"Non-Abelian Compton Effect" on Spin-One Targets

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Scattering of isovector photons by spin-1 targets is considered in detail. To first order in photon frequency, three new theorems are obtained. Two of these are analogous to the Cabibbo-Radicati theorem and the Bég theorem (for the nucleon case). The third is a new theorem involving only the quadrupole moment. We also obtain several new theorems up to second order in photon frequency by using Singh's lemma. On the basis of these results and the earlier results for spin-0 and spin-$\frac{1}{2}$ targets, various theorems up to first and second order are conjectured for arbitrary-spin targets.

I. INTRODUCTION

RECENTLY, there has been a great deal of interest in obtaining various exact low-energy results for hadron Compton scattering.[1] Besides being important as a matter of principle, these theorems give rise to a variety of sum rules if the relevant amplitudes satisfy unsubtracted dispersion relations.[2] By studying their properties with respect to saturation by a select set of states, these sum rules, on the one hand, provide a basis for investigations on the nature of dynamical symmetries[3]; on the other hand, saturation by low-lying states may give rise to various useful coupling-constant relations.

The earliest low-energy theorem for Compton scattering of photons is, of course, Thomson's zero-energy theorem for spin-0 and spin-$\frac{1}{2}$ targets. This states that the total amplitude at zero energy is entirely given by the total charge of the scatterer. A decade ago, Low[1] and Gell-Mann and Goldberger[1] proved an important theorem for Compton scattering on spin-$\frac{1}{2}$ targets. This theorem states that the entire amplitude up to first order in photon frequency ω, to second order in e, and to all orders in strong interactions is given by the static moments of the scatterer, namely, the total charge and the total magnetic moment.

Recently, Bég,[1] using the techniques invented by Low, considered the case of nucleon Compton scattering when the photons also carry a "charge" label and are associated with isovector currents of an octet satisfying current commutation relations (non-Abelian Compton scattering). He showed that in this way one can get further low-energy theorems and that they lead to sum rules like the Cabibbo-Radicati sum rule, which had been derived earlier by the usual infinite-momentum-frame method.[2] However, the low-energy-theorem approach has the advantage that, whereas the sum rules may or may not be correct, the theorems are exact.[4]

So far, all the low-energy theorems had been obtained at most up to first order in ω. This is because the "excited-state contribution" to the scattering amplitude that appeared in second order in ω was not calculable. Using current conservation, Singh[1] gave a lemma giving the precise form of this contribution. Using this lemma, several new low-energy theorems up to second order were obtained by Singh for spin-0 and spin-$\frac{1}{2}$ targets, both for the physical and the "charged" photons.

Next, the case of scattering of physical photons by spin-1 targets was taken up by Pais.[1] To first order in ω, Pais obtained three new theorems. In addition to the generalizations of two earlier theorems, he obtained a new zero-energy theorem (the Pais theorem). Using Singh's lemma, he also obtained a new quadrupole-moment theorem in second order.

The present work is a generalization of Pais's work to non-Abelian Compton scattering on spin-1 targets. To first order in photon frequency, we obtain three new theorems. Two of these are analogous to the Cabibbo-Radicati theorem and the Bég theorem (for the nucleon case). The third is a new theorem involving only the quadrupole moment. Using Singh's lemma, we obtain several new theorems up to second order in photon frequency. On the basis of these results and the earlier results for spin-0 and spin-$\frac{1}{2}$ targets, various theorems up to first and second order in frequency are conjectured for arbitrary-spin targets.

In Sec. II, we give the divergence conditions for the non-Abelian case. Section III is devoted to the evaluation of various terms occurring in our basic equation (10). The tensor decomposition of the amplitude and the crossing-symmetry requirements are discussed in Sec. IV. Section V deals with the results obtained and the possible generalizations to the case of targets with arbitrary spin. The Appendix gives the details of the spin-1 vertex function that we have used.

[1] For classical low-energy theorems, see F. Low, Phys. Rev. **96**, 1428 (1954); M. Gell-Mann and M. L. Goldberger, *ibid.* **96**, 1433 (1954). For recent work, see M. A. B. Bég, Phys. Rev. Letters **17**, 333 (1966); A. Pais, *ibid.* **19**, 544 (1967); V. Singh, *ibid.* **19**, 730 (1967); Phys. Rev. **165**, 1532 (1968). In addition, a good account of this subject may be found in A. Pais, CERN Report No. TH.816 (unpublished); M. A. B. Bég, SINBI Lectures, Copenhagen, 1967 (unpublished).

[2] S. D. Drell and A. C. Hearn, Phys. Rev. Letters **16**, 908 (1966); N. Cabibbo and L. Radicati, Phys. Letters **19**, 697 (1967); M. A. B. Bég, Phys. Rev. Letters **17**, 333 (1966).

[3] M. A. B. Bég and A. Pais, Phys. Rev. **160**, 1479 (1967).

[4] In this sense, these theorems provide a more direct test of current commutation relations than any of the standard results of current algebra (e.g., the Adler-Weisberger relation), which inevitably use the partially conserved axial-vector current hypothesis also. Note, however, that the latter uses the commutation relations of the "charges" and is thus free from the Schwinger terms, unlike the present case.

II. DIVERGENCE CONDITION

Consider the scattering of isovector photons by a spin-1 target D:

$$\gamma^\beta+D=\gamma^\alpha+D,\quad (k+p=k'+p'),$$

where α and β are the charge labels for the final and initial photons, respectively.

The amplitude for this process is

$$T^{\alpha\beta}(p',k';\,p,k)=\epsilon_\mu'(k')T_{\mu\nu}{}^{\alpha\beta}(p',k';\,p,k)\epsilon_\nu(k).\quad (1)$$

We have suppressed the spin- and isospin-wave functions of the target; $T_{\mu\nu}{}^{\alpha\beta}$ is thus a matrix in the spin and the isospin spaces of the target.

We shall choose the transverse gauge so that

$$\boldsymbol{\varepsilon}'\cdot\mathbf{k}'=\boldsymbol{\varepsilon}\cdot\mathbf{k}=0\quad (2)$$

and the physical amplitude is

$$T^{\alpha\beta}=\epsilon_m'T_{mn}{}^{\alpha\beta}\epsilon_n.\quad (3)$$

Next, we have

$$i(2\pi)^4\delta^4(p'+k'-p-k)(4V^2E_pE_{p'})^{-1/2}T_{\mu\nu}{}^{\alpha\beta}(p',k';\,p,k)$$
$$=\int d^4x d^4y\, e^{-i(k'\cdot x-k\cdot y)}\langle p'|[T\{J_\mu{}^\alpha(x),J_\nu{}^\beta(y)\}-i\rho_{\mu\nu}{}^{\alpha\beta}(\kappa)\delta^4(x-y)]|p\rangle.\quad (4)$$

The term $\rho_{\mu\nu}{}^{\alpha\beta}(\kappa)$ compensates for the noncovariant nature of the T product and ensures that $T_{\mu\nu}{}^{\alpha\beta}$ is a completely covariant object.

