The unequal-mass conspiracy relation

$$\mu^{-4}F_{RR}(s,0)=-F_{II}(s,0) \quad \text{(A10)}$$

is satisfied by the (full) factorized residues $[\gamma_I^{\pi}(0)]^2=[\gamma_R^{\pi'}(0)]^2$ for the parity-doublet solution. This relation eliminates the apparent pole in $f_{10,10}{}^t$ at $t=0$.

We have checked that the $M=1$ conspiracy is a solution to all conspiracy relations for NN, photoproduction, and Compton scattering processes and that the residues factorize.[18] It should be noted that a slight change in the kinematical singularities given in Ref. 2 is necessary to have them obey factorization. Namely, the factors $(1-t/4m^2)^{-1}$ in Eq. (1) of Ref. 2 should not be present for the singlet and uncoupled triplet amplitudes. Since this factor is very close to 1 for $0<|t|<0.5$, these NN fits are not affected.

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Reanalysis of the Lowest-Mass Negative-Parity Baryon Resonances Using the Symmetric Quark Model

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The lowest-mass negative-parity baryon resonances are reanalyzed using a systematic $SU(6)$ analysis of mass operators developed earlier in the symmetric quark model. Fourteen resonances with known spin and parity are fitted to within 15 MeV on the average, using a six-parameter mass formula. Strong mixings are found for $J^P=\frac{1}{2}^-$ and $\frac{3}{2}^-$ resonances, and thus the Gell-Mann–Okubo mass formula cannot be expected to hold for these resonances, and there is no sense in trying to group them into octets and decuplets.

I. INTRODUCTION

We reanalyze the lowest negative-parity baryon resonances using the symmetric quark model with orbital excitation. In the symmetric quark model, the wave function of the quarks is symmetric under simultaneous permutations of the orbital and $SU(6)$ degrees of freedom of the quarks. This model can be based on parafermions of order three[1] or on the three-triplet model.[2] The lowest negative-parity baryons should be in the supermultiplet with $(SU(6),L^P)=(\mathbf{70},1^-)$.[1] In the present article we use the systematic $SU(6)$ analysis of mass operators given earlier[3] and refer to A for notation and additional references.

II. DATA

We take experimental data concerning resonances in the $(\mathbf{70},1^-)$ from the phase-shift analysis done by Donnachie, Kirsopp, and Lovelace[4] and private communication from Donnachie[5] for nonstrange resonances, and from the compilation of Rosenfeld *et al.*[6] for strange resonances. There are 14 resonances with known spin and parity; these are listed in Table I, together with one Σ and two Ξ resonances whose spin and parity have not been determined.

TABLE I. Experimental data for resonances in the $(\mathbf{70},1^-)$.

Resonance		J^P	Source of data (Ref. No.)
N	$D_{13}(1520)$	$\frac{3}{2}^-$	5
	$S_{11}(1540)$	$\frac{1}{2}^-$	5
	$D_{15}(1678)$	$\frac{5}{2}^-$	4
	$D_{13}(1680)$	$\frac{3}{2}^-$	5
	$S_{11}(1710)$	$\frac{1}{2}^-$	5
Δ	$S_{31}(1635)$	$\frac{1}{2}^-$	4
	$D_{33}(1691)$	$\frac{3}{2}^-$	4
Λ	$S_{01}(1405)$	$\frac{1}{2}^-$	6
	$D_{03}(1519)$	$\frac{3}{2}^-$	6
	$S_{01}(1670)$	$\frac{1}{2}^-$	6
	$D_{03}(1690)$	$\frac{3}{2}^-$	6
	$D_{05}(1827)$	$\frac{5}{2}^-$	6
Σ	$D_{13}(1660)$	$\frac{3}{2}^-$	6
	$D_{15}(1767)$	$\frac{5}{2}^-$	6
	1690	?	6
Ξ	1815	?	6
	1930	?	6

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1 O. W. Greenberg, Phys. Rev. Letters **13**, 598 (1964).
2 Y. Nambu, in *Preludes in Theoretical Physics*, edited by A. de Shalit, M. Feshbach, and L. Van Hove (North-Holland Publishing Co., Amsterdam, 1966), pp. 133–142; M. Y. Han and Y. Nambu, Phys. Rev. **139**, B1006 (1965); A. Tavkhelidze, in *Proceedings of the Seminar on High-Energy Physics and Elementary Particles, 1965* (International Atomic Energy Agency, Vienna, 1965), pp. 763–779, and references cited therein.
3 O. W. Greenberg and M. Resnikoff, Phys. Rev. **163**, 1844 (1967). We will refer to this reference as A.
4 A. Donnachie, R. G. Kirsopp, and C. Lovelace, Phys. Letters **26B**, 161 (1968).
5 A. Donnachie (private communication).
6 A. H. Rosenfeld *et al.*, Rev. Mod. Phys. **40**, 77 (1968).

