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[23]The zero-sound velocity depends on all of the $F_l^{(1)}$'s and in a model in which $F_l^{(1)} = 0$ for $l \geqslant 2$ the theoretical value of c_0 (see Appendix B) is in good agreement with experiment. (See Ref. 24.)

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[27]A procedure analogous to the one used for examining the LR Eq. (IV. 8), could be used in the FR. However the solution would require new parameters.

[28]We do not include a term $\propto \Phi_3 e_q^3 \ln|\epsilon/\epsilon_L|$. This term requires the presence of a term $\alpha\, e_q^2/\epsilon$ on the right-hand side of (VI. 1) which does not exist; $\Phi_3 = 0$.

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Logarithmic Terms in the Specific Heat of a Normal Fermi Liquid*

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Using the properties of the mass operator and vertex function derived previously, we calculate corrections of the form $T^3 \ln T$ to the term in the specific heat C_V that is linear in the temperature. The connection of this calculation with the microscopic theory is pointed out and the limitations are discussed.

I. INTRODUCTION

In a previous paper[1] we studied the corrections to the single-particle mass operator of a normal Fermi liquid, which arise from the interaction of quasiparticles with density and spin density fluctuations at $T = 0$. For a quasiparticle with momentum q near the Fermi momentum k_F, the corrections were found to be proportional to $(q - k_F)^3 \times \ln|q - k_F|$. The coefficient of this logarithmic contribution was obtained in a completely renormalized form in terms of Landau parameters.

In this paper, which supplements I, we calculate the change in the low-temperature specific heat resulting from the above corrections to the mass operator. The calculation is performed within the phenonenological Landau theory.[2] The result is a $T^3 \ln T$ correction to the leading linear term in the specific heat.

II. ENTROPY AND SPECIFIC HEAT

In I we used the formalism of zero-temperature Green's functions to study the single-particle excitation spectrum of a normal Fermi liquid. In this scheme the concept of quasiparticles is introduced through the poles, in the complex frequency plane, of the single-particle Green's function. The real part of the solution of the pole equation determines the energy dispersion law and the imaginary part measures the quasiparticle lifetimes. Within this formalism the low-temperature equilibrium properties of a Fermi system may be calculated by carrying out a temperature expansion of the thermodynamic potential, $\Omega[G]$, given as a stationary functional of the temperature-dependent Green's function G.[3] This procedure was used by Luttinger[4] to derive the linear term in the specific heat C_V. The problem of calculating the coefficient of the $T^3 \ln T$ term rigorously turns out to be rather difficult. The trouble arises from the overlap of the momentum distributions of quasiparticles and quasiholes at nonzero temperatures.[5,6] If this overlap is neglected in the expansion of $\Omega[G]$, the resulting expression for the $T^3 \ln T$ term is found to be identical to that which one obtains starting from the Landau entropy of a normal Fermi liquid.[2] That is, it is equivalent to the specific heat computed from the entropy as given in Eq. (II.2) (below), using (I.VIII.4) for the quasiparticle energy and neglecting the temperature dependence of the Landau parameters.

The postulates of Landau's phenomenological theory of a Fermi liquid are[2]:

(*i*) The low-lying excited states (excited quasiparticles) of the interacting system are in one-to-one correspondence with those of an ideal gas.

(*ii*) The internal energy E is a functional of the quasiparticle distribution function $n_{\vec{q},\sigma}$ with

$$\delta E = \sum_{\vec{q},\sigma} \epsilon_{\vec{q},\sigma}\, \delta n_{\vec{q},\sigma} + \frac{1}{2}\sum_{\vec{q},\sigma}\sum_{\vec{q}',\sigma'} f(\vec{q},\sigma;\vec{q}',\sigma')\,\delta n_{\vec{q},\sigma}\,\delta n_{q'\sigma'}, \quad \text{(II.1)}$$

where $\epsilon_{\vec{q},\sigma} = \epsilon_{q,\sigma}$ is the energy, in an isotropic system, of a quasiparticle with momentum $q = |\vec{q}|$ and spin σ. The $f(\vec{k}_F,\sigma;\vec{k}'_F,\sigma')$ are the Landau parameters.

