for $\rho^0 p$ or for φp. (3) Although there are very large uncertainties in the forward $\varphi^0 p$ experimental cross sections, the calculated values seem to be too big by a factor of about 2.5. The predicted cross sections fall off very slowly with energy. There seems to be no way out of the problem of the φ cross section being too large with the vector-dominance model as formulated here. A modification of the mass dependence of the photon–vector-meson coupling, or possibly the introduction of an additional diagram, may be called for. (4) The vector-meson production processes are dominated by the $SU(6)_W$ **1** amplitude in varying degrees. The φp cross section is the flattest and has relatively little dependence on the other $SU(6)_W$ amplitudes. The ωp cross section falls most steeply with increasing energy, displaying the importance of the $SU(6)_W$ $\mathbf{35}_D$, $\mathbf{35}_F$, and **405** terms. The $\rho^0 p$ cross section falls at an intermediate rate.

There are scant data for the clsss PB^*, the best being that for $\gamma p \rightarrow \pi^- N^{*++}$ (see Table III). Calculated values of $d\sigma/dt$ for $\theta^*=0$ using vector dominance and the $(m_\rho/m_V)^4$ factor are listed in Table III. They differ by about 2% from values calculated with no factor. They agree with experiment at low energies, but are too large by a factor of 2 in the range 2.5–5.5 GeV.

In summary, we have demonstrated that $SU(6)_W$ may be used to make quantitative predictions for photoproduction amplitudes at low and high energies. The predicted ratios of $K^+\Sigma^0/K^+\Lambda$ forward cross sections are in good agreement with experiment. This accord demands the inclusion of a **405** amplitude, an amplitude not allowed by a simple quark model. Predicted forward vector-meson production cross sections are in good agreement for ρ and ω production, but are too big, by a factor of 2.5, for φ production. The degree of breaking of the octet purity of the photon may be determined, quantitatively, by a careful measurement of the forward φ and ω cross sections.

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s-Wave K^+p Dynamics*

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An investigation of the dynamics of low-energy s-wave K^+p scattering has been made by the semiphenomenological application of dispersion relations. Contributions to the s-wave K^+p dispersion relation from the nearby singularities due to the following processes were explicitly calculated: (a) exchange of Λ and Σ in the u channel; (b) exchange of ρ, ω, and ϕ in the t channel, treated in the narrow-width approximation; and (c) exchange of an s-wave $T=0$ π-π pair, treated as a continuum state. Other exchanges, and short-range forces, were treated phenomenologically. The amplitude in the physical region was taken from a recent phase-shift analysis of K^+p scattering data, and the necessary coupling constants for the exchange processes were obtained from experiment, supplemented by the use of $SU(3)$ symmetry when experimental data were not available. The exchange of the s wave $T=0$ π-π pair is shown to be potentially a very important term, and the present difficulties in calculating its exact size are discussed in detail.

I. INTRODUCTION

Early work on the low-energy KN system proceeded on the assumption that the dominant forces arise from the exchange of a small number of stable, or quasistable, states in the crossed channels. Examples of such possible exchanged particles are Λ, Σ, ρ, ω, Y_0^*, and Y_1^*, and various combinations have been considered by several authors.[1–3] Little definitive information can be drawn from these calculations, however, because of a variety of reasons. For example, sometimes only a limited set of particles was considered, and sometimes the number of free parameters involved exceeded the constraining capacity of the data as they existed at the time. A more serious objection is, perhaps,

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the neglect of other terms. In particular, it was pointed out by several authors[1,4] that the exchange of a nonresonant pair of pions produces a singularity in the KN partial-wave amplitude which approaches very close to the physical region and would *a priori* be expected to be of some importance. Because of the length and nearness of the cut in the complex energy plane resulting from this process, it is clear that the exchange of such a state must be considered rather carefully. Finally, there is the persistent neglect of short-range forces, which cannot be adequately represented by single-particle exchanges and, moreover, would naturally be expected to be important for the low partial waves.

Attempts to remedy some of the above defects have been made in two recent calculations. Warnock and Frye[5] have considered the exchange of a large number of stable and quasistable states as well as background terms represented by low-order polynomials in the manner of Cini and Fubini.[6] However, the partial-wave amplitudes are obtained by projection from fixed-variable dispersion relations, a procedure which is well known to give rise to divergent expressions in the physical region. Of the large number of parameters needed in this calculation very few can be determined from the presently available data. On the other hand, Martin and Spearman[7] have concentrated on carefully calculating the effect of the exchange of a nonresonant π-π pair, i.e., the force of longest range. Unfortunately, the method of producing a physical s-wave amplitude was to use the N/D technique in the Balázs approximation,[8] and apart from other difficulties in the N/D method (particularly in producing spurious zeroes of D for s-wave scattering[3]), the Balázs approximation is now known to give misleading results often.[9] Furthermore, as will be discussed later, the form of the s-wave π-π interaction used by these authors may no longer be adequate.

It is evident from the above discussion that present data are neither plentiful enough nor of sufficient accuracy to allow a determination of the parameters governing the exchange processes. In this situation, a more fruitful question to examine is the nature of the dynamics, given reasonable estimates for the necessary input parameters. This is the question that will be examined in this paper. The method used is similar to that which has met with considerable success in understanding the dynamics of the low-energy πN interaction.[10] Basically, the idea is to calculate explicitly, as far as possible, contributions due to the *nearby* singularities produced by the exchange of states in crossed channels. These can all be calculated by convergent techniques and include the exchange of an s-wave $T=0$ π-π pair, treated explicitly as a continuum state. The rest of the interaction, representing more distant singularities (i.e., forces of shorter range), are represented phenomenologically. This method requires that the physical scattering amplitude be reasonably well known and so we will apply it initially to s-wave $T=1$ (K^+p) scattering, since this amplitude is by far the best known experimentally at present.

II. KINEMATICS OF KN SCATTERING

The kinematics of spin-0–spin-$\frac{1}{2}$ scattering are standard.[11] We give in this section just those formulas which we shall need in later sections.

The three channels we shall consider are

$$K+N\to K+N,\quad (s)$$
$$\bar{K}+N\to \bar{K}+N,\quad (u)$$
$$\bar{K}+K\to \bar{N}+N,\quad (t)$$

with p_i (q_i) the initial-state four-momentum of the nucleon (kaon), and p_f (q_f) the corresponding final-state four-momentum. These three processes are described by scattering amplitudes which are functions of the usual invariants,[12,13]

$$s=(q_i+p_i)^2=(q_f+p_f)^2\,,$$
$$t=(q_i+q_f)^2=(p_i+p_f)^2\,,$$
$$u=(q_i+p_f)^2=(q_f+p_i)^2\,,$$

where momentum conservation implies

$$s+t+u=2(M^2+m^2)\,,\qquad (2.1)$$

and M and m are the masses of the nucleon and kaon, respectively.

The S matrix for these three processes may be written

$$S_{fi}=\delta_{fi}-\frac{i(2\pi)^4M\delta^{(4)}(p_i+q_i+p_f+q_f)}{(4p_i^0q_i^0p_f^0q_f^0)^{1/2}}\tau_{fi}\,,\qquad (2.2)$$

where

$$\tau_{fi}=\bar{u}(p_f)T_{fi}u(p_i)\,.$$

The problem of spin is treated by writing the T-matrix element T_{fi} in terms of two scalar invariant amplitudes A_{fi} and B_{fi}, which are assumed to satisfy the Mandelstam representation.[13] Thus,

$$T_{fi}=-A_{fi}+\tfrac{1}{2}i\gamma\cdot(q_i+q_f)B_{fi}\,.$$

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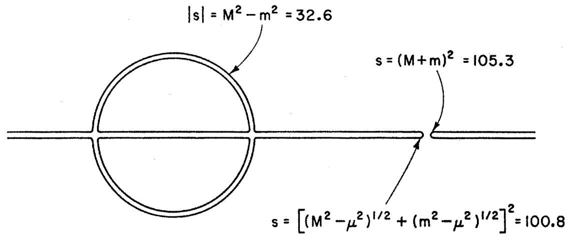

FIG. 1. Singularities of the K^+p partial wave amplitudes $f_{l\pm}(s)$ as a function of s, the square of the total c.m. energy.

The amplitudes $A(s,t)$ and $B(s,t)$ may be further decomposed into charge states $A^{\pm}(s,t)$ and $B^{\pm}(s,t)$ by

$$\begin{aligned} A(s,t) &= A^+(s,t) + A^-(s,t)(\boldsymbol{\tau}_N\cdot\boldsymbol{\tau}_K)\,, \\ B(s,t) &= B^+(s,t) + B^-(s,t)(\boldsymbol{\tau}_N\cdot\boldsymbol{\tau}_K)\,. \end{aligned} \tag{2.3}$$

A. s Channel

In the c.m. of the s channel the differential cross section may be written

$$\left(\frac{d\sigma}{d\Omega}\right)_{fi} = \sum \left| \left\langle f \left| f_1 + \frac{(\boldsymbol{\sigma}\cdot\mathbf{q}_i)(\boldsymbol{\sigma}\cdot\mathbf{q}_f)}{q_i q_f} f_2 \right| i \right\rangle \right|^2 ,$$

where the matrix is taken between two-component spinors, and the expression is summed over final spin states and averaged over initial spin states. The helicity amplitudes f_1 and f_2 are related to the invariant amplitudes A and B by[14]

$$\frac{A_s(s,t)}{4\pi} = \left(\frac{W_s+M}{E+M}\right) f_1(s,t) - \left(\frac{W_s-M}{E-M}\right) f_2(s,t)\,, \tag{2.4}$$

$$\frac{B_s(s,t)}{4\pi} = \left(\frac{1}{E+M}\right) f_1(s,t) + \left(\frac{1}{E-M}\right) f_2(s,t)\,, \tag{2.5}$$

where E is the total c.m. energy of the nucleon and $W_s = \sqrt{s}$. They may be expressed in terms of partial-wave amplitudes by the expansions

$$f_1(s,t) = \sum_{l=0}^{\infty} f_{l+}(s) P_{l+1}'(x) - \sum_{l=0}^{\infty} f_{l-} P_{l-1}'(x)\,, \tag{2.6}$$

$$f_2(s,t) = \sum_{l=1}^{\infty} [f_{l-}(s) - f_{l+}(s)] P_l'(x)\,, \tag{2.7}$$

where $x \equiv \cos\theta_s$ is the cosine of the s-channel c.m. scattering angle, related to t by

$$t = -2k^2(s)[1 - \cos\theta_s]\,, \tag{2.8}$$

and

$$f_{l\pm}(s) = \frac{\exp[2i\delta_{l\pm}(s) - 1]}{2ik(s)} \tag{2.9}$$

is the partial-wave amplitude for scattering in a state of total angular momentum $J = l \pm \frac{1}{2}$. The magnitude of the c.m. three-momentum is denoted by $k(s)$, and $\delta_{l\pm}(s)$ is the complex K^+p phase shift. The partial wave amplitudes of Eq. (2.9) may be expressed as

$$f_{l\pm}(s) = \frac{E+M}{8\pi W_s}[A_l(s) + (W_s - M)B_l(s)] + \frac{E-M}{8\pi W_s}[-A_{l\pm 1}(s) + (W_s+M)B_{l\pm 1}(s)]\,, \tag{2.10}$$

where

$$[A_l(s);B_l(s)] = \tfrac{1}{2}\int_{-1}^{1} dx \times P_l(x)[A_s(s,t);B_s(s,t)]\,. \tag{2.11}$$

Finally, if we denote by $A^T(s,t)$ the amplitude for scattering in an isospin state T, then these are given in terms of $A^{\pm}(s,t)$ by

$$A_s^0(s,t) = A^+(s,t) - 3A^-(s,t)\,, \tag{2.12}$$

and

$$A_s^1(s,t) = A^+(s,t) + A^-(s,t)\,, \tag{2.13}$$

and similarly for $B_s^T(s,t)$.

