Pion Photoproduction, NN Scattering, and Photoproduction Sum Rules*

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The differential cross sections for pion photoproduction have been examined along with those for $pn \to np$ and $p\bar{p} \to n\bar{n}$ scattering. It is found that an $M=1$ pion parity-doublet fit is consistent with both sets of data if a full dynamical zero in the $N\bar{N}\pi$ vertex function is hypothesized. If a square-root-type dynamical zero is postulated, some problems with consistency between fits arise. In the former case, the zero is at $t_0 \approx -0.05$ GeV^2. In both fits the $M=0$ ρ, A_2, and B trajectories are introduced. The possibility of an $M=1$ parity-doublet-type conspiracy for the B trajectory has also been investigated qualitatively. This assignment is suggested by a B-trajectory photoproduction finite-energy sum rule and by consistency requirements between phenomenological fits and the Bietti-Roy-Chu pion-photoproduction sum rule which predicts $t_0 \approx -0.03$ GeV^2. Additional experimental tests for an $M=1$ B trajectory are proposed.

INTRODUCTION

It has been known for some time that the differential cross sections for positive-pion photoproduction[1] show a marked forward peak very close to $t=0$, similar to the peak found in np charge exchange, with a width close to μ^2. A number of people have conjectured that an $M=1$ type conspiracy involving a pion parity doublet would prove to be successful, as it was in np charge exchange,[2] and preliminary fits have been made for small t.[3] We give an account here of a more detailed fit of photoproduction data from 2.6 to 16 GeV and t ranging up to -0.5 GeV^2. We find that the $M=1$ parity doublet (the pion π and its parity doublet partner π') provides a satisfactory explanation of the data if the ρ, A_2, and B trajectories are also included (as they were in Ref. 2). These latter trajectories are all assumed to be $M=0$ trajectories with the $BN\bar{N}$ residue vanishing at $t=0$. (The B parent trajectory is completely neglected here, i.e., we assume that it decouples completely from the $N\bar{N}$ and $\gamma\pi$ channels.) The only other known meson trajectory that could be exchanged here is the A_1 trajectory. Although the A_1 trajectory (with an $M=0$ assignment) seems necessary to fit certain resonance production data,[4] we do not include it here. Thus the fits here are consistent with the assumption of zero (or small) $A_1N\bar{N}$ and $A_1\gamma\pi$ couplings.

The question of the order of the zero in the pion residue function is also investigated (this dynamical zero is denoted here by t_0). We find that the assumption of a full zero in the vertex function is preferred over that of a square-root-type zero in the above model. Constraints involving factorization have been imposed from previous fits,[5] and a fit assuming the existence of a double zero in the pion $N\bar{N} \to N\bar{N}$ residue function has been carried out for the reactions $np \to pn$ and $p\bar{p} \to n\bar{n}$. This double zero occurs around $t_0 \approx -2.5\mu^2$ rather than $t_0 \approx -\mu^2$ as is the case if a single zero is assumed.

We have also qualitatively investigated the possibility that the B trajectory is an $M=1$ rather than an $M=0$ object. For some time people have speculated about the possibility of the B trajectory conspiring with an as yet unknown trajectory, usually denoted by ρ', from certain high-energy data.[6] Here we find evidence from two sources that this may be the case. The first is a photoproduction sum rule for the B trajectory, similar to the Bietti-Roy-Chu sum rule for the pion trajectory.[7] It was found that there was evidence from this sum rule for a conspiring pion with a zero in the pion residue at $t_0 \approx -1.5\mu^2$, qualitatively (but not exactly) consistent with phenomenological fits of the data. We perform a similar calculation using the small photoproduction isoscalar amplitudes for the B trajectory and find similar results; the B residue is small and nonvanishing at $t=0$ with a zero displaced by about $5\mu^2$. The second source comes from the pion photoproduction and NN data, relying on the Regge fits. The small $M=1$ B amplitude suggested by the sum rule seems inconsistent with the large $M=0$ B amplitude found in the fit. Further, if one demands consistency of the posi-

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[1] A. M. Boyarski, F. Bulos, W. Busza, R. Diebold, S. D. Ecklund, G. E. Fischer, J. Rees, and B. Richter, Phys. Rev. Letters **20**, 300 (1968); G. Buschhorn, P. Heide, U. Kötz, R. Lewis, P. Schmüser, and H. Skronn, Phys. Letters **25B**, 622 (1967); Z. Bar-Yam, J. de Pagter, M. Hoenig, W. Kern, D. Luckey, and L. Osborne, Phys. Rev. Letters **19**, 40 (1967).
[2] F. Arbab and J. W. Dash, Phys. Rev. **163**, 1603 (1967). In this paper the values of $g^2/4\pi$ quoted should all be multiplied by a factor $1.6=(0.389)^{-1/2}$; see also R. J. N. Phillips, Nucl. Phys. **B2**, 394 (1967).
[3] M. B. Halpern, Phys. Rev. **160**, 1441 (1967); S. Frautschi and L. Jones, Phys. Rev. **163**, 1820 (1967); J. Ball, W. Frazer, and M. Jacob, Phys. Rev. Letters **20**, 518 (1968); K. Dietz and W. Korth, Phys. Letters **26B**, 394 (1968); F. Cooper, Phys. Rev. Letters **20**, 643 (1968); D. Amati, G. Cohen-Tannoudji, R. Jengo, and Ph. Salin, Phys. Letters **26B**, 510 (1968).
[4] F. Arbab and R. Brower, Lawrence Radiation Laboratory Report (unpublished). Since an $M=0$ A_1-type conspiracy does not contribute to photoproduction at $t=0$ in leading order in s, we expect it to play a secondary role here in any case, although it may be important at large t. Further, we remark that both types of zeros in the $\pi N\bar{N}$ vertex function are compatible with the resonance-production data.
[5] F. Arbab, N. F. Bali, and J. W. Dash, Phys. Rev. **158**, 1515 (1967). Unfortunately, the total cross sections in these fits were miscalculated by a factor of 2; however, these fits were not sensitive to these cross sections (which have large errors).
[6] See, e.g., L. Sertorio and M. Toller, Phys. Rev. Letters **19**, 1146 (1967); R. F. Sawyer, *ibid.* **19**, 137 (1967).
[7] A. Bietti, P. DiVecchia, F. Drago, and M. L. Paciello, Phys. Letters **26B**, 457 (1968); D. P. Roy and Shu-Yuan Chu, University of California-Riverside Report, 1968 (unpublished).

TABLE I. Definition of $G_{ij}(t)$ and the full residues $\beta_{ij}(t)$.

Factors $G_{ij}(t)$ in Eq. (2)

$$G_{SR}(t)=\begin{Bmatrix}\alpha & (\text{Chew }\rho\text{ with }M=0)\\ \alpha & (\text{Gell-Mann }A_2\text{ with }M=0)\\ \alpha & (\text{Gell-Mann }\pi'\text{ with }M=1)\end{Bmatrix},\qquad G_{NR}(t)=\begin{Bmatrix}\alpha t & (\rho)\\ t & (A_2)\\ 1 & (\pi')\end{Bmatrix},$$

$$G_{0I}(t)=\begin{Bmatrix}\alpha t(1-t/\mu^2) & (B\text{ with }M=0)\\ \alpha(1-t/t_0) & (\pi\text{ with }M=1)\end{Bmatrix},\qquad G_{1I}(t)=\begin{Bmatrix}\alpha & (\text{Chew }A_1)\\ 1 & (\text{Gell-Mann }A_1)\end{Bmatrix}\begin{Bmatrix}t\ (\text{uncoupled}, M=0)\\ 1\ (\text{coupled}, M=0)\end{Bmatrix}$$

$X(\alpha)=(1+\alpha)\Gamma(\alpha+1)/(\sqrt{\pi})(2\alpha+1)\Gamma(\alpha+\frac{1}{2})$. Connection of the full residue functions $\beta_{ij}(t)$ with the functions $\bar{\gamma}_{ij}(t)$ in Eq. (2).

