PHYSICAL REVIEW VOLUME 175, NUMBER 5 25 NOVEMBER 1968

Quasidiffraction Scattering and Selection Rules for Spin and Parity*

L. RESNICK
Physics Department, Carleton University, Ottawa, Canada
(Received 12 July 1968)

The diffraction model for inelastic scattering is shown to lead to an approximate selection rule on the spin of a diffractively produced resonance. The model leads to parity selection only for a spinless incident particle. Some examples are discussed briefly. For the reaction $p+p\rightarrow p+N+\pi$, it is shown that the nucleon pole graphs cancel in the forward direction, leaving only the pion pole graph as the dominant one.

1. INTRODUCTION

ELASTIC diffraction scattering at high energy is characterized by a forward differential cross section independent of energy [$d\sigma/dt(s,t=0)\approx$ independent of s] and displaying a very steep forward peak. An inelastic process may be called quasidiffractive if it can be considered a two-body reaction (one of the final "bodies" may actually consist of several stable particles) and if it displays a dependence on s and t similar to that in elastic diffraction scattering.

Among the first to consider the concept of inelastic diffraction scattering in high-energy physics were Good and Walker.[1] On of the salient features of the analysis is the *coherence* of the produced inelastic system with the incoming particle. This implies that the quantum numbers of any resonance so produced must be unchanged, e.g., isotopic spin, hypercharge, G parity if applicable, etc. The mass is, of course, not the same, but Good and Walker give a criterion for coherence when $\Delta M^2\neq 0$.

In this paper we address ourselves to the question of possible restrictions on the spin and parity of a diffractively produced resonance. It is not unnatural to expect such restrictions in light of the simple coherence arguments mentioned above, and we find that approximate selection rules within a particular model do exist. If a particle of spin s_1 produces diffractively in the forward direction a two-particle resonance of spin j, then $0\leq[j]\leq[s_1]+[s_2]+[s_3]$, where s_2 and s_3 are the spins of the decay products. Here $[s]=s$ for a boson and $s-\frac{1}{2}$ for a fermion.

Recently, Goldhaber and Goldhaber[2] and Morrison[3] have proposed selection rules on the parity of a diffractively produced resonance. These relate the change in parity to the change in spin. We shall discuss the relationship between those selection rules and the ones presented here.

The present work is a direct generalization of an earlier paper[4] on an application of the Drell-Hiida (DH) model.[5] There is now a fairly extensive literature[6] on this model (or variations and applications of it). For want of a better name, we shall refer to the model as the quasidiffraction model (QDM). The notation and conventions in the following are those of Ref. 4.

2. $p+p\rightarrow p+N+\pi$

We consider in this and the next section two explicit examples, because they serve to clarify some points. The general case is considered in Sec. 4.

The first example is the reaction $p+p\rightarrow p+W$, where W is an $N\pi$ system. This is the example studied in Ref. 4, where it was concluded that the experimental peak seen at a missing mass $W=1.40$ GeV/c^2 must be at least partially attributed to dynamics in the πN system, i.e., to final-state interactions. However, an argument was made in favor of the DH process [Fig. 1(a)] as the production mechanism for this p_{11} resonance or dynamical enhancement, and the approximate spin selection rule $j=\frac{1}{2}$ was obtained. Parity selection did not follow; s_{11} was also allowed. One of the questions raised was the possible importance of nucleon

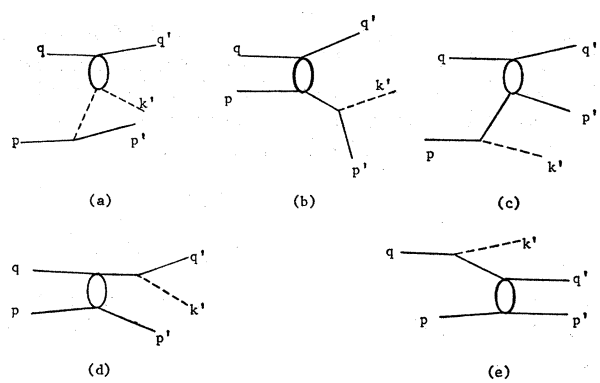

FIG. 1. Graphs for $p+p\rightarrow p+N+\pi$. (a) Drell-Hiida process; (b)–(e) nucleon pole graphs.

---

* Supported in part by a grant from the National Research Council of Canada.

[1] M. L. Good and W. D. Walker, Phys. Rev. **120**, 1857 (1960).

[2] A. S. Goldhaber and M. Goldhaber *Preludes in Theoretical Physics*, edited by A. de-Shalit *et al.* (John Wiley & Sons, Inc., New York, 1966), p. 313; see also the Berkeley-Milan-Orsay-Saclay Collaboration, in Ref. 12, Vol. I, p. 537.

[3] D. R. O. Morrison, Phys. Rev. **165**, 1699 (1968).

[4] L. Resnick, Phys. Rev. **150**, 1292 (1966).

[5] S. D. Drell and K. Hiida, Phys. Rev. Letters **7**, 199 (1961).

[6] M. M. Islam, Phys. Rev. **131**, 2292 (1963); Y. Takada and M. Bando, Progr. Theoret. Phys. (Kyoto) **33**, 657 (1965); R. T. Deck, Phys. Rev. Letters **13**, 169 (1964); U. Maor and T. A. O'Halloran, Jr., Phys. Letters **15**, 281 (1965); D. R. O. Morrison, *ibid.* **22**, 226 (1966); L. Stodolsky, Phys. Rev. Letters **18**, 973 (1967); M. Ross and L. Stodolsky, Phys. Rev. **149**, 1172 (1966). See especially E. West *et al.*, *ibid.* **149**, 1089 (1966), Appendix A; M. Ross and Y. Y. Yam, Phys. Rev. Letters **19**, 546 (1967).

pole graphs [Fig. 6 of Ref. 4 and Figs. 1(b)–1(e) here]. We resolve this problem now.