TABLE II. Calculation versus experiment for the (**70**,1⁻): Fit of 14 resonances with six-parameter mass formula. The left-hand columns are the calculated masses. The right-hand columns are experimental masses. The three experimental masses with question marks (not included in the 14 masses fitted) do not have spin and parity determined and were placed in the table in the position of the closest calculated masses. The superscript M indicates resonances mixed by more than 20% in the square of the mixing amplitude.

	^4P							^2P			
	$J=\frac{5}{2}$		$\frac{3}{2}$		$\frac{1}{2}$			$\frac{3}{2}$		$\frac{1}{2}$	
							Ω	2062		2062	
							Ξ	1950		1938	1930?
							Σ	1815		1809	
Ξ	1895		1831^M		1742		Δ	1669	1691	1669	1635
Σ	1765	1767	1630^M	1660	1639^M						
Λ	1809	1827	1792		1779		Ξ	1816^M		1801	1815?
N	1689	1678	1690	1680	1691	1710	Σ	1722^M		1682^M	1690?
							Λ	1690	1690	1689^M	1670
							N	1527	1520	1528	1540
							Λ	1527	1519	1428^M	1405

III. RESULTS

The six-parameter mass formula derived earlier was used to fit the 14 resonances with known spin and parity. The criterion of fit was to minimize $\sum(M_{\text{th}}-M_{\text{expt}})^2$; ultimately this formula should be weighted to take account of the experimental determination of the masses of different resonances. We felt that it was premature to do this for the present analysis. Since the mass formula yields the same mass for the two Δ resonances, we compared it with the average of the experimental masses. As in A, mixings among all particles with the same I, Y, and J were allowed. The nucleonic states were essentially unmixed, but in general the strange resonances were strongly mixed. All mixing parameters were determined at the same time as the masses by diagonalizing the six-parameter mass formula using a computer. The results for the masses are shown in Table II. The 14 calculated masses are on the average 15 MeV in the least-squares sense from the corresponding experimental masses. The mixing parameters are given in Table III. The mass parameters are[7]

$$({}^{21}M_1{}^1)-({}^{15}M_1{}^1)=-283.3\text{ MeV},$$
$$({}^{21}M_{35}{}^8)-({}^{15}M_{35}{}^8)=-112.8\text{ MeV},$$
$$M_{405}{}^1=197\text{ MeV},\quad M_{405}{}^8=107\text{ MeV},$$
$$M_{189}{}^1=31.5\text{ MeV},\quad M_{189}{}^8=-73.5\text{ MeV},$$
$$M_{\mathbf{L}\cdot\mathbf{s}}{}^1=71.4\text{ MeV},\quad M_{\mathbf{L}\cdot\mathbf{s}}{}^8=-91\text{ MeV}.$$

IV. REMARKS

As has been pointed out earlier,[3,8] the spin-orbit operators act only on the two-body states in the **15** of $SU(6)$. (We emphasize that this conclusion holds in a formalism in which the three-body system is properly treated using traceless coordinates.) Thus the spin-orbit operators do not (acting directly) split the $J=\frac{1}{2}$ and $\frac{3}{2}$ resonances in the $S=\frac{1}{2}$ decuplet. (The $J=\frac{1}{2}$ and $\frac{3}{2}$ Σ and Ξ resonances in this decuplet are split indirectly because the physical resonances are not pure decuplets, but are mixed with the $S=\frac{1}{2}$ and $\frac{3}{2}$ octets whose $J=\frac{1}{2}$ and $\frac{3}{2}$ members are split by the spin-orbit interaction.) The $J=\frac{1}{2}$ and $\frac{3}{2}$ Δ and Ω resonances are not split by any two-body interaction within the (**70**,1⁻).[9] The phase-

TABLE III. Mixing amplitudes for resonances in the (**70**,1⁻). The N's are essentially unmixed: The largest off-diagonal amplitude for the N's has magnitude 3×10^{-4}. We label the resonances by their experimental masses if they have been found, and by their calculated masses otherwise.

Resonance (mass) (J)	Amplitudes: ($S,SU(3)$) ($\frac{1}{2}$,1)	($\frac{1}{2}$,8)	($\frac{3}{2}$,8)
Λ(1405) ($\frac{1}{2}$)	0.87	−0.49	0.01
Λ(1670) ($\frac{1}{2}$)	0.49	0.86	0.14
Λ(1779) ($\frac{1}{2}$)	−0.07	−0.11	0.99
Λ(1519) ($\frac{3}{2}$)	0.95	−0.30	−0.03
Λ(1690) ($\frac{3}{2}$)	0.29	0.94	−0.17
Λ(1792) ($\frac{3}{2}$)	0.08	0.16	0.98
	($\frac{1}{2}$,10)	($\frac{1}{2}$,8)	($\frac{3}{2}$,8)
Σ(1639) ($\frac{1}{2}$)	−0.03	0.67	−0.74
Σ(1682) ($\frac{1}{2}$)	0.26	0.72	0.64
Σ(1809) ($\frac{1}{2}$)	−0.96	0.17	0.20
Σ(1660) ($\frac{3}{2}$)	−0.23	0.79	0.58
Σ(1722) ($\frac{3}{2}$)	0.25	0.62	−0.75
Σ(1815) ($\frac{3}{2}$)	0.94	0.03	0.33
Ξ(1743) ($\frac{1}{2}$)	0.11	0.99	0.05
Ξ(1801) ($\frac{1}{2}$)	0.19	−0.07	0.98
Ξ(1938) ($\frac{1}{2}$)	−0.98	0.10	0.19
Ξ(1816) ($\frac{3}{2}$)	−0.29	0.86	0.42
Ξ(1831) ($\frac{3}{2}$)	0.25	0.48	−0.84
Ξ(1950) ($\frac{3}{2}$)	0.92	0.14	0.36