Trajectories	Full residue	$N\bar{N}$ vertex	$\gamma\pi$ vertex
$\begin{Bmatrix}\rho\\ A_2\\ \pi'\end{Bmatrix}$	$\beta_{SR}=$	$\left[(\sqrt{X})p^{\alpha-1}\begin{Bmatrix}1\\ \sqrt{\alpha}\\ \sqrt{\alpha}\sqrt{t}\end{Bmatrix}\right]$	$\left[(\sqrt{X})k^{\alpha}(\alpha+1)^{1/2}\begin{Bmatrix}(\sqrt{\alpha})\sqrt{t}\\ \sqrt{t}\\ 1\end{Bmatrix}\right]\bar{\gamma}_{SR}$
$\begin{Bmatrix}\rho\\ A_2\\ \pi'\end{Bmatrix}$	$\beta_{NR}=$	$\left[(\sqrt{X})p^{\alpha-1}(\alpha+1)^{1/2}\begin{Bmatrix}\sqrt{\alpha}\sqrt{t}\\ \sqrt{t}\\ 1\end{Bmatrix}\right]$	$\left[(\sqrt{X})k^{\alpha}(\alpha+1)^{1/2}\begin{Bmatrix}(\sqrt{\alpha})\sqrt{t}\\ \sqrt{t}\\ 1\end{Bmatrix}\right]\bar{\gamma}_{NR}$
$\begin{Bmatrix}B\\ \pi\end{Bmatrix}$	$\beta_{0I}=$	$\left[(\sqrt{X})p^{\alpha}\begin{Bmatrix}\sqrt{t}\\ (1-t/t_0)^{1/2}\text{ or }1\end{Bmatrix}\right]$	$\left[(\sqrt{X})k^{\alpha-1}[\alpha(\alpha+1)]^{1/2}\begin{Bmatrix}(1-t/\mu^2)\\ t^{-1/2}(1-t/t_0)^{1/2}\text{ or }0\end{Bmatrix}\right]\gamma_{0I}$
$\begin{Bmatrix}A_1\\ A_1\end{Bmatrix}$	$\beta_{1I}=$	$\left[(\sqrt{X})p^{\alpha}(\alpha+1)^{1/2}\begin{Bmatrix}\sqrt{t}\\ 1\end{Bmatrix}\begin{Bmatrix}\sqrt{\alpha}\\ 1\end{Bmatrix}\right]$	$\left[(\sqrt{X})k^{\alpha}(\alpha+1)^{1/2}\begin{Bmatrix}t\\ \sqrt{t}\end{Bmatrix}\begin{Bmatrix}\sqrt{\alpha}\\ 1\end{Bmatrix}\right]\bar{\gamma}_{1I}$

tion of the zero in the pion residue function found in these fits with the Bietti-Roy-Chu sum rule, an $M=1$ B trajectory is preferred over $M=0$. Experimental tests involving $p\bar{p}\to n\bar{n}$, $\gamma n\to\pi^- p$, and $\pi N\to\omega N^*$ reactions at small t are proposed to make a quantitative determination possible.

In Sec. I we give a brief account of the pion photoproduction formalism. In Sec. II we describe the data and the fits. Section III describes the photoproduction B sum rule. Section IV is concerned with qualitative remarks designed to support an $M=1$ assignment for the B trajectory. The Appendix contains some remarks about photoproduction kinematics and conspiracies.

I. FORMALISM FOR PION PHOTOPRODUCTION

We define our s and t channels as

$$s:\gamma+N\to\pi+N\,,$$
$$t:\gamma+\pi\to\bar{N}+N\,.$$

We next define t-channel parity-conserving kinematic-singularity-free helicity amplitudes by the formulas

$$\begin{aligned}
F_1{}^t&=\frac{1}{\sin\theta_t}(f_{++,1}{}^t+f_{--,1}{}^t)\frac{1}{t-\mu^2}\,,\\
F_2{}^t&=\frac{1}{\sin\theta_t}(f_{++,1}{}^t-f_{--,1}{}^t)[t/(t-4m^2)]^{1/2}\,,\\
F_3{}^t&=\left(\frac{f_{+-,1}{}^t}{1+z_t}+\frac{f_{-+,1}{}^t}{1-z_t}\right)\frac{t^{1/2}}{t-\mu^2}\,,\\
F_4{}^t&=\left(\frac{f_{+-,1}{}^t}{1+z_t}-\frac{f_{-+,1}{}^t}{1-z_t}\right)\frac{1}{(t-\mu^2)(t-4m^2)^{1/2}}\,,
\end{aligned}\tag{1}$$

where $z_t=(s+\frac{1}{2}t-m^2-\frac{1}{2}\mu^2)/2kp=\nu/2kp$, $k=(t-\mu^2)/2t^{1/2}$, $p=\frac{1}{2}(t-4m^2)^{1/2}$. The pion contributes to $F_2{}^t$ only while sense-nonsense coupled triplet states contribute only to $F_1{}^t$. $F_3{}^t$ and $F_4{}^t$ in leading order are composed of nonsense-nonsense coupled triplet amplitudes and uncoupled triplet amplitudes, respectively.

The Reggeization of the parity-conserving amplitudes yields

$$\begin{aligned}
F_1{}^t&=\sum_i\frac{(1+\alpha_i)(1\pm e^{-i\pi\alpha_i})}{2\sin\pi\alpha_i}\bar{\gamma}_{SR}{}^i(t)G_{SR}{}^i(t)\left(\frac{\nu}{\nu_0}\right)^{\alpha_i-1},\\
F_2{}^t&=\sum_i\frac{(1+\alpha_i)(1\pm e^{-i\pi\alpha_i})}{2\sin\pi\alpha_i}\bar{\gamma}_{0I}{}^i(t)G_{0I}{}^i(t)\left(\frac{\nu}{\nu_0}\right)^{\alpha_i-1},\\
F_3{}^t&=\sum_i\frac{(1+\alpha_i)(1\pm e^{-i\pi\alpha_i})}{2\sin\pi\alpha_i}\Big[\alpha_i\bar{\gamma}_{NR}{}^i(t)G_{NR}{}^i(t)\\
&\qquad-\frac{1}{\nu}(\alpha_i-1)(t-\mu^2)(t-4m^2)\bar{\gamma}_{1I}{}^i(t)G_{1I}{}^i(t)\Big]\left(\frac{\nu}{\nu_0}\right)^{\alpha_i-1},\\
F_4{}^t&=\sum_i\frac{(1+\alpha_i)(1\pm e^{-i\pi\alpha_i})}{2\sin\pi\alpha_i}\Big[\alpha_i\bar{\gamma}_{1I}{}^i(t)G_{1I}{}^i(t)\\
&\qquad-\frac{1}{\nu}(\alpha_i-1)\frac{t-\mu^2}{t}\bar{\gamma}_{NR}{}^i(t)G_{NR}{}^i(t)\Big]\left(\frac{\nu}{\nu_0}\right)^{\alpha_i-1},
\end{aligned}\tag{2}$$

where $\nu_0\equiv1$ GeV^2.

The residue functions $\bar{\gamma}_{ij}(t)$ have been given labels descriptive of the vertices. We label the singlet, uncoupled triplet, sense coupled triplet, and nonsense coupled triplet $N\bar{N}X$ vertices by 0, 1, S, N, and the regular $[P=(-1)^J]$ and irregular $[P=(-1)^{J+1}]$ $\gamma\pi X$

vertices by R and I. The residues may contain powers of α or t depending on the ghost-killing mechanisms and $t=0$ coupling schemes,[2,5] which are denoted by $G_{ij}(t)$ (Table I). The connection of the $\bar{\gamma}_{ij}$ with factorizable residues β_{ij} is given in Table I.

The cross section in the s channel in terms of helicity amplitudes is given by

$$\frac{d\sigma}{dt}=\frac{389.5}{2\pi(s-m^2)^2}(|f_{++,1}{}^t|^2+|f_{--,1}{}^t|^2+|f_{+-,1}{}^t|^2+|f_{-+,1}{}^t|^2)\mu b\ \mathrm{GeV}^{-2}, \quad (3a)$$

or, in terms of the parity-conserving amplitudes,

$$\frac{d\sigma}{dt}=\frac{389.5}{4\pi(s-m^2)^2}\Bigg[(z_t^2-1)\left((\mu^2-t)^2|F_1{}^t|^2+\frac{4m^2-t}{-t}|F_2{}^t|^2\right)+(z_t^2+1)\left(\frac{(\mu^2-t)^2}{-t}|F_3{}^t|^2+(4m^2-t)(\mu^2-t)^2|F_4{}^t|^2\right)+4z_t[(4m^2-t)/-t]^{1/2}(\mu^2-t)^2\,\mathrm{Re}(F_3{}^{t*}F_4{}^t)\Bigg]\times\mu b\ \mathrm{GeV}^{-2}. \quad (3b)$$

At $t=0$ we get the additional constraint arising from the required analyticity properties of the amplitudes,

$$2mF_2{}^t(s,0)=\mu^2F_3{}^t(s,0). \quad (4a)$$

Note that this constraint removes the apparent singularity in $d\sigma/dt$ at $t=0$. In terms of the $M=1$ parity doublet conspiracy between the π and π' we obtain the following relation between the residue functions $\bar{\gamma}_{0I}{}^{\pi}$ and $\bar{\gamma}_{NR}{}^{\pi'}$:

$$\bar{\gamma}_{NR}{}^{\pi'}(0)=-(2m/\mu^2)\bar{\gamma}_{0I}{}^{\pi}(0). \quad (4b)$$

Moreover, the first daughter one unit below the parity doublet must have a singular residue

$$\gamma_{1I}{}^d(t)\propto-(\mu^2/t)\bar{\gamma}_{NR}{}^{\pi'}(t)\,|_{t\to0}$$

that is correlated with the π' residue to avoid a singularity of $1/t$ in $F_4{}^t(0)$. Indeed, this condition is the result of the pseudothreshold relation found by Ball, Frazer, and Jacob[3] for the use of unequal-mass baryons (see Appendix).

