Further Comments on the Description of Three-Body Associated Production in Proton-Proton Collisions through the One-Boson-Exchange Model

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(Received 29 July 1968)

The parameters of the one-boson-exchange model (including pion and kaon exchange), applied in a previous work to the reactions $pp \to YKN$, are reexamined in the light of the results from a recent experiment which presents new data on Σ production. It is found that the cutoff parameters on the momentum transfer in pion exchange must be different for Λ and Σ production. Furthermore, in the former case, the relative amounts of π and K exchange cannot be determined from experiment, provided pion exchange is assumed to be dominant. Finally, a criticism is raised about the existing estimates of resonance production in the above reactions.

1. SUMMARY OF EARLIER RESULTS

In a recently published paper[1] (hereafter referred to as I) three-body associated production in proton-proton collisions has been analyzed in the energy range up to an incident proton momentum of 8 GeV/c, where a certain amount of experimental information is available.[2–9]

The reactions were supposed to proceed via the exchange of a spinless boson (pion or kaon), as shown in the diagrams of Fig. 1. As is well known, the so-called "bare" matrix elements provided by the model (which lead to an expression for the cross section of the Chew-Low[10] type) must be modified because of several physical effects, like initial- or final-state interactions, vertex form factors, or other corrections caused by the fact that the intermediate boson is off its mass shell. As pointed out in I, the correct theoretical treatment of all these effects is severely handicapped by the fact that they generally depend strongly on the angular momentum and parity of the final YK system (for π exchange) or KN system (for K exchange). Hence the theory can be safely applied only to the production of resonances. However, in our case, where such angular momenta and parities can actually assume a variety of values, an approximate evaluation of these corrections has been sought, in order to reproduce satisfactorily at least the main features of the experimental results. On the basis of previous experience gained in the study of quasi-two-body reactions, it was proposed to lump all the corrections into a single cutoff function on the four-momentum squared Δ^2 of the exchanged particle (of mass μ)[11]:

$$F(\Delta^2)=\left(1+\frac{\Delta^2+\mu^2}{\alpha}\right)^{-1}, \qquad (1)$$

which multiplies the "bare" peripheral matrix element. In the above formula, α must be considered as an adjustable parameter, which in I was assumed for simplicity to be the same for all diagrams involving pion exchange and for all diagrams involving kaon exchange separately, but different in the two cases.

The use of the cutoff form (1) is by no means original. It was first used in the description of pion production in proton-proton collisions at energies of a few GeV.[12] A cutoff of this kind has been also introduced in some experimental analyses[6,7] in order to improve the agreement between the yields of those particular experiments and the theoretical predictions. The analysis performed

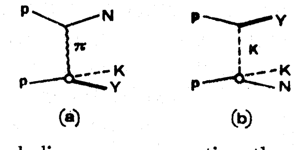

FIG. 1. Typical diagrams representing three-body associated production in proton-proton collisions through (a) pion exchange and (b) kaon exchange. p = incoming protons, N = outgoing nucleon, Y = outgoing hyperon.

[1] E. Ferrari and S. Serio, Phys. Rev. **167**, 1298 (1968).
[2] W. J. Fickinger, E. Pickup, D. K. Robinson, and E. O. Salant, Phys. Rev. **125**, 2082 (1962) (2.8 GeV/c); R. I. Louttit, T. W. Morris, D. C. Rahm, R. R. Rau, A. M. Thorndike, and W. J. Willis, *ibid.* **123**, 1465 (1961) (3.67 GeV/c).
[3] J. T. Reed, A. C. Melissinos, N. W. Reay, T. Yamanouchi, E. J. Sacharidis, S. Lindenbaum, S. Ozaki, and L. C. L. Yuan, Phys. Rev. **168**, 1495 (1968) (3.67 GeV/c).
[4] E. Bierman, A. P. Colleraine, and U. Nauenberg, Phys. Rev. **147**, 922 (1966) (5 GeV/c).
[5] G. Alexander, O. Benary, G. Czapek, B. Haber, N. Kidron, B. Reuter, A. Shapira, E. Simopoulou, and G. Yekutieli, Phys. Rev. **154**, 1284 (1967) (5.5 GeV/c).
[6] W. Chinowsky, R. R. Kinsey, S. L. Klein, M. Mandelkern, J. Schultz, F. Martin, M. L. Perl, and T. H. Tan, Phys. Rev. **165**, 1466 (1968) (6 GeV/c).
[7] I. J. Bloodworth, Oxford thesis, 1967 (unpublished); C. Caso, F. Conte, G. Tomasini, L. Casè, L. Mosca, S. Ratti, L. Tallone-Lombardi, I. Bloodworth, L. Lyons, and A. Norton, Nuovo Cimento **55**, A66 (1968) (6 GeV/c).
[8] W. Dunwoodie, W. E. Slater, H. K. Ticho, G. A. Smith, A. B. Wicklund, and S. G. Wojcicki, communication at the Thirteenth Annual International Conference on High-Energy Physics, Berkeley, California, 1966 (unpublished) (5.4 and 6.6 GeV/c: see Ref. 6 of I).
[9] M. Firebaugh, G. Ascoli, E. L. Goldwasser, R. D. Sard, and J. Wray, Phys. Rev. **172**, 1354 (1968) (7.9 GeV/c).
[10] G. F. Chew and F. E. Low, Phys. Rev. **113**, 1640 (1959).
[11] The metric is chosen in such a way that, in the physical region, $\Delta^2 > 0$.
[12] E. Ferrari and F. Selleri Phys. Rev. Letters **7**, 387 (1961).

