Determination of the Isotensor Electromagnetic Current from Pion Photoproduction*

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Some consequences of an isotensor component of the electromagnetic current are examined in simple pion photoproduction. An experiment to detect the isotensor contribution in pion photoproduction from deuterium (already suggested by several authors) is reformulated to improve the sensitivity of the test. To estimate how accurately the experiment should be performed, upper bounds on the strength of the isotensor contribution are obtained from our experimental and theoretical knowledge of pion photoproduction from the nucleon. This limit turns out to be much smaller than previously stated in the literature.

1. INTRODUCTION

The electromagnetic current $J_\mu(x)$ is usually assumed to be the sum of two terms, an isoscalar $J_\mu{}^S(x)$, and the third component of an isovector $J_\mu{}^V(x)$. From this it follows that to lowest order in the electromagnetic coupling constant only transitions with $\Delta I=0$ and 1 are possible in scattering processes initiated by electromagnetic interactions. Without questioning the validity of the Gell-Mann–Nishjima relation

$$Q=\int d^3x\,\langle A|J_0(x)|A\rangle=I_3+\tfrac{1}{2}Y\,, \tag{1.1}$$

one may ask whether the electromagnetic current has a more complicated isospin structure. This question has been raised, during the past few years, by a number of authors[1–5] who were motivated to find an explanation for the first results on the branching ratio

$$R=\Gamma(\eta\to 3\pi^0)/\Gamma(\eta\to\pi^+\pi^-\pi^0)\,. \tag{1.2}$$

They have stated that there is little experimental information on the isospin properties of the electromagnetic current and have suggested several experimental tests for detecting an isotensor component of $J_\mu(x)$. Implicit in their discussions is the assumption that there might exist a very large isotensor contribution to the electromagnetic current, which would not have been detected by present experiments. The main reason for this state of affairs is the lack of systems of isospin greater than $\frac{3}{2}$ with which to observe $\Delta I=2$ transitions unambiguously.

Grishin *et al.*[1] and Kabir and Dombey[2] recommended a measurement of the ratio of the cross sections for photoproduction of the charged and neutral $\Delta(1236)$ from a deuterium target to test for the presence of an isotensor current.

$$\gamma+d\to p+\Delta^0 \begin{array}{l}\nearrow p+(p+\pi^-)\\ \\ \searrow p+(n+\pi^0)\,,\end{array} \tag{1.3a}$$

$$\gamma+d\to n+\Delta^+ \begin{array}{l}\nearrow n+(p+\pi^0)\\ \\ \searrow n+(n+\pi^+)\,.\end{array} \tag{1.3b}$$

For pure Δ production, the isospin of the final state is restricted to 1 or 2. In the absence of an isotensor interaction, only an $I=1$ state is produced and the ratio of these cross sections is unity. As pointed out by Kabir and Dombey,[2] and emphasized by Shaw,[3] the nonresonant production of an $I=\frac{1}{2}$ final state at a mass of the $\Delta(1236)$ constitutes a serious background. This $I=\frac{1}{2}$ state is produced through isoscalar as well as isovector photons and changes the ratio of (1.3a) and (1.3b) from unity even in the absence of an isotensor interaction [see Eqs. (2.7) and (2.8)].

Shaw[3] suggested that the background could be reduced by a factor of 10 or 20 by detecting only those events in which the final state contained a neutral pion. It is known that almost all of the π^0 photoproduction cross section at the resonance proceeds through the $\Delta(1236)$. Consequently, requiring a π^0 in the final state will enrich the ratio of N^*'s in the sample.

Unfortunately, the requirement of Shaw adds a serious complication to an already difficult experiment. There is an obvious technical simplification if the experimenter is required only to detect and measure the momentum and angle of a single nucleon, using the two-body kinematics to restrict the mass of the undetected pion-nucleon system.

(1) This "simpler" type experiment is even a more sensitive test for the isotensor interaction, if one measures the difference in the cross sections for reactions (1.3a) and (1.3b) as a function of the mass of the undetected particles in the region of the $\Delta(1236)$ resonance (Sec. 2).