B. t Channel

In the t channel the invariants s and t are given by

$$s = -p^2 - q^2 - 2pq\cos\theta_t \tag{2.14}$$

and

$$t = 4(p^2 + M^2) = 4(q^2 + m^2)\,, \tag{2.15}$$

where p (q) is the magnitude of the c.m. three-momentum in the $N\bar{N}$ ($K\bar{K}$) channel, and $\cos\theta_t$ is the cosine of the t-channel c.m. scattering angle. The differential cross section may be written

$$d\sigma/d\Omega = \sum |F_{\lambda\bar{\lambda}}|^2\,, \tag{2.16}$$

where $\lambda, \bar{\lambda}$ are the helicities of the nucleon and antinucleon, respectively. If we define the amplitudes $T_{\lambda\bar{\lambda}}^J(t)$ for scattering in a state of total angular momentum J by

$$T_{\lambda\bar{\lambda}}^J(t) = -iS_{\lambda\bar{\lambda}}^J(t)\,,$$

where $S_{\lambda\bar{\lambda}}^J(t)$ is a submatrix of the S matrix of Eq. (2.2), then the relations between $F_{\lambda\bar{\lambda}}$ and $T_{\lambda\bar{\lambda}}^J$ are

$$F_{++}(s,t) = F_{--}(s,t) = \frac{1}{q}\sum_J (J+\tfrac{1}{2}) T_{++}^J(t) P_J(y)\,, \tag{2.17}$$

$$F_{+-}(s,t) = -F_{-+}(s,t) = \left(\frac{1}{q}\right)\sum_J \frac{J+\frac{1}{2}}{[J(J+1)]^{1/2}} \times T_{+-}^J(t)\sin\theta_t\, P_J'(y)\,, \tag{2.18}$$

where $y \equiv \cos\theta_t$. The helicity decompositions of the

[14] M. Jacob and G. C. Wick, Ann. Phys. (N. Y.) **7**, 404 (1959).

invariant amplitudes are then

$$A_t(s,t)=-\frac{4\pi W_t}{p}\sum_J \frac{J+\frac{1}{2}}{(pq)^{1/2}}$$

$$\times\left(T_{++}{}^J(t)P_J(y)-\frac{2M}{W_t}T_{+-}{}^J(t)\frac{yP_J'(y)}{[J(J+1)]^{1/2}}\right), \quad (2.19)$$

$$B_t(s,t)=-\frac{8\pi}{q}\sum_J \frac{J+\frac{1}{2}}{[J(J+1)]^{1/2}}\frac{T_{+-}{}^J(t)}{(pq)^{1/2}}P_J'(y), \quad (2.20)$$

where $W_t=\sqrt{t}$.

Finally, in the t channel, isospin amplitudes are given by

$$A_t{}^0(s,t)=2A^+(s,t), \quad (2.21)$$

$$A_t{}^1(s,t)=2A^-(s,t), \quad (2.22)$$

and similarly for $B_t{}^T(s,t)$.

III. SINGULARITIES AND DISPERSION RELATIONS

The singularities of the partial-wave amplitudes $f_{l\pm}(s)$ of Eq. (2.9) as a function of s were first given by MacDowell[15] and are shown in Fig. 1.[16] Exchanging a particle of mass $m_t=\sqrt{t}$ in the t channel produces four branch points in the s plane, 0, $-\infty$, and $s_\pm$, where

$$s_\pm=[(M^2-\tfrac{1}{4}t)^{1/2}\pm(m^2-\tfrac{1}{4}t)^{1/2}]^2. \quad (3.1)$$

For some range of values of t, $s_\pm$ are complex and obey

$$s_+=s_-{}^*.$$

For the exchange of either a ρ or ω meson in the t channel, $4\mu^2<t<4m^2$, and so $s_\pm$ are both real. For ϕ meson exchange, however, $s_\pm$ are complex, and hence this process contributes only to the circle and real-axis cuts to the left of the circle. Exchanging a particle of mass $m_u=\sqrt{u}$ in the u channel also produces four branch points, 0, $-\infty$, s_1, and s_2, where

$$\begin{aligned} s_2&=2(M^2+m^2)-u,\\ s_1&=(M^2-m^2)^2/u, \end{aligned} \quad (3.2)$$

and $s_2>s_1$. For $m_u{}^2<2(M^2+m^2)$ the cut along the real axis nearest the physical region may be taken between s_1 and s_2. In the spirit of the remarks made in Sec. I, we shall explicitly calculate only those contributions coming from the nearby cuts due to the exchange of Λ, Σ, ρ, ω, and ϕ, and an s-wave $T=0$ pion pair which we shall denote by $(\pi\text{-}\pi)_0$. The nearby singularities due to these processes are shown in Fig. 2.

[15] S. W. MacDowell, Phys. Rev. **116**, 774 (1959).

[16] All numerical work in this paper is in units such that $\hbar=c=m_\pi\equiv\mu=1$.

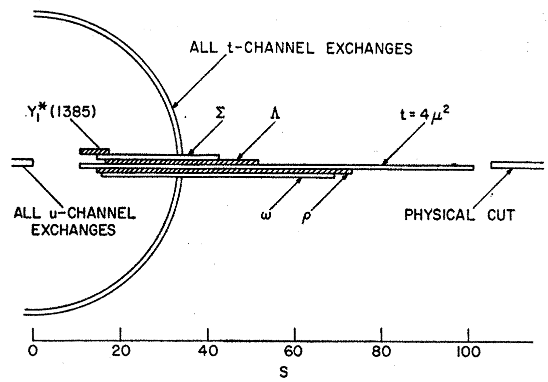

FIG. 2. Nearby singularities of the K^+p partial-wave amplitudes $f_{l\pm}(s)$ as a function of s, the square of the total c.m. energy. The arrows indicate which processes contribute to each singularity.

The dispersion relation that we shall use is, for $s\geq s_0\equiv(M+m)^2$,

$$\mathrm{Re}f_{l\pm}(s)=B_{l\pm}(s)+\frac{P}{\pi}\int_{s_0}^{\infty}ds'\frac{\mathrm{Im}f_{l\pm}(s')}{s'-s}$$

$$+\frac{1}{\pi}\int_N ds'\frac{\mathrm{Im}f_{l\pm}(s')}{s'-s}+\frac{1}{\pi}\int_D ds'\frac{\mathrm{Im}f_{l\pm}(s')}{s'-s}, \quad (3.3)$$

where $B_{l\pm}(s)$ denotes contributions from the Λ and Σ pole terms, and N and D mean that the integrals are to be evaluated over the nearby and distant cuts, respectively.

IV. Λ AND Σ EXCHANGE

We shall evaluate the contributions due to the nearby cuts

$$(M^2-m^2)^2/Y^2\leq s\leq 2M^2-Y^2+2m^2,$$

(where Y denotes the hyperon mass), produced by Λ and Σ exchange in the u channel. The pole terms in the invariant amplitudes $A(s,t)$ and $B(s,t)$ are

$$A(s,t)=-g_\Lambda{}^2\frac{(\Lambda-M)}{\Lambda^2-u}-g_\Sigma{}^2\frac{(\Sigma-M)}{\Sigma^2-u}, \quad (4.1)$$

$$B(s,t)=-g_\Lambda{}^2\frac{1}{\Lambda^2-u}-g_\Sigma{}^2\frac{1}{\Sigma^2-u}, \quad (4.2)$$

where the coupling constants are defined via the Lagrangian density

$$L=g_\Lambda[(\bar{N}\Lambda K)+\mathrm{H.c.}]+g_\Sigma[(\bar{N}\boldsymbol{\tau}\cdot\boldsymbol{\Sigma}K)+\mathrm{H.c.}]. \quad (43).$$

The denominators in Eqs. (4.1) and (4.2) may be written

$$1/(Y^2-u)\equiv 2s/(a_Y+b_Y\cos\theta_s), \quad (4.4)$$

where

$$a_Y=s^2-2s(M^2-Y^2+m^2)-(M^2-m^2)^2,$$
$$b_Y=s^2-2s(M^2+m^2)+(M^2-m^2)^2.$$

Using (4.4) in Eqs. (4.1) and (4.2), and projecting out partial waves, gives

$$[A_l(s);B_l(s)]=\sum_{Y=\Lambda,\Sigma}\int_{-1}^{1}d\cos\theta_s \times\frac{[R_u{}^Y;R_u{}'^Y]P_l(\cos\theta_s)s}{a_Y+b_Y\cos\theta_s}, \quad (4.5)$$

where

$$R_u{}^Y=-g_Y{}^2(Y-u),\quad R_u{}'^Y=-g_Y{}^2. \quad (4.6)$$

The integral in (4.5) is singular for $-1\leq\cos\theta_s\leq 1$ and it is a simple matter to show that we must pass below the pole in the $\cos\theta_s$ plane. Thus, from Eq. (4.5),

$$\mathrm{Im}[A_l(s);B_l(s)]=\pi s\sum_{Y=\Lambda,\Sigma}\left(\frac{1}{b_Y}\right)P_l\times\left(-\frac{a_Y}{b_Y}\right)[R_u{}^Y;R_u{}'^Y]. \quad (4.7)$$

Using Eq. (4.7) and (4.6) in (2.10) gives the discontinuity across the nearby hyperon cuts:

$$\mathrm{Im}f_{l\pm}(s)=-\tfrac{1}{8}W_s\sum_{Y=\Lambda,\Sigma}\left(\frac{g_Y{}^2}{b_Y}\right)\times[(E+M)(W_s+Y-2M)P_l(-a_Y/b_Y)+(E-M)(W_s-Y+2M)P_{l\pm1}(-a_Y/b_Y)]. \quad (4.8)$$

Finally, the contribution to $\mathrm{Re}f_{l\pm}(s)$ in the physical region from a given hyperon cut is given by

$$\mathrm{Re}f_{l\pm}(s)=\frac{1}{\pi}\int_{s_1}^{s_2}ds'\frac{\mathrm{Im}f_{l\pm}(s')}{s'-s}, \quad (4.9)$$

where

$$s_1=(M^2-m^2)^2/Y^2,\quad s_2=2M^2-Y^2+2m^2.$$

V. s-WAVE $T=0$ π-π EXCHANGE

As we have seen in Sec. III, the force of longest range in KN scattering, due to the exchange of a pion pair, gives rise to a singularity in the s plane which almost reaches the physical region (see Fig. 2). It is important, therefore, to treat the exchange of such a state with particular care. The framework for doing so has been given by Martin and Spearman,[7] and we will basically follow their method here. Firstly, we give the necessary notation for π-K scattering that we shall need.