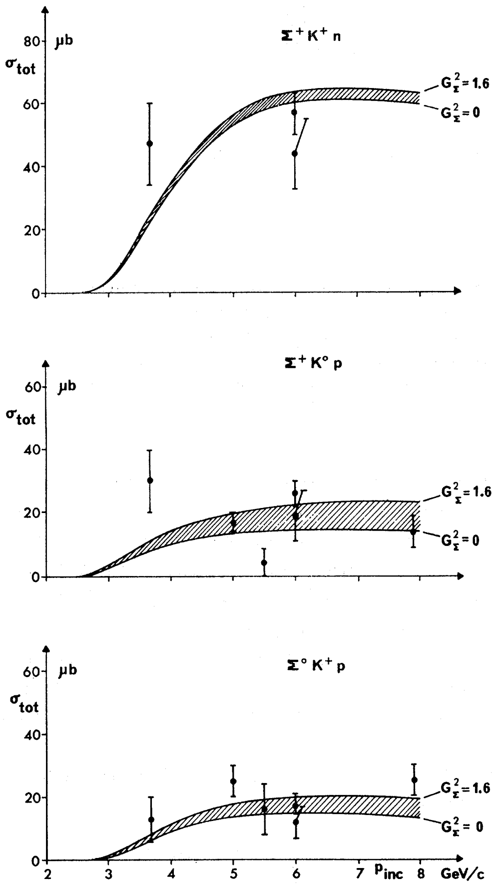

FIG. 2. Behavior of the cross section for reactions (3) as a function of incident momentum. The experimental points are taken from Refs. 2, 4–7, and 9. The curves represent the prediction of the model, and are given almost completely by pion exchange, with the cutoff parameter given by (4). The additional contributions from kaon exchange (for the coupling constant G_Σ^2 less than 1.6) are represented by the shaded regions.

in I, however, was more reliable, because it made use of *all* the data available at that time. The conclusion was reached that the one-boson-exchange model, modified as shown, was quite a powerful tool for predicting the basic features of the experiment (cross sections, angular and mass spectra), whereas it was unable to describe properly the results from more elaborate analyses, like angular correlations of any kind in the final state, including the distributions in the so-called Treiman-Yang angles.[13] Furthermore, it was shown that the most satisfactory fit to the data was obtained by an admixture of pion and kaon exchange, whose relative weights were essentially fixed by the cutoff parameters entering Eq. (1) (let us call them α_π and α_K). The values used in I were the following:

$$\alpha_\pi = 45m_\pi^2 = 0.88 \text{ GeV}^2, \quad \alpha_K = 11.2m_\pi^2 = 0.22 \text{ GeV}^2. \quad (2)$$

As far as the value of α_K is concerned, a further ambiguity is provided by the fact that the two basic coupling constants among hyperons, nucleons, and kaons, ($G_{\Lambda pK^+}{}^2/4\pi = G_\Lambda{}^2$ and $G_{\Sigma^0 pK^+}{}^2/4\pi = G_\Sigma{}^2$) have not been determined experimentally. The value of α_K given by Eq. (2) is to be used together with the value $G_\Lambda{}^2 = 4.8$, as determined from KN dispersion relations in an early work.[14] A change in $G_\Lambda{}^2$ would entail a corresponding change in α_K too.

Unfortunately, when paper I was accepted for publication, one of the most comprehensive experimental analyses (Ref. 6) was not known to the authors. Especially for the Σ channels the results of this paper were of considerable importance; in particular they were not completely consistent with the choice of cutoffs made in I.[15] Furthermore, some additional amount of information about the YNK coupling constants has become available in the meantime. All this has prompted us to pursue the study started in I.

However, the changes with respect to I do not concern the physical considerations contained in that paper, which retain their validity; they rather concern the practical side of the problem, that is, the choice of the parameters of the model.

2. Σ CHANNELS

Figure 2 reports the experimental values of the total cross section for all the reactions involving Σ production, namely,[16]

$$pp \to \Sigma^+ K^+ n, \quad (3a)$$

$$pp \to \Sigma^+ K^0 p, \quad (3b)$$

$$pp \to \Sigma^0 K^+ p. \quad (3c)$$

The additional experimental points (not reported in I) concerning reaction (3a) are of considerable interest because they are not consistent with the predictions made in I; the calculated values are too large by a factor of 2. It follows that the value of α_π given by Eq. (2) is certainly not adequate for Σ production, and must be replaced by a smaller value. The theoretical curves plotted in Fig. 2 are obtained by a new choice of this parameter, which will be now called $\alpha_\pi{}^{(\Sigma)}$:

$$\alpha_\pi{}^{(\Sigma)} = 20m_\pi{}^2 = 0.39 \text{ GeV}^2. \quad (4)$$

This choice leads to a better agreement with experiment also for reaction (3b). Since for reaction (3c) a satisfactory agreement was reached in I, one might think that the choice given by (4) should deteriorate it. However, this is not necessarily true. Indeed, the new data provided by Ref. 6 seem to indicate that the

[13] S. B. Treiman and C. N. Yang, Phys. Rev. Letters **8**, 140 (1962).

