Current Algebra and $K^+\Lambda$ Photoproduction

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(Received 16 April 1968; revised manuscript received 24 June 1968)

The available angular-distribution data on $K^+\Lambda$ photoproduction are analyzed using partial conservation of axial-vector current (PCAC) and $SU(3)\times SU(3)$ algebra of currents. The off-shell extrapolation implicit in the use of PCAC is contained in the limit (kaon four-momentum)$^2 \to 0$ which reproduces the dynamical singularities of the amplitude in detail. The results of the calculation show reasonable agreement with experiment at low energy. The behavior of the angular distribution at high energy is also discussed.

1. INTRODUCTION

RECENTLY, there have been several attempts to study the photoproduction processes using partially conserved axial-vector currents (PCAC) and $SU(3)\otimes SU(3)$ algebra of currents. These calculations[1] were mostly confined to the determination of the magnetic moments and to the discussion of low-energy theorems. Some of these calculations involve the soft-meson limit $q_\mu \to 0$ for the off-shell extrapolation that is necessary in the use of PCAC. However, this limit is known to yield conditions on the amplitude at the threshold,[2] and may not be justified for calculations in the energy regions that are appreciably away from the threshold. The purpose of this paper is to analyze the angular-distribution data on $K^+\Lambda$ photoproduction in the energy range $E_\gamma = 1000$–1200 MeV. We use PCAC and current algebra, but for the off-shell extrapolation we adopt the limit $q^2 \to 0$ instead of the usual soft-kaon limit. This is certainly a less stringent condition and more reasonable for comparison of our results with the available experimental data.[3]

Thus in the present case we are faced with the problem of the computation of the unknown "weak amplitude" term that corresponds to the process

$$V+p \to A+\Lambda ,$$

and, as discussed later, this involves the knowledge of various form factors. In view of the experimental uncertainties associated with the coupling constants that appear in the calculation, the results of the present analysis show reasonable agreement with experiment in the energy region under consideration.

The main body of our calculation has been divided into four sections. The first part of Sec. 2 is devoted to introducing the notation and the various definitions, and the second part deals with separation of the current commutator from the so-called "weak amplitude" term. In Sec. 3, we discuss a basic problem encountered in any such calculation, namely, that the amplitude does not satisfy the usual gauge-invariance requirement when the final kaon is off the mass shell.[4] Section 4, deals with the details of calculation of the weak amplitude (in the limit $q^2 \to 0$) and the subsequent computations for the angular distribution. Finally, we discuss our results and compare them with available experimental data.

2. PHOTOPRODUCTION AMPLITUDE

We define the S matrix for the process

$$\gamma(k)+p(p_1) \to K^+(q)+\Lambda(p_2) \qquad (1)$$

as follows:

$$S_{fi} = ie(2\pi)^4\delta^{(4)}(p_2+q-p_1-k) \times (M_pM_\Lambda/4E_pE_\Lambda q_0k_0)^{1/2}T_{fi}, \qquad (2)$$

where e is the electric charge and the other symbols have their usual meanings.

It must be emphasized that, when the final kaon is off the mass shell, the photoproduction amplitude has a nonvanishing divergence (as discussed in Sec. 3), and consequently one must retain all six Lorentz-invariant amplitudes in the equation

$$T_{fi} = \sum_{j=1}^{6} V_j \bar{u}_\Lambda(p_2) O(V_j) u_p(p_1) , \qquad (3)$$

where

$$\begin{aligned} O(V_1) &= i\gamma_5(\gamma\cdot\epsilon)(\gamma\cdot k) , \\ O(V_2) &= i\gamma_5(p_1+p_2)\cdot\epsilon , \\ O(V_3) &= i\gamma_5(q\cdot\epsilon) , \\ O(V_4) &= \gamma_5(\gamma\cdot\epsilon) , \\ O(V_5) &= \gamma_5(\gamma\cdot k)(p_1+p_2)\cdot\epsilon , \\ O(V_6) &= \gamma_5(\gamma\cdot k)(q\cdot\epsilon) . \end{aligned} \qquad (4)$$

Using the standard reduction technique and PCAC, the T-matrix element for the process (1), as defined by

[1] S. Fubini, G. Furlan, and C. Rossetti, Nuovo Cimento **40**, 1171 (1965); Riazuddin and B. W. Lee, Phys. Rev. **146**, B1202 (1966); S. Adler and Y. Dothan, *ibid.* **151**, 1267 (1966); S. Adler and F. Gilman, *ibid.* **152**, 1460 (1966); V. S. Mathur and L. K. Pandit, *ibid.* **147**, 965 (1966). Some accounts of earlier works on strange-particle photoproduction may be found in M. Gourdin and J. Dufour, Nuovo Cimento **27**, 1410 (1963); S. Hatsukade and H. Schnitzer, Phys. Rev. **132**, 1301 (1963); S. Hatsukade, L. K. Pandit, and A. H. Zimerman, Nuovo Cimento **34**, 819 (1964).

[2] This point is discussed in S. Adler and R. Dashen, *Current Algebras and Applications to Particle Physics* (W. A. Benjamin, Inc., New York, 1968).

[3] R. L. Anderson *et al.*, Phys. Rev. Letters **9**, 131 (1962); also C. W. Peck, Phys. Rev. **135**, B830 (1964). Experimental points are, however, reproduced from a compilation of data given by H. Thom, *ibid.* **151**, 1322 (1966).

