced reactions can take place in the Ge(Li) de-

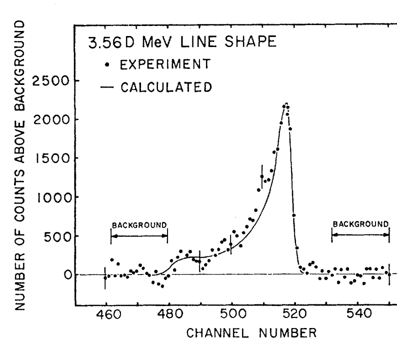

FIG. 3. γ line shape at 24.3 MeV. The method used to calculate the line shape and the deviation from the calculations near channel 510 are discussed in the text of Sec. III.

[10] Purchased from the Oak Ridge National Laboratory, Oak Ridge, Tenn.

[11] This computation was performed using a FORTRAN program written by J. J. Kolata and described by J. J. Kolata, R. Auble, and A. Galonsky, Phys. Rev. **162**, 957 (1967).

[12] G. M. Crawley and S. M. Austin, in *Proceedings of the International Nuclear Physics Conference*, edited by R. L. Becker (Academic Press Inc., New York, 1967), p. 165.

tector. Several peaks due to the former effects are observed in the spectra of Fig. 2. A possible explanation for these peaks is found in neutron or proton inelastic scattering from aluminum. Similar peaks were observed in the target-out spectra but these peaks were somewhat smaller than those observed with the target in position. Thus we are forced to ascribe part of the background to neutrons from the $^6Li(p,n)$ reaction. For this reason, background spectra measured with the target removed were not adequate and were not used in the analysis of the data.

Based on the spectra in Fig. 2, one expects that the single-escape peaks of the 3.000 and 2.975 γ rays fall in the region of interest. However, estimates based on deviations from the calculated line shape of Fig. 3 and on the relative size of the single- and double-escape peaks indicate that they amount to only 7% of the counts ascribed to the 3.56-MeV γ ray. Since the ratio of the (3.000D+2.975D) peak to the 3.56D-MeV peak varies by at most 20%, this contamination introduces an error of at most 2% in the relative cross section.

Introduction of 5 cm of Pb reduced the 3.56-MeV peak by (94 ± 6)% compared to the expected 91%. The transmission of neutrons with energies above 5 MeV is about 40%, so any contribution from neutron interactions in the detector must be small.

A correction should be applied to the data to account for the fact that an angular distribution of γ rays isotropic in the rest frame of the recoiling nucleus is anisotropic in the laboratory. An estimate of this effect has been made at 24.4 MeV, where a proton angular distribution is available and the total anisotropy is found to be about 4%. The anisotropy is expected to change only slowly with energy and not to substantially affect the relative cross section, so this correction was not made.

The data, normalized to a cross section obtained by integrating an existing angular distribution at 24.4 MeV,[12] are shown in Fig. 4. The errors shown include statistics and an allowance for difficulties in background subtraction. In addition, there is a possible error in the normalization of about 8%.

IV. THEORY

In the distorted-wave theory of inelastic scattering the transition amplitude has the form[1,2]

$$T_{fi}=\int \chi_f^{(-)*}(\mathbf{r})\langle\psi_f|V_{\rm eff}|\Psi_i\rangle\chi_i^{(+)}(\mathbf{r})d\mathbf{r}.$$

The χ_f and χ_i are distorted waves generated from an optical model using parameters which fit the elastic scattering. In a microscopic picture of the reaction ψ_f and ψ_i are shell-model states and $V_{\rm eff}$ is the effective interaction causing transitions between these states. This expression neglects particle exchange.

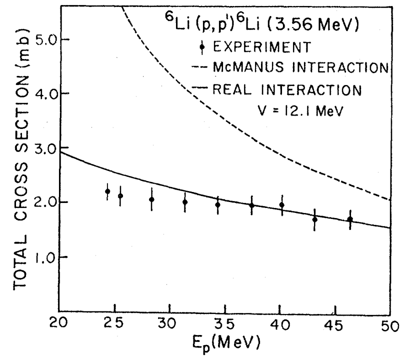

FIG. 4. Variation of the total cross section with energy. The solid curve was calculated using a real interaction with a constant strength of 12.1 MeV and a range $1/\alpha=1.0$ F. The dashed curve was obtained by using the impulse approximation interaction derived by McManus *et al.* (Ref. 5).