The basic assumption of this work is the following set of current commutation relations[5]:

$$\begin{aligned}&[J_0{}^\alpha(x),J_0{}^\beta(y)]\delta(x_0-y_0)=if^{\alpha\beta\gamma}J_0{}^\gamma(x)\delta^4(x-y),\\&[J_0{}^\alpha(x),J_n{}^\beta(y)]\delta(x_0-y_0)=if^{\alpha\beta\gamma}J_n{}^\gamma(x)\delta^4(x-y)\\&\qquad\qquad+i\partial_m[\rho_{mn}{}^{\alpha\beta}(x)\delta^4(x-y)].\end{aligned}\quad (5)$$

Partial differentiation of Eq. (4), together with Eq. (5) and the current conservation $\partial_\mu J_\mu{}^\alpha=0$, yields the following divergence conditions:

$$k_\mu'T_{\mu\nu}{}^{\alpha\beta}=T_{\nu\lambda}{}^{\alpha\beta}k_\lambda=-i(4V^2E_pE_{p'})^{1/2}f^{\alpha\beta\gamma}\times\langle p'|J_\nu{}^\gamma(0)|p\rangle.\quad (6)$$

Equations (6) are the divergence conditions for the "non-Abelian" photons. It is clear that for physical photons (which do not have any charge label) these reduce to the standard divergenceless conditions. Using Eqs. (6), one obtains the following identity:

$$k_m'T_{mn}{}^{\alpha\beta}k_n=k_4'T_{44}{}^{\alpha\beta}k_4+i(4V^2E_pE_{p'})^{1/2}f^{\alpha\beta\gamma}\times\langle p'|\tfrac{1}{2}(k_4'+k_4)J_4{}^\gamma(0)-\tfrac{1}{2}(k_m'+km)J_m{}^\gamma(0)|p\rangle.\quad (7)$$

To proceed further, we put

$$T_{\mu\nu}{}^{\alpha\beta}=U_{\mu\nu}{}^{\alpha\beta}+E_{\mu\nu}{}^{\alpha\beta},\quad (8)$$

where $U_{\mu\nu}{}^{\alpha\beta}$ is the contribution due to the target intermediate state and $E_{\mu\nu}{}^{\alpha\beta}$ is the contribution due to all the possible remaining "excited" states of the target. We shall consider intermediate states with no photons present, and thus our results will be true only to second order in electromagnetism but to all orders in strong interactions.

Next, define

$$T_{\mu\nu}{}^{\{\alpha\beta\}}=\tfrac{1}{2}(T_{\mu\nu}{}^{\alpha\beta}+T_{\mu\nu}{}^{\beta\alpha})$$

and

$$T_{\mu\nu}{}^{[\alpha\beta]}=\tfrac{1}{2}(T_{\mu\nu}{}^{\alpha\beta}-T_{\mu\nu}{}^{\beta\alpha}).\quad (9)$$

Therefore,

$$k_m'E_{mn}{}^{\{\alpha\beta\}}k_n=-k_m'U_{mn}{}^{\{\alpha\beta\}}k_n+k_4'U_{44}{}^{\{\alpha\beta\}}k_4+k_4'E_{44}{}^{\{\alpha\beta\}}k_4\quad (10a)$$

and

$$k_m'E_{mn}{}^{[\alpha\beta]}k_n=-k_m'U_{mn}{}^{[\alpha\beta]}k_n+k_4'U_{44}{}^{[\alpha\beta]}k_4+k_4'E_{44}{}^{[\alpha\beta]}k_4+i(4V^2E_pE_{p'})^{1/2}f^{\alpha\beta\gamma}\times\langle p'|\tfrac{1}{2}(k_4'+k_4)J_4{}^\gamma(0)-\tfrac{1}{2}(k_m'+k_m)J_m{}^\gamma(0)|p\rangle.\quad (10b)$$

Equations (10) are the basic equations for getting various results. The symmetric combination satisfies the same equation as satisfied by physical photons. Equation (10a) thus gives results that are trivial extensions of the results for the physical photons. However, Eq. (10b) yields a variety of new interesting theorems.

III. EVALUATION OF TERMS

Using translational invariance, the target-intermediate-state contribution is easily obtained from Eq. (4):

$$(4V^4E_pE_{p'})^{-1/2}U_{44}{}^{\alpha\beta}=\frac{\sum\langle p'|J_4{}^\alpha(0)|\mathbf{p}+\mathbf{k},\,E(\mathbf{p}+\mathbf{k})\rangle\langle\mathbf{p}+\mathbf{k},\,E(\mathbf{p}+\mathbf{k})|J_4{}^\beta(0)|p\rangle}{E_p+k_0-E(\mathbf{p}+\mathbf{k})}+\frac{\sum\langle p'|J_4{}^\beta(0)|\mathbf{p}-\mathbf{k}',\,E(\mathbf{p}-\mathbf{k}')\rangle\langle\mathbf{p}-\mathbf{k}',\,E(\mathbf{p}-\mathbf{k}')|J_4{}^\alpha(0)|p\rangle}{E_p-k_0'-E(\mathbf{p}-\mathbf{k}')},\quad (11)$$

[5] Note that we have identified the Schwinger term occurring in the commutator of time and space components of $J_\mu{}^\alpha(x)$ in Eq. (5) with the quantity $\rho_{\mu\nu}{}^{\alpha\beta}$ of Eq. (4). Thus the T-matrix element defined here differs from the collision amplitude in some equal time commutator terms. For a full discussion of this point, see L. S. Brown, Phys. Rev. **150**, 1338 (1966). Also, note that $\rho_{4\nu}{}^{\alpha\beta}=\rho_{\nu 4}{}^{\alpha\beta}=0$.

where $\sum$ stands for the spin summation over the intermediate state;

$$(4V^4E_pE_{p'})^{-1/2}U_{mn}{}^{\alpha\beta}=\frac{\sum\langle p'|J_m{}^\alpha(0)|\mathbf{p}+\mathbf{k},E(\mathbf{p}+\mathbf{k})\rangle\langle\mathbf{p}+\mathbf{k},E(\mathbf{p}+\mathbf{k})|J_n{}^\beta(0)|p\rangle}{E_p+k_0-E(\mathbf{p}+\mathbf{k})}+\frac{\sum\langle p'|J_n{}^\beta(0)|\mathbf{p}-\mathbf{k}',E(\mathbf{p}-\mathbf{k}')\rangle\langle\mathbf{p}-\mathbf{k}',E(\mathbf{p}-\mathbf{k}')|J_m{}^\alpha(0)|p\rangle}{E_p-k_0'-E(\mathbf{p}-\mathbf{k}')}. \quad (12)$$

Observe that

$$U_{\mu\nu}{}^{\alpha\beta}(p',k';p,k)=U_{\nu\mu}{}^{\beta\alpha}(p',-k;p,-k'). \quad (13)$$

These are the crossing-symmetry requirements that will be used in Sec. IV.