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[1] V. G. Grishin, V. L. Lyuboshitz, V. I. Ogievetskii, and M. I. Podgoretskii, Yadern. Fiz. **4**, 126 (1966) [English transl.: Soviet J. Nucl. Phys. **4**, 90 (1967)].
[2] N. Dombey and P. K. Kabir, Phys. Rev. Letters **17**, 730 (1966).
[3] G. Shaw, Nucl. Phys. **B3**, 338 (1967).
[4] P. P. Divakaran, V. Gupta, and G. Rajesekaran, Phys. Rev. **166**, 1792 (1968).
[5] S. L. Adler, Phys. Rev. Letters **18**, 519 (1967); **18**, 1036(E) (1967).

(2) Present experimental data on pion photoproduction from the nucleon permit one to set an upper bound on the strength of the isotensor current which is only a few percent of the isovector (Sec. 3). The bounds found in Sec. 3 imply that Shaw's experiment[3] is not sufficiently sensitive. Furthermore, the experiment suggested in Sec. 2 will yield a null result unless the difference between the cross sections for (1.3a) and (1.3b) is measured with sufficient accuracy to define energy-dependent changes in this difference which are smaller than 5–10% of the individual cross sections.

Although we do not address ourselves to the η decay, nevertheless it is interesting to note that recent results find the ratio $R=1.4\pm0.2$.[6] Expressing R as[5]

$$R=1.63\left|\frac{1-(\sqrt{\frac{2}{3}})r}{1+(\sqrt{\frac{3}{2}})r}\right|^2, \tag{1.4}$$

where r is the ratio of the isospin $I=3$ amplitude to the $I=1$ amplitude in the final state, one obtains from the recent data the ratio $r=(3.8\pm3.8)$ %, if r is assumed to be real. Therefore, if there exists an isotensor current which excites the $I=3$ final state in the η decay, it should be only a small contribution to the total electromagnetic current.

2. ISOSPIN ANALYSIS OF $\gamma+d\rightarrow 2n+\pi$

Consider the four reactions

$$\gamma+d\rightarrow n+(n+\pi^+), \tag{2.1a}$$

$$\gamma+d\rightarrow n+(p+\pi^0), \tag{2.1b}$$

$$\gamma+d\rightarrow p+(p+\pi^-), \tag{2.1c}$$

$$\gamma+d\rightarrow p+(n+\pi^0). \tag{2.1d}$$

The general transition matrix element may be written in the form

$$T=e\epsilon_\mu\langle^{\text{out}}N_1,N_2,\pi|J_\mu(0)|d\rangle, \tag{2.2}$$

where ϵ_μ is the polarization vector of the photon.

In lowest order of the electromagnetic interaction, the current operator $J_\mu(0)$ depends only on the strong interactions. Now in order to introduce in (2.2) transitions with $\Delta I=0$, 1, *and* 2 we assume the following decomposition of $J_\mu(0)$ into isospin tensors $J_\mu{}^T(0)$:

$$J_\mu(0)=J_\mu{}^0(0)+J_\mu{}^1(0)+J_\mu{}^2(0). \tag{2.3}$$

Usually terms with $T>1$ are assumed to be zero.

We are interested in describing experiments in which only a single nucleon of reaction (2.1) is detected and in deducing information about strengths of amplitudes for pure isospin states of the undetected pion-nucleon system. For convenience we will often refer to this undetected system as the χ particle. The first step is to expand the amplitudes for the reactions (2.1) in states of definite isospin, $I^\chi=\frac{1}{2}$, $\frac{3}{2}$, and $I_z{}^\chi$ of the χ particle. We denote these amplitudes by $\langle N_d;\ I^\chi, I_z{}^\chi|J_\mu i|d\rangle$, where N_d stands for the detected proton (p) or neutron (n).

