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Nucleon and Pion Form Factors and the Dipole ρ Meson*

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A model of the pion and nucleon form factors is studied in which the isoscalar nucleon form factor is dominated by monopole ω and ϕ mesons, and the isovector form factor by a dipole ρ meson. The predicted form factors are in good agreement with the data. The experimental ρ-, ω-, and ϕ-meson masses and the nucleon magnetic moments constitute the only numerical input in the model.

RECENTLY,[1] the implications of assuming that the ρ meson has a dipole structure have been investigated for the nucleon form factors and πN charge-exchange scattering. It was found that the dipole ρ meson provided an explanation of the $1/q^4$ falloff of the nucleon form factors confirmed by the new measurements performed at SLAC.[2] The πN charge-exchange scattering and a nonzero polarization were explained on the basis of a Regge dipole model of the ρ meson, which also satisfied the superconvergence relations. In the following, we shall extend our predictions of the form factors by taking into account the isoscalar contributions due to the ω and ϕ mesons. We shall assume in our model that the ω and ϕ mesons are simple poles and that the ρ meson is a dipole. A dipole Lee model has been constructed by Bell and Goebel[3] in which the dipole nature of a resonance in momentum space arises from a process of the type $\rho_1 \to \rho_2 \to 2\pi$, where the ρ_1 and ρ_2 are mass-degenerate and there is a cancellation of the single-pole residues for this process, because of a particular relationship between the coupling constants. The ρ meson may be explained in terms of such a mass degeneracy, and we adopt the attitude that the ω and ϕ mesons do not satisfy this type of degeneracy condition.[4]

The proton and neutron form factors are defined in terms of isoscalar and isovector contributions by

$$G_{E,M}{}^p = \tfrac{1}{2}(G_{E,M}{}^S + G_{E,M}{}^V)\,, \qquad G_{E,M}{}^n = \tfrac{1}{2}(G_{E,M}{}^S - G_{E,M}{}^V)\,, \tag{1}$$

where $G_E{}^p(0)=1$, $G_E{}^n(0)=0$, $G_M{}^p(0)=\mu_p=2.79$, and $G_M{}^n(0)=\mu_n=-1.91$.

We assume that the form factors obey the unsubtracted dispersion relations

$$G_{E,M}{}^S(q^2) = \frac{1}{\pi}\int_{(3m_\pi)^2}^{\infty} \frac{\mathrm{Im}G_{E,M}{}^S(q'^2)}{q'^2+q^2}\, d(q'^2)\,, \tag{2}$$

$$G_{E,M}{}^V(q^2) = \frac{1}{\pi}\int_{(2m_\pi)^2}^{\infty} \frac{\mathrm{Im}G_{E,M}{}^V(q'^2)}{q'^2+q^2}\, d(q'^2)\,. \tag{3}$$

The absorptive parts are taken to be dominated by the dipole ρ meson and single-pole ω and ϕ mesons such that

$$\mathrm{Im}G_{E,M}{}^S(q^2) = \pi R_\omega \delta(q^2 - m_\omega{}^2) + \pi R_\phi \delta(q^2 - m_\phi{}^2) \tag{4}$$

and

$$\mathrm{Im}G_{E,M}{}^V(q^2) = \pi R_\rho \frac{d}{d(q^2)} \delta(q^2 - m_\rho{}^2)\,, \tag{5}$$

where R_ω, R_ϕ, and R_ρ denote the corresponding residues. By assuming that $G_{E,M}{}^S$ is superconvergent, it follows that $R_\omega + R_\phi = 0$, and we obtain the results:

$$G_{E,M}{}^S(q^2) = \frac{G_{E,M}{}^V(0)}{(1+q^2/m_\omega{}^2)(1+q^2/m_\phi{}^2)}\,, \tag{6}$$

$$G_{E,M}{}^V(q^2) = \frac{G_{E,M}{}^V(0)}{(1+q^2/m_\rho{}^2)^2}\,. \tag{7}$$

In view of Eq. (1) and the normalization conditions at the origin, this leads us to the predictions for the pro-

