Final-State Interactions in $\eta \to 3\pi$ and $K_2{}^0 \to 3\pi$ Decays

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We study $\eta \to 3\pi$ and $K_2{}^0 \to 3\pi$ decays assuming that the nonconstancy of the matrix elements originates from the S- and P-wave pion-pion final-state interactions. The $I=J=0$ and $I=J=1$ $\pi\pi$ interactions are assumed to be dominated by the S meson (mass 700 MeV and width 400 MeV) and the ρ meson, respectively. Our analysis shows that the ρ meson swamps the S meson in describing the energy spectra. The results for the energy spectra and the branching ratios are in good agreement with recent experiments.

I. INTRODUCTION

A NUMBER of theoretical studies in $\eta \to 3\pi$ and $K_2{}^0 \to 3\pi$ have been made, but there is still an open question regarding the decay structure and it requires further investigations. The recent current-algebra approach[1,2] precludes final-state interactions[3] and leads to good results for K decay; the results for η decay, however, are not encouraging. The earlier approaches,[4–8] on the other hand, incorporated the final-state interactions to obtain the observed decay processes. Khuri and Treiman[4] assumed the dominance of S-wave $\pi\pi$ final-state effects only and used dispersion-relation methods. Later Bég and DeCelles[5] included the P-wave interactions along with the Khuri-Treiman[4] results for the S-wave part.[9] Here we reinvestigate the same problem making use of the recent experimental information[10–12] on $\pi\pi$ scattering for the description of the final-state interactions. We assume that the $I=J=0$ state of the $\pi\pi$ system is dominated by the S meson[10,12] of mass 700 MeV and width 400 MeV and the $I=J=1$ $\pi\pi$ interactions by the ρ meson. This simple model leads to reasonably good results for the branching ratios and the energy spectra for both η and K decays. Recently this model has been successfully applied by Dutta-Roy and Lapidus[13] in the context of the K_1-K_2 mass difference.

Following Ref. 5, we introduce a phenomenological parameter b which measures the amount of P wave in the final-state interactions, and we then apply the pion-pole model[14] to estimate it. Recently the influence of P waves has been investigated by Schult and Barbour[15] using the Faddeev equations without invoking the pion-pole model. Our calculations show that inclusion of only S-wave $I=0$ $\pi\pi$ interactions gives good results for the branching ratios, but fails to reproduce the experimental energy spectra. The interesting result obtained is that although the P wave cannot change the branching ratios appreciably, it swamps the S wave in describing the energy spectra. The observation that the ρ meson dominates over the S meson in the slope is consistent with the results of earlier calculations.[16]

In our analysis we have considered $I=1$ final-state interactions, neglecting the very unusual decay interaction $|\Delta I|=3$ in η decay. For the case of neutral K decay, the assumptions of the $|\Delta I|=\frac{1}{2}$ rule and CP conservation lead to $I=1$ three-pion final states. The unusual decay interaction $|\Delta I|=3$ was invoked by Adler[17] to explain the low value of the branching ratio for η decay. Now, the present experiments[18] lead to a larger value of this ratio and the assumption of $|\Delta I|=3$ interactions in η decay is less compelling; C invariance excludes the $I=2$ three-pion final state in η decay.

II. MATRIX ELEMENTS AND RESULTS

The matrix element M for $\eta \to \pi^+\pi^-\pi^0$ contains both S- and P-wave terms. Neglecting $I=2$ $\pi\pi$ scattering following Fujii,[8] we have for a simple D function in each channel, with no rescattering distortions,[19]

$$M(s,t,u)=N_0D_0^{-1}(s)+l[(s-t)D_1^{-1}(u)+(s-u)D_1^{-1}(t)], \quad (1)$$

where $D_0(s)$ is the S-wave $\pi\pi$ scattering denominator function and $D_1(u)$ and $D_1(t)$ are the corresponding P-

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[19] I. M. Barbour and R. L. Schult, Phys. Rev. **155**, 1712 (1967).