$$\begin{aligned}\langle n;\, n\pi^+|J_\mu(0)|d\rangle=&(\sqrt{\tfrac{1}{3}})\langle n;\, \tfrac{3}{2},\tfrac{1}{2}|J_\mu{}^1(0)+J_\mu{}^2(0)|d\rangle\\ &+(\sqrt{\tfrac{2}{3}})\langle n;\, \tfrac{1}{2},\tfrac{1}{2}|J_\mu{}^0(0)+J_\mu{}^1(0)|d\rangle,\end{aligned} \tag{2.4a}$$

$$\begin{aligned}\langle n;\, p\pi^0|J_\mu(0)|d\rangle=&(\sqrt{\tfrac{2}{3}})\langle n;\, \tfrac{3}{2},\tfrac{1}{2}|J_\mu{}^1(0)+J_\mu{}^2(0)|d\rangle\\ &-(\sqrt{\tfrac{1}{3}})\langle n;\, \tfrac{1}{2},\tfrac{1}{2}|J_\mu{}^0(0)+J_\mu{}^1(0)|d\rangle,\end{aligned} \tag{2.4b}$$

$$\begin{aligned}\langle p;\, p\pi^-|J_\mu(0)|d\rangle=&(\sqrt{\tfrac{1}{3}})\langle p;\, \tfrac{3}{2},-\tfrac{1}{2}|J_\mu{}^1(0)+J_\mu{}^2(0)|d\rangle\\ &-(\sqrt{\tfrac{2}{3}})\langle p;\, \tfrac{1}{2},-\tfrac{1}{2}|J_\mu{}^0(0)+J_\mu{}^1(0)|d\rangle,\end{aligned} \tag{2.4c}$$

$$\begin{aligned}\langle p;\, n\pi^0|J_\mu(0)|d\rangle=&(\sqrt{\tfrac{2}{3}})\langle p;\, \tfrac{3}{2},-\tfrac{1}{2}|J_\mu{}^1(0)+J_\mu{}^2(0)|d\rangle\\ &+(\sqrt{\tfrac{1}{3}})\langle p;\, \tfrac{1}{2},-\tfrac{1}{2}|J_\mu{}^0(0)+J_\mu{}^1(0)|d\rangle.\end{aligned} \tag{2.4d}$$

We notice that transitions to $I^\chi=\frac{3}{2}$ proceed only by the $T=1$, 2, whereas transitions to $I^\chi=\frac{1}{2}$ proceed only by the $T=0$, 1 components of the currents. The Wigner-Eckart theorem relates the amplitudes for $I_z{}^\chi=\pm\frac{1}{2}$, so that we can introduce the notation

$$\begin{aligned}-(\sqrt{\tfrac{2}{3}})A_{3/2}{}^1=&e\epsilon_\mu\langle n;\, \tfrac{3}{2},\tfrac{1}{2}|J_\mu{}^1(0)|d\rangle\\ =&-e\epsilon_\mu\langle p;\, \tfrac{3}{2},-\tfrac{1}{2}|J_\mu{}^1(0)|d\rangle,\end{aligned} \tag{2.5a}$$

$$\begin{aligned}(\sqrt{6})A_{3/2}{}^2=&e\epsilon_\mu\langle n;\, \tfrac{3}{2},\tfrac{1}{2}|J_\mu{}^2(0)|d\rangle\\ =&e\epsilon_\mu\langle p;\, \tfrac{3}{2},-\tfrac{1}{2}|J^2(0)|d\rangle,\end{aligned} \tag{2.5b}$$

$$\begin{aligned}\sqrt{3}A_{1/2}{}^0=&e\epsilon_\mu\langle n,\tfrac{1}{2},\tfrac{1}{2}|J_\mu{}^0(0)|d\rangle\\ =&-e\epsilon_\mu\langle p;\, \tfrac{1}{2},-\tfrac{1}{2}|J^0(0)|d\rangle,\end{aligned} \tag{2.5c}$$

$$\begin{aligned}(\sqrt{\tfrac{1}{3}})A_{1/2}{}^1=&e\epsilon_\mu\langle n,\tfrac{1}{2},\tfrac{1}{2}|J_\mu{}^1(0)|d\rangle\\ =&e\epsilon_\mu\langle p;\, \tfrac{1}{2},-\tfrac{1}{2}|J^1(0)|d\rangle.\end{aligned} \tag{2.5d}$$