[14] M. Lusignoli, M. Restignoli, G. A. Snow, and G. Violini, Phys. Letters **21**, 229 (1966).

[15] This point has been also briefly discussed in I, in a note added in proof.

[16] All the relevant cross sections and coupling constants to be used in the calculations can be found in E. Ferrari, Phys. Rev. **120**, 988 (1960). Note, however, that the factor ½ in Eq. (15) of this paper is wrong and should be dropped. Furthermore, as done in I, all interference terms have been neglected.

cross section for Σ^0 production is smaller than indicated by some of the previous experiments.[4,9] Therefore, on the whole the situation is definitely improved by the assumption of a stronger cutoff on pion exchange, although for some particular experiments the theoretical and experimental normalizations must necessarily disagree.

Note that the theoretical predictions for reactions (3b) and (3c) are quite uncertain for a variety of reasons. The first one is the ill-defined amount of K exchange, because of the poor knowledge of the coupling constant G_Σ^2, for which no substantial progress has been reported recently.[17] Another uncertainty comes from the fact that the low-energy behavior of the cross section for the reactions $\pi^0 p \rightarrow \Sigma^0 K^+$ and $\pi^- p \rightarrow \Sigma^0 K^0$ is quite poorly known. [These cross sections must be inserted into the pion-exchange contributions to reactions (3c) and (3b), respectively.] This holds especially for the reaction $\pi^- p \rightarrow \Sigma^0 K^0$, for which a recent accurate measurement[18] is at variance with the results obtained in earlier experiments.[19] As a matter of fact, we have used empirical fits for the above reactions different from those used in I; the new fits give a substantial weight to the accurate measurements reported in Ref. 18. It is evident that a large uncertainty is necessarily associated to the choice of such fits. This must be kept in mind when comparing theory and experiment in more detail.

On the contrary, the prediction for reaction (3a), which has guided us in the choice of the cutoff $\alpha_\pi^{(\Sigma)}$, is practically unaffected by K exchange and is not even influenced by the ambiguities in the fit of the cross section for the reaction $\pi^+ p \rightarrow \Sigma^+ K^+$ (which must be inserted into the pion-exchange contribution).[20]

In Figs. 3 and 4 we present the fits to the spectra reported in Ref. 6. Figure 3 shows the c.m. proton angular distribution in reaction (3c), Fig. 4 shows the mass spectra for all three reactions. The latter appear to be quite consistent with the experiment, except for the normalization in Fig. 4(b), where, however, for the reasons explained above, a discrepancy in normalization should not be taken too seriously. The most significant theoretical mass spectrum [the one of Fig. 4(a)] is in good agreement with the experiment. On the other hand, the distribution of Fig. 3 seems to require the presence of a certain amount of K exchange; indeed, pion exchange alone predicts too fast a decrease at large angles, and the ambiguities associated to the calculation of the pion-exchange contribution cannot appreciably modify this conclusion.

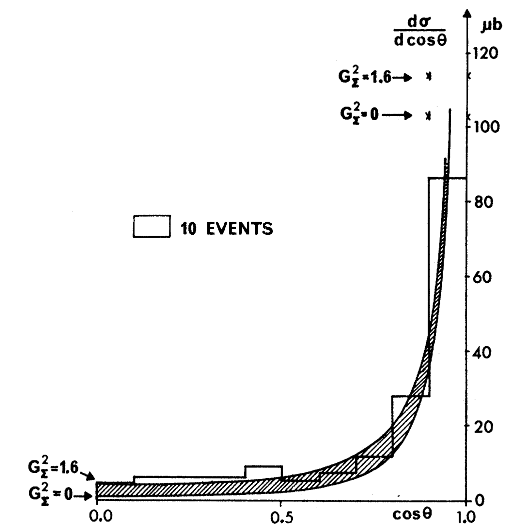

FIG. 3. Experimental c.m. angular distribution of the protons (folded around 90°) for the reaction $pp \rightarrow \Sigma^0 K^+ p$ at 6 GeV/c (from Ref. 6), compared with the predictions of the model. The shaded area has the same meaning as in Fig. 3. The average of the theoretical predictions over the first experimental bin ($0.9 \leq \cos\theta \leq 1$) is also shown.

3. Λ CHANNEL

There is only one reaction which can produce a Λ in a three-body final state, namely,

$$pp \rightarrow \Lambda K^+ p. \qquad (5)$$

It is straightforward to ask the question whether the cutoff parameter α_π ought to be reduced also for this reaction to the value given by Eq. (4). We think that the answer should be negative. Indeed, in order to preserve the correct normalization, one should make up for the decrease of the pion-exchange contribution by adding more K exchange (namely, by increasing the value of α_K). In this way, K exchange would dominate over π exchange in Λ production, and would be depressed in Σ production only because of the smallness of the coupling constant. This situation looks rather artificial, and it cannot be accepted also for other

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[17] We have retained the same K-exchange contributions to reactions (3) as calculated in I (and we have taken them always as isotropic, in the sense explained in I). In particular, the cutoff α_K is given by Eq. (2), and the upper limit on G_Σ^2 is taken as 1.6. The parameters for K exchange will be discussed more thoroughly in Sec. 3. Here we only point out that, on the basis of the latest analyses on the coupling constants (cf. Ref. 23), values of G_Σ^2 smaller than 1 are more likely than values as high as 1.6. However, no conclusive evidence has been provided on this subject so far.