We next require the spin-1 electromagnetic vertex function[6]

$$\langle p'\Lambda'|J_\mu{}^\alpha(0)|p\Lambda\rangle=(4V^2E_pE_{p'})^{-1/2}\eta_\rho{}^{(\Lambda')}(p')X_{\mu\rho\sigma}{}^\alpha(p',p)\eta_\sigma{}^{(\Lambda)}(p). \quad (14)$$

One can "unboost" the $\eta(p)$ functions and get

$$\langle p'\Lambda'|J_\mu{}^\alpha(0)|p\Lambda\rangle=(4V^2E_pE_{p'})^{-1/2}\eta_\rho{}^{(\Lambda')}(\mathbf{p}'=0)Y_{\mu\rho\sigma}{}^\alpha(p',p)\eta_\sigma{}^{(\Lambda)}(\mathbf{p}=0). \quad (15)$$

To exhibit the dependence of the vertex function on the spin vector $\mathbf{s}$ of the target explicitly, we suppress the indices ρ and σ. Then

$$\langle p'\Lambda'|J_\mu{}^\alpha(0)|p\Lambda\rangle=(4V^2E_pE_{p'})^{-1/2}\eta^{(\Lambda')}(\mathbf{p}'=0)Y_\mu{}^\alpha(p',p)\eta^{(\Lambda)}(\mathbf{p}=0), \quad (16)$$

where $Y_\mu{}^\alpha$ is a matrix in the spin and the isospin spaces of the target. The explicit expression for $Y_\mu{}^\alpha$ is given in the Appendix.

We shall be working in the laboratory frame ($\mathbf{p}=0$). We put $k_0'=\omega'$, $k_0=\omega$, and $\mathbf{k}'\cdot\mathbf{k}=\omega\omega'\cos\theta$, where θ is the angle of scattering in the laboratory frame. In this frame, we have the kinematic identity

$$\omega'=\omega[1+\omega(1-\cos\theta)/M]^{-1}. \quad (17)$$

Using this identity, we get

$$[E_p+k_0-E(\mathbf{p}+\mathbf{k})]^{-1}[2E(\mathbf{p}+\mathbf{k})]^{-1}=\frac{1}{2M\omega}\left(1+\frac{\omega}{2M}-\frac{\omega^2}{4M^2}-\frac{\omega^3}{4M^3}+O(\omega^4)\right), \quad (18)$$

$$[E_p-k_0'-E(\mathbf{p}-\mathbf{k}')]^{-1}[2E(\mathbf{p}-\mathbf{k}')]^{-1}=-\frac{1}{2M\omega}\left(1+\frac{\omega}{2M}-\frac{\omega\cos\theta}{M}-\frac{\omega^2}{4M^2}+\frac{\omega^3}{2M^3}-\frac{\omega^3\cos\theta}{4M^3}+O(\omega^4)\right). \quad (19)$$

Using Eqs. (16), (18), and (19), we obtain in a straightforward way

$$\frac{k_m'U_{mn}{}^{\{\alpha\beta\}}k_n}{\omega\omega'}=\frac{\{F_0{}^\alpha,F_0{}^\beta\}}{4M^2}\omega^2\cos\theta(3-2\cos\theta)+O(\omega^3), \quad (20)$$

$$\frac{k_m'U_{mn}{}^{[\alpha\beta]}k_n}{\omega\omega'}=\frac{[F_0{}^\alpha,F_0{}^\beta]}{4M^2}[\omega^2(1+2\cos^2\theta-3\cos\theta)+2M\omega(2\cos\theta-1)], \quad (21)$$

$$\begin{aligned}-U_{44}{}^{\{\alpha\beta\}}=&+\cos\theta\,\{F_0{}^\alpha,F_0{}^\beta\}+\omega^2(1/4M^2)\{F_0{}^\alpha,F_0{}^\beta\}\\&+\omega^2\cos\theta\,(\{F_0'^\alpha,F_0{}^\beta\}+\{F_0{}^\alpha,F_0'^\beta\}-(1/2M^2)\{F_0{}^\alpha,F_1{}^\beta\}-(1/2M^2)\{F_0{}^\beta,F_1{}^\alpha\}\\&+\tfrac{2}{3}\{F_2{}^\alpha,F_0{}^\beta\}+\tfrac{2}{3}\{F_2{}^\beta,F_0{}^\alpha\}+(5/4M^2)\{F_0{}^\alpha,F_0{}^\beta\})-\omega^2\cos^2\theta\,(1/2M^2)\{F_0{}^\alpha,F_0{}^\beta\}\\&+\cos\theta\,(\mathbf{s}\cdot\mathbf{k})^2((1/2M^2)(F_1{}^\beta F_0{}^\alpha+F_1{}^\alpha F_0{}^\beta)-(F_2{}^\beta F_0{}^\alpha+F_2{}^\alpha F_0{}^\beta)-(1/2M^2)\{F_0{}^\alpha,F_0{}^\beta\})\\&+\cos\theta\,(\mathbf{s}\cdot\mathbf{k}')^2((1/2M^2)(F_0{}^\beta F_1{}^\alpha+F_0{}^\alpha F_1{}^\beta)-(F_0{}^\beta F_2{}^\alpha+F_0{}^\alpha F_2{}^\beta)-(1/2M^2)\{F_0{}^\alpha,F_0{}^\beta\})\\&+[i\mathbf{s}\cdot(\mathbf{k}\times\mathbf{k}')/\omega]((1/M)(F_1{}^\alpha F_0{}^\beta+F_1{}^\beta F_0{}^\alpha)-(1/M)\{F_0{}^\alpha,F_0{}^\beta\})\\&+i\mathbf{s}\cdot(\mathbf{k}\times\mathbf{k}')((1/2M^2)(F_1{}^\alpha F_0{}^\beta+F_1{}^\beta F_0{}^\alpha)-(1/2M^2)\{F_0{}^\alpha,F_0{}^\beta\})\\&+i\cos\theta\,\mathbf{s}\cdot(\mathbf{k}\times\mathbf{k}')((1/2M^2)\{F_0{}^\alpha,F_0{}^\beta\}-(1/2M^2)(F_1{}^\beta F_0{}^\alpha+F_1{}^\alpha F_0{}^\beta)), \quad (22)\end{aligned}$$

where the $F_i{}^\alpha$'s are the form factors defined in the Appendix.