In Eqs. (2.4) and (2.5) the final states are assumed to be antisymmetric in the two nucleons. However, this symmetry does not affect our arguments. In terms of the reduced amplitudes defined in (2.5), the transition amplitudes for reactions (2.1) may be written

$$\begin{aligned}T_a=&e\epsilon_\mu\langle n;\, n\pi^+|J_\mu(0)|d\rangle\\ =&\sqrt{2}(A_{1/2}{}^0+\tfrac{1}{3}A_{1/2}{}^1-\tfrac{1}{3}A_{3/2}{}^1+A_{3/2}{}^2),\\ T_b=&e\epsilon_\mu\langle n;\, p\pi^0|J_\mu(0)|d\rangle\\ =&-(A_{1/2}{}^0+\tfrac{1}{3}A_{1/2}{}^1+\tfrac{2}{3}A_{3/2}{}^1-2A_{3/2}{}^2),\\ T_c=&e\epsilon_\mu\langle p;\, p\pi^-|J_\mu(0)|\rangle\\ =&\sqrt{2}(A_{1/2}{}^0-\tfrac{1}{3}A_{1/2}{}^1+\tfrac{1}{3}A_{3/2}{}^1+A_{3/2}{}^2),\\ T_d=&e\epsilon_\mu\langle p;\, n\pi^0|J_\mu(0)|d\rangle\\ =&-(A_{1/2}{}^0-\tfrac{1}{3}A_{1/2}{}^1-\tfrac{2}{3}A_{3/2}{}^1-2A_{3/2}{}^2).\end{aligned} \tag{2.6}$$

We have not bothered the reader with the details of the nucleon and photon spin. The amplitudes T_i should have a set of helicity indices λ associated with them, and we could denote them $T_i{}^\lambda$. The cross section, e.g., for $\gamma+d\rightarrow n+n+\pi^+$ would then be given by

$$\sigma=\textstyle\sum_\lambda|T_a{}^\lambda|^2.$$

Since the details of spin are not pertinent to our discussion we will continue to neglect them.

[6] I. Butterworth, in *Proceedings of the Heidelberg International Conference on Elementary Particles, 1967*, edited by H. Filthuth (Interscience Publishers, Inc., New York, 1968), p. 14.

Consider an experiment in which only a nucleon is detected and no attempt is made to identify the other particles. If we detect a neutron, we are measuring the sum of the cross sections for (2.1a) and (2.1b),

$$\sigma_n=|T_a|^2+|T_b|^2=6|\tfrac{1}{3}A_{3/2}{}^1-A_{3/2}|^2+3|A_{1/2}{}^0+\tfrac{1}{3}A_{1/2}{}^1|^2; \quad (2.7)$$

and if we detect a proton we are measuring the sum of (2.1c) and (2.1d),

$$\sigma_p=|T_c|^2+|T_d|^2=6|\tfrac{1}{3}A_{3/2}{}^1+A_{3/2}{}^2|^2+3|A_{1/2}{}^0-\tfrac{1}{3}A_{1/2}{}^1|^2. \quad (2.8)$$

We are interested in the difference between these two cross sections,

$$\Delta\sigma=\sigma_p-\sigma_n=\mathrm{Re}(8A_{3/2}{}^1A_{3/2}{}^{2*}-4A_{1/2}{}^0A_{1/2}{}^{1*}), \quad (2.9)$$

since in $\Delta\sigma$ only the interference terms between isotensor and isovector or isoscalar and isovector excitations appear.