[18] J. Anderson, F. S. Crawford, Jr., and J. C. Doyle, Phys. Rev. **152**, 1139 (1967). The same group has also performed an accurate analysis of the reaction $\pi^- p \rightarrow \Sigma^- K^+$ at the same energy (1170 MeV/c) [J. C. Doyle, F. S. Crawford, Jr., and J. A. Anderson, *ibid.* **165**, 1483 (1968)].

[19] L. B. Leipuner and R. K. Adair, Phys. Rev. **109**, 1358 (1958); F. S. Crawford, Jr., M. Cresti, R. L. Douglas, M. L. Good, G. R. Kalbfleisch, M. L. Stevenson, and H. K. Ticho, in *Proceedings of the Ninth Annual International Conference on High-Energy Physics, Kiev, 1959* (Academy of Science, IUPAP, Moscow, 1960), Vol. 1, p. 443; F. S. Crawford,Jr., R. L. Douglas, M. L. Good, G. R. Kalbfleisch, M. L. Stevenson, and H. K. Ticho, Phys. Rev. Letters **3**, 394 (1959).

[20] An ambiguity in the behavior of this cross section is present only at high energy, where the experimental points known today (see Ref. 24c of I) are not consistent with each other. This energy region, however, has little influence on the theoretical spectra for momenta of the incident protons in reactions (3) up to 6 GeV/c.

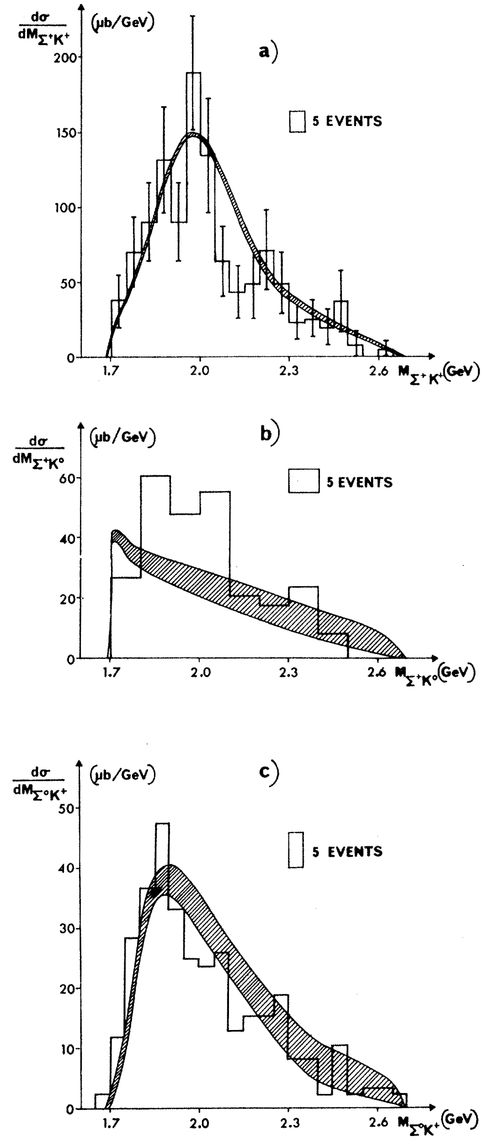

FIG. 4. Experimental ΣK mass distributions for reactions 3(a)–3(c) at 6 GeV/c (from Ref. 6), compared with the predictions of the model. Lower curves, pure pion exchange; shaded areas, additional contributions from kaon exchange for the coupling constant $G_\Sigma^2 \leq 1.6$ (value giving the upper curves).

reasons. Indeed, as discussed in I, a dominant K-exchange contribution to Λ production would not correctly predict the mass spectra and would not explain the Λ polarization experimentally observed.

Thus we think it much more meaningful physically to release the requirement that the value of α_π be the same for Λ and Σ production, and to introduce two different cutoffs, $\alpha_\pi^{(\Lambda)}$ and $\alpha_\pi^{(\Sigma)}$, into the model, which now contains three free parameters instead of two. As a matter of fact, the existence of a difference in the rates of falloff of the Δ^2 distributions can be directly inferred from the data reported in Ref. 6 for reactions (3a) and (4). If one assumes a dominant pion-exchange contribution for both reactions, it follows that the associated cutoffs must be different.[21]

It is obvious that the relative amount of pion and kaon exchange assumed in I can be maintained. However, since the cutoff parameters for Λ production are no longer tied to the reactions of Σ production (which also cannot tell anything significant concerning K exchange), we have tried to see whether the experimental data indicate a clear-cut amount of admixture between π and K exchange, or whether there is a certain freedom in the choice of the parameters. The discussion can be made only with reference to the data of Ref. 6 (at 6 GeV/c), which constitute a comprehensive sample: the conclusions obtained hold also for the data from other experiments, already discussed in I.