For convenience, we rewrite the invariant amplitude for Fig. 1(a) as follows:

$$M^{1a}=\left[2ig_{\pi N}{}^{1/2}(t)\frac{\sigma_{pp}{}^{1/2}}{2m}\bar{u}(q')u(q)\left(\frac{2m^2}{2m^2-\frac{1}{2}t}\right)^{1/2}\right]\times\left(\sigma_{\pi\pi}{}^{1/2}G_r\bar{u}(p')\gamma_5u(p)\frac{q'\cdot k'}{(p-p')^2-\mu^2}\right). \quad (1)$$

This is an approximation for $|t|$ very small and $|\mathbf{q}|$ very large.

Though it is not essential to the argument, we have written $\sigma_{\pi N}=\sigma_{\pi\pi}{}^{1/2}\sigma_{NN}{}^{1/2}$, where σ are the total asymptotic cross sections. In Ref. 4 the factor $[2m^2/(2m^2-\frac{1}{2}t)]^{1/2}$ does not appear, since $\frac{1}{2}t$ was already dropped compared with $2m^2$. We reinsert it here for reasons apparent below. The factor arises as follows. The amplitude for the virtual elastic diffraction scattering may be written as $M_d{}^{1a}=\bar{u}(q')u(q)A(s,t)$. We can evaluate $\frac{1}{2}\sum|M_d{}^{1a}|^2$ as before to obtain $(4\pi s^{1/2}/m)^2\times g_{\pi N}(t)(k\sigma_{\pi N}{}^{\text{tot}}/4\pi)^2$. We can also evaluate it directly to obtain $|A|^2(2m^2-\frac{1}{2}t)/2m^2$. Equating the two expressions gives $|A|$ or $A\approx i|A|$ [Re$A(s,t\approx0)$ is neglected]. Also, $k\sqrt{s}=[(q'\cdot k')^2-m^2\mu^2]^{1/2}\approx q'\cdot k'$. The result is Eq. (1). Note that $q'\cdot k'$ is the inner product of the two *physical* momenta in the virtual diffraction scattering (in this case the final lines). This follows from the ansatz of the model, that the elastic diffraction scattering is to be taken from experiment; off-shell effects are relegated to possible form factors, which have been neglected in writing Eq. (1).

We are now ready to write expressions for the nucleon pole graphs in the same approximation. For Fig. 1 (b), the virtual diffraction scattering is that of $pp\rightarrow pp$. Since the diffraction bubble is to be treated as a scalar "particle" coupling to the nucleon lines (in Regge language, the Pomeranchon has the quantum numbers of the vacuum),[6a] we make the ansatz that

$$M^{1b}=\bar{u}(q')u(q)\bar{u}(p')\gamma_5G_r\frac{\not{p}'+\not{k}'+m}{(p'+k')^2-m^2}u(p)B$$

and B is to be evaluated similarly to the previous diagram. If $p'+k'$ were on shell, we would write $M_d{}^{1b}=\bar{u}(q')u(q)\bar{u}(p')u(p'+k')B$, and now

$$\tfrac{1}{4}\sum|M_d{}^{1b}|^2=\frac{|B|^2(2m^2-\frac{1}{2}t)^2}{4m^4}=\left(\frac{4\pi\sqrt{s_1}}{2m^2}\right)^2g_{pp}(t)\left(\frac{k_1\sigma_{pp}{}^{\text{tot}}}{4\pi}\right)^2.$$

This allows us to evaluate $B\approx i|B|$. Here $s_1=(q+p)^2$ and k_1 is the corresponding c.m. momentum, so that $k_1\sqrt{s_1}\approx q\cdot p$. We note that this is what we should expect—it is the inner product of the two physical lines of the diffracting system (in this case the initial lines of the pp scattering system) that is the relevant quantity. In M^{1b}, we may also use the Dirac equation to set $\bar{u}(p')\gamma_5(\not{p}'+m)=0$. M^{1c} is very similar, except that the physical lines are now q' and p', and $q'\cdot p'$ replaces $q\cdot p$. By taking $\sigma_{np}{}^{\text{tot}}=\sigma_{pp}{}^{\text{tot}}$ and $g_{np}(t)=g_{pp}(t)$, we may ignore isospin (the final W system may be $p\pi^0$ or $n\pi^+$) and write for the sum of Figs. 1 (b) and 1 (c)

$$M^{1b}+M^{1c}=\left(2ig_{pp}{}^{1/2}(t)\frac{\sigma_{pp}{}^{1/2}}{2m}\bar{u}(q')u(q)\frac{2m^2}{2m^2-\frac{1}{2}t}\right)\times\left[\frac{\sigma_{pp}{}^{1/2}}{2m}G_r\bar{u}(p')\gamma_5\not{k}'u(p)\times\left(\frac{q\cdot p}{(p'+k')^2-m^2}+\frac{q'\cdot p'}{(p-k')^2-m^2}\right)\right]. \quad (2)$$

In an exactly analogous way,

$$M^{1d}+M^{1e}=\left(2ig_{pp}{}^{1/2}(\Delta^2)\frac{\sigma_{pp}{}^{1/2}}{2m}\bar{u}(p')u(p)\frac{2m^2}{2m^2-\frac{1}{2}\Delta^2}\right)\times\left[\frac{\sigma_{pp}{}^{1/2}}{2m}G_r\bar{u}(q')\gamma_5\not{k}'u(q)\times\left(\frac{p\cdot q}{(q'+k')^2-m^2}+\frac{p'\cdot q'}{(q-k')^2-m^2}\right)\right]. \quad (3)$$