The gauge invariance relation giving the pion-nucleon coupling constant for π^+ photoproduction is

$$\lim_{t\to\mu^2}(t-\mu^2)[f_{++,1}{}^t(s,t)-f_{--,1}{}^t(s,t)]=-\mu^2eg. \quad (5a)$$

Hence we obtain the connection between $g^2/4\pi$ and $\bar{\gamma}_{0I}{}^{\pi}$ for π^+ photoproduction:

$$g^2/4\pi=\frac{1}{4\pi}\left(\frac{2\bar{\gamma}_{0I}{}^{\pi}(\mu^2)(1-\mu^2/t_0)}{e\pi\mu^2}\right)^2, \quad (5b)$$

where $\bar{\gamma}_{0I}{}^{\pi}(\mu^2)=a_\pi e^{b_\pi\mu^2}$ (see Table II). The relation

TABLE II. Parameters for fits with full $N\bar{N}\pi$ vertex zero at $t_0=-0.05$.

Parameters fixed from meson-nucleon scattering.[a]

$$\alpha_\rho=0.58+1.11t$$
$$\alpha_{A2}=0.5+0.86t$$
$$b_{12}{}^{\rho}/b_{11}{}^{\rho}=\bar{\gamma}_{NR}{}^{\rho}/\bar{\gamma}_{SR}{}^{\rho}=-8.8e^{0.4t}$$
$$b_{12}{}^{A2}/b_{11}{}^{A2}=\bar{\gamma}_{NR}{}^{A2}/\bar{\gamma}_{SR}{}^{A2}=3.5e^{-0.11t}$$

Parameters obtained in nucleon-nucleon fit (notation and data normalizations correspond to Table II in Ref. 2. Residue units are $\mathrm{mb}^{1/2}$.

$\chi^2=89$ for 74 points	
$\alpha_\pi=-0.025+1.25t$	$\gamma_0{}^B=-800t(\alpha_B+2)e^{10t}$
$\alpha_{\pi'}=-0.025+t$	$\gamma_0{}^{\pi}=0.919(1+t/0.05)^2e^{11t}$
$\alpha_B=-0.4+0.9t$	$\gamma_{22}{}^{\pi'}=[b_0{}^{\pi}(0)/\alpha_\pi(0)]e^{4.8t}$
$\gamma_{11}{}^{\rho}=0.35e^{-4.4t}$	$\gamma_{12}{}^{\pi'}=-68(\alpha_{\pi'})^{1/2}e^{2.2t}$
$\gamma_{11}{}^{A_2}=1.8e^{11t}$	$g^2/4\pi=14.7$

Parameters obtained in π^+ photoproduction fit.

$\chi^2=66$ for 62 points	
$\alpha_B=-0.4+0.95t$	$\bar{\gamma}_{0I}{}^{\pi}=-0.078e^{9.1t}$
$\bar{\gamma}_{SR}{}^{\rho}=0.166e^{-9.0t}$	$\bar{\gamma}_{NR}{}^{\pi'}=-(2m/\mu^2)\bar{\gamma}_{0I}{}^{\pi}(0)e^{3.7t}$
$\bar{\gamma}_{SR}{}^{A_2}=0.77e^{-1.1t}$	$\bar{\gamma}_{SR}{}^{\pi'}/\bar{\gamma}_{NR}{}^{\pi'}=1.86e^{-2.6t}$
$\bar{\gamma}_{0I}{}^{B}=-2.95e^{11t}$	$g^2/4\pi=15.4$

[a] Reference 5.

(5a) also requires a factor of $t-\mu^2$ in the B residue; otherwise the B would contribute to the pion pole.

The constraint arising from factorization on the π' residue function from nucleon-nucleon fits is given by

$$(\bar{\gamma}_{SR}{}^{\pi'}/\bar{\gamma}_{NR}{}^{\pi'})_{\mathrm{photoprod}}=(\gamma_{12}{}^{\pi'}/(\alpha_{\pi'})^{1/2}\gamma_{22}{}^{\pi'})_{NN}, \quad (6)$$

where $\gamma_{12}{}^{\pi'}$ and $\gamma_{22}{}^{\pi'}$ are the same functions listed in Table II of Ref. 2.

Finally, we remark on an amusing connection between the cross section calculated from the gauge-invariant Born term and that calculated by using the $M=1$ π-π' conspiracy, assuming $t_0=\mu^2$. Namely, for small t and large s the Regge contribution is equivalent to the Born approximation. Satisfying the normalization condition and the conspiracy condition with the residues $\beta_\pi(t)\approx\frac{1}{2}eg(1+t/\mu^2)$ and $\beta_{\pi'}(t)\approx\frac{1}{2}eg$, one obtains

$$\left(\frac{d\sigma}{dt}\right)_{\mathrm{Regge}}\approx\frac{389.5}{4\pi(s-m^2)^2}\left(\left|\frac{\beta_\pi(t)}{1-t/\mu^2}\right|^2+|\beta_{\pi'}(t)|^2\right)\mu\mathrm{b}\ \mathrm{GeV}^{-2}=\frac{389.5}{4\pi(s-m^2)^2}\frac{e^2g^2}{2}\frac{1+(t/\mu^2)^2}{(1-t/\mu^2)^2}\times\mu b\ \mathrm{GeV}^{-2}=\left(\frac{d\sigma}{dt}\right)_{\mathrm{Born}}. \quad (7)$$

II. DATA AND FITS FOR $\gamma p\to\pi^+p$, $np\to pn$, AND $p\bar{p}\to n\bar{n}$ SCATTERING

A. Data

The photoproduction data used[1] were positive pion photoproduction data at 2.6, 2.7, 3.4, 3.7, 5, 8, 11, and 16 GeV/c lab momentum. Reliable high-energy nega-

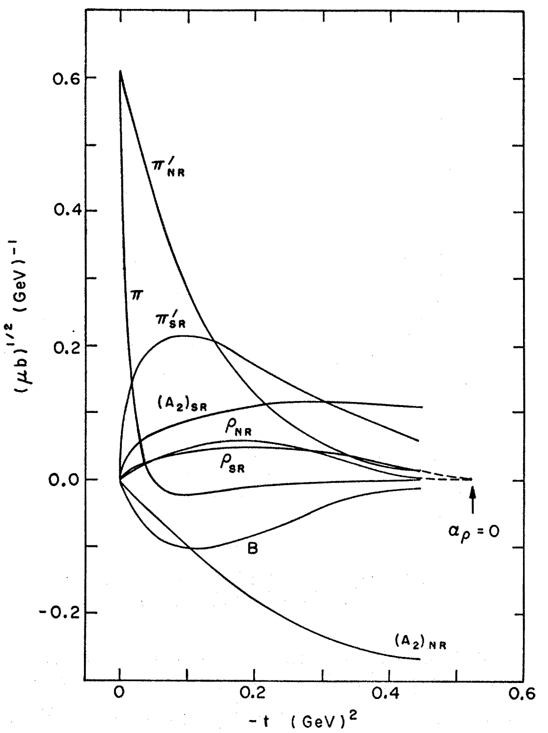

FIG. 1. Real parts of the π^+ photoproduction amplitudes at 8 GeV/c for full $N\bar{N}\pi$ vertex zero $t_0=-0.05$. To leading order,

$$d\sigma/dt|\pi^+{}_{\text{photoprod}}=2|\pi\pm B|^2+2|\pm\rho_{SR}+(A_2)_{SR}+\pi'_{SR}|^2+2|\pm\rho_{NR}+(A_2)_{NR}+\pi'_{NR}|^2.$$

tive-pion photoproduction data are scarce; we used only one point at 3.4 GeV/c, $t=-0.37$ GeV^2 as a constraint. We have included data up to $t=-0.5$ GeV^2, consistent with the NN fits. We have included the possibility of systematic errors quoted by the experimentalists on the order of $\pm5\%$. In all, 62 photoproduction data points were used.

The $np\to pn$ and $p\bar{p}\to n\bar{n}$ data were described in Ref. 2. In all, 74 data points were used.