The difference $\Delta\sigma$ has the important property that the χ particle in the isospin-$\frac{3}{2}$ state will be seen only by the interference of the isovector and isotensor amplitudes. Therefore, a measurement of $\Delta\sigma$ as a function of the mass W of the χ particle in the region of $W=1236$ MeV should provide a particularly sensitive test of the isotensor contribution to the electromagnetic current. There will, of course, be a contribution to $\Delta\sigma$ from the $I^\chi=\frac{1}{2}$ state, but we expect it to vary slowly with W so that a bump in $\Delta\sigma$ around $W=1236$ MeV should be a clear indication for the presence of the isotensor excitation. Since around the $\Delta(1236)$ the isovector $A_{1/2}{}^1$ is at most of the order $\frac{1}{10}$ of $A_{3/2}{}^1$, the difference (2.9) is sensitive to an isotensor interaction $A_{3/2}{}^2$ even if it is as small as $\frac{1}{10}$ of the isoscalar term $A_{1/2}{}^0$.

One should realize that the experiment does not distinguish whether there are one or two pions in the final state. Since the two-pion threshold is 20 to 30 MeV below the mass of the $\Delta(1236)$, there will be a small additional contribution to the background. Also if the isotensor term is smaller than the isoscalar, which may be expected from our discussion in Sec. 2, then it is necessary to consider the higher-order electromagnetic effects. These radiative corrections can also produce $I=2$ final states and may limit the sensitivity of the experiment if they can not be calculated reliably.

Finally, we would like to mention that according to Shaw[3] one should measure the ratio $|T_b|^2/|T_d|^2$ of reaction (2.1b) and (2.1d). To first order in the isoscalar and isotensor terms this ratio becomes

$$r=|T_b|^2/|T_d|^2=1+4\frac{\mathrm{Re}(A_{3/2}{}^2\sqrt{2}-A_{1/2}{}^0)(A_{3/2}{}^1\sqrt{2}-A_{1/2}{}^1)^*}{2|A_{3/2}{}^1-A_{1/2}{}^1|^2}+\cdots. \quad (2.10)$$

Here one sees that not only the isotensor amplitude will produce a deviation from $r=1$ but also the isoscalar amplitude. Consequently, one is limited in sensitivity to $A_{3/2}{}^2=O(A_{1/2}{}^0)$.

3. LIMITS ON ISOTENSOR CURRENT FOLLOWING FROM PHOTOPRODUCTION

A. General Considerations

As mentioned in the Introduction, the analysis of single-pion photoproduction provides an upper limit for the strength of the isotensor interaction. We will first discuss the results following from π^0 production. Then we give a discussion of the limits that can be set by analyzing the difference between π^+ and π^- production from deuterium.

Assuming that the electromagnetic current has the structure given in (2.3), one obtains for an arbitrary helicity amplitude

$$(\sqrt{\tfrac{1}{2}})H^{\pi\pm}=H^0\pm H^-+H^2=H^0\pm\tfrac{1}{3}H^{1/2}+(\mp\tfrac{1}{3}H^{3/2}+H^2), \quad (3.1a)$$

$$H^{\pi^0}=H^0\pm H^+-2H^2=H^0\pm\tfrac{1}{3}H^{1/2}-2(\mp\tfrac{1}{3}H^{3/2}+H^2). \quad (3.1b)$$

[In (3.1b) the upper or lower sign applies for the reaction with the proton or neutron, respectively.]

In (3.1) we have used the conventional notation of CGLN[7] for the isoscalar excitation H^0 and the two isovector excitations $H^\pm$, and introduced by H^2 the new isotensor term leading to a final state with isospin $\frac{3}{2}$. In the second lines of Eqs. (3.1a) and (3.1b) we have expanded the isovector parts $H^\pm$ into their contributions leading to final states with isospin $\frac{1}{2}$ and $\frac{3}{2}$. These have been denoted by $H^{1/2}$, $H^{3/2}$. Note that in this second form Eqs. (3.1a) and (3.1b) are of the form (2.6). This is true because in (2.6) the detected nucleon plays only the role of a spectator for the isospin decomposition. Therefore, if we subtract the cross sections for photoproduction at the neutron from those for photoproduction at the proton we obtain a relation analogous to (2.9),

$$\Delta\sigma'=\sigma_{p\pi^-}+\sigma_{n\pi^0}-\sigma_{n\pi^+}-\sigma_{p\pi^0}=\mathrm{Re}(8H^{3/2}H^{2*}-4H^0H^{1/2*}). \quad (2.9')$$

However, from the experimental point of view the difference (2.9) is easier to obtain, and therefore it is preferred for the detection of a possible isotensor interaction.