A. Kinematics of π-K Scattering

The kinematics of π-K scattering are standard.[17] The S matrix is of the form

$$S_{fi}=\delta_{fi}-\frac{i(2\pi)^4\delta^{(4)}(p_i+q_i+p_f+q_f)}{(16p_i{}^0q_i{}^0p_f{}^0q_f{}^0)^{1/2}}T_{fi}, \quad (5.1)$$

and the decomposition of the T matrix into s-channel partial waves may be written

$$T_s(s,t)=-8\pi W_s\sum_{l=0}^{\infty}(2l+1)P_l(\cos\theta_s)f_l(s), \quad (5.2)$$

where

$$f_l(s)=\frac{\exp[2i\delta_l(s)]-1}{2ip(s)}. \quad (5.3)$$

To specify isospin, the T matrix is written

$$T(s,t)=\delta_{\beta\alpha}T^+(s,t)+\tfrac{1}{2}[\tau_\beta,\tau_\alpha]T^-(s,t), \quad (5.4)$$

where α and β are the pion isospin indices, and in the s channel the amplitudes for scattering in a pure isospin state are given by

$$T_s{}^{1/2}(s,t)=T^+(s,t)+2T^-(s,t), \quad (5.5)$$

$$T_s{}^{3/2}(s,t)=T^+(s,t)-T^-(s,t). \quad (5.6)$$

In the t channel, $\pi\pi\to K\bar{K}$, we shall denote by q the magnitude of the c.m. three-momentum of the kaon, and by κ the corresponding quantity for the pion. Then

$$t=4(q^2+m^2)=4(\kappa^2+\mu^2), \quad (5.7)$$

where $\mu\equiv m_\pi$. The partial-wave decomposition is now

$$T_t(s,t)=\sum_{l=0}^{\infty}(2l+1)(\kappa q)^l g_l(t)P_l(y), \quad (5.8)$$

with its inverse

$$g_l(t)=\tfrac{1}{2}(\kappa q)^{-l}\int_{-1}^{1}dyT_t(s,t)P_l(y), \quad (5.9)$$

where $y\equiv\cos\theta_t$. The factor $(\kappa q)^l$ is introduced, following Frazer and Fulco,[18] to remove a kinematical threshold zero. Finally, isospin amplitudes in the t channel are given by

$$T_t{}^0=(\sqrt{6})T^+,\quad T_t{}^1=2T^-. \quad (5.10)$$

B. Unitarity for the Process $K\bar{K}\to N\bar{N}$

We wish to calculate the discontinuity in the partial-wave amplitudes $f_{l\pm}(s)$ across the nearby cuts due to $(\pi$-$\pi)_0$ exchange. If we recall the definition of the physical amplitude,

$$A_t(s,t)=\lim_{\epsilon\to0+}A_t(s,t+i\epsilon),$$

[17] See, e.g., M. Gourdin, Y. Noirat, and Ph. Salin, Nuovo Cimento 18, 651 (1960).

[18] W. R. Frazer and J. R. Fulco, Phys. Rev. 117, 1609 (1960).

then using the reality conditions

$$A_t^*(s,t+i\epsilon)=A_t(s,t-i\epsilon)\,,$$
$$B_t^*(s,t+i\epsilon)=B_t(s,t-i\epsilon)\,,$$

in Eqs. (2.19) and (2.20), we have

$$A_t(s,t+i\epsilon)-A_t(s,t-i\epsilon)=-\frac{4\pi W_t}{p}\sum_J\frac{J+\frac{1}{2}}{(pq)^{1/2}}\times\biggl([T_{++}^J(t)-T_{++}^J(t)^\dagger]P_J(y)-\frac{2M}{W_t}[T_{+-}^J(t)-T_{+-}^J(t)^\dagger]\frac{yP_J'(y)}{[J(J+1)]^{1/2}}\biggr)\quad(5.11)$$

and

$$B_t(s,t+i\epsilon)-B_t(s,t-i\epsilon)=\frac{8\pi}{q}\sum_J\frac{J+\frac{1}{2}}{[J(J+1)]^{1/2}}\times[T_{+-}^J(t)-T_{+-}^J(t)^\dagger]\frac{P_J'(y)}{(pq)^{1/2}}.\quad(5.12)$$

The 2π contribution to the unitarity condition for $T_{\lambda\bar{\lambda}}^J(t)$ is given by

$$T_{\lambda\bar{\lambda}}^J(t)-T_{\lambda\bar{\lambda}}^J(t)^\dagger=iG_{\lambda\bar{\lambda}}^J(t)T^J(t)^\dagger\,,\quad(5.13)$$

where $T^J(t)=-iS^J(t)$ is a submatrix of the π-K S matrix of Eq. (5.1), and $G_{\lambda\bar{\lambda}}^J(t)$ are helicity amplitudes for the process $\pi\pi\to N\bar{N}$. In terms of the helicity amplitudes $f_\pm^J(t)$ introduced by Frazer and Fulco,[18] the amplitudes $G_{\lambda\bar{\lambda}}^J(t)$ are given by

$$G_{++}^J(t)=\left(\frac{2}{W_t}\right)\left(\frac{\kappa}{p}\right)^{1/2}(p\kappa)^Jf_+^J(t)\,,\quad(5.14)$$

$$G_{+-}^J(t)=\left(\frac{\kappa}{p}\right)^{1/2}(p\kappa)^Jf_-^J(t)\,.\quad(5.15)$$

The amplitudes $T^J(t)$ are given in terms of the amplitudes $g_J(t)$ of Eq. (5.9) by

$$T^J(t)=-\frac{(q\kappa)^{1/2}}{4\pi W_t}(q\kappa)^Jg_J(t)\,.\quad(5.16)$$

If we now use Eqs. (5.13)–(5.16) in Eqs. (5.11) and (5.12), we can derive the following expressions for the discontinuities in the invariant amplitudes $A_t(s,t)$ and $B_t(s,t)$:

$$\mathrm{Im}A_t(s,t)=\frac{1}{p^2W_t}\sum_J(J+\tfrac{1}{2})(pq^*)^J\kappa^{2J+1}\times\biggl(f_+^J(t)P_J(y)-\frac{M}{[J(J+1)]^{1/2}}\times yP_J'(y)f_-^J(t)\biggr)g_J^*(t)\,,\quad(5.17)$$

$$\mathrm{Im}B_t(s,t)=-\frac{1}{pqW_t}\sum_J\frac{J+\frac{1}{2}}{[J(J+1)]^{1/2}}\times(pq^*)^J\kappa^{2J+1}f_-^J(t)g_J^*(t)P_J'(y).\quad(5.18)$$

To treat isospin we use Eqs. (2.21), (2.22), and (5.10). Thus,

$$\mathrm{Im}A_t^+(s,t)=\frac{3}{p^2W_t}\sum_J(J+\tfrac{1}{2})(pq^*)^J\times(\kappa)^{2J+1}f_+^{J+}(t)g_J^+(t)^*P_J(y)\quad(5.19)$$

and

$$\mathrm{Im}B_t^+(s,t)=0.\quad(5.20)$$

The superscript now refers to the (+) charge combination, and the sum in (5.19) is taken over even J values only because of the restriction implied by G parity. Since we are interested only in the exchange of the $J=0$ state, Eq. (5.19) becomes

$$\mathrm{Im}A_t^+(s,t)=\frac{3\kappa}{2p^2W_t}f_+^{0+}(t)g_0^+(t)^*.\quad(5.21)$$

In the elastic region, $4\mu^2\leq t\leq 16\mu^2$, the amplitudes $f_+^{0+}(t)$ and $g_0^+(t)$ have the same phase, and this is also equal to the phase of the D function[19] for π-π scattering in a state having $J=0$ and $T=0$. Thus we may write

$$f_+^{0+}(t)g_0^+(t)^*=-\frac{\mathrm{Im}f_+^{0+}(t)D(t)g_0^+(t)}{\mathrm{Im}D(t)},\quad(5.22)$$

and using (5.22) in (5.21) gives

$$\mathrm{Im}A_t^+(s,t)=-\frac{3\kappa}{2p^2W_t}\frac{\mathrm{Im}f_+^{0+}(t)}{\mathrm{Im}D(t)}D(t)g_0^+(t).\quad(5.23)$$

Provided that the equality of phases used to derive Eq. (5.23) is approximately valid outside the elastic region, we may use Eq. (2.11) to project out partial waves from (5.23) (noting that since $s+i\epsilon$ crosses to $t-i\epsilon$ an extra minus sign is thereby introduced). Substituting the result into Eq. (2.10) gives for the discontinuity in the $T=1$ direct channel partial-wave amplitudes

$$\mathrm{Im}f_{l\pm}(s)=\frac{-3}{64\pi W_sk^2}\int_{4\mu^2}^{-4k^2(s)}dt\times\left[\frac{\kappa}{p^2W_t}\frac{\mathrm{Im}f_+^{0+}(t)}{\mathrm{Im}D(t)}D(t)g_0^+(t)\right]H_{l\pm}(s,t)\,,\quad(5.24)$$

where

$$H_{l\pm}(s,t)=(E+M)P_l(1+t/2k^2)-(E-M)P_{l\pm1}(1+t/2k^2).\quad(5.25)$$

[19] G. F. Chew and S. Mandelstam, Phys. Rev. **119**, 467 (1960).