B. Parametrization of Pion Photoproduction Fit

The most important part of the parametrization of the pion residue function is the zero at t_0. If one makes the assumption that the zero in the $N\bar{N}\to N\bar{N}$ pion residue is a single zero (i.e., a square-root-type zero in the $N\bar{N}\pi$ vertex), then the square-root zero must propagate throughout all vertex functions of the form $X\bar{Y}\pi$. If, however, we assume that there is a full zero in the $N\bar{N}\pi$ vertex, and thus a double zero in the $N\bar{N}\to N\bar{N}$ pion residue, only reactions involving $N\bar{N}$ need have the zero. (Of course there is nothing to prevent any other $X\bar{Y}\pi$ vertex function from having such a zero, but it is then not required to be in any specific place.) While the origin and full content of the zero is not well understood, it seems to have some connections with the hypothesis of partially conserved

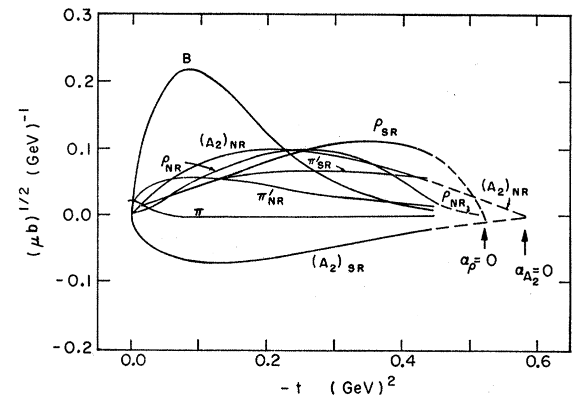

FIG. 2. Imaginary parts of the π^+ photoproduction amplitudes at 8 GeV/c for full $N\bar{N}\pi$ vertex zero $t_0=-0.05$.

axial-vector current,[8] and recent work of Toller[9] indicates that, on group-theoretic grounds, the hypothesis of a full zero in the $N\bar{N}\pi$ vertex function is preferred over that of a square-root-type zero. Since earlier fits to NN scattering assumed the square-root-type vertex zero, we have fit the NN data with the full $N\bar{N}\pi$ vertex-function-zero hypothesis and find that the zero is then required to be at around $t_0=-2.5\mu^2$ rather than at $-\mu^2$. The photoproduction pion residue function has a single zero in any case (we assume nothing about the $\gamma\pi\pi$ vertex in the full-vertex-zero case). We find that consistent fits to all data can be obtained with the full-vertex-zero hypothesis but that some discrepancy exists between the values of $g^2/4\pi$ obtained in the NN and photoproduction fits if the square-root vertex zero is assumed.

The parametrization of all trajectories and residue functions was made consistent with meson-nucleon and nucleon-nucleon fits.[2,5] The π, π', ρ, and A_2 trajectories were considered fixed and the B trajectory slope was assumed unknown. Factorization from meson-nucleon fits constrained the ρ and A_2 residues, which were taken to have the Chew and Gell-Mann ghost-killing mechanisms at $\alpha=0$, respectively. Thus this fit violates ρ-A_2 exchange degeneracy in this respect. The π' was made to choose nonsense at $\alpha=0$, and its residues were constrained through factorization with the NN fit and the conspiracy equation. Altogether three couplings (ρ, A_2, B), five exponentials, one trajectory (B) slope, and the zero t_0 were used as variables. In addition, the 2.6, 5, 8, 11, and 16 GeV/c data were allowed to have systematic errors of less than $\pm7\%$.

C. Fits

Photoproduction fits for both cases of a full vertex zero and a square-root-type vertex zero were obtained. The parameters obtained in the former case are listed in Table II. The amplitudes are pictured in Figs. 1 and

[8] S. Mandelstam, Phys. Rev. **168**, 1884 (1968).

[9] M. Toller, invited talk at the Fifth Gables Conference, 1968 (unpublished).

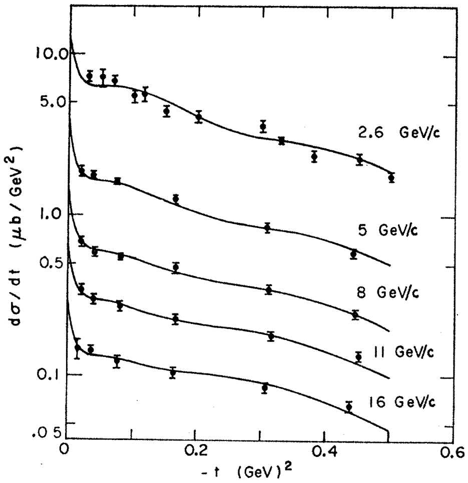

FIG. 3. π^+ photoproduction fit. Curves have been multiplied by 0.99, 1.03, 1.03, 0.97, and 0.93, respectively, for 2.6, 5, 8, 11, and 16 GeV/c.

2, and the fit itself is pictured in Figs. 3 and 4. Although little effort has been made to test the nonuniqueness of the fits, it is probably true that they are not unique, so that these parameters should not be regarded quantitatively too seriously. We note in passing that the small ρ amplitudes found here seem consistent with the result of small $\gamma\pi\rho$ coupling found in photoproduction dispersion-relation calculations.

A fit to the np-pn and $p\bar{p}$-$n\bar{n}$ data was obtained with the assumption of a full $N\bar{N}\pi$ vertex zero, and these parameters are also presented in Table II. The notation used is that of Ref. 2. The amplitudes are pictured in Figs. 8 and 9. Fits with the square-root zero at various locations were also obtained, and will be discussed below.

The best photoproduction fit for the square-root vertex-zero case was obtained with $\chi^2=73$ for 62 points and a value of $g^2/4\pi=16.8$, in some disagreement with the value of $g^2/4\pi=13$ obtained in the NN fit for this value of t_0 ($t_0=-0.027$). Fits with larger values of t_0 tend to decrease $g^2/4\pi$ for photoproduction faster than $g^2/4\pi$ for NN scattering, so that the farther out we move t_0 the closer we come to consistency. However, for $t_0=-0.034$ we obtain $g^2/4\pi=15$ and 11.7, respectively, for γp and NN scattering; we cannot move t_0 farther out and retain an acceptable value of $g^2/4\pi$ for NN scattering. On the other hand, moving the zero in to $t_0=-0.018$ only raises $g^2/4\pi$ to 15.7 in NN scattering. Thus some inconsistency seems to exist. This discrepancy may, however, not be serious, since we cannot be sure that there are no other $M=1$ conspiring parity doublets (e.g., B-ρ'). If there were, one could put a zero at some $t_B\neq 0$ into the B residue function, and the data could then be fit with a wide range of values for t_0 since the coupling of the pion would then no longer be constrained at $t=0$. We discuss this point more fully in Sec. IV.

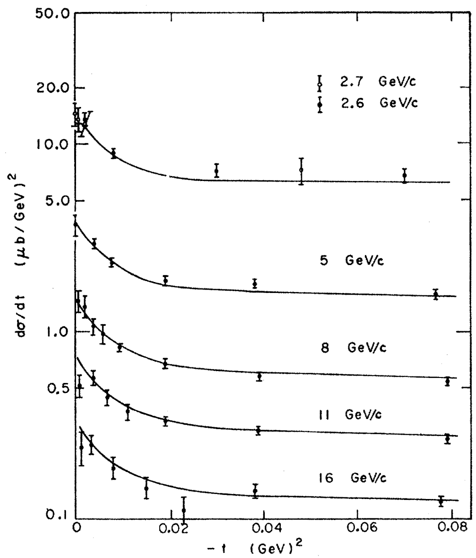

FIG. 4. Small-t region for π^+ photoproduction fit with the same normalization factors.

The best photoproduction fit for the full-vertex-zero case was obtained with $\chi^2=66$ for 62 points (not significantly different from the previous case). With a value of $t_0=-0.05$, nearly equal values of $g^2/4\pi\doteq 15.4$ and 14.7 were obtained for photoproduction and NN scattering, respectively. Thus problems of consistency do not seem to arise if the zero is assumed to be a full vertex zero.

The value of the π^-/π^+ cross-section ratio at 3.41 GeV/c, $t=-0.37$ GeV^2 is measured to be $0.73/2.1=0.35$. We obtain $\sigma(\pi^-)/\sigma(\pi^+)=0.87/1.5=0.57$ for both types of zeros, giving a total χ^2 of about 3 for the π^- and π^+ cross sections in each case.

The photoproduction data can be fit well only out to about $t=-0.5$ GeV^2 with the models assumed here. Past this point, the data show a break which we do not quantitatively reproduce. This break may be related to the structure in the $p\bar{p}\to n\bar{n}$ cross sections past $t=-0.5$, which the NN fits could not quantitatively describe. It is possible that the inclusion of other trajectories (e.g., an $M=1$ ρ' or some amount of A_1) could be used to affect quantitative reproduction of the data.

III. *B* TRAJECTORY PHOTOPRODUCTION SUM RULE

We begin by writing the sum rule, a positive-moment sum rule for the even-ν part of the t-channel photoproduction amplitude which contains the B (but not the π) trajectory. This t-channel amplitude is proportional to the photon isoscalar amplitude which in CGLN[10] notation is (we use CGLN's ν in this section)

$$\tilde{A}(\nu,t)\equiv A_1^{(0)}(\nu,t)+tA_2^{(0)}(\nu,t)\,. \qquad (8)$$

[10] G. F. Chew, M. L. Goldberger, F. E. Low, and Y. Nambu, Phys. Rev. **106**, 1345 (1957), hereafter referred to as CGLN.