We have obtained the perhaps surprising result that there is no way of determining how much pion exchange and how much kaon exchange is present in Λ production, provided that the former gives the dominant contribution. In other words, if the choice of the cutoffs is made in such a way that the total cross section comes out right, all possible admixtures (from pure pion exchange to the admixture used in I) give satisfactory fits to all spectra. One must also keep in mind the additional ambiguity about the off-shell reactions (in the 4-particle vertices of the diagrams shown in Fig. 1) being "anisotropic" or "isotropic" (in the sense exhaustively discussed in I). This ambiguity affects one or the other of the two possible contributions, according to the kind of spectrum analyzed.

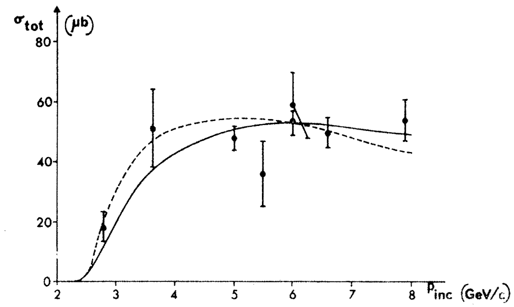

FIG. 5. Behavior of the cross section for reaction (5) as a function of incident momentum. The experimental points are taken from Refs. 2, 4–9. Dashed curve, prediction of the model for pure pion exchange (with a cutoff parameter $\alpha_\pi^{(\Lambda)} = 180 m_\pi^2$); solid curve, prediction of the model for an admixture of pion and kaon exchange, with the cutoff parameters given by (6).

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[21] At this point, it is interesting to recall the empirical determination, made in Ref. 6, of the cutoffs on pion exchange. At different stages, two independent determinations are made of a cutoff parameter playing the same role as our α_π; in both cases, the values obtained for Λ and Σ production turn out to be different. We refer to the parameter α appearing in Eq. (1) of Ref. 6, which is 0.35 ± 0.05 GeV^2 for Σ's and 0.53 ± 0.03 GeV^2 for Λ's (more or less, of the order of the values chosen by us); and to the parameter A introduced after Eq. (3) of the above paper, which is 5.0 GeV^2 for Λ's and 1.8 GeV^2 for Σ's (thus, much larger than our values, even in the hypothesis of complete absence of K exchange). Note that both determinations yield a cutoff effect which is stronger in Σ production than in Λ production.

The results of our analysis are presented in the figures which follow. In all of them, only the extreme cases have been plotted, namely, pure pion exchange (with a cutoff $\alpha_\pi^{(\Lambda)}=180m_\pi^2=3.53$ GeV^2) and an admixture very close to the one used in I, with the following values of the cutoff parameters[22]:

$$\alpha_\pi^{(\Lambda)}=50m_\pi^2=0.98\ \text{GeV}^2;$$
$$\alpha_K=11.2m_\pi^2=0.22\ \text{GeV}^2. \quad (6)$$

However, this value of α_K (the same as used in I) must now be used together with the value $G_\Lambda^2=6$ of the ΛK^+p coupling constant. We have taken this value from Davies *et al.*,[23] which brings additional evidence (with respect to the calculation of Ref. 14) in favor of a relatively low value of G_Λ^2. We have not considered the larger value of G_Λ^2 determined by Kim,[24] which would require a smaller value of α_K, much less satisfactory from the physical point of view, in our calculations. Since none of the evidences presented can be considered as conclusive, we favor the choice of G_Λ^2 that, in our case, seems to have more physical significance.

Figure 5 shows the theoretical curves and the experimental values for the total cross section; Fig. 6, for the c.m. angular distribution of the protons; Fig. 7, for the distributions in Δ^2 (cf. Ref. 22 of I); Fig. 8, for the ΛK^+ and K^+p mass distributions. In all figures, the isotropic and anisotropic cases (when occurring) have been plotted separately. Except for the spectra of Figs. 7(b) and 8(b), this ambiguity occurs for the K-exchange contribution. We can see that the curves from an admixture with isotropic K exchange give good fits, and are in general agreement with the curves from pure pion exchange. Anisotropic K exchange, contrary to what has been stated in I, is much less satisfactory.[25] In Fig. 8(b), again isotropic pion exchange is preferred (both for the admixture and for no K exchange). Finally, in Fig. 7(b) isotropic pion exchange still gives the best fit for the admixture, whereas if one takes pure pion exchange the correct behavior is not obtained by any of the two extreme hypotheses. We think that