[6] See A. Pais, CERN Report quoted in Ref. 1.

Similarly,

$$\begin{aligned}-U_{44}^{[\alpha\beta]}=&(2M/\omega+1-\cos\theta)[F_0^\alpha,F_0^\beta]+\omega\left[if^{\alpha\beta\gamma}\left(\frac{7}{2M}F_0^\gamma+4MF_0'^\gamma-\frac{2}{M}F_1^\gamma+\frac{8M}{3}F_2^\gamma\right)\right]\\
&-\omega\cos\theta\,\frac{1}{M}[F_0^\alpha,F_0^\beta]+\omega^2\left[if^{\alpha\beta\gamma}\left(-\frac{7}{4M^2}F_0^\gamma-2F_0'^\gamma+\frac{1}{M^2}F_1^\gamma-\tfrac{4}{3}F_2^\gamma\right)\right]\\
&+\omega^2\cos\theta\left[if^{\alpha\beta\gamma}\left(\frac{9}{4M^2}F_0^\gamma+2F_0'^\gamma-\frac{1}{M^2}F_1^\gamma+\tfrac{4}{3}F_2^\gamma\right)\right]-\omega^2\cos^2\theta\,(1/2M^2)[F_0^\alpha,F_0^\beta]\\
&+\frac{(\mathbf{s}\cdot\mathbf{k})^2+(\mathbf{s}\cdot\mathbf{k}')^2}{\omega}\left[if^{\alpha\beta\gamma}\left(-\frac{1}{M}F_0^\gamma+\frac{1}{M}F_1^\gamma-2MF_2^\gamma\right)\right]\\
&+[(\mathbf{s}\cdot\mathbf{k})^2+(\mathbf{s}\cdot\mathbf{k}')^2]\left[if^{\alpha\beta\gamma}\left(-\frac{1}{2M^2}F_0^\gamma+\frac{1}{2M^2}F_1^\gamma-F_2^\gamma\right)\right]\\
&+\cos\theta\,(\mathbf{s}\cdot\mathbf{k})^2\left((F_2^\alpha F_0^\beta-F_2^\beta F_0^\alpha)+\frac{1}{2M^2}(F_1^\beta F_0^\alpha-F_1^\alpha F_0^\beta)+\frac{1}{2M^2}[F_0^\alpha,F_0^\beta]\right)\\
&+\cos\theta\,(\mathbf{s}\cdot\mathbf{k}')^2\left((F_0^\alpha F_2^\beta-F_0^\beta F_2^\alpha)+\frac{1}{2M^2}(F_0^\beta F_1^\alpha-F_0^\alpha F_1^\beta)+\frac{1}{2M^2}[F_0^\alpha,F_0^\beta]\right)\\
&+i\cos\theta\,\mathbf{s}\cdot(\mathbf{k}\times\mathbf{k}')\left(\frac{1}{2M^2}(F_1^\alpha F_0^\beta-F_1^\beta F_0^\alpha)-\frac{1}{2M^2}[F_0^\alpha,F_0^\beta]\right),\end{aligned}\tag{23}$$

$$\begin{aligned}&i(4V^2E_pE_{p'})^{1/2}f^{\alpha\beta\gamma}\langle p'|\tfrac{1}{2}(k_4'+k_4)J_4^\gamma(0)-\tfrac{1}{2}(k_m'+k_m)J_m^\gamma(0)|p\rangle/\omega\omega'\\
&=\omega^{-1}(-if^{\alpha\beta\gamma}2MF_0^\gamma)+(-ij^{\alpha\beta\gamma}F_0^\gamma)+\cos\theta\,(if^{\alpha\beta\gamma}F_0^\gamma)\\
&\quad+\omega\left[-if^{\alpha\beta\gamma}\left(\frac{4F_0^\gamma}{M}+4MF_0'^\gamma-\frac{2F_1^\gamma}{M}+\frac{8M}{3}F_2^\gamma\right)\right]+\omega\cos\theta\left[if^{\alpha\beta\gamma}\left(\frac{4F_0^\gamma}{M}+4MF_0'^\gamma-\frac{2F_1^\gamma}{M}+\frac{8M}{3}F_2^\gamma\right)\right]\\
&\quad+\omega^2\left[-if^{\alpha\beta\gamma}\left(-\frac{2}{M^2}F_0^\gamma-2F_0'^\gamma+\frac{F_1^\gamma}{M^2}-\tfrac{4}{3}F_2^\gamma\right)\right]+\omega^2\cos\theta\left[-if^{\alpha\beta\gamma}\left(\frac{4}{M^2}F_0^\gamma+4F_0'^\gamma-\frac{2}{M^2}F_1^\gamma+\frac{8}{3}F_2^\gamma\right)\right]\\
&\quad+\omega^2\cos^2\theta\left[-if^{\alpha\beta\gamma}\left(-\frac{2}{M^2}F_0^\gamma-2F_0'^\gamma-\tfrac{4}{3}F_2^\gamma+\frac{F_1^\gamma}{M^2}\right)\right]+\frac{(\mathbf{s}\cdot\mathbf{k})^2+(\mathbf{s}\cdot\mathbf{k}')^2-\{\mathbf{s}\cdot\mathbf{k},\mathbf{s}\cdot\mathbf{k}'\}}{\omega}\\
&\quad\times\left[if^{\alpha\beta\gamma}\left(\frac{F_0^\gamma}{M}-\frac{F_1^\gamma}{M}+2MF_2^\gamma\right)\right]+[(\mathbf{s}\cdot\mathbf{k})^2+(\mathbf{s}\cdot\mathbf{k}')^2-\{\mathbf{s}\cdot\mathbf{k},\mathbf{s}\cdot\mathbf{k}'\}]\left[if^{\alpha\beta\gamma}\left(\frac{F_0^\gamma}{2M^2}-\frac{F_1^\gamma}{2M^2}+F_2^\gamma\right)\right]\\
&\quad+[(\mathbf{s}\cdot\mathbf{k})^2+(\mathbf{s}\cdot\mathbf{k}')^2-\{\mathbf{s}\cdot\mathbf{k},\mathbf{s}\cdot\mathbf{k}'\}]\cos\theta\left[-if^{\alpha\beta\gamma}\left(\frac{F_0^\gamma}{2M^2}-\frac{F_1^\gamma}{2M^2}+F_2^\gamma\right)\right]+\frac{\mathbf{s}\cdot(\mathbf{k}\times\mathbf{k}')}{\omega^2}f^{\alpha\beta\gamma}F_1^\gamma+\frac{\mathbf{s}\cdot(\mathbf{k}\times\mathbf{k}')}{\omega}f^{\alpha\beta\gamma}F_1^\gamma\\
&\quad-\frac{\mathbf{s}\cdot(\mathbf{k}\times\mathbf{k}')}{\omega M}\cos\theta\,f^{\alpha\beta\gamma}F_1^\gamma+\mathbf{s}\cdot(\mathbf{k}\times\mathbf{k}')(1-\cos\theta)2f^{\alpha\beta\gamma}F_1'^\gamma.\end{aligned}\tag{24}$$