The sum rule is then

$$0=\frac{1}{\pi}\int_{\nu_t}^{N}\nu\,\mathrm{Im}\tilde{A}(\nu,t)d\nu+\nu_B\frac{eg}{4M}\frac{t+\mu^2}{t-\mu^2}-\frac{1}{\pi}\bar{\gamma}_{0I}{}^B(t)R(t)\frac{N^{\alpha_B+1}}{\alpha_B+1},\quad(9)$$

where $\bar{\gamma}_{0I}{}^B(t)$ is the same residue used in the photoproduction fit,

$$-4M\nu_B=\mu^2-t,\ 4M\nu=s-u,$$

and

$$R(t)=2\alpha_B(1+\alpha_B)(2+\alpha_B)(t/\mu^2)(2m)^{\alpha_B-1}/(0.389)^{1/2}.$$

We evaluate $A(\nu,t)$ by writing its multipole expansion formally as

$$\tilde{A}(\nu,t)=\sum_i \mathfrak{M}_i(\nu,t)+\frac{eg}{4M}\frac{t+\mu^2}{t-\mu^2}\left(\frac{1}{-\nu+\nu_B}+\frac{1}{\nu+\nu_B}\right),\quad(10)$$

where the multipole sum has the (real) Born term explicitly removed. The sum $\sum_i \mathfrak{M}_i(\nu,t)$ is given in CGLN through the multipole expansion of $\mathfrak{F}_i{}^{(0)}$, where

$$\begin{aligned}\tilde{A}(\nu,t)=&(4\pi/k)[(M+E_1)/(M+E_2)]^{1/2}\mathfrak{F}_i{}^{(0)}(\nu,t)\\&-(4\pi/q)[(M+E_2)/(M+E_1)]^{1/2}\mathfrak{F}_2{}^{(0)}(\nu,t)\\&+\frac{4\pi}{q}\frac{(Wt-M\mu^2)\mathfrak{F}_3{}^{(0)}(\nu,t)}{(W-M)^2[(M+E_2)(M+E_1)]^{1/2}}\\&-(4\pi/2kWq^2)(Wt+M\mu^2)\\&\times[(M+E_2)/(M+E_1)]^{1/2}\mathfrak{F}_4{}^{(0)}(\nu,t).\quad(11)\end{aligned}$$

The final form of the sum rule is thus

$$\frac{eg}{(4M)^2}(t+\mu^2)+\frac{1}{\pi}\int_{\nu_t}^{N}\nu\,\mathrm{Im}\sum_i \mathfrak{M}_i(\nu,t)d\nu=\frac{1}{\pi}\bar{\gamma}_{0I}{}^B(t)R(t)\frac{N^{\alpha_B+1}}{\alpha_B+1}.\quad(12)$$

We use the parametrization of the multipoles given by Walker[11] to evaluate the sum $\sum_i \mathfrak{M}_i(\nu,t)$. This parametrization utilizes six resonances and a number of nonresonant parts, which are generally small.

The results of the calculation are presented in Table III, and the integrands of both the Bietti-Roy-Chu and the B-meson sum rules at $t=0$ are plotted in Fig. 5.

It is seen that the B residue is finite at $t=0$ and has a zero at $t_B\approx 5\mu^2$. The implication is that the B trajectory is an $M=1$ trajectory, conspiring with an as yet unknown trajectory usually denoted as ρ'. Before turning to the relevance of this to scattering data, we remark that the form found for the B residue suggests an analogy with the pion residue function and perhaps suggests some correlation between the two trajectories in the sense of exchange degeneracy. If the B trajectory were to pass through the B meson and through zero at $t=0$ the slope α_B' would be 0.7, which is not unreasonable.

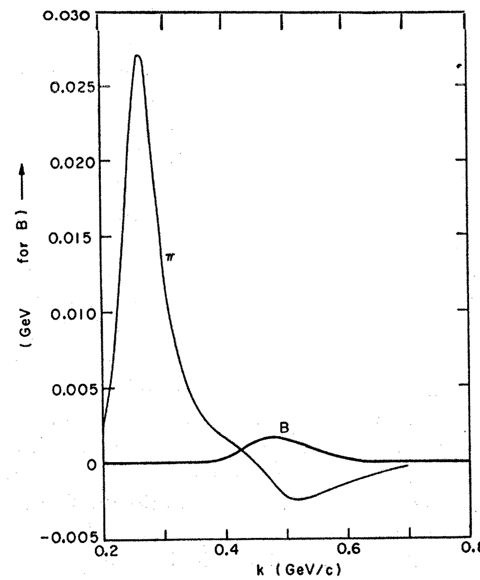

FIG. 5. Integrands at $t=0$ for the B and pion photoproduction sum rules $[(-\mu^2\nu/\pi)\ \mathrm{Im}A_1{}^{(0)}$ and $(-\mu^2/\pi)\ \mathrm{Im}\tilde{A}_1{}^{(-)}$, respectively].

We comment next on the reliability of the positive-moment sum rule. First, we note a deficiency of the B sum rule that the corresponding pion sum rule does not possess. First the Born term here is depressed by a factor $t-\mu^2$ relative to the pion sum rule, so that the inherent stability of the pion sum rule due to a large Born term is lost. Secondly, the small isoscalar amplitude is presumably not too reliably determined, since it involves cancellation of large and nearly equal resonant amplitudes for π^+ and π^- photoproduction. Thus, if there were important isoscalar resonant contributions at $k>1.2$ GeV/c the sum rule would be inaccurate. We remark, however, that the integrand is positive over the whole region $k=0.2$–1.2 GeV/c; hence to reverse the sign of the integral (thus making the B an $M=0$ trajectory), one would need to undo the total effect of

TABLE III. Results of the B meson sum rule.

Equation (12) reads $\pi^{-1}\bar{\gamma}_{0I}{}^B(t)R(t)N^{\alpha_B+1}/(\alpha_B+1)=B(t)+I(t)$, where $B(t)$ is the Born term. The power-series expansions of $I(t)$ and $B(t)$ around $t=0$ are given by

$$I(t)=-0.027-0.06t+0.06t^2,$$
$$B(t)=+0.0058+0.294t.$$

The residue is zero at $t_B\approx 0.09$.
The contributions [in units of $(-10^3\mu^2)$] to $I(t)|_{t=0}$ are given by

$P_{33}(1238)$	0.01	$S_{11}(1560)$	0.13	nonresonant	0.01
$P_{11}(1470)$	0.21	$D_{15}(1652)$	0.00		
$D_{13}(1520)$	0.21	$F_{15}(1672)$	−0.01		

[11] R. L. Walker, California Institute of Technology, Pasadena, Calif. (private communication).

the first six resonances. Since we are working with a positive-moment sum rule, this is not inconceivable. However, the convergence of the integral over the first six resonances is good even with the positive moment, so the sum rule as it presently stands converges well. Notice that the "duality concept" as advanced by Schmid and Chew,[12] whereby dominant Regge trajectories provide a semilocal average to the energy dependence of the imaginary part of the amplitude at low energies in the resonant region, does not appear to hold in this energy region, since the contribution of the first six resonances to the B sum-rule integrand produces only a wide positive bump over the whole region of integration. In fact, the Bietti-Roy-Chu sum-rule integrand is even worse, being purely positive at momenta $0.2<k<0.7$ GeV/c and negative for $0.7<k<1.2$ GeV/c (see Fig. 5). Thus photoproduction amplitudes at these energies seem to violate the Schmid duality concept, though there is no reason why it should not be valid over a larger energy region. Finally, we remark on the zero in the B residue indicated by the sum rule. The zero is caused by cancellation of the Born term that rapidly increases in t with the nearly constant integral. If we double the integral, the zero moves outward to $t_B=0.24$; if we cut the integral in half, the zero moves in to $t_B=0.06$. Since we cannot reliably estimate the errors on the integral, we cannot really be sure that the zero is not in fact at $t_B=0$ (thus indicating an $M=0$ B trajectory).

We have also investigated the possibility of evaluating the π and B residues using ordinary cutoff dispersion relations. The results are only roughly in agreement with the unsubtracted sum rules, yielding $M=1$ π and B residues without any zeros and with magnitudes at $t=0$ larger than those of the finite-energy sum rules by an order of magnitude. However, the cutoff dispersion relation is satisfied very nearly by the Born term and roughly by the resonances, so that the calculation of the Regge term is inherently inaccurate.

IV. IMPLICATIONS FOR SCATTERING DATA AND THE PION SUM RULE

The actual existence of an $M=1$ B trajectory cannot conclusively be established from experimental evidence. As we have shown, an $M=0$ B trajectory is certainly compatible with the existing data. We argue, however, that an $M=1$ B trajectory is also compatible and perhaps preferred by existing data, but that exhaustive fits using it would be inappropriate until measurements at small t are made of the high-energy cross sections for the processes $p\bar{p}\to n\bar{n}$ and $\gamma n\to\pi^- p$. These measurements should serve to determine the existence of an $M=1$ B trajectory in a model where only the π and B trajectories have $M=1$, since the π-B and π'-ρ' interference terms change sign between the processes

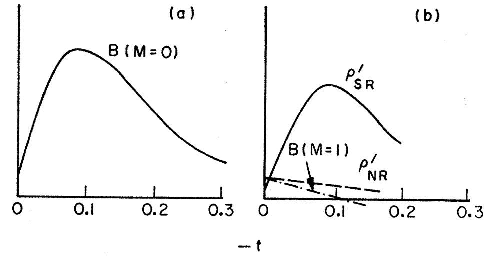

FIG. 6. Conjectured simulation of $M=0$ B amplitude found in photoproduction fit with $M=1$ B, ρ' amplitudes.