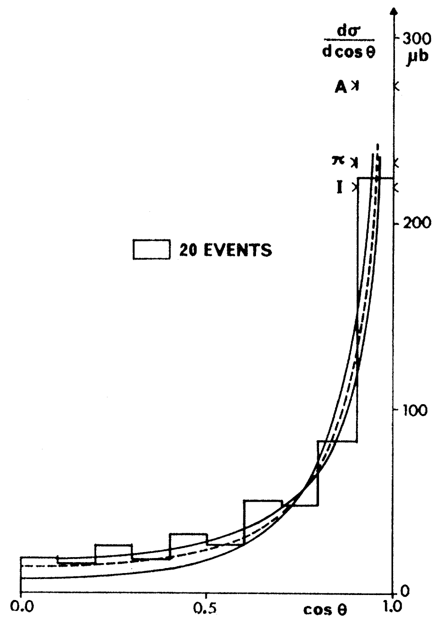

FIG. 6. Experimental c.m. angular distribution of the protons (folded around 90°) for reaction (5) at 6 GeV/c (from Ref. 6), compared with the prediction of the model. Dashed curve, pure pion exchange (with $\alpha_\pi^{(\Lambda)}=180m_\pi^2$); solid curves, admixture of pion and kaon exchange, with the cutoff parameters given by (6). The two curves correspond to the assumption of anisotropic or isotropic off-shell K^+p scattering (as explained in I) in the K-exchange contribution. The anisotropic case corresponds to the upper curve near $\cos\theta=1$. The average of the theoretical predictions in the first experimental bin is also shown.

it could be reproduced by assuming a somewhat "intermediate" offshell $\pi p\rightarrow\Lambda K$ angular distribution, forward-peaked but not so much as the experimental one (as measured in π^-p collisions). This conclusion is at variance with the result of the analysis performed in Ref. 6, which suggests that the angular distribution for $\pi^-p\rightarrow\Lambda K^0$ is always nearly the same, whether the incident pion be real or virtual.

4. SUMMARY OF CONCLUSIONS

On the basis of the recent experimental results, which have provided substantial information for the Σ channels, we have reevaluated the parameters of the one-boson-exchange model presented in I. We have been forced to assumed different cutoff parameters for pion exchange in Λ and Σ production. In the latter case, pion exchange, with the cutoff parameter given by (4), provides the bulk of the effect: A satisfactory over-all fit to the various reactions can be achieved. The channels are rather insensitive to kaon exchange; however, the presence of some amount of K exchange (obtained by assuming the same cutoff parameter as in I and a coupling constant G_Σ^2 of order 1) would provide slightly better fits.

For the Λ channel, instead, no definite conclusions about the relative amounts of pion and kaon exchange can be reached. We are in favor of an admixture of the

[22] It is obvious that the same α_K must be used both for Λ and Σ production: indeed, we have calculated the K-exchange contribution to the Σ's with the same α_K as given by (6). If one changes the admixture of π and K exchange in Λ production, by taking more π and less K exchange, then the K-exchange contribution to the Σ channels will be correspondingly reduced. However, this does not represent a problem, because the K-exchange contribution to Σ production is small, and it is poorly determined anyway because of the uncertainty in the coupling constant G_Σ^2.

[23] G. H. Davies, N. M. Queen, M. Lusignoli, M. Restignoli, and G. Violini, Nucl. Phys. **B3**, 616 (1967). This paper concludes that G_Λ^2 should lie in the interval (6 ± 2).

[24] J. K. Kim, Phys. Rev. Letters **19**, 1074 (1967); **19**, 1079 (1967). Kim's result is $G_\Lambda^2=(16.0\pm2.5)$.

[25] The histograms of Fig. 8(a) show a broader low-mass peak than predicted by our theoretical curves. This happens because we have inserted only one resonant peak in our fit for the $\pi^-p\rightarrow\Lambda K^0$ cross section. A second resonant peak, following immediately, would definitely improve the agreement. However, the experimental data for the above reaction are quite uncertain in the mass region around 1.85 GeV (corresponding to incident momenta around 1.3 GeV/c), and we have preferred not to "guess" any resonance here.