Finally, let us calculate $E_{44}^{\alpha\beta}$. According to Singh's lemma,

$$E_{44}^{\alpha\beta}=k_m'k_n[\Lambda_{mn}^{\alpha\beta}(k';k)+\Lambda_{nm}^{\beta\alpha}(-k;-k')]\tag{25}$$

$$=k_m'k_n[\Lambda_{mn}^{\alpha\beta}(0,0)+\Lambda_{nm}^{\beta\alpha}(0;0)]+O(\omega^3),\tag{26}$$

where $\Lambda_{mn}^{\alpha\beta}(0;0)$ is a pure numerical tensor. Thus

$$\Lambda_{mn}^{\alpha\beta}(0;0)=\delta_{mn}A^{\alpha\beta}+\epsilon_{mnl}s_lB^{\alpha\beta}+\{s_m,s_n\}C^{\alpha\beta},\tag{27}$$

$$E_{44}^{\{\alpha\beta\}}=\omega^2\cos\theta\,A^{\{\alpha\beta\}}+\{\mathbf{s}\cdot\mathbf{k}',\mathbf{s}\cdot\mathbf{k}\}C^{\{\alpha\beta\}},\tag{28}$$

$$E_{44}^{[\alpha\beta]}=\mathbf{s}\cdot(\mathbf{k}'\times\mathbf{k})B^{[\alpha\beta]},\tag{29}$$

where A, B, and C are unknown constants.

Having obtained all the terms appearing on the right-hand side of Eqs. (10a) and (10b), one needs the tensor decomposi ion of $E_{mn}{}^{\alpha\beta}$ in order to get various low-energy theorems. This we now proceed to do.

IV. TENSOR DECOMPOSITION OF $E_{mn}{}^{\alpha\beta}$

Following Pais,[6] we write down the "complete minimal basis" for $E_{mn}{}^{\alpha\beta}$ using P and T invariance:

$$E_{mn}{}^{\alpha\beta}(\mathbf{k}',\mathbf{k},\mathbf{s})=E_{mn}{}^{i}(\mathbf{k}',\mathbf{k},\mathbf{s})E_i{}^{\alpha\beta}(\omega',\omega). \tag{30}$$

Up to first order in ω, only three basis functions[7] are required;

$$\begin{aligned} i=1:&\quad \delta_{mn},\\ i=2:&\quad \epsilon_{mnl}s_l,\\ i=3:&\quad \{s_m,s_n\}-\tfrac{4}{3}\delta_{mn}. \end{aligned} \tag{31}$$

Up to second order in ω, 14 more basis functions can contribute. We list these below:

$$\begin{aligned}
i=4:&\quad k_mk_n+k_m'k_n',\\
i=5:&\quad k_mk_n'-\mathbf{k}'\cdot\mathbf{k}\delta_{mn},\\
i=6:&\quad k_m'k_n,\\
i=7:&\quad \delta_{mn}\mathbf{s}\cdot(\mathbf{k}\times\mathbf{k}')+\mathbf{k}'\cdot\mathbf{k}\epsilon_{mnl}s_l,\\
i=8:&\quad \epsilon_{mnl}[k_l(\mathbf{s}\cdot\mathbf{k})+k_l'(\mathbf{s}\cdot\mathbf{k}')],\\
i=9:&\quad \epsilon_{mnl}[k_l(\mathbf{s}\cdot\mathbf{k}')+k_l'(\mathbf{s}\cdot\mathbf{k})],\\
i=10:&\quad k_m(\mathbf{s}\times\mathbf{k})_nk_m'(\mathbf{s}\times\mathbf{k}')_n-(m\leftrightarrow n),\\
i=11:&\quad k_m(\mathbf{s}\times\mathbf{k}')_n+k_m'(\mathbf{s}\times\mathbf{k})_n\\
&\qquad\qquad -(m\leftrightarrow n)-2\mathbf{k}'\cdot\mathbf{k}\epsilon_{mnl}s_l,\\
i=12:&\quad \delta_{mn}[(\mathbf{s}\cdot\mathbf{k})^2+(\mathbf{s}\cdot\mathbf{k}')^2],\\
i=13:&\quad \delta_{mn}\{\mathbf{s}\cdot\mathbf{k},\mathbf{s}\cdot\mathbf{k}'\}-\mathbf{k}'\cdot\mathbf{k}\{s_m,s_n\},\\
i=14:&\quad k_m\{s_n,\mathbf{s}\cdot\mathbf{k}\}+k_n'\{s_m,\mathbf{s}\cdot\mathbf{k}'\},\\
i=15:&\quad k_n\{s_m,\mathbf{s}\cdot\mathbf{k}\}+k_m'\{s_n,\mathbf{s}\cdot\mathbf{k}'\},\\
i=16:&\quad k_m\{s_n,\mathbf{s}\cdot\mathbf{k}'\}+k_n'\{s_m,\mathbf{s}\cdot\mathbf{k}\}-2\mathbf{k}'\cdot\mathbf{k}\{s_m,s_n\},\\
i=17:&\quad k_n\{s_m,\mathbf{s}\cdot\mathbf{k}'\}+k_m'\{s_n,\mathbf{s}\cdot\mathbf{k}\}.
\end{aligned} \tag{32}$$