It is usual[1–3,5,6] to write

$$V_{\rm eff}=\sum_i t_{ip},$$

where t_{ip} is the scattering amplitude of the projectile from the ith target nucleon and the sum is over the active nucleons of the target nucleus. Further, t_{ip} is approximated[2,5,6] by the expression

$$t_{ip}=V_{00}+V_{10}\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_p+V_{01}\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_p+V_{11}(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_p)(\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_p),$$

where $\boldsymbol{\sigma}_i$ and $\boldsymbol{\sigma}_p$ are the spin operators for the target nucleon and the projectile and $\boldsymbol{\tau}_i$ and $\boldsymbol{\tau}_p$ are the analogous isospin operators. This expression does not include the tensor and spin-orbit forces known to be present in the free nucleon-nucleon interaction. Thus one expects to find cases where $V_{\rm eff}$ is not sufficient to account for the phenomena. Tensor forces have been invoked to explain the results of studies of $^{14}C(p,n)$-^{14}N(g.s.) and $^{14}N(p,p')^{14}N$(2.311 MeV) reactions.[7,13]

The selection rules for this process are obtained from the following relationships:

$$\mathbf{J}_f-\mathbf{J}_i=\mathbf{J},\quad \mathbf{T}_f-\mathbf{T}_i=\mathbf{T},\quad \mathbf{S}_i-\mathbf{S}_f=\mathbf{S},\quad \mathbf{J}=\mathbf{L}+\mathbf{S},$$
$$(\pi_f)(\pi_i)=(-1)^L,$$

where the transition is $(J_i^\pi,T_i)\rightarrow(J_f^\pi,T_f)$. J, L, and S are the total, orbital, and spin angular momentum transferred in the reaction. T is the transferred isospin. Since the proton has spin and isospin equal to one-half, we have

$$S=0,1;\quad T=0,1.$$

The transition in ^6Li is $(1^+,0)\rightarrow(0^+,1)$, so that $J=1$; $L=0$, 2; $S=1$; $T=1$. In the formulation of the theory outlined above, the subscripts on the V's are the transferred spin and isospin, so only $V_{ST}=V_{11}$ can contribute to the reaction.

The data have been compared to two forms of V_{11}. In the first of these V_{11} is taken to be real and to have

[13] V. A. Madsen (private communication).

TABLE I. Optical-model parameters[a] at 24.4 MeV.

V	r_0	a	W_d	r_0'	a'	V_s
44.8 MeV	1.13 F	0.62 F	5.87 MeV	1.12 F	0.68 F	7.40 MeV

[a] Notation that of G. R. Satchler, Nucl. Phys. **A92**, 273 (1967), with $r_s = r_0$, $a_s = a$.

the form

$$V_{11} = Ve^{-\alpha r}/\alpha r.$$

Earlier analysis[6] has shown that $1/\alpha = 1.0$ F is near the optimum range and this value is used throughout the analysis. The second form of V_{11} is obtained[5] from a fit of the nucleon-nucleon scattering amplitude to a Yukawa shape. This impulse approximation interaction is complex and both the ranges and strengths vary with energy.

The nuclear wave functions were taken to be *LS*-coupled harmonic-oscillator wave functions with an oscillator parameter, $b = 1.90$ F, chosen to fit the most recent electron scattering results.[14] Form factors[2] for the reaction were calculated and were then inserted into a distorted-wave approximation code[15] which allows the use of spin-orbit potentials. Optical-model potentials obtained from a fit made to elastic scattering at 24.4 MeV[12] using the optical-model search code ABACUS[16] are shown in Table I. The real potential was scaled to other energies using the relationship

$$V(E) = V(24.4) - 0.33(E - 24.4).$$

The calculated total cross sections depend weakly on V and are changed by only 2.9% at 50.0 MeV if one completely neglects this energy dependence. Setting the spin-orbit part of the optical potential to zero and leaving the other parameters unchanged increased the total cross section by 4.5% at 50.0 MeV.