[7] The N_i introduced in Eqs. (4) and (7) of A are, in order (and in MeV), 1003, −211.2, 16.8, 17.2, −79.25, −14.7, 5.6, −18.76, 4.2, and −13. The parameters N_0 through N_3 are determined, as in A, by particles and resonances in the (**56**,0⁺). The parameter N_1 in A should have been −211.2 rather than −168.1.

[8] R. H. Dalitz, in *Proceedings of the Second Hawaii Topical Conference in Particle Physics 1967*, edited by S. Pakvasa and S. F. Tuan (The University of Hawaii Press, Honolulu, 1968), pp. 327–466.

[9] Tensor forces vanish acting on the Δ's since it has $S=\frac{1}{2}$. Thus tensor forces will not split the Δ's as was incorrectly stated in A. We thank G. Karl (private communication) for pointing this out.

shift analysis gives a splitting of 54 MeV between the $J=\frac{1}{2}$ and $\frac{3}{2}$ Δ's. Some deviation from the phase-shift mass values can be tolerated, and it is generally the case that small perturbations produce large shifts in levels that are initially degenerate, so we do not regard the splitting of these Δ's as a serious deficiency of our present model. We can think of three mechanisms to split these Δ's within the framework of the symmetric quark model: (1) Three-body spin-orbit forces act on the $(\mathbf{70},1^-)$, and, since the decuplet has $S=\frac{1}{2}$, give a nonzero splitting in the decuplet. (2) Configuration mixing with other L-excited Δ's which are spin-orbit split will induce a splitting for the Δ's which are predominantly in the $(\mathbf{70},1^-)$. (3) Configuration mixing with $SU(3)''$-excited Δ's will give a similar effect.[10]

Aside from the Δ's, the worst-fit resonances are $\Lambda(1405)$ and $\Sigma(1660)$ which are 23 MeV below and 30 MeV above the calculated masses of 1428 and 1630 MeV, respectively. We expect that this fit can be improved by introducing some of the noncentral two-body octet-dominant operators which have not yet been used; in particular, the spin-orbit operator in the **189**. One of us (D.R.D.) is now doing this calculation.

We chose our spin-orbit operators on the basis of the systematic $SU(6)$ analysis given earlier. This analysis shows that there is only one independent two-body unitary singlet spin-orbit operator which can act on the $(\mathbf{70},1^-)$. We have not used the spin-orbit operator $C_2^{(3)}\mathbf{L}\cdot\mathbf{S}$,[8] because it is not a two-body operator.

The lowest-mass predicted resonances which have not yet been seen are a $\Sigma(1639)$ $(\frac{1}{2}^-)$, a $\Sigma(1682)$ $(\frac{1}{2}^-)$, and a $\Sigma(1722)$ $(\frac{3}{2}^-)$. These predicted Σ resonances may be the cause of the discrepancies[6] in the branching ratios of the $\Sigma(1660)$ $(\frac{3}{2}^-)$ whose mass we calculate to be 1630. Our $\Sigma(1682)$ $(\frac{1}{2}^-)$ may be the $\Sigma(1690)$ whose spin and parity are undetermined.

[10] See O. W. Greenberg and C. A. Nelson [Phys. Rev. Letters **20**, 604 (1968)] for a discussion of $SU(3)''$-excited resonances in the three-triplet model.

V. SUMMARY

We have reanalyzed the baryon resonances in the $(\mathbf{70},1^-)$ using the mass analysis developed in A. We find a fit to 14 resonances with known spin and parity using a six-parameter mass formula. The calculated masses are on the average within 15 MeV of the experimental masses in the least-squares sense. The physical resonances are not pure states in the $(SU(3),SU(2)_S)$ basis, except for the nucleonic resonances, which are essentially unmixed. The mixing parameters which we find are used in the following article[11] to analyze the decays of these resonances. *The strong mixings in the* $(SU(3),SU(2)_S)$ *basis among the strange* $J^P=\frac{1}{2}^-$ and $\frac{3}{2}^-$ *resonances mean that the Gell-Mann–Okubo mass formula will not be valid for single octets and decuplets*, although the consequences of octet dominance are valid for the $(\mathbf{70},1^-)$ supermultiplet as a whole, *and therefore it makes no sense to try to group these particles into octets and decuplets.*

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[11] D. R. Divgi, following paper, Phys. Rev. **175**, 2027 (1968).