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## Spin Relaxation and Transport in Magnetic Alloys

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The spin susceptibility for an interacting electron gas is derived, including a fully microscopic treatment of spin relaxation and transport effects due to nonmagnetic impurities. The corresponding macroscopic equations for the paramagnet resemble the relaxation-to-equilibrium phenomenological model, with the spin relaxation time given by the spin-orbital scattering rate appearing in the self-energy, multiplied by renormalization factors. In the ferromagnetic region, the spin diffusion constant is found to be much smaller than in previous estimates. The consequences for paramagnons are briefly discussed.

### I. INTRODUCTION

A GOOD knowledge of the frequency and wave-number-dependent spin susceptibility is important for understanding many physical situations such as magnetic resonances. This susceptibility should include transport effects such as spin diffusion and spin relaxation as well as correctly describing the interaction between electrons. At present there exist phenomenological models for this susceptibility but relatively little work has been reported which critically examines the various models from microscopic principles. It is our purpose here to calculate the spin susceptibility, including a fully microscopic treatment of uniform electron exchange, spin diffusion, and relaxation by randomly distributed impurities for ferromagnetic and paramagnetic metals.

The spin susceptibility for pure metals has been derived in the random-phase approximation by Wolff<sup>1</sup> and by Izuyama, Kim, and Kubo.<sup>2</sup> In their calculations, the magnetic electrons are assumed to comprise one band and to interact with a short-range screened Coulomb potential. Our approach here is similar but we include in addition the influence of a small concentration of nonmagnetic impurities following the work

of de Gennes<sup>3</sup> on the static susceptibility. The impurities lead to a randomization of directed particle momentum due to ordinary scattering processes and hence diffusion. In addition, spin-orbital scattering processes lead to a randomization of the particle spins and thus cause spin relaxation.

The ferromagnetic spin susceptibility in the presence of ordinary scattering processes has been discussed from a phenomenological viewpoint by Hirst<sup>4</sup> and by Kaplan.<sup>5</sup> They argue that the spin diffusion constant is decreased by the action of exchange polarization but that spin diffusion should still characterize the long-wavelength spin susceptibility. However, as we show here, the long-wavelength spin susceptibility for this situation contains spin-wave propagation but essentially no diffusion. As known from previous work,<sup>6</sup> the impurity spin diffusion constant in the paramagnetic range approaches zero at the ordering point and our result shows that it vanishes throughout the ferromagnetic region except for a negligibly small term proportional to an external magnetic field.

The situation is slightly different when the effects of spin-orbital scattering are included. In addition to an infinite-wavelength broadening of the spin resonance, one finds a slight contribution to the diffusion constant proportional to the spin-orbital scattering rate. How-

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<sup>1</sup> P. A. Wolff, *Phys. Rev.* **120**, 814 (1960).

<sup>2</sup> T. Izuyama, D. J. Kim, and R. Kubo, *J. Phys. Soc. Japan* **18**, 1025 (1963).

<sup>3</sup> P. G. de Gennes, *J. Phys. Radium* **23**, 630 (1962).

<sup>4</sup> L. L. Hirst, *Phys. Rev.* **141**, 503 (1966).

<sup>5</sup> J. I. Kaplan, *Phys. Rev.* **143**, 351 (1966).

<sup>6</sup> P. Fulde and A. Luther, *Phys. Rev.* **170**, 570 (1968).

ever, this contribution is very small and does not appear to be experimentally important.

For the paramagnet we extend previous work<sup>6</sup> to include the effects of spin-orbital scattering. For long wavelengths our result is, to a good approximation, equivalent to the phenomenological relaxation-to-equilibrium model.<sup>7</sup> The spin relaxation time in this model is equivalent to the spin-orbital scattering rate which appears in the self-energy, modified by a renormalization factor. This renormalization factor tends to zero as the ferromagnetic instability is approached. From our approach, we derive the proper macroscopic equations which characterize the paramagnetic system.