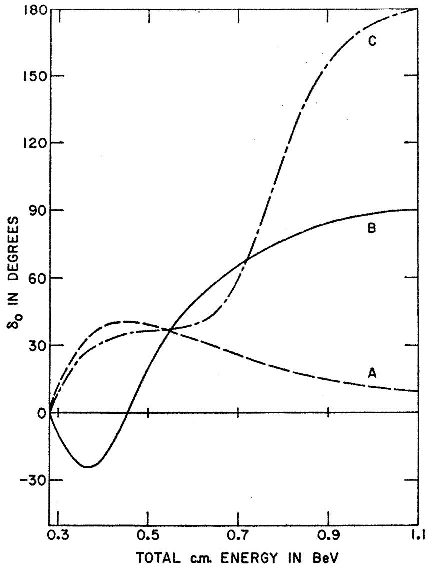

FIG. 3. Three forms for the $J=0$, $T=0$, π-π phase shift below $W_c=1$ BeV.

Finally, the contribution to $\mathrm{Re}f_{l\pm}(s)$ in the physical region from the real-axis cut due to $(\pi$-$\pi)_0$ exchange is given by

$$\mathrm{Re}f_{l\pm}(s)=\frac{1}{\pi}\int_{s_-}^{s_+}ds'\frac{\mathrm{Im}f_{l\pm}(s')}{s'-s}, \qquad (5.26)$$

where $\mathrm{Im}f_{l\pm}(s')$ is given by (5.24) and

$$s_\pm=[(M^2-\mu^2)^{1/2}\pm(m^2-\mu^2)^{1/2}]^2. \qquad (5.27)$$

Equation (5.24) could also be used to derive an expression for the contribution to $\mathrm{Re}f_{l\pm}(s)$ from the circular cut, but this would involve values of $t\geq 4m^2$ for which the approximations we have used are not expected to be valid. Furthermore, as we shall see later, our knowledge of $\mathrm{Im}f_+{}^{0+}(t)$ is restricted to small values of t. We shall consider the possible error introduced by the neglect of the circle contribution when we discuss vector meson exchange in Sec. VI.

C. $\pi\pi\rightarrow K\bar{K}$ Amplitudes

To evaluate Eq. (5.24) we need to know the value of $D(t)g_0{}^+(t)$ for $4\mu^2\leq t\leq -4k^2(s)$ and s in the range $s_-\leq s\leq s_+$, where $s_\pm$ are given by (5.27). We shall firstly write a dispersion relation for $D(t)g_0{}^+(t)$ subtracted once at $t=0$. Thus,

$$D(t)g_0{}^+(t)=D(0)g_0{}^+(0)+\frac{1}{\pi}\int_{-\infty}^{0}dt'\frac{D(t')\,\mathrm{Im}g_0{}^+(t')}{t'(t'-t)}. \qquad (5.28)$$

To calculate $\mathrm{Im}g_0{}^+(t')$ for $t'\leq 0$, which is needed in (5.28), we write a once-subtracted fixed-t dispersion relation for $T^+(s,t)$ and then project out partial waves by using Eq. (5.9). Noting, again, the s,t crossing relation, this procedure leads to the following expression:

$$\mathrm{Im}g_0{}^+(t)=4\pi\sum_{l=0}^{\infty}\frac{2l+1}{\kappa_-q_-}\times\int_{(m+\mu)^2}^{L(t)}ds\,W_sP_l(x)\,\mathrm{Im}f_l{}^+(s), \qquad (5.29)$$

where

$$L(t)=m^2+\mu^2+2q_-\kappa_--\tfrac{1}{2}t,\quad q_-=(m^2-\tfrac{1}{4}t)^{1/2},\quad \kappa_-=(\mu^2-\tfrac{1}{4}t)^{1/2}. \qquad (5.30)$$

We shall evaluate Eq. (5.29) keeping only the p-wave $K^*(890)$ and the d-wave $K^*(1400)$, representing these resonances by the following forms:

$$f_l(s)=B\gamma\kappa^{2l}/[(\omega_R-\omega)-i\gamma\kappa^{2l+1}], \qquad (5.31)$$

where $\omega=(\kappa^2+\mu^2)^{1/2}$, and B is the branching ratio into πK states. The parameters γ and ω_R are fixed by the widths and positions of the K^* resonances.[20] It should be noted in passing that evaluating (5.29) in the narrow-width approximation can give misleading results because of the form of the upper limit in the integral. It is easy to show that $\mathrm{Im}g_0{}^+(t)$ can be calculated by a convergent-partial-wave expansion only for $-32\mu^2\lesssim t\leq 0$. Thus the lower limit in the integral of Eq. (5.28) is replaced by $t'=-32\mu^2$.

The only term left to calculate in (5.28) is $g_0{}^+(0)$. This may be found by using Eq. (5.9) at $t=0$, giving

$$g_0{}^+(0)=\frac{1}{4m\mu}\int_{(m-\mu)^2}^{(m+\mu)^2}ds\,T^+(s,0), \qquad (5.32)$$

and $T^+(s,0)$ may be found from a once-subtracted forward dispersion relation,

$$T^+(s,0)=T^+[(m+\mu)^2,0]+\frac{1}{\pi}\int_{(m+\mu)^2}^{\infty}ds'\,\mathrm{Im}T^+(s',0)\times\left(\frac{1}{s'-s}+\frac{1}{s'+s-2(m^2+\mu^2)}-\frac{1}{s'-(m+\mu)^2}-\frac{1}{s'-(m-\mu)^2}\right), \qquad (5.33)$$

again using (5.31) to represent the K^* resonances. The subtraction constant in (5.33) is given by

$$T^+[(m+\mu)^2,0]=-(8\pi/3)(m+\mu)(a_0{}^{1/2}+2a_0{}^{3/2}), \qquad (5.34)$$

where $a_0{}^T$ are the s-wave π-K scattering lengths.

[20] We use the following values taken from the tables of A. H. Rosenfeld *et al.*, Rev. Mod. Phys. **40**, 77 (1968): $K^*(890)$: $m=893$ MeV, $\Gamma=49$ MeV, $B=1$; $K^*(1400)$: $m=1419$ MeV, $\Gamma=89$ MeV, $B=0.53$.

D. $J=0$, $T=0$ π-π D Function

In order to calculate the amplitudes for $\pi\pi\to K\bar{K}$ of Sec. V C, we need to know the D function for π-π scattering in a state with $J=0$ and $T=0$. If we normalize the D function to unity at $t=0$, then a suitable representation is[21]

$$D(t)=\exp\left(-\frac{t}{\pi}\int_{4\mu^2}^{\infty}dt'\frac{\delta(t')}{t'(t'-t-i\epsilon)}\right), \qquad (5.35)$$

where $\delta(t)$ is the π-π phase shift for s-wave $T=0$ scattering. Equation (5.35) enables both $D(t)$ for $-32\mu^2\leq t\leq 0$ [which is needed in Eq. (5.28)], and Im$D(t)$ for $4\mu\leq t\leq -4k^2(s)$ [which is needed in Eq. (5.24)] to be calculated, provided that we know $\delta(t)$.

At the present time there is much conflicting evidence, both experimental and theoretical, on the form of $\delta(t)$, and consequently we cannot make a definitive choice. Three of the major forms for $\delta(t)$ that have been suggested are shown in Fig. 3. Solution A exhibits no significant structure, and the phase shift is $\lesssim 40°$ for all $W_t=\sqrt{t}$. Solution B has a negative phase shift for $W_t\lesssim 450$ MeV, thereafter becoming positive. Finally, solution C has $\delta(t)\lesssim 40°$ for $W_t\lesssim 600$ MeV and above this energy exhibits a resonance at $W_t=750$ MeV of width 150 MeV. The evidence for each of these three solution types is briefly discussed in the Appendix. We will use each of them to calculate the $(\pi$-$\pi)_0$ contribution to K^+p scattering, although, for reasons that will become apparent later, most of our remarks will be restricted to the use of solution A. For all solutions $\delta(t)$ for $W_t>1.1$ BeV is set equal to its value at 1.1 BeV.

VI. VECTOR-MESON EXCHANGE

As we discussed in Sec. III, the vector mesons that we shall consider are $\rho(765)$, $\omega(783)$, and $\phi(1019)$. The ρ meson is a two-pion state and so could, in principle, be treated in the same way that the $(\pi$-$\pi)_0$ contribution was treated in Sec. V. However, the current state of the art hardly warrants such a detailed treatment, and we will therefore treat the ρ contribution, along with those of the ω and ϕ, in a narrow-width resonance approximation.

By analogy with the Frazer and Fulco[18] amplitudes $f_{\pm}^J(t)$ for $\pi\pi\to N\bar{N}$ we shall define amplitudes $h_{\pm}^J(t)$ for the process $K\bar{K}\to N\bar{N}$. In terms of the amplitudes $T_{\lambda\bar{\lambda}}^J(t)$ of Eqs. (2.17) and (2.18) these are given by

$$T_{++}^J(t)=\left(\frac{2}{W_t}\right)\left(\frac{q}{p}\right)^{1/2}(pq)^J h_+^J(t), \qquad (6.1)$$

$$T_{+-}^J(t)=\left(\frac{q}{p}\right)^{1/2}(pq)^J h_-^J(t). \qquad (6.2)$$

[21] R. Omnès, Nuovo Cimento 8, 316 (1958).