$pn\to np$, $p\bar{p}\to n\bar{n}$ and between $\gamma p\to\pi^+ n$, $\gamma n\to\pi^- p$. If the B has the quantum number $M=0$ these interference terms at $t=0$ are zero in all cases. However, for an $M=1$ assignment these interference terms would be nonzero at $t=0$. Further, the small-t behavior of the $p\bar{p}\to n\bar{n}$ reaction also provides a clear way to distinguish the type of zero in the $\pi N\bar{N}$ vertex function.

Another reaction which would be critical in determining the M quantum number of the B would be $\pi N\to\omega N^*$ near $t=0$. Notice that this reaction is the analog of the reaction $\pi N\to\rho N^*$ involving π exchange. Finally, $pn\to np$ polarization measurements near $t=0$ should affect this determination; these measurements are currently in progress.

We now consider the implications of consistency of high-energy data combined with the pion sum rule for an $M=1$ assignment for the B trajectory.

A. Photoproduction

If we take the result of the B sum rule at least as an indication of the magnitude of the B residue, there appears to be a contradiction with the fit. For $|t|>\mu^2$ the fit with $t_0=-0.03$ seems to require at least a factor of 30 times the B contribution given by the sum rule. The $M=0$ B assumed in the $\gamma p\to\pi^+ n$ fit may therefore be interpreted as simulating the effect of a small $M=1$ B amplitude together with the ρ' amplitudes. If we assume small $\pi\gamma B$ and $\pi\gamma\rho'$ couplings, a medium $N\bar{N}B$ and medium $N\bar{N}\rho'$ nonsense coupling, and a large $N\bar{N}\rho'$ sense coupling, the $M=1$ B and ρ' will very nearly simulate the $M=0$ B amplitude assumed in the photoproduction fit, being predominantly equal to the sense-nonsense ρ' amplitude which vanishes at $t=0$ (see Fig. 6).

It is possible that with different ρ or A_2 ghost-killing mechanisms (or the inclusion of some amount of $M=0$ A_1), less $M=0$ B would be required to fit the data. In any case, the π^+ photoproduction fit can surely be made consistent with an $M=1$ B-ρ' conspiracy.

Next, we consider implications of an $M=1$ B trajectory for π^- photoproduction. Assuming the existence of an $M=1$ B trajectory and the zeros indicated by the sum rules in the π and B residues, it is phenomenologically clear that more constructive π-B and $(\rho+\rho')-(\pi'+A_2)$ interferences would give better results for the fit to the π^-/π^+ ratio at moderate t. This, unfor-

[12] C. Schmid, Phys. Rev. Letters **20**, 689 (1968); and G. F. Chew and A. Pignotti, *ibid.* **20**, 1078 (1968).

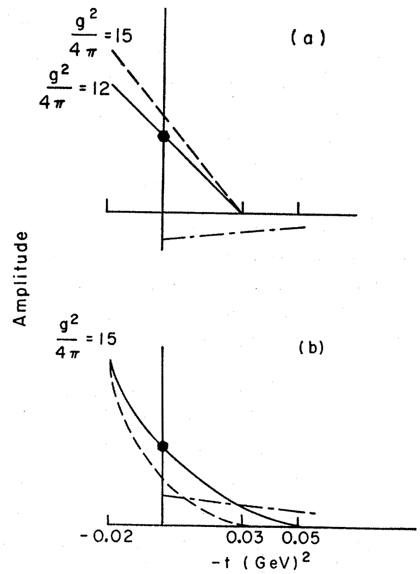

FIG. 7. Conjectured $M=1$ B amplitude and resulting π amplitude for $pn \rightarrow np$ scattering near $t=0$: ——, π amplitude with $M=0$ B assumed; - - -, π amplitude with $M=1$ B assumed; —·—, $M=1$ B amplitude. The mark on the vertical axis yields $(d\sigma/dt)(pn \rightarrow np)=1$ mb at 8 GeV/c.

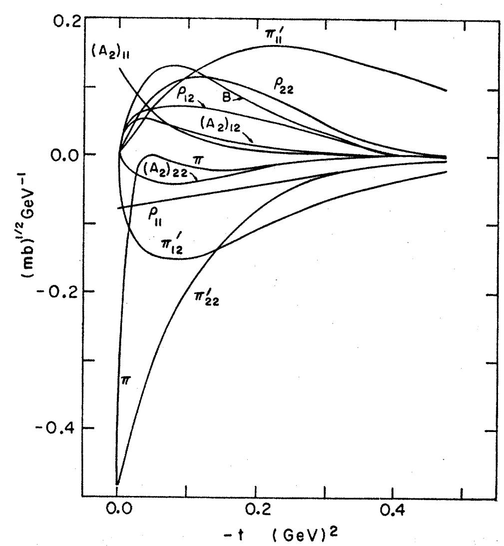

FIG. 8. Real parts of the nucleon-nucleon amplitudes at 8 GeV/c for full $N\bar{N}\pi$ vertex zero $t_0=-0.05$. To leading order,

$$d\sigma/dt|_{pn\rightarrow np,\ p\bar{p}\rightarrow n\bar{n}}=2|\pi\pm B|^2+2|\pm\rho_{11}+(A_2)_{11}+\pi'_{11}|^2 +2|\pm\rho_{22}+(A_2)_{22}+\pi'_{22}|^2+4|\pm\rho_{12}+(A_2)_{12}+\pi'_{12}|^2.$$

To leading order the coupled triplet amplitudes factorize; e.g., $\pi_{11}'\pi_{22}'=-(\pi_{12}')^2$. Note that the (12) amplitudes have additional weight in the cross sections.

tunately, does not predict that the π^-/π^+ ratio near $t=0$ would continue to be small since the ρ and A_2 terms vanish at $t=0$, and these terms are significant at moderate t. Notice that local fluctuations (i.e., maxima or minima) should occur in the π^-/π^+ ratio in the $M=1$ B model when the π or B residues vanish. Notice also that as $t\rightarrow 0$ an $M=1$ B trajectory predicts that the π^-/π^+ ratio would be different from 1, whereas the model utilized in the fit with the $M=0$ B residue vanishing at $t=0$ yields the prediction of a ratio of 1 at $t=0$. Even if the ρ' nonsense and B residues were small as indicated by the sum rule, interference with the large π and π' amplitudes would produce a noticeable effect. Hence a measurement of the π^- photoproduction cross section near $t=0$ would provide a critical test of the ρ'-B conspiracy.

Next we consider π^0 photoproduction. Ader and Capdeville and Braunschweig *et al.*[13] have fitted low-energy π^0 photoproduction data utilizing an $M=0$ B amplitude very similar in magnitude to what our $M=0$ B would yield for π^0 photoproduction at small t (e.g., $t\approx-0.1$). For higher values of t, the ρ amplitudes in our fit would simulate the B amplitudes in these fits (which did not include the ρ). Hence the conjectured simulation of the $M=0$ B by an $M=1$ $B+\rho'$ should fit the π^0 photoproduction data.

Finally, we remark that the presence or absence of polarization in $\pi^-p\rightarrow\pi^0n$ is not critical to any of these arguments, since we may always fit the polarization with a sufficiently small $\rho'\pi\pi$ residue.

B. *NN* Scattering

Next we consider implications of an $M=1$ B-ρ' conspiracy for the $pn\rightarrow np$ and $p\bar{p}\rightarrow n\bar{n}$ reactions. First, suppose that the zero in the pion vertex function ($\pi N\bar{N}$) is of the square-root type. The value of the zero $t_0=-1.5\mu^2$ is consistent in the sum rule and the photoproduction fits, but leads to some inconsistency in the NN fits, since $g^2/4\pi$ in the NN fit turned out to be rather low. However, an $M=1$ B trajectory could easily remove this discrepancy by releasing the constraint on the pion residue at $t=0$, thus allowing a higher value of $g^2/4\pi$ to be obtained in the NN fit via destructive interference of the π with the B at $t=0$ [the ρ' and π' would also interfere destructively (see Fig. 7)]. Notice that since more parameters are introduced in an $M=1$ B fit, the amount of freedom in fitting the NN data actually increases, so there is no doubt that a successful NN fit can be performed. Also note that in this case destructive interference in $pn\rightarrow np$ implies constructive interference in $p\bar{p}\rightarrow n\bar{n}$ so that the $p\bar{p}\rightarrow n\bar{n}$ cross sections should remain larger than the $pn\rightarrow np$ cross sections at $t=0$ if this square-root-type vertex-zero model is correct.