In the following we shall be only concerned with the limits on the isotensor excitation of the $\Delta(1236)$ resonance. We shall compare it with the corresponding isovector contribution, to characterize the strength of the interaction in question.

[7] G. F. Chew, M. L. Goldberger, F. E. Low, and Y. Nambu, Phys. Rev. 106, 1345 (1957).

B. π^0 Production

From a phenomenological point of view, π^0 photoproduction is the best place to isolate the large magnetic dipole excitation of the $\Delta(1236)$ resonance, which is conventionally denoted by $M_{1+}^{3/2}$.[7,8] This is because the nonresonant contributions at low energy are relatively small in π^0 production. Most suitable for our considerations are data on the total π^0 cross section $\sigma_{\rm tot}$ or results from measurements with plane-polarized γ's at $\theta=90°$ and azimuthal angle $\phi=90°$, $d\sigma_\perp/d\Omega$. Keeping in the multipole expansion of both cross sections only $l=0$ and 1 final states, one obtains

$$(k/q)\sigma_{\rm tot}{}^{\pi 0}=|E_{0+}{}^{\pi 0}|^2+|M_{1-}{}^{\pi 0}|^2+6|E_{1+}|^2+2|M_{1+}|^2+\cdots, \quad (3.2a)$$

$$\frac{k}{q}\frac{d\sigma_\perp{}^{\pi 0}}{d\Omega}(E,\theta=\phi=90°)=|E_{0+}{}^{\pi 0}|^2+|2M_{1+}{}^{\pi 0}+M_{1-}{}^{\pi 0}|^2+\cdots. \quad (3.2b)$$

Note that in (3.2a) interference terms between different multipoles drop out completely and that in (3.2b) the particularly uncertain $E_{1+}{}^{3/2}$ does not appear. Neglecting the background terms at the resonance, one obtains from (3.2)

$$2\frac{k}{q}\sigma_{\rm tot}(E_R)\approx\frac{k}{q}\frac{d\sigma_\perp}{d\Omega}(E=E_R,\theta=\phi=90°)\approx 4|M_{1+}{}^{\pi 0}|^2\approx(16/9)({\rm Im}M_{1+}{}^{3/2})^2. \quad (3.3)$$

Present theory[8] predicts (3.2) absolutely only within 20% using dispersion relations and pion-nucleon phase shifts. The failure of the theoretical prediction is caused mainly by the uncertainty of $M_{1+}{}^{3/2}$. The reason for our incomplete knowledge of $M_{1+}{}^{3/2}$ has been investigated by Engels and Schmidt.[9] It was found that the main uncertainties in $M_{1+}{}^{3/2}$ treated by dispersion theory are unknown high-energy contributions to the dispersion integrals. These terms can be lumped together in a constant, which fixes ${\rm Im}M_{1+}{}^{3/2}(E_R)$ and which has to be taken from experiment to obtain an accuracy of $M_{1+}{}^{3/2}$ better than 10%. But, of course, one might also assume that part of the discrepancy in (3.2) is caused by the neglect of the isotensor interaction.

An isotensor excitation of the $\Delta(1236)$ resonance would contribute in first order to (3.3) the amount

$$2(k/q)\Delta\sigma_{\rm tot}\approx(k/q)\Delta\sigma_\perp=-(32/3)\,{\rm Im}M_{1+}{}^{3/2}\,{\rm Im}M_{1+}{}^2. \quad (3.4)$$

From (3.3) and (3.4) follows the ratio

$$\frac{{\rm Im}M_{1+}{}^2(E_R)}{{\rm Im}M_{1+}{}^{3/2}(E_R)}=-\frac{1}{6}\frac{\Delta\sigma_\perp(E_R)}{\sigma_\perp(E_R)}=-\frac{1}{6}\frac{\Delta\sigma_{\rm tot}(E_R)}{\sigma_{\rm tot}(E_R)}. \quad (3.5)$$