[4] M. Nauenberg, Phys. Letters **22**, 201 (1966); also S. Adler and Y. Dothan, cited in Ref. 1.

Eq. (2), is given by

$$T_{fi}=\frac{q^2+m_K^2}{c_K}\left(\frac{E_pE_\Lambda}{M_pM_\Lambda}\right)^{1/2}$$

$$\times\left\{-i\int d^4x\, e^{-iq\cdot x}\delta(x_0)\langle\Lambda|[A_0(x),j_\mu(0)]|p\rangle\right.$$

$$\left.-q_\sigma\int d^4x\, e^{-iq\cdot x}\langle\Lambda|T\{A_\sigma(x)j_\mu(0)\}|p\rangle\right\}\epsilon_\mu$$

$$=T_{fi}^{(c)}+T_{fi}^{(w)}, \tag{5}$$

where $T_{fi}^{(c)}$ is the current-commutator term and $T_{fi}^{(w)}$ is the so-called weak amplitude; c_K is the PCAC constant corresponding to the strangeness-changing axial-vector current as defined by

$$\partial A_\mu/\partial x_\mu=c_K\phi_K. \tag{6}$$

Now, using the equal-time current commutation relation

$$\delta(x_0)[A_0(x),j_\mu(0)]=\alpha_K A_\mu(x)\delta^4(x), \tag{7}$$

we obtain

$$T_{fi}^{(c)}=-i\left(\frac{E_pE_\Lambda}{M_pM_\Lambda}\right)^{1/2}\frac{m_K^2}{c_K}\alpha_K\langle\Lambda|A_\mu(0)|p\rangle\epsilon_\mu, \tag{8}$$

where the off-shell extrapolation $q^2\to 0$ is implicit. The evaluation of the antisymmetric $SU(3)$ coefficient α_K is facilitated by recalling the $SU(3)$ structure of the electromagnetic current,

$$j_\mu=j_\mu^{(3)}+\tfrac{1}{3}\sqrt{3}j_\mu^{(8)},$$

whence, from the standard table of isoscalar factors,[5] we get

$$\alpha_K=-\tfrac{1}{2}\sqrt{3}. \tag{9}$$

Thus one finally obtains

$$T_{fi}^{(c)}=-\frac{1}{\sqrt{3}}\frac{m_K^2}{c_K}\bar{u}_\Lambda(p_2)$$

$$\times\left(g_A{}^\Lambda(t^2)\gamma_\mu\gamma_5-\frac{ih_A{}^\Lambda(t^2)t_\mu\gamma_5}{M_p+M_\Lambda}\right)u_p(p_1)\epsilon_\mu$$

$$=\frac{1}{\sqrt{3}}\frac{m_K^2}{c_K}\bar{u}_\Lambda(p_2)$$

$$\times\left(g_A{}^\Lambda(t^2)O(V_4)+\frac{h_A{}^\Lambda(t^2)}{M_p+M_\Lambda}O(V_3)\right)u_p(p_1), \tag{10}$$

where $t=q-k$ and $g_A{}^\Lambda(t^2)$ and $h_A{}^\Lambda(t^2)$ are, respectively, the Λ β-decay axial-vector and induced-pseudoscalar form factors evaluated at the momentum transfer t.

[5] P. McNamee and Frank Chilton, Rev. Mod. Phys. 36, 1005 (1964).

3. GAUGE INVARIANCE AND OFF-SHELL AMPLITUDE

We begin with a brief outline of the derivation of the gauge condition satisfied by the photoproduction amplitude.

If we define the total transition matrix element by

$$T_{fi}=T_\mu\epsilon_\mu, \tag{11}$$

then its divergence, which is obtained by the replacement $\epsilon_\mu\to k_\mu$, is given by

$$T_\mu k_\mu=\frac{q^2+m_K^2}{c_K}\left(\frac{E_pE_\Lambda}{M_pM_\Lambda}\right)^{1/2}\left\{-i\alpha_k\langle\Lambda|A_\mu(0)|p\rangle k_\mu\right.$$

$$\left.-q_\sigma\int d^4x\, e^{-iq\cdot x}\langle\Lambda|T[A_\sigma(x)j_\mu(0)]|p\rangle k_\mu\right\}$$

$$=\frac{q^2+m_K^2}{c_K}\left(\frac{E_pE_\Lambda}{M_pM_\Lambda}\right)^{1/2}\left\{-i\alpha_k\langle\Lambda|A_\mu(0)|p\rangle k_\mu\right.$$

$$\left.+iq_\sigma\int d^4x\, e^{-ik\cdot x}\delta(x_0)\langle\Lambda|[A_\sigma(0),j_0(-x)]|p\rangle\right\}$$

$$=\frac{\alpha_K(q^2+m_K^2)}{c_K}\left(\frac{E_pE_\Lambda}{M_pM_\Lambda}\right)^{1/2}i(q_\mu-k_\mu)\langle\Lambda|A_\mu(0)|p\rangle. \tag{12}$$