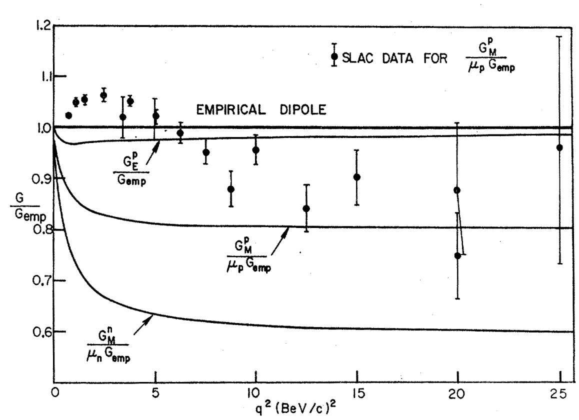

FIG. 1. Predictions of $G_E{}^p/G_{\mathrm{emp}}$, $G_M{}^p/(\mu_p G_{\mathrm{emp}})$, and $G_M{}^n/(\mu_n G_{\mathrm{emp}})$ compared with the experimental data for $G_M{}^p/(\mu_p G_{\mathrm{emp}})$ presented in Ref. 2.

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[1] R. E. Kreps and J. W. Moffat, preceding paper, Phys. Rev. **175**, 1945 (1968).

[2] W. K. H. Panofsky, in *Proceedings of The Heidelberg International Conference on Elementary Particles, 1967*, edited by H. Filthuth (North-Holland Publishing Co., Amsterdam, 1968), p. 371.

[3] J. S. Bell and C. J. Goebel, Phys. Rev. **138**, B1198 (1965).

[4] By using the alternative suggestion that an additional isovector meson ρ' is necessary to saturate current-algebra sum rules and form factors [see J. W. Moffat, Phys. Rev. Letters **20**, 620 (1968)], a prediction of $F_1{}^V(q^2)$ has been obtained by J. G. Cordes and P. J. O'Donnell, Phys. Rev. Letters **20**, 1462 (1968). This prediction of $F_1{}^V(q^2)$ is in good agreement with the data provided it is assumed that $G_E{}^n(q^2)\approx 0$. It is difficult to obtain the correct asymptotic behavior of $F_2{}^V \sim 1/q^6$ in this model just on the basis of ρ and ρ'.

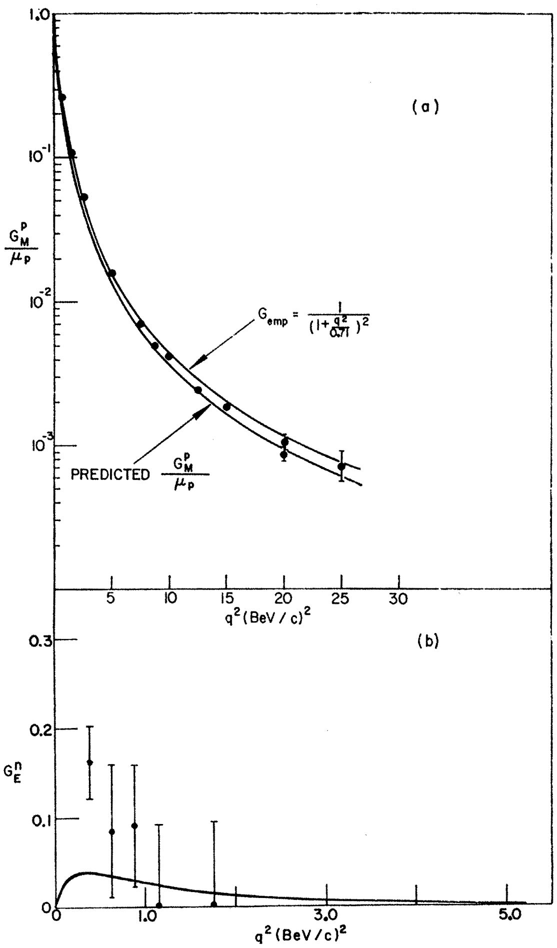

FIG. 2 (a) Prediction of $G_M{}^p/\mu_p$ compared with the SLAC data for $G_M{}^p/\mu_p$ in Ref. 2. (b) Prediction of $G_E{}^n$ compared with the neutron form-factor data shown in Ref. 5.