Using Eqs. (6.1), (6.2), (2.21), and (2.22) in Eqs. (2.19) and (2.20) for $J=1$ then gives

$$A_t^-(s,t)=-\frac{12\pi y}{\sqrt{2}}\left(\frac{q}{p}\right)[\sqrt{2}h_+^{1-}(t)-Mh_-^{1-}(t)], \qquad (6.3)$$

$$B_t^-(s,t)=\frac{12\pi}{\sqrt{2}}h_-^{1-}(t). \qquad (6.4)$$

If we now form the linear combinations

$$\Gamma_1(t)=\frac{M}{p_-^2}\left[\frac{(M^2-p_-^2)}{\sqrt{2}M}h_-^{1-}(t)-h_+^{1-}(t)\right], \qquad (6.5)$$

$$\Gamma_2(t)=\frac{1}{2p_-^2}\left[h_+^{1-}(t)-\frac{M}{\sqrt{2}}h_-^{1-}(t)\right], \qquad (6.6)$$

where $p_-^2=M^2-\frac{1}{4}t$, then Eqs. (6.3) and (6.4) may be written

$$A_t^-(s,t)=12\pi(s+\tfrac{1}{2}t-M^2-m^2)\Gamma_2(t), \qquad (6.7)$$

$$B_t^-(s,t)=-12\pi[\Gamma_1(t)+2M\Gamma_2(t)]. \qquad (6.8)$$

In the narrow-width approximation we may represent $\Gamma_i(t)$ by a sum of terms $\Gamma_i^V(t)$ (where V represents ρ, ω, and ϕ), given by

$$\Gamma_i^V(t)=\gamma_i^V/(t_V-t-i\epsilon), \qquad (6.9)$$

where $t_V=m_V^2$, m_V is the vector meson mass, and γ_i^V are constants. Using (6.9) in (6.7) and (6.8), and taking the absorptive part, gives, for a particular vector-meson contribution,

$$\mathrm{Im}A_t^{V-}(s,t)=12\pi^2\gamma_2^V(s+\tfrac{1}{2}t-M^2-m^2)\delta(t_V-t), \qquad (6.10)$$

$$\mathrm{Im}B_t^{V-}(s,t)=-12\pi^2(\gamma_1^V+2M\gamma_2^V)\delta(t_V-t). \qquad (6.11)$$

The discontinuity in the partial-wave amplitudes $f_{l\pm}$ across the real axis may be simply found following the method used in Sec. V. This gives

$$\mathrm{Im}f_{l\pm}^V(s)=\frac{3\pi(E+M)}{8W_sk^2}\{\gamma_2^V[(W_s-M)^2+\tfrac{1}{2}t_V]-\gamma_1^V(W_s-M)\}P_l(x_V)-\frac{3\pi(E-M)}{8W_sk^2}\{\gamma_2^V[(W_s+M)^2+\tfrac{1}{2}t_V]+\gamma_1^V[W_s+M]\}P_{l\pm1}(x_V), \qquad (6.12)$$

where $x_V=1+t_V/2k^2$. The contribution of a single vector meson to Re$f_{l\pm}(s)$ is then

$$\mathrm{Re}f_{l\pm}^V(s)=\frac{1}{\pi}\int_{s_-^V}^{s_+^V}ds'\frac{\mathrm{Im}f_{l\pm}^V(s')}{s'-s}, \qquad (6.13)$$

where

$$s_\pm^V=[(M^2-\tfrac{1}{4}t_V)^{1/2}\pm(m^2-\tfrac{1}{4}t_V)^{1/2}]^2. \qquad (6.14)$$

The vector mesons, like the $(\pi$-$\pi)_0$ state, also contribute to the front of the circular cut. In the case of

$(\pi\text{-}\pi)_0$ exchange this contribution was not calculated, but for the resonances we shall do so in order to have some feeling for the relative importance of contributions from the circle and the nearby real axis.

The contribution of a vector-meson exchange from the circle is given by

$$\mathrm{Re} f_{l\pm}{}^V(s) = \frac{1}{2\pi i}\oint ds' \frac{\Delta f_{l\pm}{}^V(s')}{s'-s}, \tag{6.15}$$

where

$$\Delta f_{l\pm}{}^V(s') = 2i\,\mathrm{Im} f_{l\pm}{}^V(s'),\ \mathrm{Im}s'>0$$
$$= -2i\,\mathrm{Im} f_{l\pm}{}^V(s'),\ \mathrm{Im}s'<0$$

so that if we set

$$s' = (M^2-m^2)e^{i\varphi},$$

we have, for real s,

$$\mathrm{Re} f_{l\pm}{}^V(s) = -\frac{2}{\pi}\int_0^\pi d\varphi\,\mathrm{Im}\left(\frac{s'\,\mathrm{Im} f_{l\pm}{}^V(s')}{s'-s}\right). \tag{6.16}$$

Now, from the kinematics of Sec. II,

$$\mathrm{Im} f_{l\pm}{}^V(s) = \frac{1}{32\pi s}\int_{-1}^{1} dx[H_1{}^V(s,t)P_l(x) + H_2{}^V(s,t)P_{l\pm1}(x)], \tag{6.17}$$

where

$$H_1{}^V(s,t) = [(W_s+M)^2-m^2][\mathrm{Im}A_t{}^{V-}(s,t) + (W_s-M)\,\mathrm{Im}B_t{}^{V-}(s,t)],$$
$$H_2{}^V(s,t) = [(W_s-M)^2-m^2][-\mathrm{Im}A_t{}^{V-}(s,t) + (W_s+M)\,\mathrm{Im}B_t{}^{V-}(s,t)]. \tag{6.18}$$

If we transform the integral in (6.17) by use of the relation

$$t = -2k^2(1-x),$$

and then substitute (6.17) and (6.18) into (6.16), we have

$$\mathrm{Re} f_{l\pm}{}^V(s) = -\frac{1}{8\pi^2}\int_{4\mu^2}^{t_m} dt\int_{\varphi_{\min}}^{\varphi_{\max}} d\varphi\left(\frac{1}{t_m}\right) \times \mathrm{Im}\left(\frac{H_1{}^V(s',t)P_l(1-2t/t_m)+H_2{}^V(s',t)P_{l\pm1}(1-2t/t_m)}{s'-s}\right), \tag{6.19}$$

where

$$t_m = 4(M^2\sin^2\tfrac{1}{2}\varphi + m^2\cos^2\tfrac{1}{2}\varphi), \tag{6.20}$$

$$\varphi_{\min} = 0, \qquad t_V \le 4m^2$$
$$= 2\sin^{-1}\left[\left(\frac{\tfrac{1}{4}t_- - m^2}{M^2-m^2}\right)^{1/2}\right],\quad t_V > 4m^2 \tag{6.21}$$

and $\varphi_{\max}$ is a value of φ chosen so that the partial-wave expansion into t-channel helicity amplitudes is still within its radius of convergence. Since no real-axis contributions will be explicitly calculated for $s \lesssim 10$, we will take $\varphi_{\max} = 60°$ for which $\mathrm{Re}s' \sim 15$. Finally, Eqs. (6.10) and (6.11) may be used in (6.19) to give the contribution to $\mathrm{Re} f_{l\pm}(s)$ from the front of the circle due to a single vector meson.

VII. COUPLING CONSTANTS

In Secs. IV and VI we have introduced a number of coupling constants. It is the purpose of this section to give numerical estimates for these couplings, relying as much as possible on experiment but supplementing inadequate empirical information by theory when necessary. Some of the estimates which we will make are undoubtedly rather crude and will probably change when more reliable methods of estimation become available. For this reason we will attempt to estimate errors on the coupling constants, although these errors will not actually be used in the calculation.

A. Hyperons

Early attempts to evaluate the KYN couplings from forward-dispersion relations[22] found values which were far smaller than the πNN coupling constant. Typical results were $g_\Lambda{}^2/4\pi \sim 5$, $g_\Sigma{}^2/4\pi \sim 1$, and small values of this order were also found in the analysis of kaon-photoproduction reactions.[23] However, recently a re-evaluation[24] of the KN forward dispersion relations using a new parametrization for the $\bar{K}N$ unphysical region gives $g_\Lambda{}^2/4\pi = 13.5\pm2.1$ and $g_\Sigma{}^2/4\pi = 0.2\pm0.4$. We shall use these latter values in the calculations below.

B. Vector Mesons

The vector-meson couplings $\gamma_i{}^V (i=1,2)$ may be expressed in terms of conventional field-theoretical coupling constants f_{VPP}, $f_{VN\bar{N}}{}^{(V)}$, and $f_{VN\bar{N}}{}^{(T)}$ by[25]

$$\gamma_1{}^V = (-\tfrac{2}{3})\frac{f_{VPP}f_{VN\bar{N}}{}^{(V)}}{4\pi},$$
$$\gamma_2{}^V = \left(\frac{1}{3M}\right)\frac{f_{VPP}f_{VN\bar{N}}{}^{(T)}}{4\pi}. \tag{7.1}$$

From dispersion-relation analyses of low-energy s-wave πN scattering,[10] we know that

$$f_{\rho\pi\pi}f_{\rho N\bar{N}}{}^{(V)}/4\pi = 1.43\pm0.15. \tag{7.2}$$

[22] See, e.g., G. H. Davies *et al.*, Nucl. Phys. **B3**, 616 (1967), and references therein.

[23] See, e.g., N. F. Nelipa, Nucl. Phys. **82**, 680 (1966), and references therein.

[24] J. K. Kim, Phys. Rev. Letters **19**, 1079 (1967).

[25] The corresponding Lagrangian densities are:

$$L_{VN\bar{N}}{}^{(V)} = f_{\rho N\bar{N}}{}^{(V)}\boldsymbol{\rho}^\mu\cdot(\bar{N}\gamma_\mu\boldsymbol{\tau}N) + f_{\omega N\bar{N}}{}^{(V)}\omega^\mu(\bar{N}\gamma_\mu N) + f_{\phi N\bar{N}}{}^{(V)}\phi^\mu(\bar{N}\gamma_\mu N),$$

$$L_{VN^-}{}^{(T)} = (f_{\rho N\bar{N}}{}^{(T)}/4M)(\bar{N}\sigma_{\mu\nu}\boldsymbol{\tau}N)\cdot(\partial^\mu\boldsymbol{\rho}^\nu - \partial^\nu\boldsymbol{\rho}^\mu) + (f_{\omega N\bar{N}}{}^{(T)}/4M)(\bar{N}\sigma_{\mu\nu}N)(\partial^\mu\omega^\nu - \partial^\nu\omega^\mu) + (f_{\phi N\bar{N}}{}^{(T)}/4M)(N\sigma_{\mu\nu}N)(\partial^\mu\phi^\nu - \partial^\nu\phi^\mu),$$

and

$$L_{VPP} = f_{\rho\pi\pi}\boldsymbol{\rho}^\mu\cdot(\boldsymbol{\pi}\times\partial_\mu\boldsymbol{\pi}) + if_{\omega K\bar{K}}\omega^\mu(\partial_\mu K^\dagger K - K^\dagger\partial_\mu K) + if_{\phi K\bar{K}}\phi^\mu(\partial_\mu K^\dagger K - K^\dagger\partial_\mu K) + if_{\rho K\bar{K}}\boldsymbol{\rho}^\mu\cdot(\partial_\mu K^\dagger\boldsymbol{\tau}K - K^\dagger\boldsymbol{\tau}\partial_\mu K).$$