Ascribing now part of the discrepancy of 20% in $\sigma_\perp$ or $\sigma_{\rm tot}$ to the isotensor interaction we obtain at the resonance the inequality

$$|{\rm Im}M_{1+}{}^2(E_R)|<0.03|{\rm Im}M_{1+}{}^{3/2}(E_R)|. \quad (3.6)$$

We would like to stress that (3.6) derived from π^0 production is based on the assumption that current theory can predict (3.3) within 20% and that only these 20% can be made responsible for an appreciable isotensor interaction. A violation of (3.6) would clearly upset our present understanding of the theory of $M_{1+}{}^{3/2}$.

The bound (3.6) is confirmed by a less model-dependent analysis, which the present data on the π^+/π^- ratio allow.

C. π^+/π^- Ratio

According to (3.1) the difference of the cross section for π^+ and π^- production is

$$\sigma_-=\frac{k}{q}\left(\frac{d\sigma^{\pi+}}{d\Omega}-\frac{d\sigma^{\pi-}}{d\Omega}\right)=\sum_i|H_i{}^{\pi+}|^2-|H_i{}^{\pi-}|^2 =8\sum_i{\rm Re}(H_i{}^-(H_i{}^0+H_2{}^2)^*). \quad (3.7)$$

In (3.7) the sum extends over all helicity amplitudes.

Data relevant to Eq. (3.7) are shown in Fig. 1, where excitation curves for π^+ and π^- production and the difference $(q/k)\sigma_-$ are plotted, These results are obtained from the measurement of the π^+/π^- ratio and the π^+ cross section.[10] Also shown are the theoretical predictions (with $H_i{}^2\equiv 0$) following from dispersion theory.[8] One has to realize that σ_- is a very sensitive test for the isoscalar amplitudes $H_i{}^0$, which contribute only a few percent of the π^+ and π^- cross section. Changes of the order 50% in $8\sum_i H_i{}^0H_i{}^{-*}$, which contribute 1–2$\mu$b in σ_-, are therefore to be expected. On the other hand, the differences between theory and experiment could be attributed to the $\sum_i H_i{}^2H_i{}^{-*}$ term, if one believes in the existence of the isotensor interaction. Thus one obtains as a crude upper bound

$$\sum_i|{\rm Re}H_i{}^2H_i{}^{-*}|<2/8\;\mu b=0.25\;\mu{\rm b} \quad (3.8)$$

for the region of the $\Delta(1236)$ resonance. The result (2.8) is somewhat less model-dependent than (2.6) since the upper bound (2.8) is already of the order of σ_-. Also, it is improbable that the isoscalar amplitude $H_i{}^0$ is wrong by more than 100%.

We now convert the result (3.8) into a statement on the isotensor excitation of the $\Delta(1236)$ resonance. To this end we expand $d\sigma^{\pi+}/d\Omega$ into multipoles, keeping

[8] J. Engels, A. Müllensiefen, and W. Schmidt, Phys. Rev., this issue, **175**, 1951 (1968).

[9] J. Engels and W. Schmidt, Phys. Rev. **169**, 1296 (1968).

[10] J. T. Beale, S. D. Ecklund, and R. L. Walker, Cal-Tech Report No. CTSL-42, CALT-68-108 (unpublished).