## II. PARAMAGNETIC SPIN SUSCEPTIBILITY

The spin susceptibility in the paramagnetic regime with ordinary impurities has been considered previously. We study here the addition of spin-orbital scattering into the problem, calculate the dynamical spin susceptibility, and finally determine the corresponding macroscopic spin equations. The standard diagrammatic approach is used in our calculations with both ordinary and spin-orbital scattering described by the methods of Abrikosov and Gor'kov.<sup>8</sup> The interaction between the electrons is treated in the ladder approximation. The spin-susceptibility tensor is defined in terms of a vertex function

$$\Lambda(\mathbf{p}, \omega; \mathbf{q}, \omega_0)$$

by

$$\chi(\mathbf{q}, \omega_0) = \frac{1}{2} \mu_B^2 \text{Tr} \delta i \int \frac{d^3 p}{(2\pi)^4} d\omega \Lambda(\mathbf{p}, \omega; \mathbf{q}, \omega_0). \quad (1)$$

The vertex  $\Lambda(\mathbf{p}, \omega; \mathbf{q}, \omega_0)$  is determined by the usual set of diagrams<sup>9</sup> and the corresponding integral equation

$$\begin{aligned} \Lambda(\mathbf{p}, \omega; \mathbf{q}, \omega_0) &= G(\mathbf{p}, \omega) \left[ \delta + i \bar{V}_e \int \frac{d^3 p'}{(2\pi)^4} \Lambda(\mathbf{p}', \omega'; \mathbf{q}, \omega_0) \right. \\ &\quad \left. + n_i \int \frac{d^3 p'}{(2\pi)^3} u(\mathbf{p}-\mathbf{p}') \Lambda(\mathbf{p}', \omega; \mathbf{q}, \omega_0) u(\mathbf{p}'-\mathbf{p}) \right] \\ &\quad \times G(\mathbf{p}+\mathbf{q}, \omega+\omega_0). \quad (2) \end{aligned}$$

In this equation all functions are understood to be  $2 \times 2$  matrices. The first term in this equation is the unrenormalized vertex. The second term represents the vertex correction due to the screened Coulomb interaction  $\bar{V}_e$ . The third term represents the correction due to impurities of concentration  $n_i$  and scattering ampli-

tude  $u(\mathbf{p}-\mathbf{p}')$ , which is given by

$$u(\mathbf{p}-\mathbf{p}') = a(\mathbf{p}-\mathbf{p}') + ib(\mathbf{p}-\mathbf{p}') p_F^{-2} [\mathbf{p} \times \mathbf{p}'] \cdot \delta. \quad (3)$$

The first term proportional to the unit matrix describes ordinary impurity scattering, while the second term describes spin-flip scattering due to spin-orbital interaction. In practice,  $|b(\mathbf{p}-\mathbf{p}')|^2 \ll |a(\mathbf{p}-\mathbf{p}')|^2$  and hence we will neglect terms of the order  $|b|^4$ .

The function  $G(\mathbf{p}, \omega)$  is the electronic Green's function which in the case of no external field is given by

$$G(\mathbf{p}, \omega) = [\omega - (p^2 - p_F^2)/2m + (i/2\tau) \text{sgn} \omega]^{-1}. \quad (4)$$

The lifetime  $\tau$  consists of two parts,

$$1/\tau = 1/\tau_0 + 1/\tau_1, \quad (5)$$

where

$$\begin{aligned} (\tau_0)^{-1} &= n_i \frac{1}{2} [N(0)] \int d\Omega |a(\mathbf{p}-\mathbf{p}')|^2, \\ (\tau_1)^{-1} &= n_i \frac{1}{2} [N(0)] \int d\Omega |b(\mathbf{p}-\mathbf{p}')|^2 \sin^2 \theta, \quad (6) \end{aligned}$$

with  $N(0) = m p_F / 2\pi^2$  being the density of states. It is easy to verify from Eq. (2) that the susceptibility is isotropic in the absence of a magnetic field which leads to

$$\begin{aligned} \chi(\mathbf{q}, \omega_0) &= 2\mu_B^2 N(0) \\ &\quad \times \chi_{\text{imp}}^{\text{norm}}(\mathbf{q}, \omega_0) / [1 - N(0) \bar{V}_e \chi_{\text{imp}}^{\text{norm}}(\mathbf{q}, \omega_0)], \quad (7) \end{aligned}$$

where  $\chi_{\text{imp}}^{\text{norm}}(\mathbf{q}, \omega_0)$  is the normalized susceptibility determined from Eqs. (1) and (2) with the exchange vertex correction term dropped.

