MeV. Use of $n=5.3$ and $Y_D=0$ (Y_D is the fraction of direct interaction in the total differential cross section) results in an autocorrelation function for $\epsilon=0$ calculated to be $R(0)=0.37\pm0.20$. When this result is compared with the experimental result $R(0)=0.43$, we find Y_D to be between 0 and 60%. The conclusion from this analysis is that the interaction proceeds via a compound nucleus at least 40% of the time in the $O^{16}+Li^6 \rightarrow \alpha_0+F^{18}$ reaction. A summary of these results is given in Table I.

B. Angular Distributions

The angular distributions of α particles leaving F^{18} in the ground state have been measured at seven energies (Fig. 2). At 5.70 MeV, corresponding to a peak energy in the α_0 yield, the angular distribution is forward-peaked, while at 6.22 MeV, corresponding to a valley energy in the α_0 yield, the angular distribution is not forward-peaked. For angles greater than 35°, the general character of the angular distributions at 5.50, 5.70, and 6.22 MeV varies slowly with energy. At the four higher bombarding energies (11.60, 12.20, 12.80, and 13.30 MeV), the α_0 angular distributions oscillate rapidly as a function of angle, but their general character changes slowly with energy except in the first 10° of the angular distribution. In these first 10°, the differential cross section changes rapidly as is shown in Fig. 4. These four angular distributions have been fitted by least squares with a Legendre polynomial expansion (not illustrated) with terms to tenth order. The most satisfactory fits were obtained when both even and odd terms were used in the expansion. In this case, the coefficients for odd terms were as large as those for the even terms. This suggests that interaction mechanisms other than pure compound-nucleus formation are contributing to the cross section.[23]

The deuteron angular distributions shown in Fig. 3 do not have any obvious correlation with the α_0 angular distributions at the three lowest bombarding energies of 5.50, 5.70, and 6.22 MeV.

V. CONCLUSIONS

When corrections for finite sample size were made, the value of the coherence width in the center of mass was found to be $\Gamma=0.49\pm0.40$ MeV. The expected value of Γ from systematic trends of other experimental measurements in the same range of excitation energy is $\Gamma\sim0.10$ MeV.[24] This result, plus those from other lithium-induced reactions,[5,9,11] leads to the suggestion that the values of Γ obtained from fluctuation studies of lithium-induced reactions are larger than values expected from theoretical calculations[18,19] based on the Fermi-gas model and other experimental measurements.[24]

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Effects of Using a Velocity-Dependent Potential in a Monte Carlo Simulation of Intranuclear Cascades*

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A velocity-dependent potential, of the form $V=V_0(1-\epsilon/\epsilon_{max})$ for $\epsilon<\epsilon_{max}$ and $V=0$ for $\epsilon\geq\epsilon_{max}$, was used in a Monte Carlo simulation of intranuclear cascades. By using a reasonable ϵ_{max} (150 MeV), only slightly better agreement is obtained between the calculated and experimental results than was found previously with a velocity-independent potential.

I. INTRODUCTION

IN a previous paper[1] the effects of using different nuclear-density and potential-energy distributions in a Monte Carlo simulation of intranuclear cascades were explored. Also, with each of these distributions, the calculation was carried out both with and without the refraction and reflection of cascade nucleons due to the spatial nonuniformity of the nuclear potential. Of the density distributions tested, the one which approximates a realistic nuclear-density distribution by a step function gave the best fit to experimental data. One of the surprising results of the earlier calculation was that,

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TABLE I. Values of V_0 from Eq. (2) and of the ratio of effective mass to rest mass for various values of ϵ_{max} with $\epsilon_F=42$ MeV and $B=8$ MeV.

ϵ_{max} (MeV)	V_0 (MeV)	m^*/m
60.0	166.7	0.26
100.0	86.3	0.54
150.0	69.5	0.68
200.0	63.3	0.76
300.0	58.1	0.84
400.0	55.9	0.88

for incident energies below 200 MeV and especially for medium and heavy nuclei, the agreement with experimental data was better when refraction and reflection of cascade nucleons were neglected (STEPNO model) than when these effects were included (STEP model). Two possible explanations were suggested. (1) The classical treatment of refraction and reflection overestimates the effect, and complete neglect of refraction and reflection is a better approximation to a correct, quantum-mechanical treatment; (2) the potential used is incorrect. The first possibility was investigated in the previous paper[1] and it was found that the classical calculations of transmission coefficients that include refraction and reflection reproduce the quantal calculations more faithfully than do those which neglect refraction and reflection. In the present paper, we explore the effect of replacing the velocity-independent potential used in the earlier calculations by a more realistic velocity-dependent potential.

