Renormalization of the Strangeness-Changing Axial-Vector Coupling Constants*

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Using the recently determined $KN\Lambda$ coupling constant and the multichannel effective-range analysis of the $\bar{K}N$ system, the renormalized axial-vector coupling constants $g_A{}^{\Lambda}$ for the decay $\Lambda\to p+e^-+\bar{\nu}$ and $g_A{}^{\Sigma^-}$ for the decay $\Sigma^-\to n+e^-+\bar{\nu}$ are recalculated, using the method of Adler and Weisberger. The results $|g_A{}^{\Lambda}|=0.77\pm0.1$ and $|g_A{}^{\Sigma^-}|=0.55\pm0.2$ agree well with the best-fit solution to all leptonic baryon decays based on the Cabibbo theory, but disagree with the recent experimental results from the decay angular distribution of polarized baryons. The mass-continuation problem is investigated in detail, and its effect is shown to be non-negligible.

IMMEDIATELY following the success of the original work by Adler and Weisberger[1,2] in evaluating the axial-vector renormalization in neutron β decay, several authors extended the calculation to hyperon β decay.[2,3] At that time the experimental information on kaon physics was poor. We now have meaningful determinations of the KNY coupling constants and a measure of their reliability,[4] better determination of the experimental KN total cross sections, and a multichannel effective-range analysis of the $\bar{K}N$ interaction[5] below threshold. Furthermore, the authors in Refs. 2 and 3 used different models to perform the necessary continuation in the kaon mass from zero to its physical value. Such continuation can be improtant, as we shall show, even though the analogous continuation in the pion mass is relatively unimportant in neutron β decay.[1,2] In view of the importance of the axial-vector renormalization in hyperon β decay to the $SU(3)\otimes SU(3)$ current algebra and to the Cabibbo theory of weak interactions,[6] we believe that a recalculation is called for.

We present the results of a detailed calculation of these renormalization effects in Λ and Σ β decays with the latest available experimental information on the KN system. We also carefully examine the problem of continuation in the external kaon mass.

Using standard reduction techniques and the hypothesis of $SU(3)\otimes SU(3)$ current algebra, one derives[1,2] two sum rules relating the renormalized axial-vector coupling constants, $g_A{}^{\Lambda}$ for the decay $\Lambda\to p+e^-+\bar{\nu}$ and $g_A{}^{\Sigma^-}$ for the decay $\Sigma^-\to n+e^-+\bar{\nu}$, to the off-mass shell KN scattering amplitude. These are summarized as

$$3=2(g_A{}^{\Lambda})^2+f_K{}^2I_0\,,\qquad(1a)$$

$$1=(g_A{}^{\Sigma^-})^2+f_K{}^2I_1\,,\qquad(1b)$$

corresponding to pure isospin 0 or 1 in the $\bar{K}N$ channel. where f_K is the K^+ leptonic decay constant and[7]

$$I_0=\frac{1}{\pi}\int_{(M_N+M_K)^2}^{\infty}\frac{ds}{s-M_N{}^2}\times[2\sigma_{K^-p}{}^{(0)}(s)-\sigma_{K^-n}{}^{(0)}(s)-2\sigma_{K^+p}{}^{(0)}(s)+\sigma_{K^+n}{}^{(0)}(s)]+\int_{(M_\Sigma+M_\pi)^2}^{(M_N+M_K)^2}\frac{8M_N ds}{(s-M_N)^2}\,\mathrm{Im}A_0{}^{(0)}(s)\,,\qquad(2a)$$

$$I_1=\frac{1}{\pi}\int_{(M_N+M_K)^2}^{\infty}\frac{ds}{s-M_N{}^2}[\sigma_{K^-n}{}^{(0)}(s)-\sigma_{K^+n}{}^{(0)}(s)]+\int_{(M_\Lambda+M_\pi)^2}^{(M_N+M_K)^2}\frac{8M_N ds}{(s-M_N{}^2)^2}\,\mathrm{Im}A_1{}^{(0)}(s)\,.\qquad(2b)$$

The superscript (0) indicates that the external kaon mass is zero. Under the above assumptions, these equations are exact but have no physical content unless the zero-mass amplitudes can be related to the physical scattering amplitude. Once this is done, the integrals over physical KN amplitudes can be evaluated with an accuracy of 5–10%, so that the mass continuation presents the only ambiguity.