It is convenient to introduce the function

$$\begin{aligned} J(\omega; \mathbf{q}, \omega_0) &= [2\pi N(0)]^{-1} \int \frac{d^3 p}{(2\pi)^3} G(\mathbf{p}, \omega) \\ &\quad \times G(\mathbf{p}+\mathbf{q}, \omega+\omega_0) \\ &= \frac{i}{2v_F q} \ln \frac{K(\omega) + K(\omega+\omega_0) + q}{K(\omega) + K(\omega+\omega_0) - q}, \quad (8) \end{aligned}$$

where

$$K(\omega) = i/2l + (\text{sgn} \omega) (p_F^2 + 2m\omega)^{1/2} \quad (9)$$

and  $l = v_F \tau$  is the mean free path. Equation (2) is then solved by the following ansatz:

$$\begin{aligned} \Lambda(\mathbf{p}, \omega; \mathbf{q}, \omega_0) &= G(\mathbf{p}, \omega) G(\mathbf{p}+\mathbf{q}, \omega+\omega_0) \\ &\quad \times [\Lambda_\sigma \delta + \Lambda_p (\mathbf{p}/p_F^2) (\delta \cdot \mathbf{p})], \quad (10) \end{aligned}$$

with the result

$$\begin{aligned} \Lambda_\sigma \delta + \Lambda_p \mathbf{p} (\delta \cdot \mathbf{p}) p_F^{-2} &= \delta + (\Lambda_\sigma / \tau_0) J(\omega) \delta + (\Lambda_p / \tau_0') J(\omega) \delta \\ &\quad + (\Lambda_p / \tau_0'') J(\omega) \mathbf{p} (\delta \cdot \mathbf{p}) p_F^{-2} - (\Lambda_\sigma / \tau_1) J(\omega) \mathbf{p} (\delta \cdot \mathbf{p}) p_F^{-2}. \quad (11) \end{aligned}$$

In deriving this equation, we have made use of the

<sup>7</sup> B. Giovannini, M. Peter, and S. Koidé, Phys. Rev. **149**, 251 (1966), and references therein.

<sup>8</sup> A. A. Abrikosov and L. P. Gor'kov, Zh. Eksperim. i Teor. Fiz. **35**, 1558 (1958); **42**, 1088 (1962) [English transl.: Soviet Phys.—JETP **8**, 1090 (1959); **15**, 752 (1962)].

fact that only the  $s$ -wave part of the vertex is important for  $ql \ll 1$ .<sup>9</sup> The additional lifetimes  $\tau_0'$ ,  $\tau_0''$  appearing in Eq. (11) are defined by

$$(\tau_0')^{-1} = n_s \frac{1}{4} [N(0)] \int d\Omega |a(\mathbf{p}-\mathbf{p}')|^2 \sin^2\theta, \quad (12)$$

$$1/\tau_0'' = 1/\tau_0 - 3/\tau_0'.$$

For the sake of simplicity we take  $a(\mathbf{p}-\mathbf{p}')$  and  $b(\mathbf{p}-\mathbf{p}')$  to be a constant. Then  $\tau_0' = 3\tau_0$  and  $(\tau_0'')^{-1} = 0$ . Using Eq. (11), we find

$$\Lambda_\sigma = [1 - \tau_0^{-1} J(\omega) (1 - (3\tau_1)^{-1} J(\omega))]^{-1},$$

$$\Lambda_p = -[J(\omega)/\tau_1] [1 - \tau_0^{-1} J(\omega) (1 - (3\tau_1)^{-1} J(\omega))]^{-1}, \quad (13)$$

and

$$\chi_{\text{imp}}^{\text{norm}}(\mathbf{q}, \omega_0) = i \int_{-\infty}^{+\infty} d\omega' \frac{J(\omega') [1 - (3\tau_1)^{-1} J(\omega')]}{1 - \tau_0^{-1} J(\omega) [1 - (3\tau_1)^{-1} J(\omega')]} \quad (14)$$

The procedure for handling this integral is the same as used previously (Ref. 6) and consists in dividing the region of integration into three different parts:

- (a)  $-\infty < \omega < -\omega_0$ ,
- (b)  $-\omega_0 < \omega < 0$ ,
- (c)  $0 < \omega < \infty$ .