II. DESCRIPTION OF VELOCITY-DEPENDENT POTENTIAL MODEL

In the present calculation, the velocity-dependent potential that is investigated is

$$\begin{aligned} V &= V_0(1-\epsilon/\epsilon_{max}) \quad \text{for} \quad \epsilon < \epsilon_{max}, \\ V &= 0 \qquad\qquad\quad \text{for} \quad \epsilon \geqslant \epsilon_{max}, \end{aligned} \tag{1}$$

where ϵ is the kinetic energy of the particle, ϵ_{max} is the kinetic energy of the particle above which the potential vanishes, and V_0 is the potential of the particle when it has no kinetic energy. V_0 is negative and is chosen such that the sum of the potential and kinetic energies of a particle at the top of the Fermi distribution is the negative of the binding energy B:

$$V_0=-(\epsilon_F+B)/(1-\epsilon_F/\epsilon_{max}), \tag{2}$$

where ϵ_F is the Fermi kinetic energy.

There is only one free parameter in Eqs. (1) and (2), that is, ϵ_{max}. From optical-model-potential analyses,[2,3] V vanishes above 300 MeV and is almost negligible at 200 MeV. In Table I, values of V_0 and ϵ_{max}, along with the corresponding values of m^*/m (the ratio of effective mass to rest mass), are tabulated. Most theoretical treatments[4] of nuclear matter lead to values of m^*/m in range 0.6–1.0 and, although these results may not be directly applicable to finite nuclei, they together with the optical-model analyses make ϵ_{max} values of less than about 100 MeV in our formulation seem unreasonable.

In the calculations, total energy and momentum are conserved in each nucleon-nucleon collision. With the velocity-dependent potential given by Eq. (1), however, the relative kinetic energy in the c.m. system does not remain constant during a collision. The Monte Carlo choices of the momenta after a collision were based on the angular distributions corresponding to the relative kinetic energy before the collision. Relativistic kinematics were used throughout.

It should be mentioned that an improved density distribution was used in this calculation (STEPNW) which fits the Fermi distribution better and also is more flexible than that used in STEP. In Fig. 1, comparisons are made among the three distributions, Fermi, STEP, and STEPNW. Clearly the STEPNW distribution is an improvement over the STEP distribution.

III. COMPARISON AMONG MODELS; EXPERIMENTAL RESULTS

It was found in the previous calculation that, although the STEP model has a better theoretical basis than does STEPNO, the calculated results using the STEPNO model are in better agreement with experimental data than those using the STEP model. In particular, the STEP model leads to overestimates of residual excitation energies and underestimates of cascade-particle emission. In the present calculation, with the velocity-dependent potential described by Eq. (1), particles with kinetic energy above ϵ_{max} experience no potential and hence are neither refracted nor reflected, while particles with kinetic energy below ϵ_{max} experience a potential that

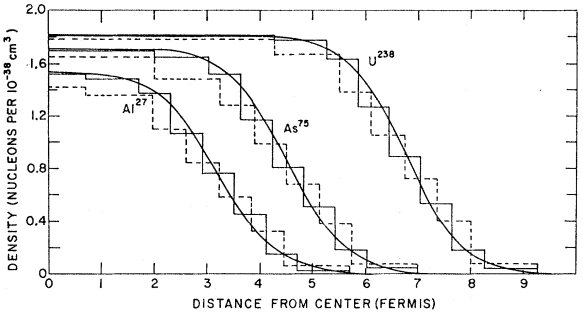

FIG. 1. Comparison between new and old density distributions for Al^{27}, As^{75}, and U^{238}. Solid curve, Fermi distribution from R. Hofstadter, Ann. Rev. Nucl. Sci. **7**, 295 (1957); dashed histogram, density distribution used in Ref. 1; solid histogram, new density distribution. In the latter, the distance between the points where the density is 90 and 10% of the central density is divided into four steps each 0.6 F wide. The radius remains unchanged at $1.07A^{1/3}+2.5$ F.