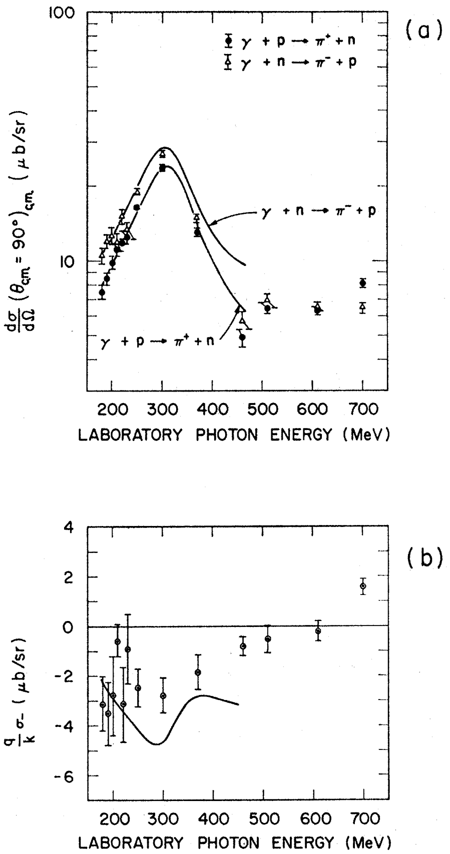

FIG. 1. (a) Center-of-mass differential cross section for π^+ and π^- photoproduction. The experimental points are obtained from measurements of the π^+/π^- ratio from deuterium and the π^+ cross sections from hydrogen (Ref. 10). The theoretical curves were calculated from the photproduction dispersion theory (Ref. 8). (b). The difference between the cross sections for π^+ and π^- production shown in Fig. 1 (a).

$$(q/k)\sigma^- = (d\sigma/d\Omega)(\gamma p \to \pi^+ n) - (d\sigma/d\Omega)(\gamma n \to \pi^- p).$$

again in only $l=0$ and 1 terms:

$$\frac{d\sigma^{\pi\pm}}{d\Omega}(E,\theta=90°) = |E_{0+}{}^{\pi\pm}|^2 + |M_{1-}{}^{\pi\pm}|^2 + \tfrac{5}{2}|M_{1+}{}^{\pi\pm}|^2 + \tfrac{9}{2}|E_{1+}{}^{\pi\pm}|^2 + 3\,\mathrm{Re}E_{1+}{}^{\pi\pm}(M_{1-}{}^{\pi\pm} - M_{1+}{}^{\pi\pm})^* + \mathrm{Re}M_{1+}{}^{\pi\pm}M_{1-}{}^{\pi\pm *} + \cdots. \quad (3.9)$$

The correction to σ_- in first order of the isotensor excitation of $\Delta(1236)$ just at the resonance is then

$$|\Delta\sigma_-(E_R)| = (20/3)|\mathrm{Im}M_{1+}{}^{3/2}| \times |\mathrm{Im}(M_{1+}{}^2 - \tfrac{3}{5}E_{1+}{}^2)|, \quad (3.10)$$

where we also neglected $E_{1+}{}^{3/2}$, which is of the order $-\frac{1}{10}$ of $M_{1+}{}^{3/2}$. Using $\mathrm{Im}M_{1+}{}^{3/2}(E_R) = 3.75\ 10^{-2}\lambda\!\!\!^-$,[8] and the inequality (3.8) for $\Delta\sigma_-$, we obtain

$$\left|\frac{\mathrm{Im}(M_{1+}{}^2 - \frac{3}{5}E_{1+}{}^2)}{\mathrm{Im}M_{1+}{}^{3/2}}\right|_{E=E_R} < \frac{3}{20}\,\frac{2\,\mu\mathrm{b}}{(3.75)^2 \times 10^{-4}\lambda\!\!\!^{-2}} = 1.1\%. \quad (3.11)$$

The upper limit (3.11) is of the same order of magnitude as obtained in Sec. 3 B.

4. SUMMARY

We conclude that present experimental information on pion photoproduction places an upper bound on the strength of the isotensor interaction for exciting the $\Delta(1236)$ resonance of a few percent of the strength of the isovector. Therefore, the interaction in question yields contribution in pion photoproduction, which is at most of the order of the isoscalar term. The isoscalar term itself is not quantitatively well defined by present experiments.

Most of the experiments suggested in the literature[1–4] for detecting the isotensor interaction will give a null result unless they are carried out to high accuracy. Those experiments that involve photoproduction of the $\Delta(1236)$ should be examined carefully, because the nonresonant contributions will generally prevent one from achieving sufficient sensitivity to the isotensor term. The experiment described in Sec. 2 avoids the problems of the nonresonant background to a high degree, but would have to be a precision measurement because of the assumed smallness of the isotensor interaction.

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