Crossing symmetry implies

$$E_{mn}{}^{\alpha\beta}(k',k;\,\mathbf{s})=E_{nm}{}^{\beta\alpha}(-k,\,-k';\,\mathbf{s}). \tag{33}$$

This gives

$$E_i{}^{\alpha\beta}(\omega',\omega)=\eta_iE_i{}^{\beta\alpha}(-\omega,\,-\omega'), \tag{34}$$

where

$$\begin{aligned}\eta_i&=+1,\quad \text{for}\quad i=1,\,3,\,4,\,5,\,6,\,12\text{ to }17\\ &=-1,\quad \text{for}\quad i=2,\,7\text{ to }11.\end{aligned}$$

In writing the above basis, we have not imposed the transversality condition [Eq. (2)]. If we now impose it, we find that $E_{mn}{}^i$ for $i=4$, 6, 15, and 17 vanish.

Using Eq. (34), it is easy to expand $E_i(\omega',\omega)$. Thus if

$$E_i(\omega',\omega)=+E_i(-\omega,\,-\omega'),$$

then

$$E_i(\omega,'\omega)=E_i(0,0)+e_{i1}M(\omega'-\omega)+e_{i2}(\omega'+\omega)^2+O(\omega^3),$$

and if

$$E_i(\omega',\omega)=-E_i(-\omega,\,-\omega'),$$

then

$$E_i(\omega',\omega)=e_i(\omega'+\omega)+O(\omega^3). \tag{35}$$

It is important to note that the expansion of E_i's [Eq. (35)] is made possible because the E_i's have neither kinematical nor dynamical singularities. The absence of kinematical singularities is "built in" in the procedure of getting the "complete minimal" basis.[6] It is for removing the dynamical singularity (namely, the target pole) that the separation of the complete amplitude T_{mn} into U_{mn} and E_{mn} is necessary. The next singularity will be a cut arising from a photon in the intermediate state. This would, however, contribute to the amplitude only to order e^4, which is why all our results are true only to the order e^2 but are true to all orders in strong interactions.[8]

Finally, we give the left-hand side of Eqs. (10a) and (10b):

$$\begin{aligned}
&k_m'E_{mn}{}^{\{\alpha\beta\}}k_n/\omega\omega'\\
&=\cos\theta\, E_1{}^{\{\alpha\beta\}}(0,0)+\omega^2E_6{}^{\{\alpha\beta\}}(0,0)+\omega^2\cos\theta\,[-e_{11}{}^{\{\alpha\beta\}}+4e_{12}{}^{\{\alpha\beta\}}+2E_4{}^{\{\alpha\beta\}}(0,0)]+\omega^2\cos^2\theta\, e_{11}{}^{\{\alpha\beta\}}\\
&\quad+[(\mathbf{s}\cdot\mathbf{k})^2+(\mathbf{s}\cdot\mathbf{k}')^2]\,2E_{17}{}^{\{\alpha\beta\}}(0,0)+[(\mathbf{s}\cdot\mathbf{k})^2+(\mathbf{s}\cdot\mathbf{k}')^2]\cos\theta\,\{E_{12}{}^{\{\alpha\beta\}}(0,0)+2E_{14}{}^{\{\alpha\beta\}}(0,0)\}\\
&\quad-[\mathbf{s}\cdot(\mathbf{k}\times\mathbf{k}')/\omega](2e_2{}^{\{\alpha\beta\}})-\mathbf{s}\cdot(\mathbf{k}\times\mathbf{k}')e_2{}^{\{\alpha\beta\}}/M+\mathbf{s}\cdot(\mathbf{k}\times\mathbf{k}')\cos\theta\, e_2{}^{\{\alpha\beta\}}/M\\
&\quad+(\{\mathbf{s}\cdot\mathbf{k},\mathbf{s}\cdot\mathbf{k}'\}/\omega^2)E_3{}^{\{\alpha\beta\}}(0,0)+(\{\mathbf{s}\cdot\mathbf{k},\mathbf{s}\cdot\mathbf{k}'\}/M\omega)E_3{}^{\{\alpha\beta\}}(0,0)-(\{\mathbf{s}\cdot\mathbf{k},\mathbf{s}\cdot\mathbf{k}'\}/M\omega)\cos\theta\, E_3{}^{\{\alpha\beta\}}(0,0)\\
&\qquad+\{\mathbf{s}\cdot\mathbf{k},\mathbf{s}\cdot\mathbf{k}'\}(4e_{32}{}^{\{\alpha\beta\}}-e_{31}{}^{\{\alpha\beta\}}+2E_{15}{}^{\{\alpha\beta\}}(0,0))+\{\mathbf{s}\cdot\mathbf{k},\mathbf{s}\cdot\mathbf{k}'\}\cos\theta\,(e_{31}{}^{\{\alpha\beta\}}),
\end{aligned} \tag{36}$$

$$\begin{aligned}
&k_m'E_{mn}{}^{[\alpha\beta]}k_n/\omega\omega'\\
&=\omega\cos\theta\, 2e_1{}^{[\alpha\beta]}-\omega^2\cos\theta\, e_1{}^{[\alpha\beta]}/M+\omega^2\cos^2\theta\, e_1{}^{[\alpha\beta]}/M\\
&\quad+(\{\mathbf{s}\cdot\mathbf{k},\mathbf{s}\cdot\mathbf{k}'\}/\omega)2e_3{}^{[\alpha\beta]}+\{\mathbf{s}\cdot\mathbf{k},\mathbf{s}\cdot\mathbf{k}'\}e_3{}^{[\alpha\beta]}/M-\{\mathbf{s}\cdot\mathbf{k},\mathbf{s}\cdot\mathbf{k}'\}\cos\theta\, e_3{}^{[\alpha\beta]}/M\\
&\quad-[\mathbf{s}\cdot(\mathbf{k}\times\mathbf{k}')/\omega^2]E_2{}^{[\alpha\beta]}(0,0)-[\mathbf{s}\cdot(\mathbf{k}\times\mathbf{k}')/M\omega]E_2{}^{[\alpha\beta]}(0,0)+[\mathbf{s}\cdot(\mathbf{k}\times\mathbf{k}')/M\omega]\cos\theta\, E_2{}^{[\alpha\beta]}(0,0)\\
&\qquad+\mathbf{s}\cdot(\mathbf{k}\times\mathbf{k}')(e_{21}{}^{[\alpha\beta]}-4e_{22}{}^{[\alpha\beta]}-2E_{10}{}^{[\alpha\beta]}(0,0))-\mathbf{s}\cdot(\mathbf{k}\times\mathbf{k}')\cos\theta\,(e_{21}{}^{[\alpha\beta]}).
\end{aligned} \tag{37}$$

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[7] $E_{mn}{}^i$ for $i=3$ is somewhat different from the basis listed by Pais. In our case, it is an irreducible spin-two operator. With this change, the quadrupole moment Q appears only in $E_3{}^{[\alpha\beta]}$ and not in $E_1{}^{[\alpha\beta]}$. I am indebted to Professor V. Singh for pointing this out to me.