Now $SU(3)$ symmetry with f-type coupling leads to the relation $f_{\rho\pi\pi}=2f_{\rho K\bar{K}}$, and this equality is verified to within 12% from Regge-pole phenomenology which gives $f_{\rho K\bar{K}}=(0.56\pm0.06)f_{\rho\pi\pi}$.[26] Using this latter value in (7.2) gives $f_{\rho K\bar{K}}f_{\rho N\bar{N}}^{(V)}/4\pi=0.80\pm0.17$, and hence

$$\gamma_1^\rho=-0.53\pm0.11. \qquad (7.3)$$

From work on the nucleon isovector form factors at *low and moderate* momentum transfers[27] we also know that $f_{\rho N\bar{N}}^{(T)}=-3.7f_{\rho N\bar{N}}^{(V)}$. Thus $f_{\rho K\bar{K}}f_{\rho N\bar{N}}^{(T)}/4\pi=-2.96\pm0.62$, and hence

$$\gamma_2^\rho=-0.15\pm0.03. \qquad (7.4)$$

To obtain estimates for the ω and ϕ coupling constants we shall use a combination of $SU(3)$ invariance and information about the nucleon isoscalar form factors. In the vector-dominance model of the electromagnetic interactions of hadrons, the hadronic electromagnetic current is viewed as a superposition of phenomenological vector-meson fields

$$-J_\mu^{\mathrm{el}}(x)=\frac{m_\rho^2}{2\gamma_\rho}\rho_\mu^0(x)+\frac{m_\omega^2}{2\gamma_\omega}\omega_\mu(x)+\frac{m_\phi^2}{2\gamma_\phi}\phi_\mu(x). \qquad (7.5)$$

From ρ-meson photoproduction on heavy nuclei[28] $\gamma_\rho^2/4\pi=0.42\pm0.07$, which agrees well with the value obtained[29] from the decay $\rho^0\to e^+e^-$, i.e., $\gamma_\rho^2/4\pi=0.47\pm0.12$. We shall use the mean of these numbers, i.e., $\gamma_\rho^2/4\pi=0.43\pm0.06$. It is interesting to compare the prediction of the hypothesis of a ρ meson universally coupled to the isospin current with our estimates. Universality predicts $\gamma_\rho=f_{\rho N\bar{N}}^{(V)}=\frac{1}{2}f_{\rho\pi\pi}$. The coupling $f_{\rho\pi\pi}$ may be found by using second-order perturbation theory to derive an expression for the decay width $\rho^0\to\pi^+\pi^-$. Using the ρ parameters of Rosenfeld *et al.*[30] gives $f_{\rho\pi\pi}^2/4\pi=2.43\pm0.27$, and using this value in (7.2) then gives

$$\tfrac{1}{2}f_{\rho\pi\pi}=2.76\pm0.17, \quad f_{\rho N\bar{N}}^{(V)}=3.25\pm0.54,$$

which agree quite well with our estimate for γ_ρ of 2.32 ± 0.16.

If the photon is a member of an $SU(3)$ octet then

$$\gamma_\omega=(-\sqrt{3}/\sin\theta)\gamma_\rho, \quad \gamma_\phi=(\sqrt{3}/\cos\theta)\gamma_\rho,$$

where θ is the ω-ϕ mixing angle. Using $\theta=38°$ and our previous estimate for γ_ρ gives $\gamma_\omega=-6.43\pm0.44$ and $\gamma_\phi=5.15\pm0.36$. To relate to experiment we may use (7.5) and our previous Lagrangians to derive expressions for the absorptive parts of the nucleon isoscalar form factors $\mathrm{Im}F_1{}^S(t)$ and $\mathrm{Im}F_2{}^S(t)$. The most extensive form-factor fits to date have been performed by Chan *et al.*,[31] who work in terms of the electric and magnetic scalar form factors $G_{\mathrm{ES}}(t)$ and $G_{\mathrm{MS}}(t)$. Chan *et al.*[31] give

$$G_{\mathrm{ES}}{}^\omega(0)=1.214, \quad G_{\mathrm{ES}}{}^\phi(0)=-0.714,$$
$$G_{\mathrm{MS}}{}^\omega(0)=1.093, \quad G_{\mathrm{MS}}{}^\phi(0)=-0.653. \qquad (7.6)$$

Using these values, and our previous estimates for γ_ω and γ_ϕ, gives $f_{\omega N\bar{N}}^{(V)}=-15.94\pm1.09$, $f_{\omega N\bar{N}}^{(T)}=-1.88\pm0.13$, $f_{\phi N\bar{N}}^{(V)}=-7.62\pm0.53$, and $f_{\phi N\bar{N}}^{(T)}=-0.89\pm0.06$. The coupling constant $f_{\omega K\bar{K}}$ is given in $SU(3)$ symmetry by

$$f_{\omega K\bar{K}}=-\sin\theta\, f_{\omega_8 K\bar{K}}+\cos\theta\, f_{\omega_1 K\bar{K}}, \qquad (7.7)$$

where ω_1 and ω_8 denote the singlet and octet members of the ω-ϕ complex. In the symmetry limit $f_{\omega_1 K\bar{K}}=0$ and hence

$$f_{\omega K\bar{K}}=-\sin\theta\, f_{\omega_8 K\bar{K}}=-\tfrac{1}{2}\sqrt{3}\sin\theta\, f_{\rho\pi\pi}.$$

Using $f_{\rho\pi\pi}=5.52\pm0.34$ gives $f_{\omega K\bar{K}}=-3.00\pm0.18$. The coupling constant $f_{\phi K\bar{K}}$ can be found in a similar way and yields $f_{\phi K\bar{K}}=3.73\pm0.23$. However, the decay $\phi\to K\bar{K}$ is physically allowed and so we may also calculate $f_{\phi K\bar{K}}$ from the decay parameters. This procedure gives $f_{\phi K\bar{K}}=4.37\pm0.51$, which is compatible with the previous estimate. We will use $f_{\phi K\bar{K}}=4.37$ in the actual calculations. Finally, combining all the above results gives

$$\gamma_1^\omega=-2.54\pm0.33, \quad \gamma_2^\omega=0.022\pm0.003, \qquad (7.8)$$

and

$$\gamma_1^\phi=1.77\pm0.33, \quad \gamma_2^\phi=-0.015\pm0.003. \qquad (7.9)$$

We will conclude this section with a remark on the ρ-meson coupling constants. It is clear that the procedure for determining the ω and ϕ couplings could, in principle, also be used to obtain the ρ couplings to the nucleon by considering the isovector form factors. Using the results of Chan *et al.*[31] we find $f_{\rho N\bar{N}}^{(V)}=5.6\pm0.9$ and $f_{\rho N\bar{N}}^{(T)}=-32.7\pm2.2$, compared with our earlier estimates of $f_{\rho N\bar{N}}^{(V)}=3.3\pm0.5$ and $f_{\rho N\bar{N}}^{(T)}=-12.0\pm2.0$. The situation regarding the former estimates is, unfortunately, very unclear. It is well known that in order to achieve a "dipole" type of fit to the form factors a second pole, in addition to the ρ, is required with a mass $m_{\rho'}\sim1$ BeV, and although theoretical arguments have been given in its favor[32] it is difficult to see why it has not been seen experimentally,[33] since there seems no reason why it should not be as strongly produced (e.g., in photoproduction reactions) as the ρ meson. In the numerical work which follows we shall use the estimates given in Eqs. (7.3) and (7.4).

[26] V. Barger and M. Olsson, Phys. Rev. Letters **18**, 294 (1967).
[27] J. S. Ball and D. Y. Wong, Phys. Rev. **130**, 2112 (1963); T. D. Spearman, *ibid.* **129**, 1847 (1963).
[28] J. G. Asbury *et al.*, Phys. Rev. Letters **20**, 227 (1968).
[29] H. Joos, DESY Report No. 67/43 (unpublished).
[30] A. H. Rosenfeld *et al.*, Rev. Mod. Phys. **40**, 77 (1968).
[31] L. H. Chan *et al.*, Phys. Rev. **141**, 1298 (1966).
[32] See, e.g., J. Moffat, Phys. Rev. Letters **20**, 620 (1968).
[33] J. G. Asbury *et al.*, Phys. Rev. Letters **19**, 869 (1967); A. Wehman *et al.*, *ibid.* **18**, 929 (1967); B. D. Hyams *et al.*, Phys. Letters **24B**, 634 (1967).

VIII. PHYSICAL REGION

Early experimental work[34] on K^+p scattering deduced the very interesting fact that the interaction was a pure s-wave repulsion for kaon laboratory momenta in the range $k_L \lesssim 650$ MeV/c. Recently, a fairly extensive phase-shift analysis[35] of all existing K^+p elastic data in the momentum range $k_L \leq 1.5$ BeV/c has confirmed the dominance of the s waves below ~ 800 MeV/c. Above 800 MeV/c several solutions were found which fitted the data equally well, but an interesting feature of all these solutions is the fact that the s-wave phase shifts are rather similar. In particular, this means that we have a fair idea of $\mathrm{Im} f_{0+}(s)$ in the region 800 MeV/$c \leq k_L \leq 1500$ MeV/c. It was found[35] that for all the acceptable solutions $\mathrm{Im} f_{0+}(s)$ passes through a broad maximum around $k_L \sim 1$ BeV/c, then falls off slowly. If we define

$$C_0 = k(\bar{s})\,\mathrm{Im} f_{0+}(\bar{s}),$$

where $\bar{s}$ is the square of the total c.m. energy at a laboratory momentum of 1.5 BeV/c, and fit a high-energy "tail" to $\mathrm{Im} f_{0+}(s)$ of the form

$$\mathrm{Im} f_{0+}(s) = C_0/k(s),$$

then we should be able to make a reasonable estimate of the rescattering integral in the dispersion relation (3.3). Since we only know $\mathrm{Re} f_{0+}(s)$ uniquely below 800 MeV/c we shall confine the evaluation of the dispersion relation to this region.

IX. CALCULATIONAL PROCEDURE AND RESULTS

The contributions to $\mathrm{Re} f_{0+}(s)$ in the physical region due to the nearby singularities were calculated by the methods of Secs. IV–VI. We shall discuss firstly the contributions due to hyperon and vector-meson exchanges.