The entropy per unit volume, S, can then be written as[8]

$$S = -k_B \sum_{\vec{q},\sigma}[n_{q,\sigma}\ln n_{q,\sigma} + (1-n_{q,\sigma})\ln(1-n_{q,\sigma})] \; . \quad \text{(II.2)}$$

The equilibrium function $n_{q,\sigma}$ which follows from (II.1) and (II.2) and the requirement that the number of quasiparticles equal the number of particles is

$$n_{q,\sigma} = \{\exp[\beta(\epsilon_{q,\sigma}(T) - \mu(T))] + 1\} \equiv n(T, \epsilon_{q,\sigma}(T), \mu)\;, \quad \text{(II.3)}$$

where $\beta^{-1} = k_B T$, and $\mu(T)$ is the chemical potential. $\epsilon_{q,\sigma}(T)$ in (II.3) is a functional of $n_{q,\sigma}(T)$ and hence is a function of T.

The specific heat at constant volume V obtained from (II.2) and (II.3) is

$$C_v = T\frac{\partial S(T,\mu)}{\partial T}\bigg|_V = T\frac{\partial S}{\partial T}\bigg|_{\mu,V} + T\frac{\partial S}{\partial \mu}\bigg|_{T,V}\frac{\partial \mu}{\partial T}\bigg|_V . \quad \text{(II.4)}$$

As $T\to 0$, $S \sim T$ and[4] $\partial\mu/\partial T|_V \sim T$. Since we shall neglect terms of $O(T^3)$ we drop the second term on the right-hand side of (II.4) and hereafter use $\mu(T=O) = \mu$.

For spin-independent forces, the quasiparticle energy does not depend on σ. We write

$$\epsilon_{q,\sigma} - \mu = e_q + \delta\epsilon_q(T), \quad \text{(II.5)}$$

where $e_q = v_F(q-k_F)$, $v_F = k_F/m^*$, and m^* is the effective mass. The variation of the entropy is then

$$\delta S = (2/T)\textstyle\sum_q e_q[\partial n(T,e_q)/\partial e_q]\delta\epsilon_q(T). \quad \text{(II.6)}$$

We shall split $\delta\epsilon_q(T)$ into two parts

$$\delta\epsilon_q(T) = \delta_q\epsilon_q + \delta_T\epsilon_q, \quad \text{(II.7)}$$

where $\delta_q\epsilon_q$ is the correction to e_q at zero temperature and $\delta_T\epsilon_q$ is a temperature-dependent correction which vanishes as $T\to 0$. Correspondingly, δS is split into two terms

$$\delta S = \delta_q S + \delta_T S, \quad \text{(II.8)}$$

where now

$$\delta_q S = (2/T)\textstyle\sum_{\vec{q}} e_q[\partial n(T,e_q)/\partial e_q]\delta_q\epsilon_q\,, \quad \text{(II.9)}$$

$$\delta_T S = (2/T)\textstyle\sum_{\vec{q}} e_q[\partial n(T,e_q)/\partial e_q]\delta_T\epsilon_q\,. \quad \text{(II.10)}$$

III. EVALUATION OF $\delta_q S$

The correction $\delta_q\epsilon_q$ at $T=0$ is given by $\delta_q E$ from Eq. (I.VIII.4) apart from a possible regular term proportional to e_q^2. Hence

$$\delta_q\epsilon_q = \gamma e_q^2 + \frac{1}{(v_F k_F)^2}(\Phi_0+\Phi_1)e_q^3\ln\left|\frac{e_q}{\epsilon}\right| . \quad \text{(III.1)}$$

Here $\epsilon_L = v_F k_L$ and $0 < k_L \ll k_F$. The quantities Φ_0 and Φ_1 are determined by Landau parameters from (I.VI.8) and (I.VII.18). The coefficient γ is unknown; however, the $\gamma e q^2$ term in (III.1) does not contribute to $\delta_q S$ due to the symmetry of $\partial n_q/\partial e_q$ around $q = k_F$. Thus we find

$$\delta_q S = 2\frac{1}{(v_F k_F)^2}(\Phi_0+\Phi_1)\frac{1}{T}\int\frac{d^3q}{(2\pi)^3}$$

$$\times[\partial n(T,e_q)/\partial e_q]e_q^4 \ln|e_q/\epsilon_L| \qquad (\text{III.}2)$$