(We can regard the above expressions as effectively giving the rules for diagrams that include a virtual diffraction scattering, or the exchange of an "inelastic Pomeranchon."[6a,7])

In Ref. 4, it was shown that, at $\theta=0$,

$$k'\cdot q'[\mu^2-(p-p')^2]^{-1}\approx|\mathbf{q}'|/2\epsilon,\quad \epsilon=E_q-E_q{}'.$$

This implied that at $\theta=0$, M^{1a} can contribute only to $j=\frac{1}{2}$, that is, s_{11} and p_{11} partial waves in the W system if no form factors are introduced. With the presence of a form factor, this selection rule is weakened but is

[6a] *Footnote added in proof.* The Pomeranchon has, of course, a spin given by $\alpha(t)\simeq1$ for $t\simeq0$. So, strictly, a vector coupling should be introduced, and terms like $q\cdot k'$ and $q'\cdot k'$, etc., would then enter. It is easy to show, however, that all these are equivalent to $q'\cdot k'$ [say, in Fig. 1(a)], using the same approximations as above. The coupling constants are then determined by demanding that the asymptotic elastic scattering fit experiment. But this is precisely equivalent to the prescription actually used. The vector nature of the Pomeranchon is therefore completely accounted for by the factors $q'\cdot k'$, etc., that are explicitly exhibited, and no further energy or angle dependence can come from the Pomeranchon exchange. So the diffraction bubble (or Pomeranchon) must be treated as a scalar in subsequent analysis.

[7] These are in essence the same rules and prescription given in the last two papers in Ref. 6. However, Ross and Yam do not explicitly consider off-shell fermions. West *et al.* do consider nucleon poles, but introduce an extra positive-energy projection operator. Their diagrams therefore cancel only in the static limit, while in the present prescription they cancel to all orders of p'/m.

still approximately true.[8] (In the numerical calculation, a suppression in the cross section of several orders of magnitude with $F=1$ becomes a suppression of a factor ≈ 10 with "reasonable" F.)

We now observe that $(p'+k')^2-m^2=(p+q-q')^2-m^2 = t+2p\cdot(q-q')$ and again, at $\theta=0$, $t\approx 0$ and $2p\cdot(q-q') \approx 2p\cdot q\epsilon/|\mathbf{q}|$, so that $q\cdot p/[(p'+k')^2-m^2]\approx |\mathbf{q}|/2\epsilon$. Likewise,

$$(p-k')^2-m^2=(p'+q'-q)^2-m^2$$
$$=t-2p'\cdot(q-q')\approx -2p'\cdot q\epsilon/|\mathbf{q}|\approx -2p'\cdot q'\epsilon/|\mathbf{q}|$$

at $\theta=0$, so that $q'\cdot p'/[(p-k')^2-m^2]\approx -|\mathbf{q}|2\epsilon$. We therefore find that though M^{1b} and M^{1c} are individually as large as M^{1a} (on doing the numerical calculations, they are seen to be even larger), nevertheless they cancel exactly in the leading approximation. M^{1b} and M^{1c} can contribute only in the higher order that has already been neglected in evaluating M^{1a}.

For $M^{1d}+M^{1e}$, the cancellation is not exact, but the sum is about 15% of the individual terms. This is for forward scattering at 20 GeV/c. However, in calculating the cross section, isotopic spin and the angular integration come in in such a way as to give a further suppression of these graphs by about a factor of 10. Thus $M^{1d}+M^{1e}$, the pion bremsstrahlung graphs, can also be safely neglected. We should remark that Figs. 1(d) and 1(e) must be included for the particular case of identical particles such as $p+p\to p+W$. However, for the reaction $A+B\to A+W$ and $A\neq B$, B is uniquely associated with W and only the first three diagrams of Fig. 1 need be considered.

There are other pole graphs as required by the Pauli principle, but these are all small. The exchange graph to Fig. 1(a) would be the "isobar" graph, already considered in Ref. 4, and other graphs would require a large momentum transfer at the "diffraction" bubble, and so would be outside the diffraction region and small.

We therefore conclude that at high energies and forward angles the only important pole graph is that of Fig. 1(a), the DH diagram (or, equivalently, the Lorentz-transformed graph, a fast incident proton producing a fast forward W). From the point of view of dispersion relations, we may look at these various diagrams as the pole terms in the various channels of the five-point function $N+N\to N+N+\pi$. To a first approximation, the unknown contributions of the cuts can be included as form factors. These would have two effects. The very strong calcellations between graphs would no longer occur, since we would have $(|\mathbf{q}|/2\epsilon) \times[F^{1b}((p'+k')^2)-F^{1c}((p-k')^2)]$ appearing in $M^{1b}+M^{1c}$, and even if $F^{1b}=F^{1c}$, the arguments are different. Secondly, as in the case of M^{1a} itself, form factors can have an angular dependence that breaks the selection rule $j=\frac{1}{2}$. However, we would still expect that Fig. 1(a) would be the dominant diagram and $j=\frac{1}{2}$ the dominant

[8] See also L. Stodolsky (Ref. 6).

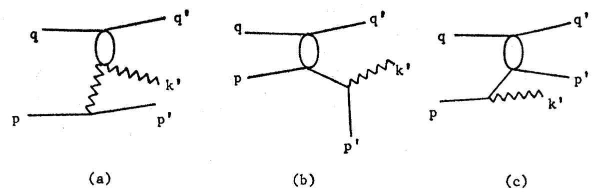

FIG. 2. Graphs for $p+p\to p+N+\rho$, corresponding to those in Fig. 1.

spin, at large energies and small angles, for "reasonable" form factors.