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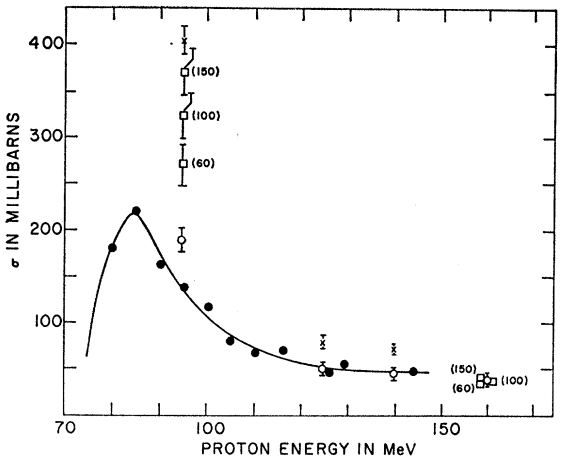

FIG. 2. Excitation function for the reaction $Bi^{209}(p,8n)Po^{202}$. The solid points are the experimental data of Ref. 6. The open circles and crosses are the results of the STEPNO and STEP calculations of Ref. 1; the squares were calculated with the velocity-dependent potential, with the ϵ_{max} values indicated.

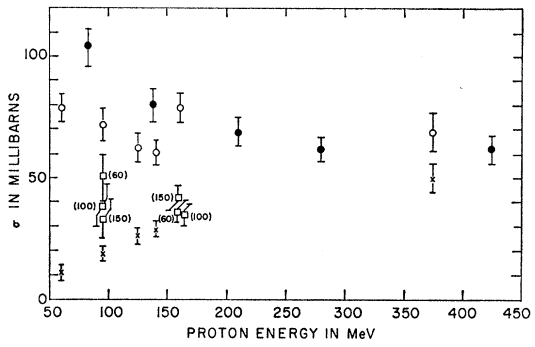

FIG. 3. Excitation function for a (p,pn) reaction. The solid points are the experimental data of Ref. 7 for production of Au^{196} from Au^{197}. The open circles and crosses are the results of the STEPNO and STEP calculations of Ref. 1; the squares were calculated with the velocity-dependent potential, with the ϵ_{max} values indicated. All the calculated results are for the reaction $Bi^{209}(p,pn)Bi^{208}$.

is smaller in absolute value than that in STEP and concomitantly should be refracted and reflected less. This effect of the velocity-dependent potential in diminishing the refraction and reflection would, in itself, lead to an increase in the emission of secondary cascade particles accompanied by a decrease in the excitation energy of the residual nucleus and thus yield results that would be expected to be closer to those from STEPNO. On the other hand, the velocity-dependent potential leads, on the average, to a lower kinetic energy for cascade particles and thus, because of the behavior of nucleon-nucleon scattering cross sections, to a smaller mean free path within nuclear matter for these particles. This effect lowers the emission of secondary cascade particles and leads to higher excitation energies for residual nuclei. Thus the velocity-dependent potential leads to two effects which oppose each other.

The results obtained with the velocity-dependent potential generally fall between those of STEP and STEPNO. However, with reasonable values of ϵ_{max} ($\gtrsim 150$ MeV), they are closer to STEP.

No comparisons with experimental data on differential cross sections for outgoing particles are presented here since the discrepancies between different experimenters are as large as the differences between different model calculations.[1,5]

In the following, we compare calculated excitation functions of 95- and 160-MeV incident photons on Bi^{209} with those measured by Brun and Lefort[6] ($p+Bi^{209}$) and by Yule and Turkevich[7] ($p+Au^{197}$). In Figs. 2 and 3, using three values of ϵ_{max}, the excitation functions of $Bi^{209}(p,8n)Po^{202}$ and $Bi^{209}(p,pn)Bi^{208}$ are compared with the corresponding experimental data [in Fig. 3, the experimentally measured excitation function is $Au^{197}(p,pn)Au^{196}$], and also with the excitation functions calculated from STEP and STEPNO. More details on the calculation are given in Ref. 1. In general, the lower the value of ϵ_{max} used, the better is the agreement between the calculated and experimental excitation functions. However, even with an ϵ_{max} of 60 MeV (which corresponds to an unrealistic effective mass ratio m^*/m of about 0.25), the results are not as close to the experimental data as are the STEPNO calculations. With a reasonable ϵ_{max} such as 150 MeV, the results calculated with the velocity-dependent potential are only slightly better than those calculated from STEP. At an incident energy of 160 MeV the results appear to be very insensitive to the value of ϵ_{max} and very close to those obtained with STEP.

For incident energies above ϵ_{max}, an obvious consequence of the present model is the absence of any refraction or reflection for the incident nucleon and for any cascade particle with kinetic energy greater than ϵ_{max}. Emission of high-energy secondaries at backward angles, which in Ref. 1 was shown to be a characteristic feature of the STEP model subject to experimental test, is thus improbable with the velocity-dependent potential.

IV. SUMMARY

It has been shown that the introduction of a velocity-dependent potential consistent with optical-model analyses of nuclear-reaction data does not lead to greatly improved agreement between intranuclear cascade calculations and experimental results.

ACKNOWLEDGMENT

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