Changing integration variables from q to e_q via

$$q^2dq = m^* k_F[1+O(e_q)]de_q ,$$

and introducing the dimensionless variable $x=\beta e_q$, we obtain

$$\delta_q S = \xi\frac{k_B^4}{\pi^2}\left(\frac{m^*}{k_F}\right)^3(\Phi_0+\Phi_1)T^3\ln\left|\frac{T}{T_L}\right| + O(T^3), \qquad (\text{III.}3)$$

where $\epsilon_L = k_B T_L$ and

$$\xi = \int_{-\infty}^{\infty} dx\,[dn(x)/dx]x^4 .$$

Defining T_F^*, the effective Fermi temperature, by $k_B T_F^* = k_F^2/2m^*$ we finally obtain the entropy per mole

$$\delta_q S = -\frac{21\pi^4}{120}R(\Phi_0+\Phi_1)\left(\frac{T}{T_F^*}\right)^3\ln\left|\frac{T}{T_L}\right| + O(T^3) , \qquad (\text{III.}4)$$

where R is the gas constant.

IV. EVALUATION OF $\delta_T S$

In order to calculate $\delta_T S$ from (II. 10) we have to find the temperature-dependent correction to the quasiparticle energy, $\delta_T\epsilon_q$. Following Landau's procedure, the variation of ϵ_q is related to the variation of the distribution function by

$$\delta_T\epsilon_q = \sum_{q'} f^{(1)}(\vec{q},\vec{q}\,')\delta_T n_{q'} , \qquad (\text{IV.}1)$$

where $f^{(1)}(\vec{q},\vec{q}\,')$ is the spin-symmetric part of $f(\vec{q},\sigma;\vec{q},\sigma')$. The distribution function varies with temperature in two ways:

(i) Through the explicit dependence on T, i.e.,

$$\delta_T n(T,\epsilon_q) \equiv n(T,\epsilon_q(T=0)) - n_q^0, \quad \text{where } n_q^0 = \Theta[-\epsilon_q(T=0)] , \qquad (\text{IV.}2)$$

and Θ is the unit step function.

(ii) through the temperature dependence of the quasiparticle energy

$$\delta_T n(0,\epsilon_q(T)) \equiv (\partial n_q^0/\partial\epsilon_q)\delta_T\epsilon_q . \qquad (\text{IV.}3)$$

Thus the net variation of n_q is

$$\delta_T n_q = \delta_T n(T,\epsilon_q) + (\partial n_q^0/\partial e_q)\delta_T\epsilon_q . \qquad (\text{IV.}4)$$

Insertion of (IV. 4) into (IV. 1) leads to an integral equation for $\delta_T\epsilon_q$

$$\delta_T\epsilon_q = \sum_{q'} f^{(1)}(\vec{q},\vec{q}\,')[\delta_T n(T,\epsilon_{q'}) + (\partial n_{q'}^0/\partial\epsilon_{q'})\delta_T\epsilon_{q'}] . \qquad (\text{IV.}5)$$

The solution of this equation can be written as

$$\delta_T\epsilon_q = \sum_{\vec{q}\,'}\psi^{(1)}(\vec{q},\vec{q}\,')\delta_{T'} n(T,e_q') \qquad (\text{IV.}6)$$

where the resolvent $\psi^{(1)}$ satisfies

$$\psi^{(1)}(\vec{q},\vec{q}\,') = f^{(1)}(\vec{q},\vec{q}\,') + \sum_{\vec{q}\,''} f^{(1)}(\vec{q},\vec{q}\,'')(\partial n_{q''}^0/\partial\epsilon_{q''})\psi^{(1)}(\vec{q}\,'',\vec{q}\,'). \qquad (\text{IV.}7)$$

The function $f^{(1)}(\vec{q},\vec{q}\,')$ is the second functional derivative of the internal energy with respect to δn_q: Thus it will, in general, be temperature-dependent and so will $\psi^{(1)}$ Since the form of this dependence is not known, we will neglect it.