Independently of the cancellations between different graphs, we observe as a general feature of Figs. 1(a)–1(c) the cancellation in dependence on the off-shell mass between the denominator of the propagator and the kinematic factor arising from the diffraction scattering.

3. $p+p\to p+N+\rho$

The pion of Sec. 2 is replaced with a ρ, so that the diagrams corresponding to Figs. 1(a)–1(c) are replaced with Figs. 2(a)–2(c). The corresponding (d) and (e) graphs are not examined because they are not expected to be important for reasons mentioned in Sec. 2.

The ρNN coupling will be taken to be minimal (no Pauli terms), so that the γ_5 of Eqs. (1)–(3) is replaced with $\gamma\cdot\epsilon$, where ϵ_μ is the polarization 4-vector of the ρ. By direct analogy with the procedure in Sec. 2, we obtain

$$M^{2a}=\left[2ig_{\rho N}{}^{1/2}(t)\frac{\sigma_{pp}{}^{1/2}}{2m}\bar{u}(q')u(q)\left(\frac{2m^2}{2m^2-\frac{1}{2}t}\right)^{1/2}\right] \times\left(\sigma_{\rho p}{}^{1/2}G_{\rho NN}\bar{u}(p')\gamma\cdot\epsilon u(p)\frac{-|\mathbf{q}|}{2\epsilon}\right), \quad (4)$$

$$M^{2b}+M^{2c} =\left[2ig_{pp}{}^{1/2}(t)\frac{\sigma_{pp}{}^{1/2}}{2m}\bar{u}(q')u(q)\left(\frac{2m^2}{2m^2-\frac{1}{2}t}\right)\right] \times\left(\frac{\sigma_{pp}{}^{1/2}}{2m}G_{\rho NN}\bar{u}(p')u(p)2\epsilon\cdot(p-p')\frac{-|\mathbf{q}|}{2\epsilon}\right). \quad (5)$$

The cancellation that occurred in the previous case is not a general feature. If one attempts to treat the production of particles with spin>0 in the QDM, the DH diagram is not the only important one. One must consistently include the other pole terms as well, as pointed out by Ross and Yam.[6]

We can learn one more thing from Eqs. (4) and (5). In making a partial-wave analysis, it is convenient to take the direction of $\mathbf{p}^W$ as the polar axis. Then $\bar{u}(p')$ will transform as $\mathfrak{D}^{1/2}$ except for parity doubling (since the relevant polar angle is linearly related to $p\cdot p'$). In the case of the π, that is all there was, so that we obtained $j=\frac{1}{2}$, or only s_{11} and p_{11} waves were allowed.

Now, however, $\gamma\cdot\epsilon$ and $\epsilon\cdot(p-p')$ will transform as mixtures of $\mathfrak{D}^0$ and $\mathfrak{D}^1$, so that, in addition to $j=\frac{1}{2}$, we shall have contributions to $j=\frac{3}{2}$. An equivalent way of seeing this is to make a helicity decomposition, for example. Because of the ϵ, the projection onto angular momentum j of the W system will be

$$\int_{-1}^{+1} dz\, H[\alpha P_{j+\frac{3}{2}}(z)+\beta P_{j+\frac{1}{2}}(z)+\gamma P_{j-\frac{1}{2}}(z)+\delta P_{j-\frac{3}{2}}(z)],$$

where H is a scalar invariant in the decomposition of the general amplitude into scalars×spinors: tensors having the correct transformation properties. But in the QDM at $\theta=0$, H to first approximation is independent of z (H is essentially $2|\mathbf{q}|/\epsilon$). Therefore only $j=\frac{1}{2}$ and $\frac{3}{2}$ waves survive. (In Ref. 4, the terms in α and δ were missing for the pion case, so that only $j=\frac{1}{2}$ survived.)

These results indicate an obvious generalization, which will now be discussed.

4. SPIN SELECTION RULE IN THE GENERAL CASE

We want now to consider the reaction $A+B\rightarrow A'+C+D$ where the $C+D$ (or W) system is produced quasi-diffractively. This may be in a reaction where A is a stationary target suffering a small recoil, while the fast incident particle B produces the fast $C+D$ system at forward angles. Alternatively, B may be at rest, $C+D$ emerges with relatively low momentum, while the momentum transfer between the scattered A $(=A')$ and the incident A is very small. For convenience we shall retain the second picture.

The relevant diagram is Fig. 3. In Fig. 4, the lower part of Fig. 3 is expanded in the three pole contributions. By the requirement that the virtual diffraction scattering be elastic, the diffraction bubble may connect only lines corresponding to the same particle. The propagators are therefore $[(p_1-p_2)^2-p_3^2]^{-1}$, $[(p_1-p_3)^2-p_2^2]^{-1}$, and $[(p_2+p_3)^2-p_1^2]^{-1}$, except for possible factors in the numerators arising from spin. Using momentum conservation, the propagator denominators are, respectively, $(p_3+q'-q)^2-p_3^2$, $(p_2+q'-q)^2-p_2^2$, and $(p_1+q-q')^2-p_1^2$. At $\theta=0$, and with $t=(q'-q)^2\approx 0$, they are very nearly $p_3\cdot q'(-2\epsilon/|\mathbf{q}|)$, $p_2\cdot q'(-2\epsilon/|\mathbf{q}|)$, and $p_1\cdot q(2\epsilon/|\mathbf{q}|)$. These are precisely the correct factors to cancel the factors $p_3\cdot q'$, $p_2\cdot q'$, and $p_1\cdot q$ arising from the diffraction scattering in the upper part of each diagram. Therefore, except for the explicit contributions from the spins, and from form factors, there is no dependence on z, where z is the cosine of the angle between $\mathbf{p}_2^W$ $(=-\mathbf{p}_3^W)$ and $\mathbf{p}_1^W$ in the $C+D$ c.m. frame. We can put this another way. If we ignore form factors, then Fig. 4 represents the coupling of a scalar "particle" (the diffraction bubble) and three particles B, C, and D having spins s_1, s_2, and s_3 such that there is no angular dependence coming from the dynamics. The only angular dependence comes from the kinematics