Adler[1] has suggested possibly the most convincing model for this continuation. First, it is reasonable to assume that $\mathrm{Im}A^{(0)}(s)$ contains a factor $K^2(0)$, where $K(q^2)$ is the kaon form factor for the KNY vertex. This is true for all diagrams where the zero-mass kaon legs join a baryon leg or a baryon loop. Second, the threshold behavior of the partial-wave amplitude $\mathrm{Im}A_l{}^{(0)}(s)$ is known to contain a factor $(p_{\mathrm{off}})^{2l}$, where p_{off} is the c.m. momentum of a zero-mass kaon and a nucleon. Both of these features are satisfied if we

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2 W. I. Weisberger, Phys. Rev. **143**, 1302 (1966).

3 D. Amati, C. Bonchiat, and J. Nuyts, Phys. Letters **19**, 59 (1965); C. A. Levinson and I. J. Muzinich, Phys. Rev. Letters **15**, 715 (1965); **15**, 842(E) (1965); L. K. Pandit and J. Schechter, Phys. Letters **19**, 56 (1965).

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5 Jae Kwan Kim, Phys. Rev. Letters **19**, 1074 (1967).

6 N. Cabibbo, Phys. Rev. Letters **10**, 531 (1963).

7 We use the normalization $\sigma_{\bar{K}N}(\nu)=(4\pi/k)\,\mathrm{Im}T(\nu)$, where k is the laboratory momentum of the kaon.

TABLE I. Contributions to the integrals in Eq. (2) in F^2.

	Unphysical region	Physical region	Total
$I_0/K^2(0)$	1.44	3.12	4.56
$I_1/K^2(0)$	0.56	1.18	1.74

assume[8]

$$\mathrm{Im}A_l^{(0)}(s)=K^2(0)(p_{\mathrm{off}}/p_{\mathrm{on}})^{2l}\,\mathrm{Im}A_l(s)\,, \qquad (3)$$

where $A_l(s)$ is the physical KN partial-wave amplitude. (All quantities are evaluated at the same value of s.) The form factor, in this model, is eliminated using the Goldberger-Treiman relation in the form[9]

$$f_K=\frac{M_\Lambda+M_N}{G_{KN\Lambda}K(0)}g_A{}^\Lambda\,, \qquad (4)$$

where $G_{KN\Lambda}$ is the $KN\Lambda$ strong-coupling constant.

The numerical evaluation of Eqs. (2) was done separately for different energy regions. In the unphysical region and for low-energy $\bar{K}N$ scattering (up to $k=500$ MeV/c), Kim's[5] effective-range parametrization was used. For low-energy KN scattering (up to $k=300$ MeV/c) the effective-range parametrization of Goldhaber *et al.*[10] was used. In the region from 500 MeV/c for $\bar{K}N$ and 300 MeV/c for KN up to 200 BeV/c, the most recent cross-section measurements available were used.[11] For the known Y^* resonances in the $\bar{K}N$ system, we separate the $\bar{K}N$ total cross section into a resonance part, given by the parameters of the resonance,[12] and background. We modify only the resonance contributions by our mass-continuation prescription, Eq. (3). Above 20 BeV/c, we assume $\sigma_{\bar{K}N}-\sigma_{KN}\propto k^{-1/2}$. Contributions from this asymptotic region depend only on the power law and the known cross section at 20 BeV/c, and are all small.

[8] If the external-mass dependence is given by the direct-channel Born terms, Eq. (3) is multiplied by an additional factor close to 1, reflecting the fermion nature of the baryons. Even if the crossed-channel Born terms are included, Eq. (3) is expected to be modified by less than 20%, depending on the interaction. See, for example, F. T. Meiere, Phys. Rev. **159**, 1462 (1967), especially the Appendix.

[9] An alternative form is $f_K=(M_\Sigma+M_N)g_A{}^{\Sigma^-}/\sqrt{2}G_{\Sigma KN}K(0)$ which could be used consistently in Eq. (1b). However, in view of the smallness of the $G_{\Sigma KN}$ and the large error, we believe that it is better to use the other form.