[13] J. P. Ader and M. Capdeville, CERN Report No. TH 803, 1967 (unpublished); M. Braunschweig, W. Braunschweig, D. Husmann, K. Lubelsmeyer, and D. Schmitz, University of Bonn Report No. 1-038, 1968 (unpublished). Very recent preliminary π^0 photoproduction data at 6, 11, and 16 GeV/c seem to indicate a different energy dependence than that of the usual $\omega+B$ model [D. Ritson *et al.*, Stanford Linear Accelerator Center, Stanford (private communication)]. This could be due to small, but nonzero, ρ amplitudes combined with flatter $M=0$ B or $M=1$ B-ρ' trajectories than those usually assumed, and/or a change in the ω residue parametrization usually employed to give the dip at about $t=-0.6$ GeV^2 at low energies.

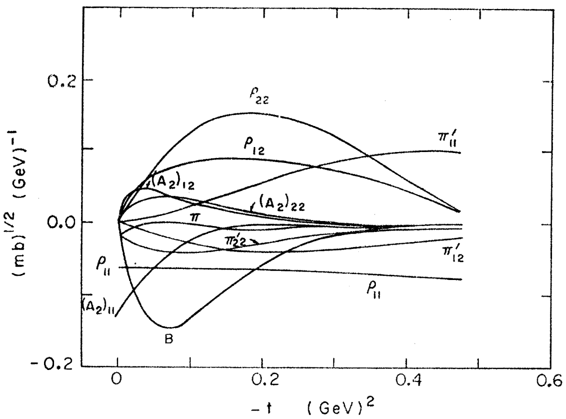

FIG. 9. Imaginary parts of the nucleon-nucleon amplitudes at 8 GeV/c for full $N\bar{N}\pi$ vertex zero $t_0=-0.05$.

Suppose now that the $\pi N\bar{N}$ vertex zero is a full zero. The value of this zero required to fit both photoproduction and NN data is $t_0\approx-2.5\mu^2$. This value is not consistent with the pion sum rule[14] but could be made consistent if the $M=1$ B trajectory were present. Moving the pion zero to $t_0=-1.5\mu^2$ would lower the $t=0$ contribution of the pion in the NN fits significantly (assuming fixed $g^2/4\pi$). The extra contribution needed in the $pn\to np$ cross section could then easily be provided by constructive interference of the B with the π, and the ρ' with the π' (see Fig. 7). Thus, in this case of a full vertex zero, the interference in $p\bar{p}\to n\bar{n}$ would be destructive so that there should actually be a dip in the $p\bar{p}\to n\bar{n}$ cross section for $|t|<0.02$ GeV^2 (i.e., the $pn\to np$ and $p\bar{p}\to n\bar{n}$ cross sections should cross over).

To summarize, if the pion photoproduction sum rule is correct, the existing NN data seem to favor the existence of an $M=1$ B trajectory regardless of the type of zero in the $\pi N\bar{N}$ vertex function. If the pion sum rule is yielding misleading results, there is no preference from NN scattering for an $M=1$ B trajectory since it could be that a full $N\bar{N}\pi$ vertex zero at $t=-0.05$ would be consistent with the sum rule. The existence of higher resonances with large γN couplings could well change these sum-rule results. In particular, measurements of the total cross section up to 2.6 GeV/c (where our Regge fits begin to work) would provide information on the γN partial widths of these resonances.

Note added in proof. Very recent data of Heide *et al.*, [DESY Report, 1968 (unpublished)] on the ratio of π^- to π^+ photoproduction on deuterons does not give conclusive support to an evading B meson, because of the experimental uncertainty in this ratio plus the theoretical uncertainty in the separate π^- and π^+ final-state corrections. We note that the inverse reaction $\pi^-p\to n\gamma$ provides a method of obtaining the π^- photoproduction cross sections, which is independent of any theoretical deuteron models.

[14] That the consistency problem is not trivial can be seen in the following way. First, the pion sum rule is highly stable due to the large Born term; doubling the effect of the resonances only moves the zero to $t_0=-0.04$ GeV^2. Secondary, a double zero NN fit for this value of t_0 yields $g^2/4\pi\approx17.5$, which is already too high. It is of course possible that other trajectories could change the position of the zero in the pion residue function, but the only candidate is the A_1 daughter, which is expected via kinematics to be small near $t=0$.

ACKNOWLEDGMENTS

We wish to thank R. L. Walker for his preliminary multipole fits to low-energy photoproduction data. We also thank G. Chew, J. D. Jackson, and F. Arbab for helpful conversations, and P. Di Vecchia for pointing out an error in an earlier version of the paper.

APPENDIX: KINEMATIC SINGULARITIES, CONSPIRACY RELATIONS, AND GAUGE INVARIANCE

The kinematic singularities for photoproduction and Compton scattering have been previously derived from the connection of helicity amplitudes with invariant amplitudes utilizing gauge invariance. There has been some confusion as to whether this method agrees with the methods using Lorentz invariance or crossing matrices as the photon mass is taken to zero. Since this question has been dealt with extensively by Gotsman and Maor, we shall present only an outline of our procedure with several new observations.[15–19]

There is complete agreement on the kinematical factors for the process $\gamma\pi\to\bar{N}_1N_2$ with all unequal masses $m_\gamma\neq0$, μ, M_1, M_2, respectively. We start with this expression[16,18,19] for the parity-conserving amplitudes free of all kinematical singularities:

$$\begin{aligned}
\tilde{F}_1&=\frac{1}{\sin\theta_t}(f_{++,1}{}^t+f_{--,1}{}^t)\frac{t^{1/2}}{\mathcal{T}(t-\Delta^2)^{1/2}}\,,\\
\tilde{F}_2&=\frac{1}{\sin\theta_t}(f_{++,1}{}^t-f_{--,1}{}^t)\frac{t^{1/2}}{(t-4M^2)^{1/2}}\,,\\
\tilde{F}_3&=\left(\frac{f_{+-,1}{}^t}{1+z_t}+\frac{f_{-+,1}{}^t}{1-z_t}\right)\frac{t}{\mathcal{T}(t-\Delta^2)^{1/2}}\,,\\
\tilde{F}_4&=\left(\frac{f_{+-,1}{}^t}{1+z_t}-\frac{f_{-+,1}{}^t}{1-z_t}\right)\frac{t}{(t-4M^2)^{1/2}}\,,\\
\tilde{F}_5&=f_{++,0}{}^t\,\mathcal{T}(t-\Delta^2)^{1/2}\,,\\
\tilde{F}_6&=\frac{1}{\sin\theta_t}f_{+-,0}{}^t\frac{t^{1/2}}{(t-4M^2)^{1/2}}\,,
\end{aligned}\tag{A1}$$

[15] J. S. Ball and M. Jacob, Nuovo Cimento **54**, 620 (1968).
[16] L. L. Wang, Phys. Rev. **142**, 1187 (1966).
[17] E. Gotsman and U. Maor, Phys. Rev. **171**, 1495 (1968).
[18] J. D. Jackson and G. E. Hite, Phys. Rev. **169**, 1248 (1968). Using the results of this paper, J. D. Jackson (private communication) has shown that Eqs. (A3) are *all* the threshold and pseudothreshold relations. Relation (a) at $4M^2$ is ignored because of its great distance from the physical region, and relation (b) is of no consequence since it involves only zero-helicity photons. Of course, this relation is of interest for ρ production when $m_\gamma=m_\rho$. See also J. W. Dash [Lawrence Radiation Laboratory Report 18377 (unpublished)] for a more complete discussion of photoproduction and NN kinematics.
[19] H. Stapp, Phys. Rev. **160**, 1251 (1967).

where

$$\mathcal{T}=\{[t-(m_\gamma+\mu)^2][t-(m_\gamma-\mu)^2]\}^{1/2},$$
$$M=\tfrac{1}{2}(m_1+m_2),$$

and

$$\Delta=m_1-m_2.$$

The quantities $\tilde{F}_5$ and $\tilde{F}_6$ are amplitudes with zero helicity for the massive photon.

In the unequal-mass case, the helicity amplitudes are analytic at $t=0$, since no pseudothreshold or boundary of the physical region coincides with this point. The above factors of $\sqrt{t}$ are to cancel the half-angle factors at $t=0$. (Note that $z_t\rightarrow 1$ as $t\rightarrow 0$.) Since both $\tilde{F}_3$ and $\tilde{F}_4$ only depend on $f_{-+,1}{}^t$ at $t=0$ we have the relation

$$2M\tilde{F}_4(s,0)=\Delta(\mu^2-m_\gamma{}^2)\tilde{F}_3(s,0). \qquad \text{(A2)}$$

This is a conspiracy relation that is satisfied in a nontrivial way by $M=1$ parity doublets. Such relations are the only conspiracy relations present in the all unequal-mass case. For equal masses in the initial or final state, $z_t\rightarrow 0$ as $t\rightarrow 0$; and for equal masses in initial *and* final states $z_t\propto s$ at $t=0$, so that the conspiracy relations cannot arise from the half-angle factors $(z_t\pm 1)^{|\lambda\pm\mu|/2}$ for these cases.