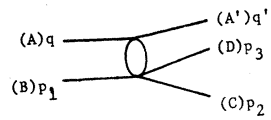

FIG. 3. Diagram for $A+B\rightarrow A'+C+D$.

of the coupling of the three spins. This leads immediately to the selection rules. [In the following, "minimal coupling" is assumed. For three spinless scalar particles, this is equivalent to a basic interaction of the form $G\Psi_1\Psi_2\Psi_3$. Extra derivatives such as $G'(\partial^\mu\Psi_1)(\partial_\mu\Psi_2)\Psi_3$ are equivalent to invariant scalar functions of the momenta in momentum space, and may conveniently be absorbed into the form factors.]

We consider first the case where all the s_i are integers (only bosons occur). The wave function for a particle of spin s can conveniently be described by an s-rank symmetric, traceless, divergenceless tensor $\Psi_{\mu_1\cdots\mu_s}{}^s$:

$$\Psi_{\mu_1\cdots\mu_i\cdots\mu_j\cdots\mu_s}{}^s=\Psi_{\mu_1\cdots\mu_j\cdots\mu_i\cdots\mu_s}{}^s,$$
$$g^{\mu_1\mu_2}\Psi_{\mu_1\mu_2\mu_3\cdots\mu_s}{}^s=\partial^{\mu_1}\Psi_{\mu_1\mu_2\cdots\mu_s}{}^s=0.$$

Each index transforms like a vector, so that the spatial part transforms under $\mathfrak{D}^1$ and the largest representation of the rotation group contained in Ψ^s is $\mathfrak{D}^s$. (Ψ^s transforms precisely like $\mathfrak{D}^s$ only in its own rest frame. The Lorentz frame required for the projection of the partial wave is the W frame, the c.m. frame of $C+D$, and this is generally not the rest frame of any of B, C, or D individually.) Because the Pomeranchon has momentum $q-q'=p_2+p_3-p_1$ by momentum conservation, only p_1, p_2, and p_3 need be considered. Consider index 1 of Ψ^{s_3}, say. $(p_3)^{\mu_1}\Psi_{\mu_1}{}^{s_3}\cdots=0$, while $(p_2)^{\mu_1}\Psi_{\mu_1}{}^{s_3}\cdots=W\Psi_0{}^{s_3}\cdots$, using the subsidiary condition and $\mathbf{p}_2{}^W=-\mathbf{p}_3{}^W$. This transforms like $\mathfrak{D}^0$. $(p_1)^{\mu_1}\Psi_{\mu_1}{}^{s_3}\cdots$ will transform as a mixture of $\mathfrak{D}^0$ and $\mathfrak{D}^1$, since $\mathbf{p}_1{}^W$ is fixed as polar axis. Therefore the highest representation is $\mathfrak{D}^{s_3}$, contained in $(p_1)^{\mu_1}\cdots(p_1)^{\mu_s}\Psi_{\mu_1\ldots\mu_s}{}^{s_3}$. Similarly, the highest representation from Ψ^{s_2} is $\mathfrak{D}^{s_2}$, and from Ψ^{s_1} is $\mathfrak{D}^{s_1}$. In the last case, this comes from coupling with either p_2 or p_3, and from the rotation properties of the spatial parts of these momenta rather than the $\Psi_{\mu_1}{}^{s_1}\cdots$ which have no dependence on $p_2{}^W$. The amplitude for Fig. 4 is trilinear in Ψ^{s_1}, $\Psi^{\dagger s_2}$, and $\Psi^{\dagger s_3}$, so that the highest representation that can occur is contained in $\mathfrak{D}^{s_1}\otimes\mathfrak{D}^{s_2}\otimes\mathfrak{D}^{s_3}$, that is, $\mathfrak{D}^{s_1+s_2+s_3}$. This leads immediately to $j\leq s_1+s_2+s_3$, where j is the total angular momentum

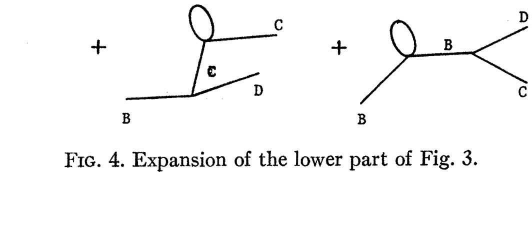

FIG. 4. Expansion of the lower part of Fig. 3.

in the W frame, i.e., is the total "spin" of the $C+D$ "system." The minimum j is zero because, in general, the inner products will also contain scalar parts. We thus conclude $0\leq j\leq s_1+s_2+s_3$.