The contributions from regions (a) and (c) give the static susceptibility plus terms of order  $q^2/p_F^2$  which are dropped. The frequency dependence arises from region (b). Combining these we find the result to be

$$\chi_{\text{imp}}^{\text{norm}}(\mathbf{q}, \omega_0) = 1 + \frac{i\omega_0 J(0; \mathbf{q}, \omega_0) [1 - (3\tau_1)^{-1} J(0; \mathbf{q}, \omega_0)]}{1 - \tau_0^{-1} J(0; \mathbf{q}, \omega_0) [1 - (3\tau_1)^{-1} J(0; \mathbf{q}, \omega_0)]}, \quad (15)$$

where  $J(0; \mathbf{q}, \omega_0)$  is obtained from Eq. (9) and is given by

$$J(0; \mathbf{q}, \omega_0) = (i/2v_F q) \ln[(\omega_0 + i/\tau + v_F q)/(\omega_0 + i/\tau - v_F q)]. \quad (16)$$

Before we discuss this susceptibility it is worthwhile to consider the effects of a static external magnetic field. When the field is switched on, say, along the  $z$  axis, the electronic Green's function must be written as

$$G(\mathbf{p}, \omega) = \frac{1}{2}(1 + \sigma_3) G_+(\mathbf{p}, \omega) + \frac{1}{2}(1 - \sigma_3) G_-(\mathbf{p}, \omega), \quad (17)$$

where

$$G_\sigma(\mathbf{p}, \omega) = [\omega - (p^2/2m) + (p_{F\sigma}^2/2m) + (i/2\tau) \text{sgn}\omega]^{-1} \quad (18)$$

and  $p_{F\sigma}$  is the radius of the Fermi sphere of spin  $\sigma$ . The spin susceptibility is then no longer isotropic but separates into longitudinal and transverse components. In terms of the vertex given by Eq. (10) this means that two independent elements  $\Lambda_{\sigma^{zz}}$ ,  $\Lambda_{\sigma^{+-}}$  and correspondingly  $\Lambda_p^{zz}$ ,  $\Lambda_p^{+-}$  characterize the vertex  $\Lambda$ . Using this separation in Eq. (2), one finds that the longitudinal susceptibility is unaffected by the external magnetic field to order  $(p_{F+} - p_{F-})^2/p_F^2$ . The transverse susceptibility is found to be described by Eqs. (13) with  $J(\omega)$  replaced by

$$J_{+-}(\omega; \mathbf{q}, \omega_0) = [2\pi N(0)]^{-1} \int \frac{d^3p}{(2\pi)^3} G_+(\mathbf{p}, \omega) \times G_-(\mathbf{p}+\mathbf{q}, \omega+\omega_0) = \frac{i}{2v_F q} \ln \frac{K_+(\omega) + K_-(\omega+\omega_0) + q}{K_+(\omega) + K_-(\omega+\omega_0) - q}, \quad (19)$$

where

$$K_\sigma = i/2l + (\text{sgn}\omega) (p_{F\sigma}^2 + 2m\omega)^{1/2}. \quad (20)$$

The corresponding transverse spin susceptibility is then given by Eq. (7) together with

$$\chi_{\text{imp}}^{\text{norm}}(\mathbf{q}, \omega_0) = 1 + \frac{i\omega_0 J(0; \mathbf{q}, \omega_0 - \Delta) [1 - (3\tau_1)^{-1} J(0; \mathbf{q}, \omega_0 - \Delta)]}{1 - (\tau_0)^{-1} J(0; \mathbf{q}, \omega_0 - \Delta) [1 - (3\tau_1)^{-1} J(0; \mathbf{q}, \omega_0 - \Delta)]}. \quad (21)$$