From (12), on reimposing PCAC, one obtains after a little calculation

$$T_\mu k_\mu=[\alpha_K(q^2+m_K^2)/(t^2+m_K^2)]\times g_\gamma(t^2)\bar{u}_\Lambda(p_2)\gamma_5u_p(p_1), \tag{13}$$

which reproduces the usual on-shell gauge condition $T_\mu k_\mu=0$ for $q^2=-m_K^2$. However, the difficulty of this nonvanishing divergence caused by the off-shell final kaon is inherent in any such current-algebra calculation. It can be eliminated altogether if we envisage a smooth extrapolation to the physical amplitude from the limit $m_K^2\to 0$, corresponding to the production amplitude of a zero-mass kaon. This conjecture is supported by the calculations of Berman[6] and of Roy.[7]

4. CALCULATION OF WEAK AMPLITUDE AND ANGULAR DISTRIBUTION

It is obvious that under a K-pole dominance the photoproduction amplitude receives a vanishing con-

[6] S. M. Berman, Phys. Rev. Letters 18, 1081 (1967).
[7] P. Roy, Phys. Rev. 162, 1644 (1967); 172, 1849(E) (1968).

tribution from the weak-amplitude term,

$$T_{fi}^{(w)}=-\frac{m_K^2}{c_K}\left(\frac{E_pE_\Lambda}{M_pM_\Lambda}\right)^{1/2}q_\sigma\int d^4x\, e^{-iq\cdot x}\times\langle\Lambda|T\{A_\sigma(x)j_\mu(0)\}|p\rangle\epsilon_\mu\,, \quad (14)$$

in the soft-kaon limit $q\to 0$, and consequently suppresses the dynamical details of the process to a considerable extent. However, an extrapolation from the limit $q^2=0$ reproduces all the dynamical singularities of the amplitude, and hence is certainly a better representation of the facts.

In order to calculate the weak amplitude we saturate the complete set of intermediate states with p, Λ, Σ, and Y_1^* single-particle states. The $\frac{1}{2}^-$ resonance Y_0^* is excluded because of the lack of experimental data on its electromagnetic form factor. The reasons for including only these low-lying states are discussed below:

(a) $N_{\frac{3}{2},\frac{3}{2}}^{*+}(1238)$ (in the direct channel) does not contribute, because of isospin conservation at the strong vertex $N^*K\Lambda$.

(b) The $N^{*+}(1400;\frac{1}{2}^+)$ and $N^*(1570;\frac{1}{2}^-)$ can, in principle, contribute to our weak amplitude. Their contributions, however, involve the unknown vertices $N^*(\frac{1}{2})^\pm N\gamma$ and $N^*(\frac{1}{2})^\pm K\Lambda$.[8] In view of the uncertainties in these couplings we have not attempted to estimate these contributions.

(c) The possible higher-spin resonances have also have also been neglected on similar grounds.

We now write the matrix elements of the vector and the axial-vector currents, where Y_1^* is treated as a stable spin-$\frac{3}{2}$ Rarita-Schwinger particle:

$$\begin{aligned}\langle p|j_\mu^{\text{em}}|p\rangle&=i\bar{u}_p\Gamma_\mu{}^{pp\gamma}u_p\,,\\ \langle\Lambda|j_\mu^{\text{em}}|j\rangle&=i\bar{u}_\Lambda\Gamma_\mu{}^{\Lambda j\gamma}u_j,\quad j=\Lambda,\Sigma \qquad (15)\\ \langle\Lambda|j_\mu^{\text{em}}|Y_1^*\rangle&=(iC_3/m_\pi)^{\Lambda Y_1^*\gamma}\bar{u}_\Lambda\nabla_{\mu\nu}\gamma_5u_\nu\,,\end{aligned}$$

where $C_3{}^{\Lambda Y_1^*\gamma}$ is the dominant one of the three gauge-invariant couplings obtained by Gourdin and Salin[9] and

$$\Gamma_\mu=F_1\gamma_\mu+iF_2\sigma_{\mu\nu}k_\nu,\quad \nabla_{\mu\nu}=k\delta_{\mu\nu}-\gamma_\mu k_\nu.$$

For the relevant matrix elements of the axial-vector current we take

$$\langle j|A_\sigma|p(p)\rangle=i\bar{u}_j\{g_A(j)\gamma_\sigma\gamma_5+[i(p_j-p)_\sigma/2M_p]\gamma_5h_A(j)\}u_p,\quad j=\Lambda,\Sigma.\quad (16)$$

Obviously the induced pseudoscalar form factor does not contribute to the weak amplitude in the limit $q^2\to 0$.

Finally, following Schnitzer,[10] we note that there are four linearly independent form factors for the p-Y_1^* axial-vector vertex:

$$\begin{aligned}\langle Y_1^*(p_n)|A_\sigma|p(p_1)\rangle=\bar{u}_\nu(p_n)[g_A(Y_1^*)\delta_{\nu\sigma}&+g_2t_\nu(\gamma_\alpha\epsilon_{\alpha\rho\lambda\sigma}p_{1\rho}t_\lambda\gamma_5)+g_3\epsilon_{\nu\alpha\beta\gamma}\epsilon_{\gamma\rho\tau\sigma}p_{1\alpha}p_{i\rho}t_\beta t_\tau\\&+g_4t_\nu(t^2p_{1\sigma}-t_\sigma p_1\cdot t)]u_p(p_1)\,,\quad (17)\end{aligned}$$

with $t=p_1-p_n$. If we note that $p_n=p_1-q$, i.e., $t=q$, it is clear from (17) that the coefficients of g_2, g_3, and g_4 are transverse to q and do not contribute to the weak amplitude (14). Accordingly, we proceed by retaining terms involving $g_A(Y_1^*)$ only.[11]

Equations (15)–(17) take care of a proton pole in the direct channel and Λ, Σ, and Y_1^* exchanges in the u channel.