The contributions from the nearby cuts due to Λ and Σ exchanges were calculated using the method of Sec. IV with the coupling constants of Sec. VII. These terms are small and had we used, instead of the coupling constants of Sec. VII, the smaller values of Ref. 22 the result, as far as the over-all dynamical picture is concerned, would remain unchanged. This fortunate situation for s-wave scattering is due to a strong cancellation in the discontinuity across the nearby cut, and has been observed previously for πN scattering[10,36] and $\pi\Lambda$ scattering.[37] Values for the Λ contribution at various points in the physical region are given in Table I. The Σ contribution is negligible.

The contributions from the real-axis cuts and the front of the circular cut due to vector-meson exchanges were calculated using the methods of Sec. VI and the coupling constants of Sec. VII. The ρ and ω contributions are both positive and of the same order of magnitude, whereas the ϕ contribution is extremely small. This smallness is a direct reflection of the fact that $m_\phi > 2m$, and hence this vector meson does not give rise to a cut along the nearby real axis, unlike both the ω and ρ (see Fig. 2). For the latter exchanges most of their contributions come from the real-axis cuts and only a small part is due to the front of the circle. This result is encouraging because it means that neglecting the front of the circle, as is done when calculating the contribution due to $(\pi\text{-}\pi)_0$ exchange, should produce only a small error. Values for the ρ and ω contributions at various points in the physical region are shown in Table I. The contribution due to ϕ exchange is negligible.

The only other process to be discussed is $(\pi\text{-}\pi)_0$ exchange, and this requires special care. We shall start by discussing the results obtained by using the nonresonant π-π phase shift labeled A in Fig. 3. Using these phase shifts a D function was constructed from Eq. (5.35) and used in Eq. (5.24). The amplitudes $\mathrm{Im} f_+^{0+}(t)$ for $\pi\pi \to N\bar{N}$ were taken from the work of Hamilton and co-workers[10] on the low-energy π-N interaction. Since the π-π phases of set A are very similar to those actually found by Hamilton *et al.* it is clear that this is a self-consistent procedure. The other amplitude needed in Eq. (5.24) is $g_0^+(t)$ describing $\pi\pi \to K\bar{K}$. The method of calculation of this amplitude has been given in Sec. V C but here there is a difficulty, because to evaluate Eq. (5.33) we need to know the value of the s-wave π-K scattering lengths in the combination $a_0^+ = \frac{1}{3}(a_1^{1/2} + 2a_0^{3/2})$.

It might be thought that as a first approximation one could neglect this term, and such a view would be borne out by the predictions of current-algebra calculations which give $a_0^+ = 0$.[38] However, explicit numerical calculation shows that even a quite small value of a_0^+ can change the value of the $(\pi\text{-}\pi)_0$ contribution appreciably, and, in particular, if $a_0^+ < 0$ then cancellation with the integral in Eq. (5.33) can considerably reduce the $(\pi\text{-}\pi)_0$ term. This is an important point and we will

TABLE I. Values of the nearby left-hand-cut contributions due to Λ, ρ, and ω exchange, together with values of the rescattering integral and $\mathrm{Re} f_{0+}(s)$. Contributions due to Σ and ϕ are negligible. All contributions are to be multiplied by 10^{-3}.

s	k_L (MeV/c)	Λ	ρ	ω	Rescattering	$\mathrm{Re} f_{0+}$
105.3	0	−8.2	20.8	37.0	183.2	−198.00
115	335	−5.8	14.6	23.8	179.4	−200.6
125	500	−4.3	10.8	15.8	164.5	−193.6
135	635	−3.2	8.3	10.8	142.2	−179.9
145	765	−2.4	6.5	7.3	115.3	−163.0

[34] S. Goldhaber *et al.*, Phys. Rev. Letters **9**, 135 (1962).
[35] A. T. Lea, B. R. Martin, and G. C. Oades, Phys. Rev. **165**, 1770 (1968).
[36] B. R. Martin, Phys. Rev. **162**, 1448 (1967).
[37] B. R. Martin, Phys. Rev. **138**, B1136 (1965).
[38] See, e.g., A. P. Balachandran, M. G. Gundzik, and F. Nicodemi, in *Boulder Lectures in Theoretical Physics* (Gordon and Breach, Science Publishers, Inc., New York, 1967), Vol. 9B.

return to it later. The actual value of a_0^+ is, unfortunately, not known. For example, Martin and Vick[39] from a dynamical calculation of πK scattering found a_0^+ in the range 0.024–0.011; Martin and Spearman[7] found $a_0^+=-0.13\pm0.03$; and Conforto *et al.*[40] in a phenomenological analysis of πK final states in $p\bar{p}$ annihilations found $a_0^+\sim-0.07$. The evidence seems to suggest that $a_0^+<0$, but clearly more work is needed before this conclusion can be considered certain. We shall consider the effect of varying a_0^+ in the range $-0.05\leq a_0^+\leq 0.05$.

Values for the $(\pi\text{-}\pi)_0$ contribution at threshold using the nonresonant π-π phases (set A) and taking $a_0^+=-0.05$, 0, and 0.05 are shown in Table II. The importance of the term involving a_0^+ is clearly evident. To see the effect of changing the form of the input π-π phase shifts, we have repeated the above calculation with sets B and C in Fig. 3. Set B still bears some resemblance to the second π-π solution found by Hamilton *et al.*[10] and thus we might hope that using their values of $\text{Im}f_+^{0+}(t)$ is still reasonably self-consistent. The same cannot be said for the use of set C, however, and the values of the $(\pi\text{-}\pi)_0$ contribution obtained with this set are almost certainly underestimated. Results of the above calculations are shown in Table II. It is clear from Table II that $(\pi\text{-}\pi)_0$ exchange can give rise to a large term in the low-energy physical region, even if the s-wave $T=0$ π-π phase shift is nonresonant.

The question to which we must now turn is whether the forces calculated above can give rise to a consistent dynamical picture and, in particular, is a large $(\pi\text{-}\pi)_0$ term compatible with the known energy dependence of the s-wave K^+p amplitude?

From the discussion of Sec. VIII we can use the results of the K^+p phase-shift analysis of Lea, Martin, and Oades[35] to find $\text{Re}f_{0+}(s)$ and the rescattering integral below $\sim$800 MeV/c. In practice we shall use solution $S_{11}P_{11}D_{13}$ of group II of Ref. 35. Values of $\text{Re}f_{0+}(s)$ and the rescattering integral are given in Table I. Using the exchange forces calculated as described above, we form the quantity

$$\Delta(s)=\text{Re}f_{0+}(s)-\frac{P}{\pi}\int_{s_0}^{\infty}ds'\frac{\text{Im}f_{0+}(s')}{s'-s}-\sum_E \text{Re}f_{0+}{}^E(s)\,,\quad(9.1)$$

where E stands for the nearby exchange forces, i.e., ρ, ω, ϕ, $(\pi\text{-}\pi)_0$, Λ, and Σ. We shall fit $\Delta(s)$ with a form chosen to represent the unknown, neglected, parts of the interaction. Firstly, to represent the distant singularities (i.e., very short-range forces) we shall use a single pole of the form $\Gamma/(\bar{s}_1-s)$. Since this pole is to represent contributions from the rear of the circle and the real axis to the left of the origin, we shall set (arbitrarily) $\bar{s}_1=-M^2$. The other major term which has been neglected is that due to the exchange of hyperon resonances in the u channel. The nearby singularities due to these processes consist of a series of short cuts on the real axis between the points

$$s_1=2(M^2+m^2)-m^{*2}$$

and

$$s_2=(M^2-m^2)^2/m^{*2}\,,$$

where m^* is the mass of the resonance. The rightmost of such cuts is due to the $Y_1^*(1385)$ and is shown in Fig. 2. Although these cuts are somewhat further to the left than the other nearby singularities, they could still give terms with a strong net-energy dependence. Nevertheless we have not attempted to calculate their contributions explicitly because, at present, there exists a rather large number of hyperon resonances the parameters of which are by no means all well established. We shall represent these neglected parts of the interaction by a form $(a+bs)/(\bar{s}_2-s)^2$ which allows more freedom than a single pole. The parameter $\bar{s}_2$ is set equal to 10, which is roughly centered in the midst of the nearby cuts due to the hyperon resonances.

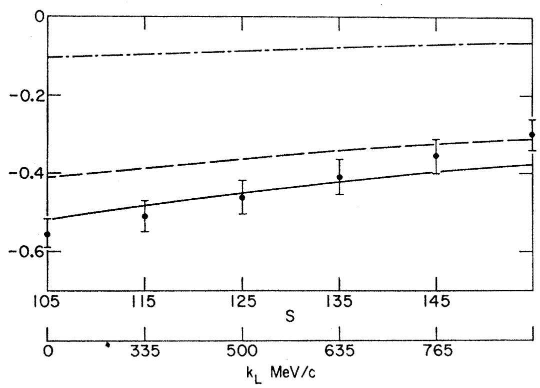

FIG. 4. Fit to $\Delta(s)$ for the π-π phase-shift set A and $a_0^+=0$: dash, pole term; dash-dot, $(a+bs)/(\bar{s}_2-s)^2$; solid line, sum.

Using the π-π phase shifts A and setting $a_0^+=0$, we have attempted to find parameters a, b, and Γ which give a reasonable fit to $\Delta(s)$. No detailed seraching has been made, but Fig. 4 shows the results obtained with $a=10$, $b=-9$, and $\Gamma=60$. The error bars on $\Delta(s)$ represent the errors due to $\text{Re}f_{0+}(s)$ and the rescattering integrals *only*, and do *not* take account of possible errors in the exchange forces. The fit is quite reasonable.

TABLE II. Values of the $(\pi\text{-}\pi)_0$ contribution at threshold for the three sets of s-wave $T=0$ π-π phase shifts shown in Fig. 3, and for $a_0^+=-0.05$, 0, and 0.05. All contributions are to be multiplied by 10^{-3}.

Value of a_0^+	s-wave $T=0$ π-π phase-shift solution A	B	C
−0.05	69.4	125.3	366.4
0	124.1	167.6	473.9
0.05	178.7	209.9	581.5

[39] A. D. Martin and L. L. J. Vick, Nuovo Cimento **39**, 905 (1965).
[40] B. Conforto *et al.*, Nucl. Phys. **B3**, 469 (1967).