wave terms. In Eq. (1) we have taken the numerator function N in the usual N/D form of the amplitude to be a constant. The parameter b in (1) measures the amount of P wave and s, t, u are defined as usual,

$$\begin{aligned} s&=(q_++q_-)^2=(m-1)^2-2mT_0\,,\\ t&=(q_-+q_0)^2=(m-1)^2-2mT_+\,,\\ u&=(q_0+q_+)^2=(m-1)^2-2mT_-\,, \end{aligned} \tag{2}$$

where q_+, q_-, and q_0 are the four-momenta of π^+, π^-, and π^0, respectively, the T's are the kinetic energies of the corresponding particles, and m is the mass of the decaying particle in pion mass units.

Expanding $D_1^{-1}(u)$ and $D_1^{-1}(t)$ and retaining terms up to first order in $(s-s_c)$ we get, from (1),

$$M(s)=N_0D_0^{-1}(s)+3b(s-s_c)\,, \tag{3}$$

where s_c is the central point of the Dalitz plot, $s_c=\frac{1}{3}m^2+1$. The fact that the matrix element for $\eta\to\pi^+\pi^-\pi^0$ is a function of T_0 only [Eq. (3)] is also justified from experiments.

For evaluating $N_0D_0^{-1}(s)$, we assume that the S-wave $\pi\pi$ interaction in the $I=0$ state is represented as a dipion particle S.[10–13] The amplitude for S-wave $I=0$ $\pi\pi$ scattering is given by

$$N_0D_0^{-1}(s)=\left(\frac{s}{s-4}\right)^{1/2}\Big/[\cot\delta_0(s)-i]\,, \tag{4}$$

where $\delta_0(s)$ is the S-wave $I=0$ $\pi\pi$ phase shift.

Taking the Breit-Wigner form, we write

$$N_0D_0^{-1}(s)=\frac{\gamma}{\{s_r-s-i\gamma[(s-4)/s]^{1/2}\}}\,, \tag{5}$$

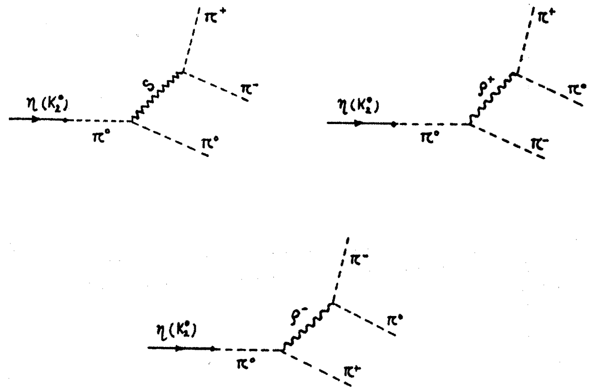

FIG. 1. Feynman diagrams for one-pion-pole model for $\eta(K_2{}^0)\to\pi^+\pi^-\pi^0$, (a) with the S meson in the intermediate state; (b) and (c) with the ρ mesons in the intermediate state.

where

$$\gamma=\frac{\Gamma_f s_r}{(s_r-4)^{1/2}}\,, \tag{6}$$

$s_r=m_S{}^2$ (the square of the mass of the S-wave $I=0$ two-pion component at resonance), and Γ_f is the full width of the S resonance.

The slope parameter ϵ of the energy spectrum of the odd pion in $\pi^+\pi^-\pi^0$ decay is defined by

$$|M(T_0)|^2=|\bar{M}|^2[1+\epsilon(T_0-T_c)/m]\,, \tag{7}$$

where $|\bar{M}|$ is the average value of $|M(T_0)|$, evaluated at the midpoint of the Dalitz plot, $T_c=\frac{1}{3}m-1$.