However, if the weak amplitude is viewed, within the framework of current algebra, as representing the process

$$V+p\to A+\Lambda\,,$$

it may receive a contribution from a t-channel K^* exchange. This involves the unknown decay width $Q_A\to K^*+\gamma$ which so far has not been observed. In any case, one may use $SU(3)$ symmetry and try to relate it to the $A_1\to\rho+\gamma$ width, which is also not known experimentally. Also, the unknown vertex $\langle K^*|A_\sigma|\gamma\rangle$, when dominated by the K pole, has the structure $g_\sigma F$, where F is some suitable form factor that has a pole at the kaon mass. Hence $q_\sigma\langle K^*|A_\sigma|\gamma\rangle$, which occurs in (14), vanishes in the limit $q^2\to 0$. However, note that the t-channel pole in the total amplitude is partly taken care of by the current commutator.[12]

The contributions of the intermediate states p, Λ,

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[8] The $N^*(\frac{1}{2})^-K\Lambda$ coupling may be obtained under the following assumptions: (a) The $N^*(\frac{1}{2})^-$ belongs to an $SU(3)$ octet, and (b) the d/f ratio for this coupling is the same as for the $NN\pi$ coupling. Under these assumptions $g_{N^*(1/2)^-K\Lambda}{}^2/4\pi\sim 0.02$. The $N^*(\frac{1}{2})^-N\gamma$ coupling involves in the simplest approximation the knowledge of the $N\to N^*(\frac{1}{2})^-$ transition dipole moment. In view of the uncertainties in these couplings, we have not attempted to estimate this contribution. This problem needs further investigation. The $N^*(\frac{1}{2})^+N\gamma$ vertex is also not known, and, furthermore, one finds that the $N^*(\frac{1}{2})^+K\Lambda$ coupling is difficult to estimate at present.

[9] M. Gourdin and Ph.Salin [Nuovo Cimento **27**, 309 (1963)] show that out of the three gauge-invariant couplings, only one makes the dominant contribution, and only this has been retained in the calculation.

[10] H. Schnitzer, Phys. Rev. **158**, 1471 (1967).

[11] See also C. Albright and L. Liu, Phys. Rev. Letters **13**, 673 (1964). It is interesting to note that this analysis of the N^* production by neutrinos also seems to favor the retention of only one form factor. For example, compare the curves e′, f′, and g′ of Fig. 2 in Albright and Liu's paper, which are obtained by retaining only $F_1{}^A$ (corresponding to g_A in our notation) and give better agreement with experiment. See also S. N. Biswas, Aditya Kumar, and R. P. Saxena, *ibid.* **17**, 268 (1966), where they discuss the decuplet contribution to nonleptonic Λ decays, incorporating only the g_A-type term, and obtain good agreement with experiment. A similar conclusion also follows from I. M. Zheleznykh, Phys. Letters **11**, 251 (1964). Here also, good agreement with experiment is obtained by retaining only one axial-vector form factor.

[12] J. J. Sakurai, Phys. Rev. Letters **17**, 552 (1966).

Σ, and Y_1^* are given, respectively, by[13]

$$T_{fi}^{(s)}(p)=\frac{m_K^2}{c_K}\bar{u}_\Lambda(p_2)ig_A(\Lambda)q\gamma_5\frac{p_2+iM_p}{(p_1+k)^2+M_p^2}i\Gamma_\mu^{pp\gamma}u_p(p_1)\epsilon_\mu\,,$$

$$T_{fi}^{(u)}(j)=-\frac{m_K^2}{c_K}\bar{u}_\Lambda(p_2)\Gamma_\mu^{\Lambda j\gamma}\epsilon_\mu\frac{(p_1+iM_j)}{(p_2-k)^2+M_j^2}g_A(j)q\gamma_5u_p(p_1)\,,$$

$$T_{fi}^{(u)}(Y_1^*)=\frac{m_K^2}{c_K}\bar{u}_\Lambda(p_2)\frac{iC_3^{\Lambda Y_1^*\gamma}}{m_\pi}\nabla_{\mu\nu}\gamma_5\frac{(p_1-q)+iM_*}{(p_1-q)^2+M_*^2}P_{\nu\sigma}q_\sigma g_A(Y_1^*)u_p(p_1)\epsilon_\mu\,,$$

where the superscripts (s) and (u) indicate the direct and the crossed channel, $j=\Lambda$, Σ, and

$$P_{\nu\sigma}=\delta_{\nu\sigma}-\tfrac{1}{3}\gamma_\nu\gamma_\sigma+(i/3M_*)[\gamma_\nu(p_1-q)_\sigma-\gamma_\sigma(p_1-q)_\nu]+(2/3M_*^2)(p_1-q)_\nu(p_1-q)_\sigma\,.$$