If two of the three particles are fermions, the result is nearly the same. A spin-s fermion wave function Ψ^s can be written[9]

$$\Psi_{\mu_1\cdots\mu_{[s]}}{}^{s}(p,\lambda)=\sum_{\lambda_1\lambda_2} C([s]\tfrac{1}{2}s;\lambda_1\lambda_2\lambda)\times\Psi_{\mu_1\cdots\mu_{[s]}}{}^{[s]}(p,\lambda_1)\Psi^{1/2}(p,\lambda_2),$$

where C is the appropriate Clebsch-Gordan coefficient and $\Psi^{1/2}$ is a spin-$\frac{1}{2}$ wave function. $[s]=s-\frac{1}{2}$ is the greatest integer contained in s. Then the two spin-$\frac{1}{2}$ wave functions in the trilinear product $\bar{\Psi}^{s_2}\bar{\Psi}^{s_3}\Psi^{s_1}$ must combine in the usual invariant way $\bar{\Psi}^{1/2}\Psi^{1/2}$. This will at most give a parity doubling but will not affect the maximum j allowable (see, e.g., Sec. 2). The remaining part of the argument proceeds as in the boson case, except that s is replaced with $[s]$. The boson and fermion case can be included together in the rule

$$0\leq[j]\leq[s_1]+[s_2]+[s_3].$$

5. PARITY SELECTION RULE

We wish now to discuss the possibility of a selection rule in parity. The connection between the model discussed here and the diffraction production of resonances discussed in Ref. 2 is illustrated in Fig. 5. Figure 5(a) represents the diffraction production of a resonance of mass W in the forward direction, from an incident particle of mass m. Figure 5(b) represents the model discussed here, where the two constituent particles (momenta p_2 and p_3) are diffractively produced, and rescatter resonantly (at energy W). This latter picture is also discussed by Morrison.[3,6]

We make no attempt to include such final-state interactions. What is assumed is that if a dynamical resonance at mass W exists in a given state, and if this (two-particle) state can be produced via Fig. 3, then the resonance is produced as in Fig. 5(b). What has not been proved is that the spin selection rule that follows from Fig. 3, where p_2 and p_3 are free lines, also holds in Fig. 5(b), where p_2 and p_3 are in principle virtual lines. Stated another way, the proposed spin selection rule for the resonance W requires also that the lines p_2 and p_3 in Fig. 5(b) may be treated as essentially on the mass shell.

Since Fig. 5(b) is a more detailed model of the general process in Fig. 5(a), it is not surprising that it contains more information (in the way of a spin selection rule). However, we have seen that, in general, for a given spin j of W, W can have either parity. This is at variance with the existence of a parity selection rule.

Let us first consider the case where p_1 is spinless.

[9] P. Carruthers, Phys. Rev. 152, 1345 (1966).

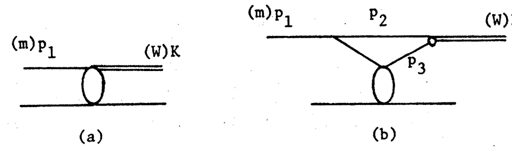

FIG. 5. Diffraction production. (a) Model discussed in Ref. 2; (b) model discussed in this paper and in Refs. 3 and 6.

Then in Fig. 4, the parity of the initial state is $\eta_1(-1)^j$, where η_1 is the intrinsic parity of B and j is the total angular momentum. By parity and angular momentum conservation, the parity of the final state $C+D$ is also $\eta_1(-1)^j$, where j is the "spin" of the $C+D$ system (or resonance W). But this means that the parity of the produced resonance W is uniquely determined by its spin j and by η_1, and is in complete agreement with the result of Goldhaber and Goldhaber. There is parity selection when the incident particle is spinless.

Now let p_1 have spin $s_1>0$. Then in Fig. 4, the initial state is not completely determined by j. For each j, there will in general be $2s_1+1$ values of l possible $(\mathbf{j}=\mathbf{l}+\mathbf{s}_1)$, and the parity will be $\eta_1(-1)^l$, which can have both signs. For a given allowed spin j and either parity of the produced resonance W, the transition is allowed by the appropriate choice of l (even or odd). This freedom is always possible whenever $s_1>0$, and we conclude that there is no parity selection when the incident particle has nonzero spin. This is for the specific model of Fig. 5(b).

This latter result can also be illustrated by a simple example, within the general framework of Fig. 5(a). Suppose that p_1 and W each have spin $\frac{1}{2}$ and there are no other spins. Let the momentum of W be K. Then the general amplitude for Fig. 5(a) is $H_1\bar{u}(K)u(p_1)$ for equal intrinsic parities and $H_5\bar{u}(K)\gamma_5u(p_1)$ for opposite parities. The H's are scalar invariants. The point is that $\bar{u}(K)\gamma_5u(p_1)$ vanishes for $\mathbf{K}\|\mathbf{p}_1$ only if $K^2(=W^2)=p_1{}^2(=m^2)$. In the inelastic case under discussion, even at exactly $\theta=0°$ neither $\bar{u}u$ nor $\bar{u}\gamma_5u$ is zero,[9] and both parities are allowed. Of course, $\bar{u}\gamma_5u/\bar{u}u\rightarrow0$ as $W^2\rightarrow m^2$, so that parity selection in this case is approximately recovered if W is close to m. For $W=1.40$ GeV and $m=0.94$ GeV,

$$\left|\frac{\bar{u}(K)\gamma_5u(p_1)}{\bar{u}(K)u(p_1)}\right|=\left(\frac{E_K-W}{E_K+W}\right)^{1/2}=0.20$$

at 20 GeV/c incoming momentum.

We conclude that parity selection at 0° is rigorous in both the general picture [Fig. 5(a)] and this particular model [Fig. 5(b)] in the case of a spinless incident particle. For an incident particle with spin>0, Fig. 5(b) leads to no parity selection *a priori*; both parities are allowed. Figure 5(a) may lead to a weaker parity selection rule, the approximation depending on the masses, energies, and spins involved.