The frequency  $\Delta$  appearing in  $\chi_{\text{imp}}^{\text{norm}}(\mathbf{q}, \omega_0)$  is the energy required to flip a particle spin in the action of the molecular field,  $(p_{F+}^2 - p_{F-}^2)/2m$ , which in the paramagnetic region is given by  $\Delta = \omega_L [1 - N(0) \bar{V}_c]^{-1}$ , where  $\omega_L$  is the free-electron spin-resonance frequency. In the limit  $ql \gg 1$  and  $\tau\Delta \ll 1$ , the susceptibility has the asymptotic form

$$\chi_{+-}(\mathbf{q}, \omega_0) = 2\mu_B^2 N(0) [1 - N(0) \bar{V}_c + (i\pi\omega_0/2v_F q) N(0) \bar{V}_c]^{-1}, \quad (22)$$

which is the result given by the free-electron picture at  $q \ll p_F$  and  $\omega_0/qv_F \ll 1$ . The absence of polarization effects result from the assumption  $\tau\Delta \ll 1$ . If this restriction is relaxed the situation is more complicated and the logarithm in Eq. (16) must be retained. In the small momentum range  $ql \ll 1$  a straightforward expansion of Eq. (21) leads to the result

$$\chi_{+-}(\mathbf{q}, \omega_0) = \frac{2\mu_B^2 N(0)}{1 - N(0) \bar{V}_c} \frac{i\gamma [1 - N(0) \bar{V}_c] [1 - i\tau\Delta ((1 - i\Gamma)/(1 + \Gamma^2)) (1 - 1/\tau_1\gamma)] - \omega_L}{i\gamma (1 - N(0) \bar{V}_c) + \omega_0 - \omega_L}, \quad (23)$$

<sup>9</sup> K. Maki and P. Fulde, Phys. Rev. **140**, A1586 (1965).

where  $\gamma$  is given by

$$\gamma = 4/3\tau_1 + Dq^2[(1-i\Gamma)/(1+\Gamma^2)], \quad (24)$$

$\Gamma = N(0)\bar{V}_c\tau\Delta$ , and  $D = \frac{1}{3}rv_F^2$ . In the absence of an external field Eq. (23) reduces to

$$\chi_{+-}(\mathbf{q}, \omega_0) = \frac{2\mu_B^2 N(0)}{1-N(0)\bar{V}_c} \times \frac{(4/3\tau_1 + Dq^2)[1-N(0)\bar{V}_c]}{(4/3\tau_1 + Dq^2)[1-N(0)\bar{V}_c] - i\omega_0}. \quad (23')$$

Equation (23) is only valid near the pole of  $\chi_{+-}(\mathbf{q}, \omega_0)$ . It is seen from the structure of Eq. (23) that the damping and diffusion effects vanish as the ferromagnetic instability is approached. We note from Eq. (24) that for very pure metals the reduction of the diffusion constant as function of magnetic field can be quite important.

It is interesting to consider the macroscopic equations which correspond to the spin susceptibility described above. For the moment we assume that no static external magnetic field is applied and consider the finite-field case later. We introduce the angular distribution function

$$f(\Omega; \mathbf{q}, \omega_0) = i[4\pi^3 N(0)]^{-1} \int d\omega dp p^2 G(\mathbf{p}, \omega) G(\mathbf{p}+\mathbf{q}, \omega+\omega_0) \times [\Lambda_\sigma(\mathbf{p}, \omega, \omega_0) + \frac{1}{3}\Lambda_p(\mathbf{p}, \omega, \omega_0)]. \quad (25)$$

From Eq. (1) it follows that the susceptibility is the angular average of this quantity:

$$\chi(\mathbf{q}, \omega_0) = 2\mu_B^2 N(0) \langle f(\Omega) \rangle. \quad (26)$$

The equation for  $f(\Omega)$  is obtained from the vertex equation (2) by multiplying both sides by a factor  $i[4\pi^3 N(0)]^{-1}$  and doing the integrals over frequency and  $|\mathbf{p}|$ . This leads to the result

$$f(\Omega; \mathbf{q}, \omega_0) = [1 + N(0)\bar{V}_c \langle f(\Omega) \rangle] \left[ 1 - \frac{\omega_0}{\omega_0 - v_F q x + i/\tau} \times \frac{1 - (3\tau_1)^{-1} J(0)}{1 - (\tau_0)^{-1} J(\omega) (1 - (3\tau_1)^{-1} J(\omega))} \right], \quad (27)$$

where

$$x = \cos(\mathbf{p} \cdot \mathbf{q} / |\mathbf{p}| |\mathbf{q}|).$$

The quantity  $J(0)$  is eliminated in this equation by relating it to  $\langle f(\Omega) \rangle$ , with the result

$$(\omega_0 - qv_F x) f(\Omega) = -qv_F x [1 + N(0)\bar{V}_c \langle f \rangle] + (4i/3\tau_1) [1 - \langle f \rangle (1 - N(0)\bar{V}_c)] - (i/\tau) [f(\Omega) - \langle f \rangle]. \quad (28)$$