ton and neutron form factors

$$G_E{}^{p,n}(q^2)=\frac{1}{2}\left(\frac{1}{(1+q^2/m_\omega{}^2)(1+q^2/m_\phi{}^2)}\pm\frac{1}{(1+q^2/m_\rho{}^2)^2}\right) \qquad (8)$$

and

$$\frac{G_M{}^{p,n}(q^2)}{\mu_p}=\frac{1}{2}\left(\frac{1+\mu_n/\mu_p}{(1+q^2/m_\omega{}^2)(1+q^2/m_\phi{}^2)}\pm\frac{1-\mu_n/\mu_p}{(1+q^2/m_\rho{}^2)^2}\right), \qquad (9)$$

where the plus and minus signs refer to the proton and neutron form factors, respectively.

The only numerical input in our predictions (8) and (9) is the experimental meson masses and nucleon magnetic moments. Our predictions of the nucleon electric and magnetic form factors are displayed in Fig. 1 as the ratios of the predicted $G_E{}^p$, $G_M{}^p/\mu_p$, and $G_M{}^n/\mu_n$ to the empirical dipole fit

$$G_{\rm emp}(q^2)=\frac{1}{(1+q^2/0.71)^2}. \qquad (10)$$

Also shown is the ratio of $G_M{}^p/\mu_p$ obtained at SLAC[2] to the empirical dipole fit.

In Fig. 2, we compare the predicted $G_M{}^p/\mu_p$ and $G_E{}^n$ to the experimental data.[2,5] The experimental points displayed do not include the systematic error (estimated to be $<6\%$), and they assume[6] $G_E{}^p=G_M{}^p/\mu_p$. This latter assumption can introduce an error $\sim5\%$, and an estimate shows that relaxing this condition shifts the experimental $G_M{}^p/\mu_p$ into better agreement with our prediction.

The asymptotic limits of the form factors for large q^2 can all be expressed in the form a^2/q^4. We find that

$$q^4G_E{}^p \xrightarrow[q^2\to\infty]{} \tfrac{1}{2}(m_\rho{}^4+m_\omega{}^2m_\phi{}^2)=(0.71)^2. \qquad (11)$$

Also, $q^4G_M{}^p/\mu_\rho$ tends to $(0.64)^2$ at large q^2, and $q^4G_E{}^n$ tends to $(0.37)^2$. We stress that these results follow just from the experimental masses, as seen in other reactions, and the measured magnetic moments. For the slope of the neutron form factor at $q^2=0$, we obtain

$$\{[d/d(q^2)]G_E{}^n(q^2)\}_{q2=0}=0.0144\ \mathrm{F}^2,$$

which is $\sim25\%$ smaller than the experimental value

$$\{[d/d(q^2)]G_n{}^E(q^2)\}_{q2=0}=0.0193\pm0.0004\ \mathrm{F}^2$$

obtained from the scattering of electrons and thermal neutrons.[7] This prediction of the neutron form-factor slope corresponds to a predicted root-mean-square radius of the neutron $r_n=0.29$ F. The predicted root-mean-square radius of the proton is $r_p=0.83$ F. In order

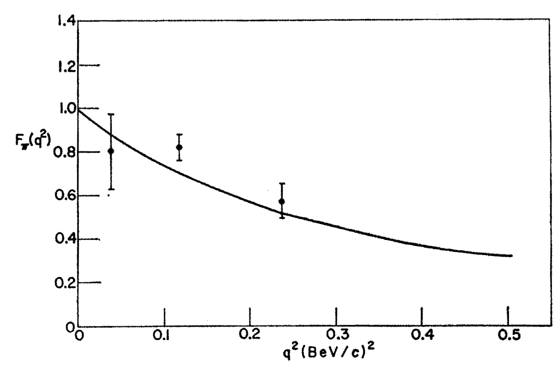

FIG. 3. Comparison of our prediction for F_π with the experimental data in Ref. 8.

[5] K. W. Chen, J. R. Dunning, Jr., A. A. Cone, N. F. Ramsey, J. K. Walker, and R. Wilson, Phys. Rev. **141**, 1267 (1966).