Now a straightforward calculation shows that the above expressions lead to

$$\begin{aligned}T_{fi}&=T_{fi}^{(c)}+T_{fi}^{(w)}\\&\equiv T_{fi}^{(c)}+T_{fi}^{(s)}(p)+\sum_{j=\Lambda,\Sigma,Y_1^*}T_{fi}^{(u)}(j)\\&=\sum_{j=1}^{6}V_j\bar{u}_\Lambda(p_2)O(V_j)u_p(p_1)\,,\end{aligned}$$

where

$$\begin{aligned}V_1=&\frac{m_K^2}{c_K}\frac{g_A(\Lambda)}{2p_1\cdot k}\left\{[(M_p+M_\Lambda)^2-2p_2\cdot q]\frac{\mu_p}{2M_p}-(M_p+M_\Lambda)\right\}-\frac{m_K^2}{c_K}\sum_{j=\Lambda,\Sigma}\frac{g_A(j)}{2p_2\cdot k+M_\Lambda^2-M_j^2}\\&\times[2p_1\cdot q+(M_p+M_\Lambda)(M_p+M_j)]\frac{\mu_{\Lambda j}}{M_\Lambda+M_j}+\frac{m_K^2C_3^{\Lambda Y_1^*\gamma}g_A(Y_1^*)}{c_K[2p_2\cdot k+(M_\Lambda^2-M_*^2)]}\\&\times\{3M_*^2q\cdot k+p_2\cdot k[2p_1\cdot q+M_*(2M_\Lambda+M_p-M_*)]-M_*(M_*+M_\Lambda)(M_\Lambda+M_p)(2M_*-M_\Lambda+M_p)\}\,,\end{aligned}\quad\text{(18a)}$$

$$V_2=\frac{m_K^2}{c_K}\left(\frac{g_A(\Lambda)}{2p_1\cdot k}(M_p+M_\Lambda)\frac{\mu_p}{2M_p}-\frac{C_3^{\Lambda Y_1^*\gamma}g_A(Y_1^*)}{2p_2\cdot k+M_\Lambda^2-M_*^2}q\cdot k\right),\quad\text{(18b)}$$

$$V_3=\frac{m_K^2}{c_K}\left(\frac{g_A(\Lambda)}{2p_1\cdot k}(M_p+M_\Lambda)\frac{\mu_p}{2M_p}+\frac{C_3^{\Lambda Y_1^*\gamma}g_A(Y_1^*)}{2p_2\cdot k+M_\Lambda^2-M_*^2}(p_1+p_2)\cdot k+\frac{h_A^\Lambda(t^2)}{\sqrt{3}(M_p+M_\Lambda)}\right),\quad\text{(18c)}$$

$$\begin{aligned}V_4=&\frac{m_K^2}{c_K}g_A(\Lambda)\left((2p_2\cdot q+M_p^2-M_\Lambda^2)-(M_p+M_\Lambda)2p_1\cdot k\frac{\mu_p}{2M_p}\right)+\frac{m_K^2}{c_K}2p_2\cdot k\sum_{j=\Lambda,\Sigma}\frac{g_A(j)(M_j+M_p)}{2p_2\cdot k+M_\Lambda^2-M_j^2}\frac{\mu_{\Lambda j}}{M_\Lambda+M_j}\\&+\frac{m_K^2}{\sqrt{3}c_K}g_A^\Lambda(t^2)+\frac{m_K^2g_A(Y_1^*)C_3^{\Lambda Y_1^*\gamma}}{3c_K(2p_2\cdot k+M_\Lambda^2-M_*^2)M_*^2}\{(M_\Lambda+M_*)3M_*^2q\cdot k\\&+2p_2\cdot k[(M_\Lambda+M_*)p_1\cdot q-M_*2p_2\cdot k+M_*^2(M_\Lambda+M_p+2M_*)\\&-\tfrac{1}{2}M_*(M_\Lambda-M_p)(M_*+2M_\Lambda+M_p)]\}\,,\end{aligned}\quad\text{(18d)}$$

$$\begin{aligned}V_5=&\frac{m_K^2}{c_K}\frac{g_A(\Lambda)}{2p_1\cdot k}(M_p+M_\Lambda)\frac{\mu_p}{2M_p}-\frac{m_K^2}{c_K}\sum_{j=\Lambda,\Sigma}\frac{(M_p+M_j)g_A(j)}{2p_2\cdot k+M_\Lambda^2-M_j^2}\frac{\mu_{\Lambda j}}{M_\Lambda+M_j}+\frac{m_K^2C_3^{\Lambda Y_1^*\gamma}g_A(Y_1^*)}{3M_*^2c_K(2p_2\cdot k+M_\Lambda^2-M_*^2)}\\&\times[2M_*p_2\cdot k-(M_\Lambda+M_*)p_1\cdot q+\tfrac{1}{2}M_*(M_\Lambda-M_p)(2M_\Lambda+M_*+M_p)-M_*^2(M_\Lambda+M_p+2M_*)]\,,\end{aligned}\quad\text{(18e)}$$

[13] The spin-$\frac{3}{2}$ propagator appearing in $T_{fi}^{(u)}(Y_1^*)$ is taken from M. Gourdin and Ph.Salin, Nuovo Cimento **27**, 193 (1963).