The susceptibility calculated from this equation agrees

with the one calculated from Eqs. (7) and (21) except for terms of the order  $(\tau_0/\tau_1)^2$ , which is the desired accuracy. If  $\tau_1^{-1} = 0$ , Eq. (28) is identical to the linearized Boltzmann equation for the spin susceptibility in the presence of exchange and ordinary impurity scattering, which can be deduced from Silin<sup>10</sup> together with the proper solution of the collision integral. Our result above represents the proper extension to include the presence of spin-orbital interaction.

Passing on to the case when a static magnetic field is applied along the  $z$  axis, the same procedure yields the following equation for the transverse spin susceptibility:

$$(\omega_0 - \Delta - qv_F x) f(\Omega) = - (qv_F x + \Delta) [1 + N(0)\bar{V}_c \langle f \rangle] + (4i/3\tau_1) [1 - \langle f \rangle (1 - N(0)\bar{V}_c)] - (i/\tau) [f(\Omega) - \langle f \rangle]. \quad (29)$$

The corresponding susceptibility agrees with the one calculated from Eqs. (1) and (2) to the desired accuracy. The longitudinal susceptibility is found to satisfy Eq. (28), in agreement with our previous discussion.

### III. FERROMAGNETIC SPIN SUSCEPTIBILITY

In this section, we study the susceptibility transverse to the direction of the spontaneous magnetization [ $N(0)\bar{V}_c > 1$ ]. Repeating the procedure of Sec. II for the ferromagnet we again must break up the vertex equation into longitudinal and transverse components as discussed in the paragraph following Eq. (16). Substituting this vertex into Eq. (2) one obtains the same equations as (7) and (14), with  $J(\omega')$  replaced by Eq. (19). The two Fermi momenta  $p_{F+}$ ,  $p_{F-}$  are obtained by the self-consistency condition

$$(2m)^{-1} (p_{F+}^2 - p_{F-}^2) = \omega_L + (\bar{V}_c/6\pi^2) (p_{F+}^3 - p_{F-}^3), \quad (30)$$

together with the conservation of total particle number. It is permissible to break the region of integration in Eq. (14) into three different parts, as done for the paramagnetic case; however, it is important to include properly the nonlinear splitting of the Fermi surface while doing the integrals.

Consider first the case with no spin-orbital interaction. Then, in the static limit, one can perform the integration of Eq. (14) to order  $q^2/p_F^2$  by making a change of variables from  $\omega$  to

$$y = [(p_{F+}^2 + 2m\omega)^{1/2} + (p_{F-}^2 + 2m\omega)^{1/2}] \operatorname{sgn}\omega.$$

One obtains the result

$$\chi^{+-}_{\text{imp}}{}^{\text{norm}}(\mathbf{q}, 0) = (2p_F)^{-1} \{ [(3y_0^2 + \delta^2)/3y_0] - \frac{1}{3}q^2 [(5y_0^2 - \delta^2)/5y_0^3] + O(q^4) \} \equiv \chi^{+-}_{\text{imp}}{}^{\text{norm}}(0, 0) (1 - \alpha q^2), \quad (31)$$

<sup>10</sup> V. P. Silin, Zh. Eksperim. i Teor. Fiz. **33**, 1227 (1957) [English transl.: Soviet Phys.—JETP **6**, 945 (1958)].

where we have introduced the curvature  $\alpha$  and taken

$$\begin{aligned} \gamma_0 &= \rho_{F+} + \rho_{F-}, \\ \delta &= \rho_{F+} - \rho_{F-}. \end{aligned} \quad (32)$$