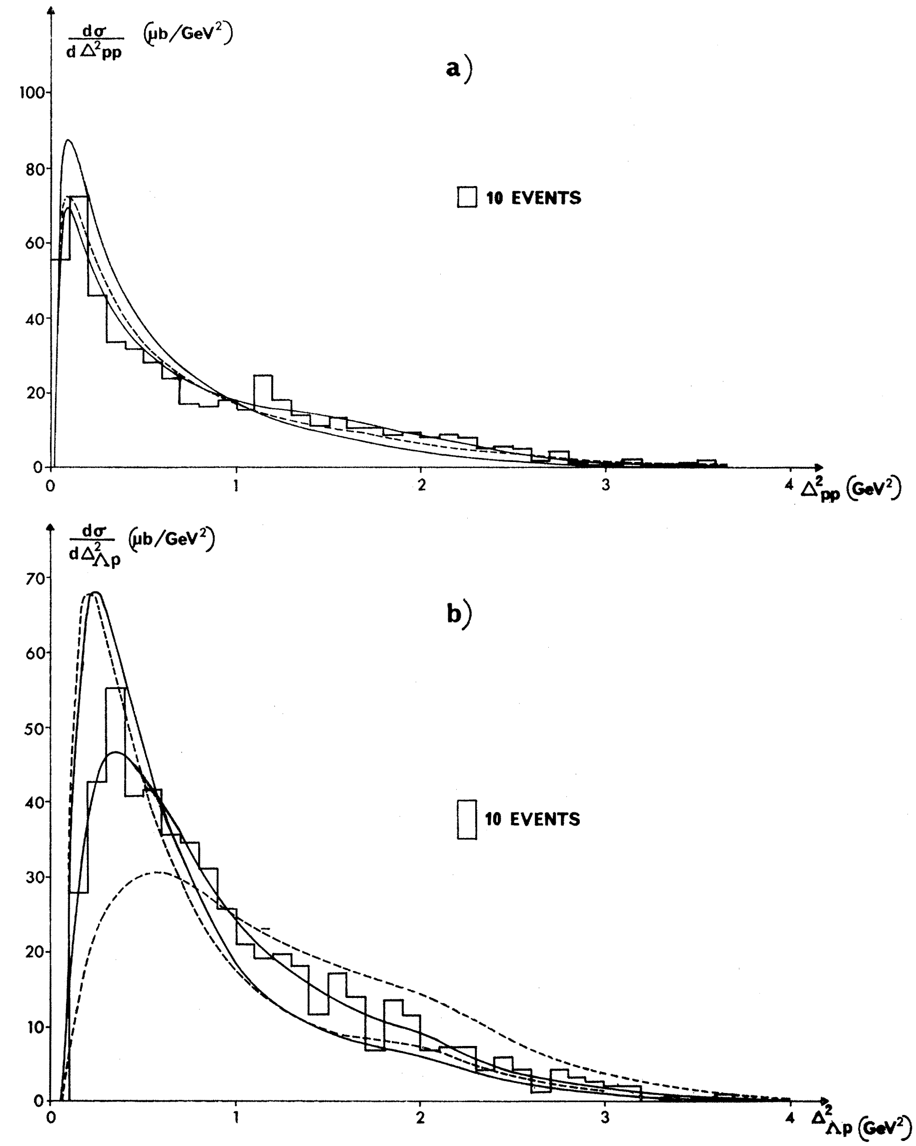

FIG. 7. (a) Experimental distribution in the momentum transfer Δ_{pp}^2 (defined as the smaller of the two momentum transfer between the final proton and the initial ones) in reaction (5) at 6 GeV/c (from Ref. 6). The curves represent the predictions of the model and have the same meaning as in Fig. 6. (b) The same for the distribution of the momentum transfer $\Delta_{\Lambda p}^2$ between the Λ and the initial protons. Here the anisotropic and isotropic cases occur for pion exchange. In both (a) and (b), the anisotropic cases correspond to the higher curves at the peak.

two mechanisms, with the cutoff parameters given by (6). However, pure pion exchange (with $\alpha_\pi^{(\Lambda)}=180m_\pi^2$) cannot be ruled out, nor any of the intermediate admixtures. Indeed, the various fits obtained by changing the admixture of pion and kaon exchange, provided that the total normalization is preserved, are generally not distinguishable within the experimental uncertainties.

Our best fits are generally obtained when the off-shell angular distributions in the four-particle vertices of the diagrams of Fig. 1 are taken as isotropic.[26] In this way, as already remarked in I, the K-exchange contribution plays essentially the role of a (non-phase-space) background term, which might perhaps be reproduced also by assuming other mechanisms of interaction (e.g., double Regge-pole exchange). For pion exchange, it is more difficult to justify the hypothesis of isotropy (or, in different words, to understand why the insertion of the experimental angular distribution for $\pi^- p \to \Lambda K^0$ gives poor predictions for the spectra sensitive to $\Delta_{\Lambda p}^2$). We think that this point deserves further theoretical study.

[26] With the present choice of the parameters of the model, isotropic pion exchange leads to better fits also for the spectra considered in I, contrary to what was said there.

5. PRODUCTION OF NUCLEON ISOBARS

We close with a comment on the statement, contained in Ref. 6, that in reactions (3a) and (5) a sizable fraction of events (38 and 54%, respectively) show the formation of nucleon isobars [the $T=\frac{3}{2}$ $N^*(1920)$, and the $T=\frac{1}{2}$ $N^*(1688)$, respectively], which subsequently decay into the hyperon-kaon system. It is true that the humps