[10] S. Goldhaber, W. Chinowsky, G. Goldhaber, W. Lee, T. O'Halloran. T. F. Stubbs, G. M. Pjerrou, D. H. Stork, and H. K. Ticho, Phys. Rev. Letters **9**, 135 (1962); V. J. Stenger, W. E. Slater, D. H. Stork, H. K. Ticho, G. Goldhaber, and S. Goldhaber, Phys. Rev. **134**, B1111 (1964).

[11] R. J. Abrams *et al.*, Phys. Rev. Letters **19**, 259 (1967); **19**, 678 (1967); J. D. Davies *et al.*, *ibid.* **18**, 62 (1967); R. L. Cool *et al.*, *ibid.* **16**, 1228 (1966); **17**, 102 (1966); W. Galbraith *et al.*, Phys. Rev. **138**, B913 (1965); A. Fridman and A. Michalon, Nuovo Cimento **48A**, 344 (1967).

[12] A. H. Rosenfeld, N. Barash-Schmidt, A. Barbaro-Galtieri, L. R. Price, P. Söding, C. Sohl. M. Ross, and W. Willis, Rev. Mod. Phys. **40**, 77 (1968).

TABLE II. g_A/g_V.

	This paper	Experimental fit to Cabibbo theory	Angular-distribution experiments
$n\rightarrow p+e^-+\bar{\nu}$	-1.23 ± 0.21	-1.25	-1.25 ± 0.04
$\Lambda\rightarrow p+e^-+\bar{\nu}$	-0.63 ± 0.08	-0.75	$-1.14_{-0.33}^{+0.23}$
$\Sigma^-\rightarrow n+e^-+\bar{\nu}$	0.55 ± 0.20	0.25	$0.05_{-0.32}^{+0.23}$

The results are summarized in Table I. Using the value $G_{KN\Lambda}{}^2/4\pi=13\pm3$,[4] we obtain[13]

$$|g_A{}^\Lambda|=+0.77\pm0.1\,,\qquad |g_A{}^{\Sigma^-}|=+0.55\pm0.2\,. \qquad (5)$$

According to the Cabibbo theory[6] all axial-vector coupling constants can be written in terms of two $SU(3)$ constants f and d. Thus,

$$(g_A{}^\Lambda)^2=\tfrac{1}{6}(g_A{}^N)^2(3-2\alpha)^2\,,\qquad (g_A{}^{\Sigma^-})^2=(g_A{}^N)^2(2\alpha-1)^2\,, \qquad (6)$$

where $g_A{}^N=f+d$ is the renormalized axial-vector coupling constant in neutron β decay and $\alpha=d/(f+d)$. Our results give[14]

$$g_A{}^N=-1.23\,,\quad \alpha=+0.73\,, \qquad (7)$$

compared with the latest experimental values[15,16] of -1.25 and 0.60, respectively. More detailed comparison with experiment is possible. The most meaningful one at the present time is with the leptonic decay rate. Assuming the Cabibbo theory with $\sin\theta_A=0.26$, our results[13] yield

$$(\Lambda\rightarrow p+e^-+\bar{\nu})/(\text{all }\Lambda)=(0.82\pm0.12)\times10^{-3}$$

and

$$(\Sigma^-\rightarrow n+e^-+\bar{\nu})/(\text{all }\Sigma^-)=(1.95\pm0.55)\times10^{-3}\,,$$

compared with the experimental values[15] $(0.88\pm0.15)\times10^{-3}$ and $(1.25\pm0.17)\times10^{-3}$, respectively. Thus, individually, $g_A{}^\Lambda$ is in good agreement but $g_A{}^{\Sigma^-}$ is somewhat large. Angular-distribution measurements can give the ratio g_A/g_V directly but the experimental situation[16,17] is not as clear as for the total decay rate. The results are in Table II. The experimental fit to

[13] The quoted error is statistical, arising primarily from the uncertainty in $G_{KN\Lambda}$ and does not include systematic error from the mass continuation.

[14] Equation (7) gives two solutions, the other solution $|g_A{}^N|=0.66$ and $\alpha=0.08$ disagrees with the experimental results badly, which we discard.