[8] Low-energy theorems up to order e^4 have recently been obtained in the framework of S-matrix theory by S. M. Roy and V. Singh (to be published).

V. RESULTS AND DISCUSSION

One can now read off all the low-energy theorems by comparing coefficients of various terms. We state these below.

Symmetric case. First-order theorems:

$$E_1^{\{\alpha\beta\}}(\omega)=\{F_0^\alpha,F_0^\beta\}+O(\omega^2)\,, \tag{S1}$$

$$E_2^{\{\alpha\beta\}}(\omega)=(i/M)[\{F_0^\alpha,F_0^\beta\}-(F_1^\alpha F_0^\beta+F_1^\beta F_0^\alpha)]\omega\,, \tag{S2}$$

$$E_3^{\{\alpha\beta\}}(\omega)=0+O(\omega^2)\,. \tag{S3}$$

Second-order theorems:

$$e_{11}^{\{\alpha\beta\}}=0\,, \tag{S4}$$

$$e_{31}^{\{\alpha\beta\}}=0\,, \tag{S5}$$

$$E_{12}^{\{\alpha\beta\}}(0,0)+2E_{14}^{\{\alpha\beta\}}(0,0) =(1/2M^2)(F_0^\alpha F_1^\beta+F_0^\beta F_1^\alpha)+(F_0^\alpha F_2^\beta+F_0^\beta F_2^\alpha) -(1/2M^2)\{F_0^\alpha,F_0^\beta\}\,. \tag{S6}$$

Antisymmetric case. First-order theorems:

$$E_1^{[\alpha\beta]}(\omega)=((2/3M)[F_0^\alpha,F_0^\beta]+4M[F_0^\alpha,F_0'^\beta] -(2/3M)[F_0^\alpha,F_1^\beta])\omega\,, \tag{A1}$$

$$E_2^{[\alpha\beta]}(\omega)=-f^{\alpha\beta\gamma}F_1^\gamma+O(\omega^2)\,, \tag{A2}$$

$$E_3^{[\alpha\beta]}(\omega)=-if^{\alpha\beta\gamma}(2MF_2^\gamma+F_0^\gamma/M-F_1^\gamma/M)\omega\,. \tag{A3}$$

Second-order theorems:

$$e_{21}^{[\alpha\beta]}=i((1/2M^2)(F_1^\beta F_0^\alpha-F_1^\alpha F_0^\beta) +(1/2M^2)[F_0^\alpha,F_0^\beta])+2f^{\alpha\beta\gamma}F_1'^\gamma\,. \tag{A4}$$

Theorems (S1), (S2), (S3), and (S6) are, of course, trivial extensions of those obtained by Pais earlier for physical Compton scattering. In addition, as has been shown by Pais, (S1), (S2), and (S3) are generalizable to arbitrary spin in terms of the gyromagnetic ratio g (see the Appendix).

The remaining theorems are the new theorems for spin-1 targets. However, most of them are analogous to theorems for the spin-$\frac{1}{2}$ case. We proceed to discuss them now.

(i) Theorem (A2) is the zero-order low-energy theorem for non-Abelian Compton effect that is exactly analogous to Bég's theorem for the nucleon case, which reads

$$E_2^{[\alpha\beta]}(\omega)=-2f^{\alpha\beta\gamma}F_1^\gamma,\quad \text{for spin-}\tfrac{1}{2}\text{ targets.}$$

Generalization of this theorem to arbitrary spin seems obvious, if we introduce the g factor. We therefore conjecture

$$E_2^{[\alpha\beta]}(\omega)=-gf^{\alpha\beta\gamma}F_1^\gamma,\quad \text{for arbitrary spin.}$$

(ii) Theorem (A3) is a new theorem peculiar to the spin-1 case. The most interesting feature of this theorem is that the three form factors F_0, F_1, and F_2 occurring in this combine to give a *pure quadrupole-moment term.*

(iii) Theorem (A1) is analogous to the Cabibbo-Radicati theorem for the nucleon case. The amplitude in question requires for its determination not only the charge and the magnetic moment F_0 and F_1, but also the charge distribution through F_0'. Explicitly, for the nucleon case, the theorem reads

$$E_1^{[\alpha\beta]}=((1/2M)[F_0^\alpha,F_0^\beta]+4M[F_0^\alpha,F_0'^\beta] -(1/M)[F_0^\alpha,F_1^\beta])\omega\,.$$

Note that the explicit form for the spin-1 case is not identical to the spin-$\frac{1}{2}$ case even when one uses the g factor. We are therefore as yet unable to conjecture the generalization of this theorem for arbitrary spin.

(iv) The second-order theorem (S4) is exactly analogous to Singh's theorem for the spin-$\frac{1}{2}$ case. Theorem (S5) is peculiar to the spin-1 situation. These, together, satisfy the following generalized second-order theorem first conjectured by Singh[1]: Let

$$\mathrm{Tr}(\epsilon_m' T_{mn}^{\{\alpha\beta\}}\epsilon_n)/\mathrm{Tr}(1) =T_1^{\{\alpha\beta\}}(\omega',\omega)\boldsymbol{\varepsilon}'\cdot\boldsymbol{\varepsilon}+T_2^{\{\alpha\beta\}}(\omega',\omega)(\boldsymbol{\varepsilon}'\cdot\mathbf{k}\boldsymbol{\varepsilon}\cdot\mathbf{k}'-\mathbf{k}'\cdot\mathbf{k}\boldsymbol{\varepsilon}'\cdot\boldsymbol{\varepsilon})\,,$$

where the trace is over the spin states, and let

$$T_1^{\{\alpha\beta\}}(\omega',\omega)=T_1^{\{\alpha\beta\}}(0,0)+t_1^{\{\alpha\beta\}}M(\omega'-\omega) +(\omega'+\omega)^2t_2^{\{\alpha\beta\}}\,;$$

then

$$t_1^{\{\alpha\beta\}}=0\,.$$

In the same way, theorem (A4) is analogous to Singh's corresponding theorem for the spin-$\frac{1}{2}$ case.