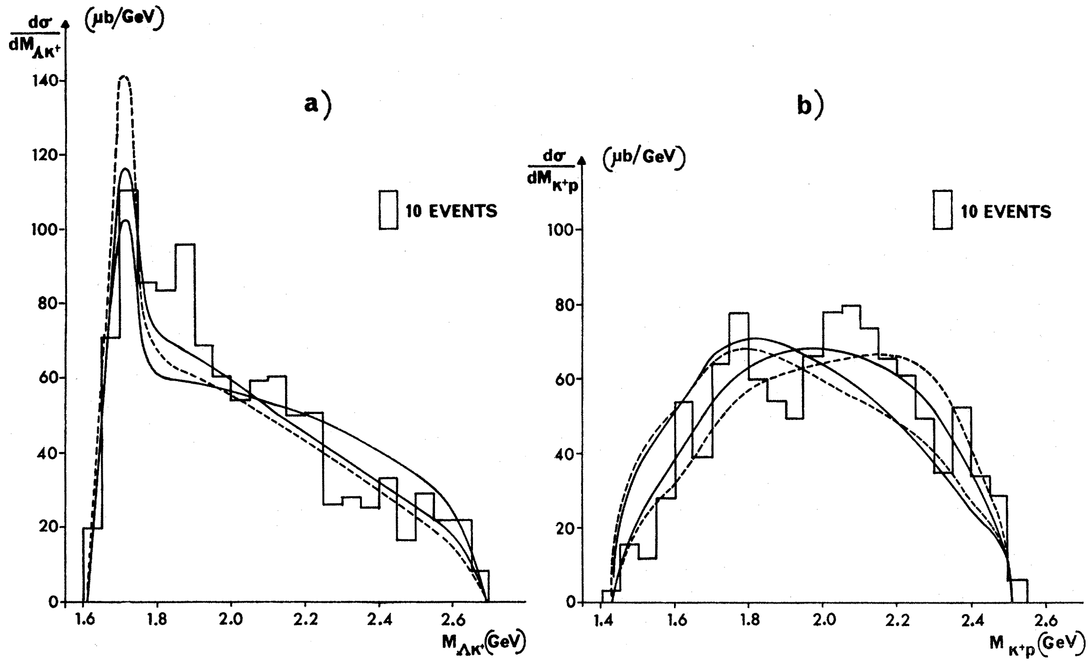

FIG. 8. Experimental distributions (a) in the ΛK^+ mass, (b) in the K^+p mass in reaction (5) at 6 GeV/c (from Ref. 6), compared with the predictions of the model. The curves have the same meaning as in Figs. 6 and 7. In both (a) and (b), the anisotropic cases correspond to the higher curves at low masses.

in the $\pi p \to YK$ cross sections, which are again found in the mass spectra of Figs. 4(a) and 8(a), reflect the presence of such resonances. But we think that actual formation of isobars must occur quite less frequently than stated in Ref. 6. Indeed, from the data given in the latter paper, one can infer the following values of the cross sections for $pp \to NN^*$ ($N^* \to YK$):

$$\sigma(pp \to pN^{*+}(1688);\, N^{*+} \to \Lambda K^+) = 29 \pm 6\ \mu\text{b}\,,$$
$$\sigma(pp \to nN^{*++}(1920);\, N^{*++} \to \Sigma^+ K^+) = 22 \pm 6\ \mu\text{b}. \quad (7)$$

However, one should keep in mind that the strange-particle decays of nucleon isobars have quite small branching ratios, still unknown, but very likely to lie below 1%.[27] Taking an upper limit of 1% for them, we can obtain lower limits on the cross sections for the total production of the same isobars in pp collisions, and also for the production of isobars decaying into the elastic channel ($N\pi$ system). We get

$$\sigma(pp \to pN^{*+}(1688)) \geqslant 2.9 \pm 0.6\ \text{mb}\,,$$
$$\sigma(pp \to pN^{*+}(1688);\, N^* \to n\pi^+,\, p\pi^0) \geqslant 1.9 \pm 0.4\ \text{mb}\,, \quad (8)$$
$$\sigma(pp \to nN^{*++}(1920)) \geqslant 2.2 \pm 0.6\ \text{mb}\,,$$
$$\sigma(pp \to nN^{*++}(1920);\, N^{*++} \to p\pi^+) \geqslant 1.1 \pm 0.3\ \text{mb}\,,$$

if we take elastic branching ratios of 65 and 50%, respectively.[27]

The experimental data for reactions (8) in the energy range around 6 GeV/c are rather scanty. From the compilation by Alexander *et al.*[28] we learn that the total production rate of $N^{*+}(1688)$ has been measured as 0.70±0.18 mb at 4.5 GeV/c, 1.02±0.17 mb at 5.5 GeV/c, and 0.5±0.1 mb at 6.0 GeV/c. At 5.5 GeV/c, the cross section for production of the same isobar decaying into the elastic channel is 0.54±0.22 mb.[5] For the $N^{*++}(1920)$, we have only the cross section for the production of the isobar decaying into the elastic channel,[5] which amounts to 0.32±0.16 mb. Although all the errors quoted are quite large, and furthermore the estimates of the cross sections depend on the manipulation of the background, we see that the experimental figures are all definitely lower than the values given by (8). Therefore, we conclude that the actual production of nucleon isobars in our reactions is likely to have been overestimated in Ref. 6. In our opinion, this should discourage any attempts to apply the theory as developed for quasi-two-body reactions (absorption model) to samples of data taken in the regions of the experimental mass peaks.[28a]

[27] See, e.g., A. H. Rosenfeld, N. Barash-Schmidt, A. Barbaro-Galtieri, L. R. Price, P. Söding, C. G. Wohl, M. Roos, and W. J. Willis, Rev. Mod. Phys. **40**, 77 (1968).

[28] G. Alexander, O. Benary, and U. Maor, Nucl. Phys. **B5**, 1 (1968).

[28a] *Note added in proof.* The most controversial point in the above argument is the correct evaluation of the branching ratios for the decays of the N^*'s into YK. We think that the most reliable way to evaluate them is to use the results of phase-shift analysis applied to the reactions $\pi^- p \to \Lambda^0 K^0$ and $\pi^+ p \to \Sigma^+ K^+$. We know that such analyses have been presented at the Fourteenth International Conference on High Energy Physics (Vienna, 1968); unfortunately, we do not have numerical results available at the moment.