[15] W. Willis *et al.*, Phys. Rev. Letters **13**, 291 (1964); N. Brene, L. Veje, M. Roos, and C. Cronström, Phys. Rev. **149**, 1288 (1966); C. Carlson, *ibid.* **152**, 1433 (1966).

[16] L. K. Gershwin, M. Alston-Garnjost, R. O. Bangerter, A. Barbaro-Geltieri, F. T. Solmitz, and R. D. Tripp, Phys. Rev. Letters **20**, 1270 (1968).

[17] G. Conforto, in *Proceedings of the International School at Hercegovni, Yugoslavia, 1965*, edited by M. Nikolic (Secretariat du Département de Physique Corpusculaire, Centre de Recherches Nucléaires, Strasbourg-Cronenbourg, France, 1965).

Cabibbo theory refers to the best fit of seven baryon decay points to the Cabibbo theory.[16]

Finally, in order to see how critically our results depend on the specific model used for the mass continuation, we evaluate Eq. (2) for several simpler models. The only important differences lie in whether one makes the mass continuation for fixed values of $\sqrt{s}$ (the total c.m. energy) or for fixed values of ν (the lab energy of the kaon). These are related by $s=2M_N\nu+M_N^2+q^2$, where $\sqrt{(q^2)}$ is the external mass of the kaon. One extreme case is to assume $\mathrm{Im}A^{(0)}=K^2(0)\,\mathrm{Im}A$ for the same value of s. This decreases I_0 and I_1, giving $|g_A{}^\Lambda=0.86$ and $|g_A{}^{\Sigma^-}|=0.63$. The other extreme case[2] is to assume $\mathrm{Im}A^{(0)}=\mathrm{Im}A$ for the same value of ν and use the empirical value of f_K. This gives $|g_A{}^\Lambda|=0.53$ and $|g_A{}^{\Sigma^-}|=0.06$. All other combinations of the Goldberger-Treiman relation and correction factors gave results lying between these extremes. Thus the extrapolation in the kaon mass is more model-dependent than extrapolation in the pion mass, but for reasons mentioned earlier we are confident that the model used is a realistic one.

In conclusion, with the recent better-determined experimental results on kaon physics, one can evaluate numerically the two Adler-Weisberger–type sum rules for strangeness-changing currents very accurately. The results that we obtained here agree well with the best-fit solution to all leptonic baryon decays.[15,16] But if we compare them with the latest experimental results determined from the decay angular distribution of polarized hyperons,[16,17] our $(g_A{}^\Lambda)^2$ is small and $(g_A{}^{\Sigma^-})^2$ large. Using a different approximation for the mass continuation will not improve the results, since it either increases both or decreases both $(g_A{}^\Lambda)^2$ and $(g_A{}^{\Sigma^-})^2$. A better experimental determination on $g_A{}^\Lambda$ and $g_A{}^{\Sigma^-}$ will clear up this point.

Erratum

Sum Rules for the Axial-Vector Coupling Constant Renormalization in β Decay, STEPHEN L. ADLER [Phys. Rev. **140**, B736 (1965); **149**, 1294(E) (1966)].

1. In the first line of Eq. (62), $M_\pi{}^2$ should read $(M_\pi{}^{i,f})^2$. In Eq. (65), $f_{lJI}{}^B(W,0,0)$ should read $f_{lJI}{}^B(W,0,M_\pi)$. I wish to thank G. E. Brown, A. M. Green, B. H. J. McKellar, and R. Rajaraman for pointing out these errors.

2. A factor of $|\mathbf{k}|/|\mathbf{k}^0|$ was omitted in Eqs. (72), (73), and (77). Equation (72) should read

$$\sigma_{0\pi}{}^{l,I}(s)=(|\mathbf{k}|/|\mathbf{k}^0|)K^{NN\pi}(0)^2(|\mathbf{k}^0|/|\mathbf{k}|)^{2l}\sigma_\pi{}^{l,I}(s),$$

and Eqs. (73) and (77) are corrected by making the substitution $ds\rightarrow(|\mathbf{k}|/|\mathbf{k}^0|)ds$. Making the correction increases the magnitude of the scattering length a_0 required to saturate the sum rule.