This result is well known from the work of Nagaoka.<sup>11</sup> To discuss the properties of the low-frequency susceptibility, we can restrict ourselves to  $\omega_0 \ll \Delta$ , the spin-flip energy in the molecular field. To this approximation we find the spin susceptibility by integrating Eq. (14), keeping terms linear in  $\omega_0$  but dropping terms of order  $(\rho_{FL})^{-1}$ , to be

$$\begin{aligned} \chi^{+-}_{\text{imp}}{}^{\text{norm}}(\mathbf{q}, \omega_0) &= \chi^{+-}_{\text{imp}}{}^{\text{norm}}(\mathbf{q}, 0) + \delta\omega_0/3v_F\gamma_0^2 + (\omega_0/v_F\delta) \\ &\times [1 + \frac{1}{3}q^2(i/l - \delta)^{-2}] / [1 + (iq^2/3\delta l)(i/l - \delta)^{-2}]. \end{aligned} \quad (33)$$

By inserting this susceptibility into Eq. (7) and looking for the poles one finds the following solution:

$$\begin{aligned} \omega_0 &= \omega_L + Bq^2, \\ B &= \alpha\Delta - (i\omega_L/\Delta)(\bar{V}_c\gamma_0^2/24\pi^2)[(1 - i\delta l)/(1 + \delta^2 l^2)]. \end{aligned} \quad (34)$$

The first term is the ordinary quadratic spin-wave excitation spectrum for pure systems which has been discussed extensively by Izuyama, Kim, and Kubo.<sup>2</sup> The second term represents the effects of impurities and is seen to vanish if the external field is set equal to zero. The reduction of the diffusion constant by  $(1 + \delta^2 l^2)^{-1}$  has been found previously by Hirst<sup>4</sup> and by Kaplan.<sup>5</sup> However, the additional factor of the form  $\omega_L/\delta$  which we obtain makes the diffusion constant vanishingly small. It should be noted here that we have ignored the effects of magnetization on the scattering rate [see Eq. (6)]. When the magnetization is small,  $\delta \lesssim 0.1\rho_F$ , this procedure is rigorously justified. We also use it for large  $\delta$ , in order to avoid unphysical spin damping.

In the case that spin-orbital interaction is present the calculations are done in analogy to the case without spin-orbital scattering. One finds that Eqs. (34) are replaced by

$$\begin{aligned} \omega_0 &= \omega_1 + B_1q^2, \\ \omega_1 &= \omega_L(1 - 4i/3\delta l_1), \\ B_1 &= B - \alpha\Delta 4i/3\delta l_1. \end{aligned} \quad (35)$$

<sup>11</sup> Y. Nagaoka, Progr. Theoret. Phys. (Kyoto) **28**, 1033 (1962).

As expected, spin-orbital scattering leads to damping of the infinite-wavelength resonance and also gives a diffusion constant proportional to the spin-flip scattering rate. It is estimated to be of order  $l/l_1$  smaller than the results of Hirst<sup>4</sup> and of Kaplan<sup>5</sup> and is probably too small to be observed in most metals.

#### IV. DISCUSSION

We have studied microscopically the one-band spin susceptibility of an electronic system in the presence of exchange interaction and spin-orbital scattering due to randomly distributed scattering centers. For the paramagnetic state a linearized Boltzmann equation was derived from the microscopic vertex equation. It turned out that the relaxation time and diffusion constant of the susceptibility tend to zero as the ferromagnetic instability is approached. In an external magnetic field the diffusion constant was found to be field-dependent if the mean free path in the sample is very large. In the ferromagnetic state the diffusion constant was found many orders of magnitude smaller than in the previous investigations.

It should be noted that the presence of spin-orbital scattering has drastic effects on the small-momentum, small-frequency portion of the spin-correlation function. In particular, the maximum in the corresponding spectral function at zero momentum occurs at a finite frequency  $\omega_{\text{max}} = (\tau_s')^{-1}$ , where  $\tau_s'$  is the total spin relaxation time given by Eq. (23). As is well known from studies of nearly ferromagnetic systems,<sup>12</sup> the free energy involves an integration of the spectral function times a Bose factor over frequency. At low temperatures  $T < \omega_{\text{max}}$  the Bose factor cuts off the integration before the peak in the spectral function is reached. Thus the thermal excitations sample only the small-frequency region in which this spectral function can be expanded in a power series in  $\omega$ . As a consequence one finds only terms linear and cubic in temperature in the specific heat. Previous considerations<sup>6</sup> have shown that the linear term is unaffected by mean free path.

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<sup>12</sup> W. Brenig, H. J. Mikeska, and E. Riedel, Z. Physik **206**, 439 (1967).