In addition to this there are the threshold and pseudothreshold relations[18]

$$\text{(a)}\qquad 2M\tilde{F}_1+\tilde{F}_3=O(t-4M^2),$$
$$\text{(b)}\qquad \Delta\tilde{F}_2-\tilde{F}_4=O(t-\Delta^2),$$
$$\text{(c)}\qquad \tilde{F}_5+\Delta(4pk\cos\theta_t)\tilde{F}_6=O(t-\Delta^2), \qquad \text{(A3)}$$
$$\text{(d)}\qquad \sqrt{2}(m_\gamma\pm\mu)\tilde{F}_6-\tilde{F}_4=O(t-(m_\gamma\pm\mu)^2),$$
$$\text{(e)}\qquad \sqrt{2}\tilde{F}_5+(m_\gamma\pm\mu)(4pk\cos\theta_t)\tilde{F}_2=O(t-(m_\gamma\pm\mu)^2),$$

where

$$4tk^2=[t-(m_\gamma-\mu)^2][t-(m_\gamma+\mu)^2]$$

and

$$4tp^2=(t-\Delta^2)(t-4M^2).$$

In order to take the limit to equal-mass baryons $(\Delta=m_1-m_2=0)$ we consider the pseudothreshold relation (b). From the limit we see immediately that $\tilde{F}_4\propto t$ as $t\rightarrow 0$, so that $F_4{}^t=[1/t(t-\mu^2)]\tilde{F}_4$ is analytic at $t=0$ as given in the text [Eq. (1)]. For the $M=1$ π-π' conspiracy this relates the residue of π' for the nonsense amplitude to the first daughter in the coupled triplet state.

By expanding relation (b) in a Taylor series about $t=0$, evaluating at $t=\Delta^2$, and comparing the first-order term with conspiracy relation (A2), we obtain the photoproduction conspiracy relation

$$2m\tilde{F}_2(s,0)=(\mu^2-m_\gamma{}^2)\tilde{F}_3(s,0), \qquad \text{(A4)}$$

analogous to the Volkov-Gribov relation of NN scattering. (Notice that for $m_\gamma=m_\rho$ this applies to ρ production.)

Now let us consider the limit of zero mass for the photon. As Gotsman and Maor noted, the normal and pseudothreshold relations (d) imply a new factor of $t-\mu^2$ for $\tilde{F}_4$, but for $\tilde{F}_2$ the situation is more interesting. Since $\tilde{F}_5$ is an amplitude for a zero-helicity massive photon plus π having a transition to the singlet $N\bar{N}$ state, one may expect a pion pole in this amplitude. Except for the case of π^0 photoproduction where charge conjugation does not permit the pion pole, one cannot argue that as $m_\gamma\rightarrow 0$, $\tilde{F}_2$ becomes proportional to $t-\mu^2$. Rather, one obtains the normalization conditions for the pion pole contributing to $F_2{}^t$. This condition and the $t-\mu^2$ factor in $\tilde{F}_4$ is the full content of gauge invariance for the t-channel helicity amplitudes of photoproduction.

We consider in more detail the limit of the threshold and pseudothreshold relations (d) and (e). To satisfy these equations we must demand that the photon-zero helicity amplitudes $\tilde{F}_5$ and $\tilde{F}_6$ do not diverge in the limit of zero photon mass. Clearly the difference of the relations (d) yields a factor of $t-\mu^2$ for $\tilde{F}_4$.

For the pion pole term we assume the usual Regge form

$$\tilde{F}_5=\frac{\beta(t)}{t-\mu^2}\tfrac{1}{2}(1+e^{-i\pi\alpha_\pi})\nu^{\alpha_\pi}+R(t,\nu), \qquad \text{(A5)}$$

where $\beta(t)$ is a smooth function of t with no kinematical zeros and $R(t,\nu)$ is regular for $t\approx\mu^2$. Using perturbation theory to obtain the coupling of the pion pole exactly at $t=\mu^2$ (definition of charge, if you like) one has the condition

$$\beta(\mu^2)=\tfrac{1}{4}\sqrt{2}egm_\gamma(2\mu+m_\gamma)(2\mu-m_\gamma)\mu^2. \qquad \text{(A6)}$$

To leading order in m_γ the relations (e) become

$$(2eg/\pm 2m_\gamma\mu)m_\gamma\mu^4+\sqrt{2}R(\mu^2,\nu)\pm 2\nu\mu\tilde{F}_2(\mu^2,s)=O(m_\gamma). \qquad \text{(A7)}$$

Again the difference of the two relations (A7) gives the required result:

$$\tilde{F}_2(s,\mu^2)=-\frac{1}{2}\frac{eg}{s-m^2}\mu^2. \qquad \text{(A8)}$$

Thus the theory of photoproduction for the massless photon can be achieved as a smooth limit of the theory of the massive photon with the use of the known analyticity properties of helicity amplitudes.

Next we consider the Compton scattering amplitudes for $\gamma\pi^\pm\rightarrow\gamma\pi^\pm$. The result is that the amplitudes F_{RR} and F_{II} are analytic, where

$$F_{RR}=\left(\frac{f_{10,10}{}^t}{1+z_t}+\frac{f_{10,-10}{}^t}{1-z_t}\right)\frac{t}{(t-\mu^2)^2},$$
$$F_{II}=\left(\frac{f_{10,10}{}^t}{1+z_t}-\frac{f_{10,-10}{}^t}{1-z_t}\right)t. \qquad \text{(A9)}$$

The unequal-mass conspiracy relation

$$\mu^{-4}F_{RR}(s,0) = -F_{II}(s,0) \qquad \text{(A10)}$$

is satisfied by the (full) factorized residues $[\gamma_I^{\pi}(0)]^2 = [\gamma_R^{\pi'}(0)]^2$ for the parity-doublet solution. This relation eliminates the apparent pole in $f_{10,10}{}^t$ at $t=0$.

We have checked that the $M=1$ conspiracy is a solution to all conspiracy relations for NN, photoproduction, and Compton scattering processes and that the residues factorize.[18] It should be noted that a slight change in the kinematical singularities given in Ref. 2 is necessary to have them obey factorization. Namely, the factors $(1-t/4m^2)^{-1}$ in Eq. (1) of Ref. 2 should not be present for the singlet and uncoupled triplet amplitudes. Since this factor is very close to 1 for $0<|t|<0.5$, these NN fits are not affected.

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Reanalysis of the Lowest-Mass Negative-Parity Baryon Resonances Using the Symmetric Quark Model

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The lowest-mass negative-parity baryon resonances are reanalyzed using a systematic $SU(6)$ analysis of mass operators developed earlier in the symmetric quark model. Fourteen resonances with known spin and parity are fitted to within 15 MeV on the average, using a six-parameter mass formula. Strong mixings are found for $J^P=\frac{1}{2}^-$ and $\frac{3}{2}^-$ resonances, and thus the Gell-Mann–Okubo mass formula cannot be expected to hold for these resonances, and there is no sense in trying to group them into octets and decuplets.

I. INTRODUCTION

We reanalyze the lowest negative-parity baryon resonances using the symmetric quark model with orbital excitation. In the symmetric quark model, the wave function of the quarks is symmetric under simultaneous permutations of the orbital and $SU(6)$ degrees of freedom of the quarks. This model can be based on parafermions of order three[1] or on the three-triplet model.[2] The lowest negative-parity baryons should be in the supermultiplet with $(SU(6),L^P) = (\mathbf{70},1^-)$.[1] In the present article we use the systematic $SU(6)$ analysis of mass operators given earlier[3] and refer to A for notation and additional references.

II. DATA

We take experimental data concerning resonances in the $(\mathbf{70},1^-)$ from the phase-shift analysis done by Donnachie, Kirsopp, and Lovelace[4] and private communication from Donnachie[5] for nonstrange resonances, and from the compilation of Rosenfeld *et al.*[6] for strange resonances. There are 14 resonances with known spin and parity; these are listed in Table I, together with one Σ and two Ξ resonances whose spin and parity have not been determined.

TABLE I. Experimental data for resonances in the $(\mathbf{70},1^-)$.

Resonance		J^P	Source of data (Ref. No.)
N	$D_{13}(1520)$	$\frac{3}{2}^-$	5
	$S_{11}(1540)$	$\frac{1}{2}^-$	5
	$D_{15}(1678)$	$\frac{5}{2}^-$	4
	$D_{13}(1680)$	$\frac{3}{2}^-$	5
	$S_{11}(1710)$	$\frac{1}{2}^-$	5
Δ	$S_{31}(1635)$	$\frac{1}{2}^-$	4
	$D_{33}(1691)$	$\frac{3}{2}^-$	4
Λ	$S_{01}(1405)$	$\frac{1}{2}^-$	6
	$D_{03}(1519)$	$\frac{3}{2}^-$	6
	$S_{01}(1670)$	$\frac{1}{2}^-$	6
	$D_{03}(1690)$	$\frac{3}{2}^-$	6
	$D_{05}(1827)$	$\frac{5}{2}^-$	6
Σ	$D_{13}(1660)$	$\frac{3}{2}^-$	6
	$D_{15}(1767)$	$\frac{5}{2}^-$	6
	1690	?	6
Ξ	1815	?	6
	1930	?	6

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