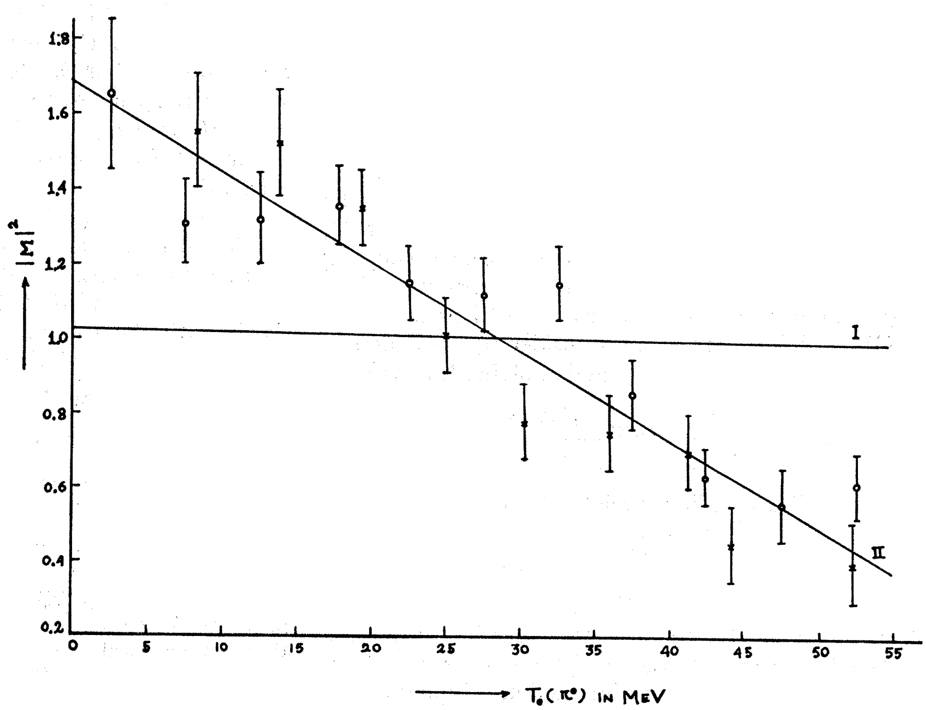

FIG. 2. The predicted K-decay spectrum compared to the experimental values of Ref. 20 (circles) and Ref. 21 (crosses), considering (I) only the S-wave $l=0$, (II) the S wave $l=0$ and the P-wave (ρ width$=125$ MeV which corresponds to $b_K=0.054\ \mu^{-2}$) $\pi\pi$ final-state interactions.