The results of these calculations are shown in Figs. 4 and 5. The solid curve in Fig. 4 is calculated from a real interaction of a Yukawa shape with $1/\alpha = 1.0$ F and $V = 12.1$ MeV. The data appear to be consistent with a constant value of V_{11} for energies above 34 MeV.

The dashed curve of Fig. 4 is the excitation function predicted by the impulse approximation.[5] There are no adjustable parameters in this calculation. The ratio of the cross section predicted by this theory to the experimental cross section decreases from 2.9 at 25 MeV to 1.4 at 45 MeV.

V. SUMMARY OF INFORMATION ON V_{11}

The results of this and other pertinent experiments are summarized in Fig. 5. The V_{11} were obtained by forcing the calculated cross section to match the experimental cross section. A range $1/\alpha = 1.0$ F was used for all these calculations. An estimate of the equivalent strength for other values of the range may be obtained by using the approximate empirical relationship[17]

$$V(\alpha)/V(\alpha') = (\alpha/\alpha')^n, \quad n \simeq 2.7.$$

For example, for $1/\alpha = 1.4$ F, V_{11} is obtained by dividing the values in Fig. 5 by 2.48. More accurate estimates can be obtained only by performing detailed calculations.

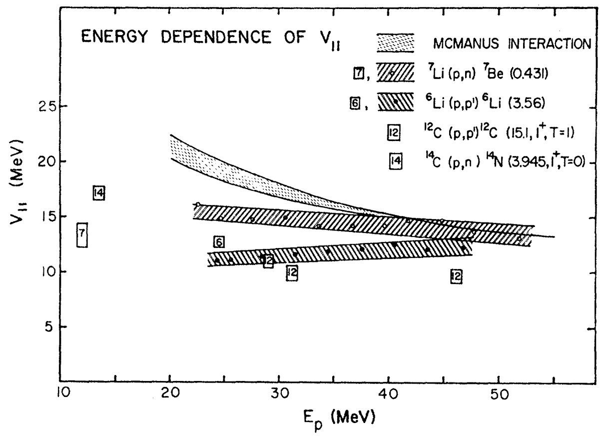

FIG. 5. Energy dependence of V_{11}. See the text of Secs. V and VI for a discussion of these results.

We now discuss the relevant experiments in turn.

A. Present Experiment: ^{6}Li$(p,p')^6$Li(3.56 MeV)

The solid circles correspond to the data points of Fig. 4, while the width of the cross-hatched area is an indication of the experimental errors. The single point at 24.4 MeV was taken from Ref. 18.

B. ^{7}Li$(p,n)^7$Be(0.431 MeV)

The data shown on the cross-hatched area between 23 and 52 MeV are from Ref. 4, and the single point at 12 MeV is taken from Ref. 19. These cross sections were measured by γ-ray techniques similar to those described in this paper. Four amplitudes can contribute to this $(\frac{3}{2}^-,\frac{1}{2}) \rightarrow (\frac{1}{2}^-,\frac{1}{2})$ transition. However, a single one of these, $(LSJT) = (0111)$, contributes about 95% of the cross section, so the reaction is sensitive only to $V_{ST} = V_{11}$. V_{11} was corrected for the small contribution of V_{01} through the amplitude (2021). This correction decreased V_{11} by about 0.5 MeV for a value of $V_{01} = 20$ MeV.[6,7]

LS wave functions with a harmonic-oscillator parameter, $b = 1.72$ F, were used to describe the nucleus. The rms radius for this value of b agrees with recent electron scattering data.

[14] L. R. Suelzle, M. R. Yearian, and H. Crannell, Phys. Rev. **162**, 992 (1967).
[15] This code was written by R. Haybron and T. Tamura and was modified for the Sigma 7 computer by J. J. Kolata.
[16] E. H. Auerbach, Brookhaven National Laboratory Report No. BNL-6562 (unpublished).
[17] F. Petrovich and H. McManus (private communication).
[18] S. M. Austin and G. M. Crawley, Phys. Letters **27B**, 570 (1968).
[19] P. Paul, S. M. Austin, and S. S. Hanna (to be published).