Why do most of these theorems so easily generalize to the case of arbitrary spin? The reason for this can be traced by studying the vertex functions, say, for the spin-$\frac{1}{2}$ and spin-1 targets. The following important properties can be noted by comparing the expressions given in the Appendix: (a) Up to first order, the form of the vertex $\langle p'|J_m(0)|p\rangle$ is identical for both cases if expressed in terms of the g factor; (b) the coefficient of $\mathbf{s}\cdot(\mathbf{p}\times\mathbf{p}')$ occurring in the vertex $\langle p'|J_0(0)|p\rangle$ is again identical for both cases if expressed in terms of the g factor; and (c) the coefficient of $\mathbf{q}^2$ occurring in the vertex $\langle p'|J_0(0)|p\rangle$ is not identical in the two cases even in terms of g.

The three first-order theorems (S1), (S2), and (S3) given by Pais, the Bég theorem (A2) for the antisymmetric case, and the second-order theorems of Singh owe their generalizations in terms of g to properties (a) and (b) stated above. These theorems are independent of the coefficient of $\mathbf{q}^2$ occurring in the $\langle p'|J_0(0)|p\rangle$ vertex.

However, the amplitude given by the Cabibbo-Radicati theorem depends on this coefficient of $\mathbf{q}^2$, and that is why the Cabibbo-Radicati theorem does not appear to be quite the same in the two cases. It is, of course, possible that the form of vertex $\langle p'|J_0(0)|p\rangle$ may after all be generalizable to higher spins in a less obvious way. One possibility is that there might be generalized Sachs form factors in terms of which this vertex may be expressible in identical forms in the two cases. If such a generalization of the vertex function exists, we shall, of course, get a generalized Cabibbo-Radicati theorem also.

This aspect of the problem and explicit proofs of the various conjectures made for arbitrary spin will be reported elsewhere.

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APPENDIX

A. Electromagnetic Vertex for a Spin-$\frac{1}{2}$ Particle

$$\langle p'|J_\mu(0)|p\rangle=(M^2/V^2E_pE_{p'})^{1/2}\bar{u}(p')\times[\gamma_\mu F_0(q^2)+i\sigma_{\mu\nu}q^\nu f_1(q^2)]u(p),\quad q=p'-p.$$

Here $F_0(0)=e$, $f_1(0)=(e/2M)(\mu-1)$, and μ is the total magnetic moment. Define

$$F_1(0)=e\mu=2Mf_1(0)+F_0(0).$$

One can now "unboost" $\bar{u}(p')$ and $u(p)$ by using

$$u(\mathbf{p})=\frac{(\not{p}+M)u(0)}{[2M(E+M)]^{1/2}},\quad \bar{u}(\mathbf{p}')=\bar{u}(0)\frac{\not{p}'+M}{[2M(E+M)]^{1/2}}.$$

This gives

$$(V^2E_pE_{p'}/M^2)^{1/2}\langle p'|J_m(0)|p\rangle =\xi^\dagger\{(1/2M)[P_mF_0(0)+i(\mathbf{s}\times\mathbf{q})_m2F_1(0)]\}\xi+O(p^3),$$

where ξ is the two-component spinor, $\mathbf{s}=\frac{1}{2}\boldsymbol{\sigma}$, and $P=p'+p$. Similarly,

$$(V^2E_pE_{p'}/M^2)^{1/2}\langle p'|J_0(0)|p\rangle =\xi^\dagger\bigg(F_0(q^2)+i\frac{F_0(0)-2F_1(0)}{2M^2}\mathbf{s}\cdot(\mathbf{p}\times\mathbf{p}') +\frac{F_0(0)}{4M^2}(\mathbf{p}^2+\mathbf{p}'^2)+\frac{F_0(0)-2F_1(0)}{2M^2}\tfrac{1}{4}\mathbf{q}^2\bigg)\xi+O(p^4).$$

B. Electromagnetic Vertex for a Spin-1 Particle

We give here the explicit expression for Y_μ defined in Sec. III (for more details, see Ref. 6):

$$Y_m/2M=(1/2M)[P_mF_0(0)+i(\mathbf{s}\times\mathbf{q})_mF_1(0)]+O(p^3),$$

$$Y_0=\{F_0(q^2)-[(\mathbf{s}\cdot\mathbf{q})^2-\tfrac{2}{3}\mathbf{q}^2]F_2(0)\times[E(\mathbf{p})+E(\mathbf{p})']+(F_0(0)-F_1(0))M\times[i\mathbf{s}\cdot(\mathbf{p}\times\mathbf{p}')+\mathbf{q}^2-(\mathbf{s}\cdot\mathbf{q})^2],$$

where

$$F_0(0)=e,\quad F_1(0)=e\mu,$$
$$F_2(0)=(e/2M^2)(Q+\mu-1).$$

Here μ is the magnetic moment in units $e/2M$ and Q is the quadrupole moment in units e/M^2.

To see its similarity with the nucleon vertex, we rewrite

$$\frac{Y_0}{2M}=F_0(q^2)+i\frac{F_0(0)-F_1(0)}{2M^2}\mathbf{s}\cdot(\mathbf{p}\times\mathbf{p}') +\frac{F_0(0)}{4M^2}(\mathbf{p}^2+\mathbf{p}'^2)+\frac{F_0(0)-F_1(0)}{2M^2}\tfrac{1}{3}\mathbf{q}^2 +(\text{a term involving only } Q).$$

It is then clear that Y_m is completely generalizable in terms of the gyromagnetic ratio $g=\mu/s$, The coefficient of $\mathbf{s}\cdot(\mathbf{p}\times\mathbf{p}')$ occurring in $\langle p'|J_0(0)|p\rangle$ is also the same in terms of g. However, the coefficient of $\mathbf{q}^2$ is different in the two cases.

Finally, up to third order, in the laboratory frame ($\mathbf{p}=0$),

$$\mathbf{Y}=\{F_0(q^2)-[(\mathbf{s}\cdot\mathbf{q})^2-\tfrac{2}{3}\mathbf{q}^2]F_2(0)\}\mathbf{P}+i(\mathbf{s}\times\mathbf{q})F_1(q^2) +[(F_0(0)-F_1(0))/2M^2][\mathbf{q}^2-(\mathbf{s}\cdot\mathbf{q})^2]\mathbf{P}.$$