The resolvent $\psi^{(1)}$ at $T=0$ is related to real part, $\tilde{\Gamma}^{(1)}$ of the vertex function considered in I. To obtain this relationship, we note that in analogy with (IV. 6), using Galilean invariance,[2] we can write

$$\partial\epsilon_q/\partial\vec{q} = \vec{q}/m + \sum_{\vec{q}\,'}\psi^{(1)}(\vec{q},\vec{q}\,')(\partial n_{q'}^0/\partial q')\vec{q}\,'/m \qquad (\text{IV.}8)$$

at $T=0$. On the other hand, from (I. VIII. 1), the Ward identities (I. V. 1c) and (I. V. 1d), and using a well-known renormalization procedure, we find at $T=0$.

$$\partial E_q/\partial\vec{q} = \vec{q}/m - (1/V)\sum_{\vec{q}'} a_q a_{q'} \tilde{\Gamma}^{k(1)}(\vec{q}, E_q; E_{q'}, E_{q'})\delta(E_{q'})\vec{q}'/m \tag{IV. 9}$$

in the notation of I. Here $a_q = [1-\partial M(\vec{q}, E_q)/\partial E_q]^{-1}$ is the q-dependent renormalization coefficient and $\partial n_q^0/\partial E_q = -\delta(E_q)$. Since $E_q = \epsilon_q$ at $T=0$, we deduce,[9] by comparing (IV. 8) and (IV. 9),

$$V\psi^{(1)}(\vec{q}, \vec{n}'k_F) = a_q a_F \tilde{\Gamma}^{k(1)}(\vec{q}, \epsilon_q; \vec{n}'k_F, 0). \tag{IV. 10a}$$

From (IV.10a) we infer

$$V\psi^{(1)}(\vec{q}, \vec{q}') = a_q a_{q'} \tilde{\Gamma}^{k(1)}(\vec{q}, \epsilon_q; \vec{q}', \epsilon_{q'}) + X(\vec{q}, \vec{q}') . \tag{IV. 10b}$$

$\psi^{(1)}$, $\tilde{\Gamma}^{k(1)}$, and X are symmetric in $\vec{q}$ and $\vec{q}'$. From (IV. 10a) $X(\vec{q}, \vec{q}')$ must vanish if $\vec{q}$ and/or $\vec{q}'$ is on the Fermi surface. Since we do not have further information about X, we shall neglect it. Using (IV. 11) in (IV. 6) and substituting this in (II. 10) yields

$$\frac{1}{V}\delta_T S = -\frac{2}{T}\sum_{j=1}^{2} C_{1j}\int\frac{d^3q}{(2\pi)^3}\int\frac{d^3q'}{(2\pi)^3}\int\frac{\partial n(T,e_q)}{\partial e_q} e_q a_q a_{q'} \delta_T n(T, \epsilon_{q'}) \lim_{\vec{k}\to 0} \tilde{\Gamma}^{(j)}(\vec{q}-\vec{q}', \epsilon_q - \epsilon_{q'}; \vec{k}+\vec{q}', \epsilon_{q'}, \vec{q}', \epsilon_{q'}) , \tag{IV. 11}$$

where use has been made of the crossing symmetry relation (I. IV. 2a) ($C_{11} = \frac{1}{2}$, $C_{12} = \frac{3}{2}$).

The leading term of (IV. 11), namely that proportional to $T^3 \ln T$ arises from the energy-momentum transfer variables $\vec{q}-\vec{q}'$, $\epsilon_q - \epsilon_{q'}$ in $\tilde{\Gamma}^{(j)}$. The dependence of $\tilde{\Gamma}^{(j)}$ on the deviations of the second and third variables from the Fermi surface leads to higher-order terms in T.[10] For the same reason a_q and a_q' can be replaced by a_F, the value on the Fermi surface.[11] Thus we have

$$\frac{1}{V}\delta_T S = -\frac{2}{T}\sum_{j=1}^{2} C_{1j}\,\frac{m^* k_F}{2\pi^2}\int d\epsilon\int d\epsilon' \int\frac{d\vec{n}}{4\pi}\int\frac{d\vec{n}}{4\pi}\frac{\partial n(T,\epsilon)}{\partial\epsilon}\delta_T n(T,\epsilon') \lim_{\vec{n}'\to\vec{n}} \tilde{T}^{(j)}(\vec{q}-\vec{q}', \epsilon-\epsilon'; \vec{n}', \vec{n}) + \cdots , \tag{IV.12}$$

where we have made the variable transformations $\vec{q} = \vec{n}(\epsilon/v_F + k_F)$, $\vec{q}' = \vec{n}'(\epsilon'/v_F + k_F)$; $\vec{n}'$ and $\vec{n}$ are unit vectors. In (IV. 12)

$$\tilde{T}^{(j)} = (m^* k_F a_F 2/2\pi^2)\tilde{\Gamma}^{(j)}$$

is the renormalized vertex function.