$$V_6=\frac{m_K^2}{c_K}\frac{g_A(\Lambda)}{2p_1\cdot k}(M_p+M_\Lambda)\frac{\mu_p}{2M_p}+\frac{m_K^2}{c_K}\sum_{j=\Lambda,\Sigma}\frac{g_A(j)(M_p+M_j)}{2p_2\cdot k+M_\Lambda^2-M_j^2}\frac{\mu_{\Lambda j}}{M_\Lambda+M_j}+\frac{m_K^2C_3^{\Lambda Y_1^*\gamma}g_A(Y_1^*)}{3M_*^2c_K(2p_2\cdot k+M_\Lambda^2-M_*^2)}$$
$$\times[(M_*+M_\Lambda)p_1\cdot q-2M_*p_2\cdot k-\tfrac{1}{2}M_*(M_\Lambda-M_p)(2M_\Lambda+M_*+M_p)+M_*^2(M_p-2M_\Lambda-M_*)]. \quad (18f)$$

All the masses occurring in the above equations are understood to be measured in units of the pion mass. Now the differential cross section in the c.m. frame is given by

$$\frac{d\sigma}{d\Omega}=\frac{M_pM_\Lambda}{(4\pi W)^2}\frac{|\mathbf{q}|}{|\mathbf{k}|}\frac{1}{4}X, \quad (19)$$

where $|\mathbf{q}|$ and $|\mathbf{k}|$ are the outgoing and incoming c.m. momenta, W is the total c.m. energy, and

$$X=\sum_{i,j=1}^{6}V_iV_j\,\mathrm{Tr}[\gamma_4O_\mu^\dagger(V_i)\gamma_4\Lambda_\Lambda^{(+)}(p_2)O_\mu(V_j)\Lambda_p^{(+)}(p_1)]. \quad (20)$$

Evaluating the traces, one obtains

$$\begin{aligned}X=&(1/4M_\Lambda M_p)\{16V_1^2(k\cdot p_1)(k\cdot p_2)-4V_2^2(p_1+p_2)^2(p_1\cdot p_2+M_pM_\Lambda)\\
&+4V_3^2m_K^2(p_1\cdot p_2+M_pM_\Lambda)-8V_4^2(p_1\cdot p_2-2M_pM_\Lambda)+8V_5^2(p_1+p_2)^2(k\cdot p_1)(k\cdot p_2)-8V_6^2m_K^2(p_1\cdot k)(p_2\cdot k)\\
&-8V_1V_2[2(k\cdot p_1)(p_2\cdot p_1)+M_p^2(k\cdot p_2)-M_\Lambda^2(k\cdot p_1)+M_pM_\Lambda k\cdot(p_1+p_2)]\\
&-8V_1V_3[(q\cdot k)(p_1\cdot p_2)-(k\cdot p_2)(q\cdot p_1)+(k\cdot p_1)(q\cdot p_2)+M_pM_\Lambda(q\cdot k)]\\
&+16V_1V_4(M_pk\cdot p_2+2M_\Lambda k\cdot p_1)+16V_1V_5M_\Lambda(k\cdot p_1)k\cdot(p_1+p_2)+16V_1V_6M_\Lambda(q\cdot k)(k\cdot p_1)\\
&-8V_2V_3(p_1+p_2)\cdot q(p_1\cdot p_2+M_pM_\Lambda)-8V_2V_4[(M_p-M_\Lambda)(p_2\cdot p_1)+M_pM_\Lambda(M_p-M_\Lambda)]\\
&-8V_2V_5(p_1+p_2)^2(M_pk\cdot p_2-M_\Lambda k\cdot p_1)-8V_2V_6(p_1+p_2)\cdot q(M_pk\cdot p_2-M_\Lambda k\cdot p_1)-8V_3V_4(M_pq\cdot p_2-M_\Lambda q\cdot p_1)\\
&-8V_3V_5(p_2+p_1)\cdot q(M_pk\cdot p_2-M_\Lambda k\cdot p_1)+8V_3V_6m_K^2(M_pk\cdot p_2-M_\Lambda k\cdot p_1)\\
&+8V_4V_5[(M_pM_\Lambda-M_p^2)k\cdot p_2+(M_pM_\Lambda-M_\Lambda^2)k\cdot p_1]+8V_4V_6[(q\cdot p_2)(k\cdot p_1)-(q\cdot k)(p_1\cdot p_2)\\
&+(q\cdot p_1)(k\cdot p_2)+M_pM_\Lambda q\cdot k]+16V_5V_6(p_1+p_2)\cdot q(k\cdot p_2)(k\cdot p_1)\}, \quad (21)\end{aligned}$$

where the external kaon has been extrapolated back onto the mass shell.

The explicit t dependence of the axial-vector form factor contained in Eq. (19) is taken to be

$$g_A^\Lambda(t^2)=\frac{g_A^\Lambda(O)}{1+t^2/M_X^2}\equiv\frac{g_A(\Lambda)}{1+(q-k)^2/M_X^2}. \quad (22)$$

Because of the lack of reliable data, we use the Goldberger-Trieman relation to relate the axial-vector and the induced pseudoscalar form factors in the following manner:

$$h_A^\Lambda(t^2)\approx-g_A^\Lambda(O)(M_\Lambda+M_p)^2/(t^2+M_K^2). \quad (23)$$

Although the application of the Goldberger-Treiman relation is questionable, it is not expected to affect the result significantly, at least in the domain of low momentum transfer where the contribution of the induced pseudoscalar term is known to be sufficiently small.