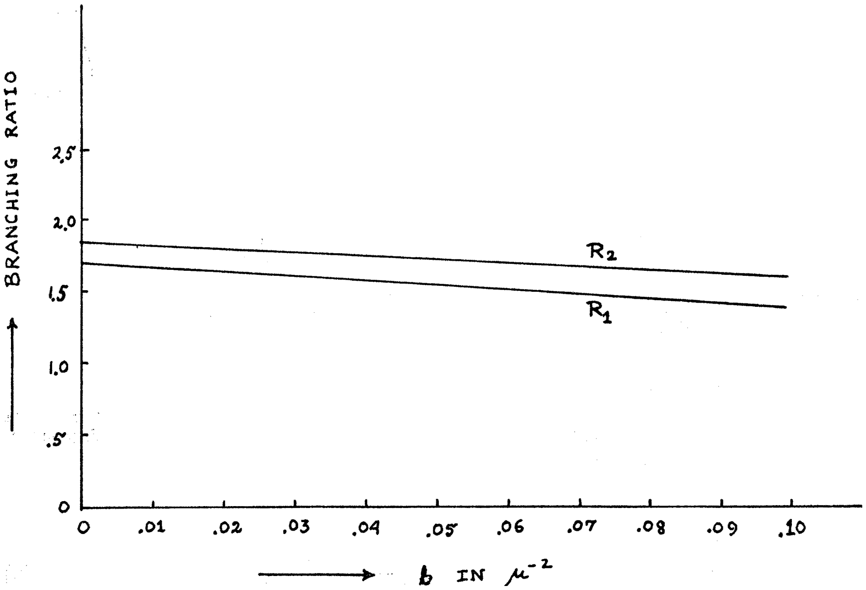

FIG. 3. The branching ratio $R_1=\Gamma(\eta\to 3\pi^0)/\Gamma(\eta\to\pi^+\pi^-\pi^0)$ and $R_2=\Gamma(K_2^0\to 3\pi^0)/\Gamma(K_2^0\to\pi^+\pi^-\pi^0)$ as a function of the P-wave parameter b.

Thus from (3), making use of (5) and (7), we have

$$\epsilon=-\frac{4m^2s_c\gamma^2}{s_c(s_r-s_c)^2+\gamma^2(s_c-4)}\left(\frac{s_r-s_c}{\gamma^2}-\frac{2}{s_c^2}\right)-12bm^2\frac{(s_r-s_c)}{\gamma}. \quad (8)$$

For the case of K_2^0 decay, we replace the η mass by the K mass, assuming that the final-state interactions determine the decay structure. Therefore, (8) gives

$$\epsilon_\eta=-1.4-234.5b,$$
$$\epsilon_K=-0.5-205.7b, \quad (9)$$

where we have used $m_S=700$ MeV and $\Gamma_f=400$ MeV.[10,12]

Next we consider the pion-pole model[14] to calculate the S-wave $I=0$ and P-wave $\pi\pi$ interactions and estimate b. In the pion-pole-model approximation one assumes that the $K\to\pi$ and $\eta\to\pi$ transitions are the dominant weak and electromagnetic processes (Fig. 1), and in this way we eliminate the unknown coupling constant $g_{\eta(K)S\pi}$. There will be no ρ^0 contribution since π^0-ρ^0 cannot couple to $I=1$ or 3 [the Clebsch-Gordan (CG) coefficients are zero]. Considering the diagrams of Figs. 1(a), 1(b), and 1(c), and applying Eq. (14), we get

$$\epsilon=-\frac{4m^2}{m_S^2-s_c}-18\sqrt{2}\frac{f_{\rho\pi\pi}^2}{g_{S\pi\pi}^2}\frac{m^2(m_S^2-s_c)}{m_\rho^2-s_c}, \quad (10)$$

where the $S\pi\pi$ coupling is related to the S width by

$$\frac{g_{S\pi\pi}^2}{4\pi}=\frac{4\Gamma_f m_S^2}{(m_S^2-4)^{1/2}}=4\gamma. \quad (11)$$

The first term of (10) gives the contribution for the S wave ($\epsilon_\eta^0=-3.3$ and $\epsilon_K^0=-2.6$) and the second term is due to the P-wave $\pi\pi$ interactions. Equating the P-wave terms from (8) and (10), we can find b:

$$b=\frac{3}{\sqrt{2}}\frac{f_{\rho\pi\pi}^2}{g_{S\pi\pi}^2}\frac{\gamma}{m_\rho^2-s_c}. \quad (12)$$

Substituting (11) in (12) we find that b is independent of the S-meson width and mass and is directly proportional to the ρ width. Hence (8) together with (6) shows that the P-wave contribution to the slope increases with the ρ width and decreases with the S-meson width. Comparing (9) with the present experimental values[20–22] $\epsilon_\eta=-11.1\pm1.0$ and $\epsilon_K=-11.5\pm1.0$, we see that only the S-wave $I=0$ term cannot produce the odd-pion spectrum (Fig. 2 I). The shape of the spectrum is reproduced (Fig. 2 II) when the P wave is also incorporated into it. For $b_\eta=0.056\ \mu^{-2}$ and $b_K=0.054\ \mu^{-2}$, which correspond to the ρ width 125 MeV,[23] we find from (16) that $\epsilon_\eta=-14.5$ and $\epsilon_K=-11.6$.