Next we notice that the integral over $\vec{n}'$ in (IV. 12) is of the same type as that considered in Sec. IV of I. Referring to that discussion, we reproduce the result (IIV.3)

$$\sum_{j=1}^{2} C_{1j}\int\frac{d\vec{n}'}{4\pi}\lim_{\vec{n}\to\vec{n}'} T^{(j)}(\vec{q}-\vec{q}', \epsilon-\epsilon'; \vec{n}, \vec{n}') = \left(\frac{\epsilon-\epsilon'}{v_F k_F}\right)^2 (\phi_0 + \phi_1 + \lambda_0)\ln\left|\frac{\epsilon-\epsilon'}{\epsilon_L}\right| \tag{IV.13}$$

$$+ \text{regular terms in } \epsilon \text{ and } \epsilon',$$

with $\varphi_2 = 0$ and ϕ_0, ϕ_1, and λ_0 given in (I.VII.18). Inserting (IV.13) into (IV.12) and transforming to the variables $x = \beta\epsilon$, $x' = \beta\epsilon'$, one obtains

$$\frac{1}{V}\delta_T S = -\frac{1}{\pi^2}\left(\frac{m^*}{k_F}\right)^3 (\phi_0 + \varphi_1 + \lambda_0) k_B{}^4 T^3 \ln\left|\frac{T}{T_L}\right| \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dx' x(x-x')^2 \frac{\partial n(x)}{\partial x}\delta_T n(x') + O(T^3). \tag{IV. 14}$$

Integrating by parts on x' we find

$$\int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dx' x(x-x')^2 \frac{\partial n(x)}{\partial x}\delta_T n(x') = \left[\int_{-\infty}^{\infty} dx\, x^2 \frac{\partial n(x)}{\partial x}\right]^2 = \frac{\pi^4}{9} \tag{IV. 15}$$

Thus $\delta_T S$ is

$$\delta_T S = -(\pi^4/24) R(\phi_0 + \phi_1 + \lambda_0)(T/T_F{}^*)^3 \ln|T/T_L| . \tag{IV. 16}$$

From (II. 4), (III. 4), and (IV. 16), we find the specific heat per mole, including the linear term, to be

$$\frac{C_V}{RT}=\frac{\pi^2}{2T_F{}^*}\left[1-\frac{\pi^2}{20}\Phi\left(\frac{T}{T_F{}^*}\right)^2\ln\left|\frac{T}{T_L}\right|\right]+O(T^2)\ , \quad \Phi=21(\Phi_0+\Phi_1)+5(\Phi_0+\Phi_1+\lambda_0)\ , \tag{IV.17}$$

where Φ_0 and Φ_1 are determined by ϕ_0 and ϕ_1 through (I. VI. 8). The result (IV. 17) is identical to that reported earlier.[6] We refer to Ref. 6 for a comparison with experiments.

The coefficient Φ of the logarithmic term in the specific heat (IV. 17) includes contributions of incoherent density and spin density fluctuations and collective modes from both the long-wave length region and the region of momenta near $2k_F$. The latter enter through λ_0. Examining the entries in Table 2, Appendix B of I, one finds that in He^3 the spin fluctuations, "paramagnons," dominate. This provides support for approximations which consider paramagnon effects exclusively.[12]

V. DISCUSSION

In conclusion, we make the following comments:

(1) The coefficient Φ in (IV. 17) is not exact since it does not include contributions from the temperature dependence of the Landau parameters[13] and possibly from X from (IV. 11). Brenig, Mikeska, and Riedel[14] have carried out a temperature expansion of the thermodynamic potential $\Omega[G]$ for a selected class of diagrams corresponding to the shielded potential approximation (SPA). If the expansion for the SPA is performed with the assumptions made in the present work, one finds, instead of Φ_{SPA} corresponding to Φ of (IV. 17), a coefficient of $\Phi_{SPA}'=\frac{1}{3}\Phi_{SPA}$. This reduction arises from "boson-like" terms in the expansion of $\Omega[G]$, terms which appear first in third-order perturbation theory. However, it should be pointed out that the factor of $\frac{1}{3}$ in Ref. 14 depends on their identification of the zero-temperature Landau parameters with the bare potential.