5. NUMERICAL RESULTS AND DISCUSSIONS

For $f_K=c_K/m_K^2$, where c_K is defined by Eq. (6), we use the value given by Ref. 6:

$$f_K=1.14\pm0.03,$$

as estimated from the information available from K_{l3} and $K\to\mu\nu$ decays.

The Λ β-decay axial-vector form factor is taken to be

$$g_A(\Lambda)=0.68\pm0.07,$$

an estimate given by Willis *et al.*,[14] which is known to be relatively free of model uncertainties. From the above value of $g_A(\Lambda)$ the form factor $g_A(\Sigma)$ is calculated using the estimate due to Brene *et al.*[15] We get

$$g_A(\Sigma)=0.23\pm0.08.$$

In the absence of reliable experimental data, $g_A(Y_1^*)$ is computed in the limit of exact $SU(3)$.[16] Thus

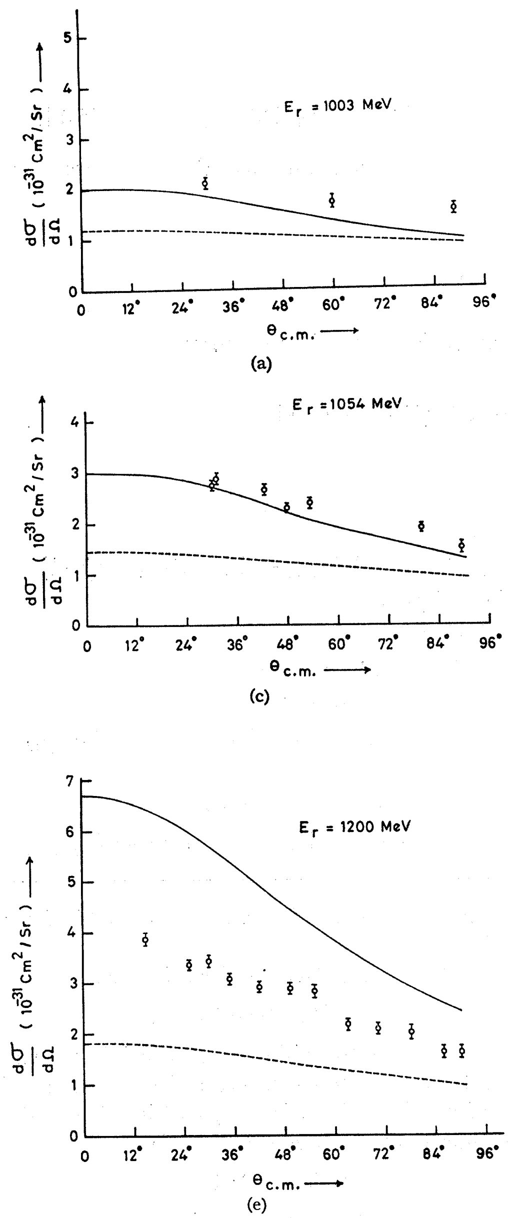
$$g_A(Y_1^*)=0.45\pm0.04.$$

We now focus our attention on the electromagnetic form factors.

The p and Λ magnetic moments are quite well known[17,18]:

$$\mu_p=1.79,\quad \mu_\Lambda=-0.69$$

[14] W. Willis *et al.*, Phys. Rev. Letters **13**, 291 (1964).
[15] N. Brene, L. Veje, M. Roos, and C. Cronstrom, Phys. Rev. **149**, 1288 (1966).
[16] The numbers quoted here for $g_A(\Lambda)$ and $g_A(\Sigma)$ are actually taken from Ref. 7.
[17] A. H. Rosenfeld *et al.*, Rev. Mod. Phys. **37**, 633 (1965).
[18] The number quoted here for μ_Λ is due to H. R. Rubinstein, F. Scheck, and R. H. Socolow, Phys. Rev. **154**, 1608 (1967). The Λ magnetic moment is in fact an average of several experiments given in Ref. 17.



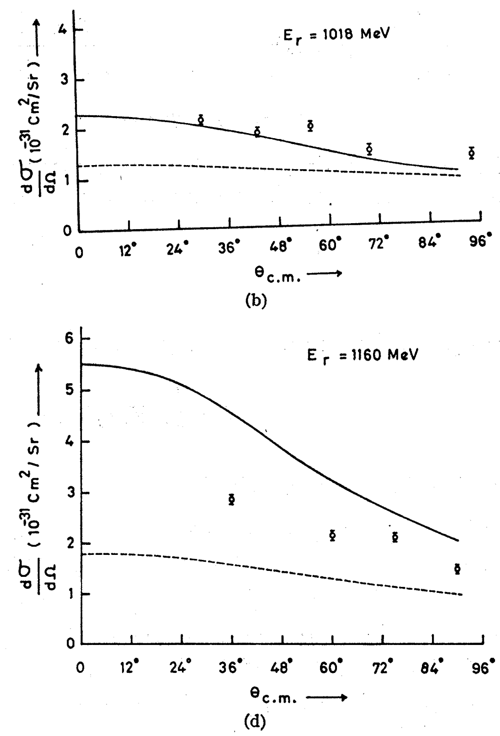


FIG. 1. Angular distribution at $E_\gamma = 1003$, 1018, 1054, 1160, and 1200 MeV. The solid curves show our results and the dashed curves are the results of the soft-kaon calculation. The experimental points are reproduced from Ref. 2.