6. DISCUSSION

It is worthwhile to repeat again the approximate validity of the spin selection rule. First, it arises in a definite model of how the "resonance" is produced. Second, it holds rigorously only at 0° scattering angle and for the pole terms. If the scattering angle is small but not zero or if form factors (to account for continuum contributions and nonminimal coupling) are included, the rule is more or less approximately valid. And of course the rule arises within the approximations made in the QDM itself ($\text{Re}M_d \approx 0$, $\sigma \approx$ constant in energy, etc.).

This being said, the rule should still be reliable and useful within its range of validity. The cancelling out of any "dynamic" dependence on z was seen to be a general feature of a graph like Fig. 3, not just of a particular component pole graph. This fact depended on the diffraction bubble (Pomeranchon) connecting only similar lines, corresponding to elastic (though virtual) diffraction scattering. Now, at first glance this restriction may seem unwarranted. After all, we are dealing with a model for quasidiffraction scattering, so that (consistently) inelastic diffraction scattering should be allowed in the virtual process as well. The justification for ignoring it is simply that experimentally the quasidiffraction cross section is small [of the order of 5% of the elastic in the reaction $p+p \to p+N^*(1400)$],[10] so that we are consistently neglecting corrections of higher order. If a resonance of spin j has several two-body decay modes i with spins s_3^i and s_2^i, the spin selection rule would generalize to read: The resonance can be produced quasidiffractively only if its spin satisfies $0 \le [j] \le \max_i([s_1]+[s_2^i]+[s_3^i])$. The QDM makes no statement about genuine three-body resonances.

We now briefly discuss a few examples of the selection rules, using also the requirement that the produced resonance have the same baryon number, strangeness, isospin, and G parity (where applicable) as the assumed incident particle. This requirement is of course implicit in the model.

In p-p collisions, the spin selection rule allows the quasidiffractive production of the p_{11} 1400-MeV πN resonance, but forbids (or suppresses) the $d_{13}(1525)$,[10a] $d_{15}(1670)$, and $f_{15}(1688)$ resonances. The production of the $s_{11}(1570)$ and $s_{11}(1700)$ is allowed, there being no parity selection. But the masses here are already higher than the region of "kinematic" peaking that occurs in the DH model and the resonant amplitudes themselves may not be too large. For these reasons, as already remarked by Morrison,[11,12] these resonances may be difficult to observe. Therefore the fact that on the basis of present data these resonances do not appear to be diffractively produced is not a serious objection to the model. There is an enhancement in the $p\pi^+\pi^-$ mass near 1500 MeV in 28-GeV p-p collisions.[13] This seems to be a candidate for the diffractive dissociation of $p \to \Delta^{++}\pi^-$. The spin selection rule allows the angular momentum of the $\Delta^{++}\pi^-$ state to be $\frac{1}{2}$ or $\frac{3}{2}$, so that the allowed quantum numbers are $\frac{1}{2}^\pm$ and $\frac{3}{2}^\pm$.

In π collisions, to the extent that the $A_1(1080)$ is a $\pi\rho$ resonance that is produced diffractively, the spin selection rule requires the spin of the A_1 to be 0 or 1. Since it is produced from a spinless pion, parity selection is valid, and the QDM allows 0^- or 1^+. On the other hand, the $A_2(1300)$, having an assigned $j^p=2^+$ and decaying primarily into $\rho\pi$, $K\bar{K}$, and $\eta\pi$, is forbidden by both spin and parity selection rules to be produced diffractively.

In K collisions, the $K\pi\pi$ situation is complicated. There is an enhancement in the $K^*(890)\pi$ system around 1300 MeV (Q meson) and there seems to be evidence for the relevance of the diffraction picture.[14,15] The Q is found to be predominantly 1^+, while the selection rules require 0^- or 1^+. For an L meson near 1700 MeV decaying primarily to $K^*(1400)\pi$, the spin and parity rules require 0^-, 1^+, or 2^- [assuming a 2^+ $K^*(1400)$]. Experimentally, 2^- seems to be favored,[14] although the situation is not yet clear. On the other hand, the $K^*(1400)$ which decays primarily to $K\pi$, $K^*(890)\pi$, $K\rho$, $K\omega$, and $K\eta$ is forbidden to be produced diffractively by both the spin and parity rules. A mass enhancement near 2240 MeV in the $\bar{\Lambda}N$ system has been reported[16] and, because of the highly peripheral nature of the events, considered as a possible candidate for a diffractively produced boson resonance. In this case the selection rules from the QDM give the unique assignment 0^- for the $K^*(2240)$.

As already mentioned, whether an allowed resonance is actually seen to be produced quasidiffractively depends also on the size of the resonant amplitude, and whether the resonant mass is within the kinematically important region of the diffraction model. This involves a more detailed calculation in the individual case. The dependence of Fig. 3 on $W^2=(p_2+p_3)^2$ is nearly[8] as $(p_2^W/W)\epsilon^{-2}R(W)$. The first factor is just phase space; it is 0 at threshold ($W=m_2+m_3$) and approaches $\frac{1}{2}$ as

[10] E. W. Anderson *et al.*, Phys. Rev. Letters **16**, 855 (1966).

[10a] *Footnote added in proof*. Since the $d_{13}(1525)$ also decays to $\Delta(1236)+\pi$, this branching mode would allow $j=\frac{3}{2}^\pm$ as well as $j=\frac{1}{2}^\pm$. So the diffractive production of the d_{13} can "go" via the $\Delta+\pi$ mode. I am indebted to Dr. R. F. Peierls, Dr. T. L. Trueman, and Dr. A. H. Mueller for raising the questions discussed in Refs. 6a and 10a.

[11] Reference 3; see also D. C. Colley, in Ref. 12, Vol. I, p. 84.