(2) It is obvious from the derivation of (III. 1)[1] that this form of $\delta_q\epsilon_q$ is only valid if $e_q \ll \epsilon_L$, i.e., $q \ll k_L \ll k_F$. As was pointed out in Ref. 6, this restricts the temperature range in which (IV. 17) hold to $T \ll T_L$. A rough estimate of T_L for He^3 at 27 atm is $T_L \sim 200$ m°K. This indicates that the temperature interval for which C_V is described by (IV. 17) may not exceed 20 m°K. At lower pressures this range is somewhat larger.[6] With the Landau parameters slightly readjusted and T_L determined by emperically fitting the experiment, (IV. 17) seems valid for $T \lesssim 50$ m°K at high pressures. At higher temperatures, (IV. 17) begins to deviate appreciably from the monotonically decreasing C_v/T data.[15]

The small range over which (IV. 17) applies led Brinkman and Engelsberg[16] to question attempts at calculating the specific heat of He^3 using Landau theory. The answer to their question has two parts. First, in the model that they use, the susceptibility does not contain m^*/m [see Eq. (I. IV. 14)]. Thus to fit the experimental susceptibility they must choose an enhancement parameter corresponding to our $A_0{}^{(2)}=-2.57$, of $A_0{}^{(2)\prime}=-19$. This modification $A_0{}^{(2)}$ precludes any comparison with the Landau result. Second, Landau's theory, which leads to logarithmic terms in C_V, provides the range of its applicability. The obvious conclusion is that at higher temperatures, higher-order terms in T will appear, possibly with strongly enhanced coefficients. An indication of possible sources of such terms is provided by the discussion of Sec. IV of I. We infer that terms proportional to $e_q{}^5\ln|e_q/\epsilon_L|$ should appear in the mass operator, with coefficients containing Landau parameters to the fifth power.

Of course, an approach in which the specific heat of a nearly ferromagnetic Fermi liquid can be computed without expansions in T and $\ln T$ is very desirable. Unfortunately, except for simple models which become questionable as the instability is approached,[17] such an approach has not yet been found.

(3) One might object that, in the momentum range where (III. 1) is important, the notion of quasiparticles is no longer valid because the decay rate is proportional to $(q-k_F)^2$. However, one can show from the temperature expansion of $\Omega[G]$ that the damping will contribute at most to order T^3.

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[5]A microscopic expansion of $\Omega[G]$ was reported in Ref. 6. There the overlap between the distributions was overlooked. This corresponds to considering $\Phi[G]$ (related to the mass operator Σ via $\Sigma=\delta\Phi/\delta G$) and $\Sigma[G]$ to be the same functionals of $G^>$ and $G^<$ at $T\neq 0$ as at $T=0$. This is not the case (see Ref. 7), as can be seen from third-order diagrams.

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[9]P. Nozières and J. M. Luttinger, Phys. Rev. 127, 1423, 1431 (1962).

[10]Here we assume that $\tilde{\Gamma}^{(j)}$ in (IV. 11) can be expanded in a power series in these deviations.

[11]Actually a_q contains a term proportional to $e_q^2 \ln|e_q|$ as may be seen from its definition and the results obtained in I for the mass operator. Because of the symmetry of $\partial n_q/\partial e_q$ and the antisymmetry of $\delta_T n_q$ around $q=k_F$, these terms do not contribute to the order we consider.

[12]N. F. Berk and J. R. Schrieffer, Phys. Rev. Letters 17, 433 (1966); S. Doniach and S. Engelsberg, *ibid.* 17, 750 (1966).

[13]V. J. Emery, Phys. Rev. 170, 205 (1968). We are indebted to Dr. Emery for providing us with a preprint prior to publication.

[14]W. Brenig, H. J. Mikeska, and E. Riedel, Z. Physik 206, 439 (1967); E. Riedel, *ibid.* 210, 403 (1967).

[15]With the smaller coefficient Φ_{SPA} of Ref. 14, a fit of the data over a larger temperature interval was obtained.

[16]W. F. Brinkman and S. Engelsberg, Phys. Rev. 169, 417 (1968).

[17]See for example the last part of the Discussion in I.