(in units of the nuclear magneton). But for the transition moment we again appeal to $SU(3)$,[19] whence

$$\mu_{\Lambda\Sigma} = -\tfrac{1}{2}\sqrt{3}\mu_n .$$

For the $Y_1{}^*\Lambda\gamma$ vertex we use the relation[20]

$$\langle \Lambda | j_{\rm em} | Y_1{}^* \rangle = -\tfrac{1}{2}\sqrt{3} \langle n | j_{\rm em} | N_{3/2}{}^{*0} \rangle .$$

This follows from the U-spin invariance of the electromagnetic interactions in the absence of medium strong interactions.

We also use

$$\langle n | j_{\rm em} | N_{3/2}{}^{*0} \rangle = \langle p | j_{\rm em} | N_{3/2}{}^{*+} \rangle ,$$

which is derivable from the first-order breaking of the isospin symmetry and the isovector character of the photon.

From these we obtain

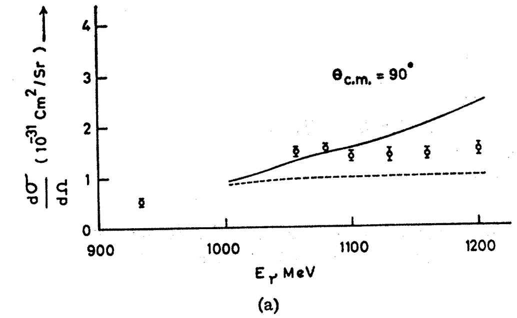
$$C_3{}^{\Lambda Y_1{}^*\gamma} = 0.31 ,$$

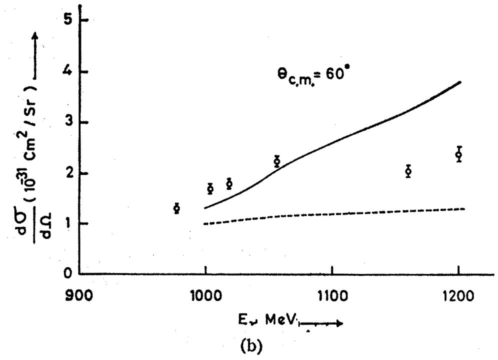
where we have used the well-known Gourdin-Salin estimate[9] based on the isobaric model for pion photoproduction.

The numerical computations have been performed in the IBM 1620 computer and the parameter M_X is

[19] S. Coleman and S. L. Glashow, Phys. Rev. Letters **6**, 423 (1961).

[20] M. Gourdin, *Unitary Symmetries and Their Application to High Energy Physics* (North-Holland Publishing Co., Amsterdam, 1967), p. 91.





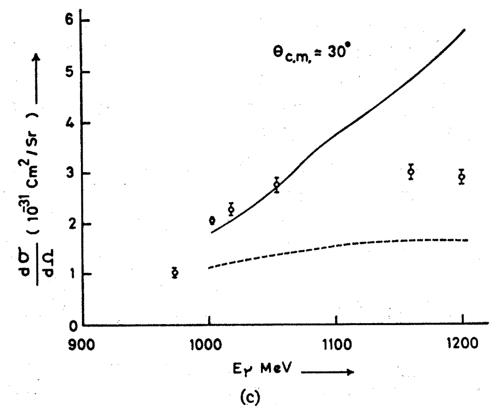


FIG. 2. Excitation spectrum at angles $\theta_{\text{c.m.}}=90°$, 60°, and 30°. The solid curves show our results, and the dashed curves are the results of the soft-kaon calculation.

varied from 800 to 1500 MeV. The best fit[21] was obtained at $M_X=850$ MeV.

In Fig. 1 we show the angular distributions, where the experimental points are taken from Ref. 3; the dashed curves are the result of the soft-kaon calculation, where the only contribution comes from the axial-vector form factor. Figure 2 shows the dependence of the c.m. differential cross section on the incident photon energy. Not many reliable data are available on the total kaon-production cross section.

The low-energy angular-distribution data fit reasonably well with experiment, but at high energy there is a significant discrepancy. We note that the soft-kaon limit ($q=0$) gives results whose variation with angle is minimal and deviates from the experimental data at all energies. Such behavior may be expected on qualitative grounds.

The observed discrepancy at high energy may be attributed to the following factors. The weak amplitude has been saturated by a few low-lying states, and, in particular, the contributions of N^* resonances in the s channel and Y_0^* in the u channel have been excluded for lack of experimental data. Further, the calculation has been performed within the framework of the off-shell $q^2\to 0$ limit, and the higher-order contributions in the kaon four-momentum, which may be important at high energies, have been neglected. The off-shell corrections also need further investigation before the use of PCAC and current algebra for the kaon may be justified. This investigation is in progress in connection with $\bar{K}N$ scattering.

ACKNOWLEDGMENTS

We are grateful to Professor S. N. Biswas for suggesting this problem and for reading the final manuscript. We also wish to thank Dr. K. Dutta for many helpful discussions.

[21] S. Adler, in Proceedings of the Argonne International Conference on Weak Interactions, 1965 [Argonne National Laboratory Report No. ANL-7130 (unpublished)], pp. 257–270. Adler has mentioned the model dependence of the parameter M_X; e.g., he obtained the best fit with $M_X=600$ MeV, whereas Ph. Salin (to be published) got the best fit with $M_X=1400$ MeV.