[12] *Proceedings of the Topical Conference on High-Energy Collisions of Hadrons* (CERN, Geneva, 1968).

[13] F. Turkot, in Ref. 12, Vol. I, p. 324.

[14] F. Bomse *et al.*, Phys. Rev. Letters **20**, 1519 (1968); J. C. Park *et al.*, *ibid.* **20**, 171 (1968). See also D. C. Colley, in Ref. 12, Vol. I, p. 72; J. C. Berlinghieri *et al.*, in Ref. 12, Vol. II, p. 172; Phys. Rev. Letters **18**, 1087 (1967).

[15] D. Denegri *et al.*, Phys. Rev. Letters **20**, 1194 (1968).

[16] G. Alexander *et al.*, Phys. Rev. Letters **20**, 755 (1968).

W gets large (compared with the masses). $R(W)$ is the explicit contribution of the spins. Since $\epsilon \simeq (W^2-m_1^2)/2m_1$, the dependence, on W is as W^{-4} for large W (if $R=1$). The result is a peaking at some intermediate value. The peak may in principle be very sharp, and near threshold ("Deck effect"). For example, if $m_1=m_2=m$ (nucleon mass) and $m_3=\mu$ (pion mass), then $\epsilon^{-2} \propto (W^2-m^2)^{-2}$, while threshold is at $W=m+\mu$, so that there is a very sharp peak just above threshold. However, this can be qualitatively modified by $R(W)$, which for $\frac{1}{2}^+ \to \frac{1}{2}^+ + 0^-$ behaves as $(W^2-m^2)^2/W^2$. Thus the net behavior in this case is as $(p_2{}^W/W)W^{-2}$, which displays a broad enhancement at $W \simeq 1400$ MeV and is $\propto W^{-2}$ for large W. Higher spins would result in an $R(W)$ involving larger powers of W, so that the peaking can not only be broadened and shifted, but even eliminated. The particular case will depend on the spins and masses involved.

ACKNOWLEDGMENTS

I am grateful to Dr. D. J. Brown and Dr. M. K. Sundaresan for some useful conversations. Part of this work was done while I was a guest at the University of Tel Aviv and I am grateful to Professor Y. Ne'eman for his kind hospitality.

PHYSICAL REVIEW VOLUME 175, NUMBER 5 25 NOVEMBER 1968

Continuous-Moment Sum Rules and Pion Conspiracy in Photoproduction

K. RAMAN*
Physics Department, Brown University, Providence, Rhode Island 02912

AND

KASHYAP V. VASAVADA
Physics Department, The University of Connecticut, Storrs, Connecticut 06268

(Received 24 July 1968)

Continuous-moment sum rules are used for examining the validity of the pion-conspiracy hypothesis in pion photoproduction. The trajectory and residue functions of the pion and conspirator trajectories are estimated.

To explain certain observed features of reactions in which pion exchange is allowed, the hypothesis of the existence of a conspirator trajectory with positive parity and with other quantum numbers the same as those of the pion has been made.[1] In particular, this has made possible an explanation of the forward peak in π^+ photoproduction.[2,3] Rough estimates of the pion and conspirator trajectory functions have been made by different authors, using finite-energy sum rules for photoproduction.[4] Recently, alternative mechanisms[5] have been proposed for explaining the observed features of pion photoproduction, and it is therefore desirable to make a further check of the conspiracy hypothesis.

We have recently used the continuous-moment sum rules[6] for determining the A_2 trajectory and residue functions in pion photoproduction.[7] As we have observed in our earlier work,[7] these sum rules provide a more reliable method of determining the Regge trajectory parameters than the finite-energy sum rules; and in this paper we use them for examining the question of pion conspiracy in pion photoproduction. Here we use these sum rules, assuming pion conspiracy, to obtain estimates for the trajectory and residue functions of the pion and the conspirator. The nature of the results thus obtained enables us to examine the validity of the pion-conspiracy hypothesis. Further, we examine in detail how the results obtained from the continuous-moment sum rules depend on the value of the moment parameter γ; this gives an estimate of the reliability of the results obtained using the available fits to the photoproduction amplitudes.[8]

* Research supported in part by the U. S. Atomic Energy Commission (Report No. NYO-2262TA-190).

[1] R. J. N. Phillips, Nucl. Phys. **B2**, 394 (1967); F. Arbab and J. Dash, Phys. Rev. **163**, 1603 (1967).

[2] A. M. Boyarski *et al.*, Phys. Rev. Letters **20**, 300 (1968).

[3] S. Drell and J. Sullivan, Phys. Rev. Letters **19**, 268 (1967); S. C. Frautschi and L. Jones, Phys. Rev. **163**, 1820 (1967); J. S. Ball, W. Frazer, and M. Jacob, Phys. Rev. Letters **20**, 518 (1968).

[4] A. Bietti, P. Di Vecchia, F. Drago, and M. L. Paciello, Phys. Letters **26B**, 457 (1968); D. P. Roy and S. Y. Chu, Phys. Rev. **171**, 1762 (1968).

[5] D. Amati, G. Cohen-Tannoudji, R. Jengo, and Ph. Salin, Phys. Letters **26B**, 510 (1968); J. Frøyland and D. Gordon, MIT Cambridge Report (to be published).

[6] Y. Liu and S. Okubo, Phys. Rev. Letters **19**, 190 (1967); A. Della Selva, L. Masperi, and R. Odorico, Nuovo Cimento **54A**, 979 (1968).

[7] K. V. Vasavada and K. Raman, Phys. Rev. Letters **21**, 577 (1968).

[8] We have used the fits to the multipole amplitudes obtained by R. L. Walker *et al.* (to be published).