C. ^{12}C$(p,p')^{12}$C(15.1 MeV)

The transition in this case is $(0^+,0) \rightarrow (1^+,1)$ and is mediated by V_{11} alone. The points at 28.05, 31.1, and 46 MeV were obtained from our analysis of total cross sections extracted from the measurements of Locard *et al.*,[20] Dickens *et al.*,[21] and Petersen *et al.*[22] The wave functions of Gillet and Vinh Mau,[23] with a harmonic-oscillator parameter $b=1.64$ F, were used in the calculation of the form factor.

D. ^{14}C$(p,n)^{14}$N(3.945 MeV)

This transition is $(0^+,1) \rightarrow (1^+,0)$ and is again a pure V_{11} transition. The result quoted is the analysis of Wong *et al.*[7,13] The value obtained for the transition to the ground state of $N^{14}(J^\pi=1^+,\ T=0)$ has not been included since the analysis is complicated by a chance cancellation[7] of the (0111) amplitude.

E. Impulse Approximation

The area labeled "McManus Interaction" was obtained in the following manner. Total cross sections were calculated using the impulse approximation interaction.[5] These calculated cross sections were treated as experimental data and the strength of the real interaction was adjusted to give the same cross section. Since the equivalence depends somewhat on the nuclear wave functions, different transitions yield a different equivalent V_{11}. The lower limit of the band was calculated for ^{6}Li(p,p') ^{6}Li(3.56 MeV) and the width of the band encompasses results calculated by Petrovich and McManus[17] for transitions in ^{12}C and ^{40}Ca.

VI. DISCUSSION

A. Exchange Effects

It is difficult to estimate the uncertainties in analyses of this sort, and one is forced to appeal to consistency among transitions in different nuclei. The present results are encouraging from this point of view. The data for the four transitions cluster about $V_{11}=12.5$ MeV and there is no evidence that V_{11} depends on energy. It happens, however, that in all of these cases, the cross section is dominated by the monopole $(L=0)$ amplitude, so these results give no information on a possible multipole dependence of V_{eff}.

Recent calculations[24] indicate that exchange processes contribute significantly to inelastic scattering and (p,n) cross sections. The exchange contribution generally interferes constructively with the direct scattering and becomes progressively more important for the higher multipoles. Since conventional calculations such as those reported in this paper neglect exchange effects, V_{eff} must increase with L to compensate for the neglected processes. For the same reason, the $L=0$ transitions considered in this paper should provide a more reliable estimate of the actual effective interactions than transitions for which $L>0$.

It is also possible that inclusion of exchange effects would reduce the small discrepancy between the ^{7}Li(p,n) and ^{6}Li(p,p') results since the negative correction to the (p,n) cross sections, which are $L=2$, would be larger at low energies where exchange effects are most important.

B. Wave Functions

The LS-coupled wave functions used in this analysis are known to describe ^{6}Li fairly well,[25] though they do not, for example, describe in detail the charge distributions derived from electron scattering.[14] We are undertaking calculations using more sophisticated intermediate-coupling wave functions. It seems likely, however, that using better wave functions would primarily affect transitions mediated by the spin-independent part V_{00} of the effective interaction since such transitions are enhanced when the wave functions have collective characteristics.[26] Collective spin-dependent phenomena are not important for these low-lying states, so for the present purpose the extreme single-particle wave functions are likely to be a good approximation.

C. Comparison with Other V_{ST}

Although V_{11} dominates the cross section for the reactions considered here, it is relatively small compared with other terms in V_{eff}. A comparison with typical values from the literature is given in Table II. No consistent studies of the energy dependence of V_{00}, V_{01}, and V_{10} are available and the values quoted are only meant to be illustrative of the range of energies above 20 MeV.

TABLE II. Comparison of the V_{ST} for energies between 20 and 60 MeV.

S,T	V_{ST} (MeV)[a]	Reference
0,0	50–100	b, c, d
0,1	20	e, f
1,0	20–50	g
1,1	10–15	Present

[a] For a 1.0-F range.
[b] Reference 26.
[c] W. G. Love, Phys. Letters 26B, 271 (1968).
[d] Reference 20.
[e] Reference 7.
[f] Reference 6.
[g] P. J. Locard and S. M. Austin (unpublished).