For $b=0$ (no P wave) a simple and straightforward calculation[5–7] gives $R_1=\Gamma(\eta\to3\pi^0)/\Gamma(\eta\to\pi^+\pi^-\pi^0)=1.73$ and $R_2=\Gamma(K_2^0\to3\pi^0)/\Gamma(K_2^0\to\pi^+\pi^-\pi^0)=1.85$, which can be compared with the recent experimental values[24] $R_1=1.5\pm0.3$ and $R_2=1.94\pm0.3$. With nonvanishing b we find that the branching ratio decreases slowly with the P-wave parameter b (Fig. 3). For ρ width$=125$ MeV, which corresponds to $b_\eta=0.056\ \mu^{-2}$

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and $b_K=0.054\ \mu^{-2}$, the quanties R_1 and R_2 are 1.52 and 1.57, respectively.[25] The reasonable agreement of the branching ratios with the experiments in the linear approximation of the matrix element was shown by Bég[26] and Wali.[27]

III. CONCLUSIONS

Treating η and K decays in a similar fashion (just interchanging η and K masses), we obtain good results for their branching ratios and their energy spectra. This shows that the final-state interactions dominate the decay structure. The interesting result obtained is that the ρ effects dominate the S in the slope. The slight deviation of the predicted spectrum from the experimental values may be due to retaining only linear terms in the matrix element.

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[25] Finally, we study the effects of low-energy S-wave $I=2$ $\pi\pi$ interactions using experimental values of the phase shift ($\delta_2{}^0\sim-15°$) given in Ref. 11. This raises the branching ratio by about 5% and the slope by about 4%.

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Pion Production in the Proton-Deuteron Interaction

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The differential cross section for the reaction $p+d\to He^3+\pi^0$ has been calculated taking Born diagrams alone. The form factor for the pd He^3 vertex is obtained. The result is compared with the available experimental result and found to be in fair agreement.

I. INTRODUCTION

RECENTLY, Melissinos and Dahanayake[1] have reported a measurement of the differential cross section for the reaction

$$p+d\to He^3+\pi^0 \tag{1}$$

at proton laboratory kinetic energy $T_p=1.515$ BeV and c.m. angle $\theta_{c.m.}=0°$. They obtain

$$d\sigma/d\Omega=(4.1_{-2}{}^{+4})\times10^{-32}\ cm^2.$$

Earlier, Harting *et al.*[2] observed the same reaction at $T_p=600$ MeV and $\theta_{c.m.}=52°$. Their result is

$$d\sigma/d\Omega=(6.1\pm2)\times10^{-30}\ cm^2.$$

Note that one result is a hundred times larger than the other one. This is essentially attributed[1] to the rapidly varying angular distribution that has been observed in other similar reactions.[3] To check this we have computed the differential cross section taking the Born diagrams (see Fig. 1). Our results also indicate rapid angular variation. Some time ago, Mathews and Deo,[4] Heinz *et al.*,[5] and Deo and Patnaik[6] computed the differential cross section for the reaction $p+p\to d+\pi^+$ with nucleon exchange and obtained many desirable results. We believe that a similar nucleon exchange also plays a dominant role in the reaction (1). However, there is another second-order diagram (Fig. 2) involving the He^3 pole that is also important, particularly at lower energy. Here we report the results of our calculation with these two Feynman diagrams. We are aware of the fact that the nucleon is far away from the mass shell and that its contribution cannot be calculated

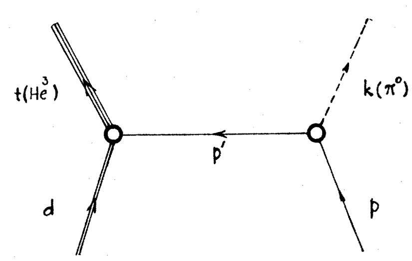

FIG. 1. Feynman diagram for single-proton exchange in $p+d\to He^3+\pi^0$.

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