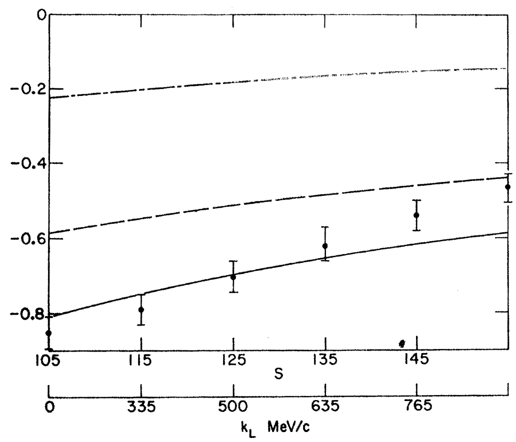

FIG. 5. Fit to $\Delta(s)$ for the π-π phase shifts set C and $a_0^+=0$: dash, pole term; dash-dot, $(a+bs)/(\bar{s}_2-s)^2$; solid line, sum.

The same thing can be done when $a_0^+=\pm 0.05$. In each case it was found that the short-range pole gives a large negative contribution and the nearby term a smaller negative contribution. The correct energy dependence is obtained by a cancellation between this latter contribution and the positive $(\pi$-$\pi)_0$ term. We have also tried the same procedure using the π-π phases B and C. Because the $(\pi$-$\pi)_0$ contribution is now far larger (see Table II) it is clear that $\Delta(s)$ is also larger and correspondingly the role of the parameters a and b becomes more important in cancelling this term. For example, we show in Fig. 5 the results for π-π phases C with $a_0^+=0$. The other parameters are $a=10$, $b=-20$, and $\Gamma=90$. Although the fit is worse than the solution of Fig. 4, nevertheless it cannot be entirely ruled out.

X. CONCLUSIONS AND OUTLOOK

We have presented an exploratory investigation of the dynamics of the s-wave K^+p system using the techniques of dispersion relations. In this final section we will try to summarize the results obtained and discuss future possible work.

Firstly, it is clear that the exchange of an s-wave $T=0$ π-π pair is *potentially* the largest force of long range, even if the s-wave $T=0$ π-π phase shift is nonresonant, and thus calculations which neglect this process *could* be seriously in error. Unfortunately, the precise size of the $(\pi$-$\pi)_0$ contribution cannot be determined at present due to (a) our lack of knowledge about the π-π phase shifts, and (b) the unknown parameter a_0^+. In particular, it may even be compatible to have a resonant π-π phase shift if a_0^+ is sufficiently negative. Compared with the $(\pi$-$\pi)_0$ term the long-range parts of the ρ, ω, ϕ, Λ, and Σ exchanges are all fairly small, the most important being the ρ and ω. In addition, it is necessary to have a fairly large net repulsion from the singularities due to hyperon resonance exchanges in the u channel, and a very large, slowly varying, short-range repulsion. The latter effect is not unexpected, and short-range repulsions of similar sizes have been found in other meson-baryon s-wave reactions.[10,37] Confirmation of whether the size of the force attributed to the hyperon resonances is a real phenomenon will have to await experimental clarification of the hyperon resonance parameters.

Before further work of this type can be attempted on the dynamics of the K^+p system, it is clear that many improvements must be made in the experimental situation. Firstly, and most important, it will be necessary to know both the s-wave $T=0$ π-π phase shift and the s-wave π-K scattering length combination a_0^+ before a reliable calculation can be made of the $(\pi$-$\pi)_0$ contribution. If it should turn out that this term is indeed small, then the relative importance of vector-meson exchange is increased and it will then be important to have reliable estimates of the vector-meson coupling constants. This remark is also true for the Λ and Σ coupling constants, since, although these Born terms are very small in s-wave scattering, they will be important for higher waves. However, it will not be possible to extend calculations of this sort to higher waves until phase-shift analyses of the type in Ref. 35 can produce unique solutions. This in turn means a greater experimental effort is required in the field of K^+p scattering. A discussion of the types of experiment needed to help resolve the present ambiguities has been given by Lea, Martin, and Oades.[35]

APPENDIX: s-WAVE $T=0$ π-π SCATTERING BELOW 1 BEV

Attempts to deduce the behavior of the s-wave $T=0$ π-π scattering amplitude in the region below 1 BeV total c.m. energy have occupied the attention of many workers, and the resulting number of suggested behaviors is very large, although none of them can be said to be wholly convincing at this time. We shall not attempt to give a critical assessment of these papers, but merely reproduce below some of the arguments that have been given in favor of the three major types of behavior which have been suggested and which we have considered in Sec. IX. The references in this Appendix are intended to be representative rather than exhaustive.

A. Nonresonant Behavior

A large number of authors have suggested forms for the s-wave $T=0$ π-π amplitude which are nonresonant below 1 BeV. Such a form was suggested by Chew and Mandelstam from their early work on the dynamics of the π-π system,[41] and a form essentially equivalent was shown by Hamilton *et al.*[42] to be compatible with the

[41] G. F. Chew and S. Mandelstam, Phys. Rev. **119**, 467 (1960).
[42] J. Hamilton, P. Menotti, G. C. Oades, and L. L. J. Vick, Phys. Rev. **128**, 1881 (1962).

dynamics of low-energy πN scattering. The phenomenological πN dispersion relation analysis of Hamilton *et al.*[42] gave an *s*-wave $T=0$ π-π scattering length $a_0 \sim 1$, and a value of this magnitude has subsequently been found by many other authors from several sources, e.g., K_{e4} decay,[43] π-π forward dispersion relations,[44] and single-pion production experiments.[45] All these results depend, to some extent, on the parametrization chosen. However, recently, Fulco and Wong[46] have shown, without specifically parametrizing the amplitude, that a nonresonant form ($\delta_0 \lesssim 50°$ for $E_\pi \lesssim 900$ MeV) with $a_0 \sim 0.8$ is the best solution compatible with a large range of experimental data as well as with forward π-π dispersion relations.

Nevertheless, claims have been made from current-algebra calculations[47] for a much smaller scattering length $a_0 \sim 0.2$. However, these calculations use, in addition to current algebra and PCAC, dynamical assumptions concerning the extrapolation of the π-π amplitude from the unphysical point given by the calculation to the physical region. These assumptions are independent of the current-algebra formulation and several authors[48] have shown that other, quite reasonable, assumptions about the extrapolation can produce a very large variety of different scattering lengths (both positive and negative) and phase-shift behaviors. It has also been shown[49] that the small scattering lengths of Weinberg[47] can still be compatible with quite large phases in the low-energy region, and since the work of Sec. IX depends not so much on the value of the amplitude at threshold (i.e., the scattering length) but on the form of the phase shift in the whole energy range below 1 BeV, this latter result is all that we really need.

B. Resonant Behavior

Many analyses of single-pion production in πN interactions have been made, some of which[50] claim evidence for an *s*-wave $T=0$ resonance variously reported in the mass region 750–900 MeV and with a width in the range 50–150 MeV. However, there seems to be little direct evidence for such a resonance.[51] Other experiments[52] have, at various times, claimed evidence for the existence of a resonance at a much lower mass $M_{\pi\pi} \sim 400$ MeV, and further evidence for such a state has been presented from a phenomenological study of backward $\pi^\pm p$ dispersion relations.[53] However, unless this low-energy resonance is extremely broad, it is difficult to see why it has not been observed in the K_{e4} decay spectrum.[43]

C. Solution with a Zero in the Physical Region

A third type of behavior which has been suggested is a phase shift which is initially negative but soon turns over, passes through zero, and becomes positive, possibly resonating at higher energies. A phase shift of this form was actually found by Hamilton *et al.*[42] from their work on πN dispersion relations but was rejected in favor of the somewhat better fit obtained by a solution of type A above. The scattering length found by Hamilton *et al.*[42] was $a_0 \sim -0.6$, and a value close to this has been found in a forward-dispersion-relation calculation by Antoniou.[44] Two other pieces of evidence for a phase shift of this type come from the backward $\pi^\pm p$ work of Lovelace *et al.*[53] and, more recently, a new analysis of low-energy single-pion production by Humble and Spearman.[54] Both Hamilton *et al.*[42] and Humble and Separman[54] find that on becoming positive the phase shift rises to a maximum nonresonant value, whereas Lovelace *et al.*[53] find a very broad resonance at $W \sim 700$ MeV. The scattering length of Humble and Spearman is $a_0 \sim -1.7$, and this value has also been found in a forward-dispersion-relation calculation by Rothe.[55] Finally, a very recent analysis of single-pion production below 750 MeV[56] also finds evidence for a negative value of δ in this region.

The threc sets of π-π phase shifts that were used in Sec. IX are shown in Fig. 3. Sets A, B, and C correspond to scattering lengths $a_0=0.6$, -0.6, and 0.6, respectively, but again we emphasize that these values are unimportant for our purposes and are only used to define the phase shift for the first few MeV above threshold.

[43] R. W. Birge *et al.*, Phys. Rev. **139**, B1600 (1965); F. A. Berends, A. Donnachie, and G. C. Oades, Nucl. Phys. **B3**, 569 (1967).
[44] See, e.g., N. G. Antoniou, Nucl. Phys. **B3**, 277 (1967).
[45] See, e.g., W. D. Walker *et al.*, Phys. Rev. Letters **18**, 630 (1967).
[46] J. R. Fulco and D. Y. Wong, Phys. Rev. Letters **19**, 1399 (1967); see also Y. Fujii, Phys. Letters **24B**, 190 (1967).
[47] S. Weinberg, Phys. Rev. Letters **17**, 336 (1966).
[48] See, e.g., J. Iliopoulos, Nuovo Cimento **52A**, 192 (1967); **53A**, 552 (1968); **54A**, 536 (1968); A. Donnachie, *ibid.* **53A**, 931 (1968); K. Kang and T. Akiba, Phys. Rev. **164**, 1836 (1967).
[49] E. P. Tryon, Phys. Rev. Letters **20**, 769 (1968); Y. Fujii and K. Hayashi, Progr. Theoret. Phys. (Kyoto) **39**, 126 (1968).
[50] See, e.g., M. Feldman *et al.*, Phys. Rev. Letters **14**, 869 (1965); V. Hagopian *et al.*, *ibid.* **14**, 1077 (1965).
[51] The evidence is discussed in Ref. 30.
[52] See, e.g., R. Del Fabbro *et al.*, Phys. Rev. **139**, B701 (1965); A. Abashian *et al.*, *ibid.* **132**, 2296 (1964).
[53] C. Lovelace, R. H. Heinz, and A. Donnachie, Phys. Letters **22**, 332 (1966).
[54] S. Humble and T. D. Spearman, Phys. Rev. **171**, 1724 (1968).
[55] H. J. Rothe, Phys. Rev. **140**, B1421 (1965).
[56] N. N. Biswas *et al.*, University of Notre Dame Report, 1968 (unpublished).