[20] P. J. Locard, S. M. Austin, and W. Benenson (unpublished).
[21] J. K. Dickens, D. A. Haner, and C. N. Waddell, Phys. Rev. 132, 2159 (1963).
[22] E. L. Petersen, I. Slaus, J. W. Verba, R. F. Carlson, and J. R. Richardson, Nucl. Phys. A102, 145 (1967).
[23] V. Gillet and N. Vinh Mau, Nucl. Phys. 54, 321 (1964).
[24] J. Atkinson and V. A. Madsen, Bull. Am. Phys. Soc. 13, 630 (1968); V. A. Madsen (private communication).
[25] See, for example, C. A. Levinson and M. K. Banerjee, Ann. Phys. (N. Y.) 2, 471 (1957).
[26] W. G. Love and G. R. Satchler, Nucl. Phys. A101, 424 (1967).

VII. CONCLUSIONS

The total cross section for the $^6Li(p,p')^6Li(3.56\text{ MeV})$ reaction was measured in the proton energy range from 24.3 to 46.4 MeV. These data and relevant results from the literature were analyzed to yield values of the spin-isospin–dependent part of the effective interaction for proton energies between 12 and 52 MeV. The results are summarized in Fig. 5. The strengths of V_{11} obtained all lie in the range from 10 to 17 MeV for a Yukawa shape with a range of 1.0 F. There is no evidence that V_{11} depends on the proton energy. The impulse approximation predicts a cross section which is substantially too large at energies below 35 MeV, but which is in good agreement with the data near 50 MeV.

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Comparison of Variational Results and Solutions of Faddeev Equation for a Local Potential*

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A comparison is made between the results obtained for a three-body system with a local Yukawa potential, using the Faddeev method and a variational technique. The result of this comparison shows that with the type of trial function used, the variational method yields upper and lower bounds which are very close to each other and consistent with the results obtained by Ball and Wong. On the other hand, our variational study rules out definitely the possibility of a ground-state collapse phenomenon claimed by Osborn, since Osborn's ground-state energy is even lower than the lower bound of our calculation when the potential depth is large.

I. INTRODUCTION

MOST calculations on the binding energies of three-body systems using the Faddeev method[1] have employed nonlocal separable potentials for the sake of simplicity. However, recently a number of calculations[2–4] with simple local potentials have been reported. The aim of the present investigation is to compare the ground-state energy of a three-body system, obtained using the Faddeev method, with the upper and lower bounds obtained from a variational calculation. This comparison should serve as a severe test for the numerical accuracy of the Faddeev solution, since, with the type of trial function and potential used, the upper and lower bounds are very close to each other; e.g., for a two-body potential with a strength that yields the deuteron binding energy, our calculation shows that the two bounds for the three-body ground-state energy are within a few percent of each other.

In particular, our purpose is to check the existence of a strange, but interesting, phenomenon reported by Osborn.[4] By solving the Faddeev equation approximately, this author has found that, when the strength of the two-body potential exceeds a certain value, the three-body system exhibits a collapse behavior, wherein the binding energy increases very sharply with the potential strength. With a variational calculation, Kok[5] has expressed some doubt about the existence of this unusual behavior, since the upper bounds he obtained do not show any sign of a sudden change with the increase of the two-body strength. However, this does not definitely rule out Osborn's collapse phenomenon, because, in Kok's calculation, a lower bound to the eigenvalue was not obtained. On the other hand, our investigation here will enable us to make a more definite conclusion regarding Osborn's observation. This is so, since it is quite clear that his result on the energy of the three-body system has to lie in the narrow gap between the upper and lower bounds of this calculation.

In Sec. II we present the expression for the upper and lower bounds on the energy, and discuss the two-

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[1] L. D. Faddeev, *Mathematical Aspects of the Three-Body Problem in the Quantum Scattering Theory* (D. Davey and Co., Inc., New York, 1965).

[2] D. Y. Wong and G. Zambotti, Phys. Rev. **154**, 1540 (1967).

[3] J. S. Ball and D. Y. Wong, Phys. Rev. **169**, 1362 (1968).

[4] T. A. Osborn, Stanford Linear Accelerator Laboratory Report No. SLAC-PUB-361, 1967 (unpublished).

[5] L. P. Kok, Trieste Internal Report IC/68/11 (unpublished).