[6] One normally assumes the equalities $G_E{}^p=G_M{}^p/\mu_p=G_M{}^n/\mu_n$ and $G_E{}^n=0$. This assumption is consistent with the data. However, since we have explicitly assumed different q^2 behavior in the isovector and isoscalar form factors, we have $G_E{}^n\neq0$ and the other equalities are also not satisfied. To the extent that $G_E{}^n$ is small, they hold approximately and our results are consistent with the data too.

[7] V. E. Kronstadt and G. R. Ringo, Phys. Rev. **148**, 1303 (1966).

to estimate the theoretical uncertainty in our prediction, we use the neutron form-factor slope at the origin as an indication that at $q^2=0$ the isovector and isoscalar form factors will have approximately a 10% uncertainty. We expect that G_E^n, being a difference of form factors, will be much more sensitive to this uncertainty than G_E^p and it is interesting to note that our predicted G_E^p is close to the empirical dipole fit [Eq. (10)], and approaches it exactly in the limit of large q^2.

Let us now consider the pion form factor. In this case, there is only an isovector contribution and we obtain

$$F_\pi(q^2)=\frac{1}{(1+q^2/m_\rho^2)^2}. \tag{12}$$

Our predicted $F_\pi(q^2)$ is displayed in Fig. 3, and it is clear that the result is consistent with the available data.[8] The predicted charge radius of the pion is $r_\pi=0.88$ F.

[8] C. W. Akerlof, W. W. Ash, K. Berkelman, C. A. Lichtenstein, A. Ramanauskas, and R. H. Siemann, Phys. Rev. 163, 1482 (1967).

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Spherical Kemmer Top*

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Kemmer's equation is modified to include the rotational contribution to the rest energy characteristic of a spherical top. The resulting representation-invariant eigenvalue equation is solved, giving the rest energy as a function of the spin. An order-of-magnitude calculation yields a mass difference for the two superposed spin states characteristic of the elementary particles.

The applicability of certain linear relativistic wave equations, with the capacity to admit a mass-spin spectrum, to a description of the elementary particles is a subject of considerable current interest.[1] In this spirit, we here treat the rest-energy eigenvalue problem deriving from Kemmer's equation,[2] modified to include a rotational contribution to the rest energy. This is a specialization of Corben's formulation[3] for a relativistic rotator to the case of a spherical top with spin 0 or 1. The calculation is independent of any particular representation of the Duffin-Kemmer matrices.

Kemmer's equation with a mass operator characteristic of the rotational energy of a spherical top is

$$(i\beta_\mu P_\mu+Mc)\psi=0, \tag{1}$$

where

$$Mc=mc+m'c\beta_{\mu\nu}\beta_{\mu\nu} \tag{2}$$

is the mass operator, $P_\mu=-i\hbar\partial_\mu$, $\beta_{\mu\nu}=(\beta_\mu,\beta_\nu)$ defines the spin operator, and the algebra of the β matrices is given by[2]

$$\beta_\mu\beta_\nu\beta_\rho+\beta_\rho\beta_\nu\beta_\mu=\beta_\mu\delta_{\nu\rho}+\beta_\rho\delta_{\mu\nu}. \tag{3}$$

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[1] H. C. Corben, Proc. Natl. Acad. Sci. US 48, 1559 (1962); 48, 1746 (1962); Phys. Rev. 134, B832 (1964); Phys. Rev. Letters 15, 268 (1965); Y. Nambu, Progr. Theoret. Phys. (Kyoto) 37, 368 (1966); Phys. Rev. 160, 1171 (1967); K. Rafanelli, Phys. Rev. 175, 1761 (1968).

[2] N. Kemmer, Proc. Roy. Soc. (London) A952, 91 (1939).

[3] H. C. Corben, *Classical and Quantum Theories of Spinning Particles* (Holden-Day Publishing Co., San Francisco, 1968), Chap. 4.

There are two parameters in (2), m and

$$m'=\hbar^2/4c^2I, \tag{4}$$

where I is the moment of inertia of the spherical rotator.[3]

Introducing the spin and boost operators

$$i\mathbf{J}=(\beta_{23},\beta_{31},\beta_{12}), \tag{5a}$$

$$i\mathbf{K}=(\beta_{14},\beta_{24},\beta_{34}), \tag{5b}$$

then in the rest system, $P_\mu=(0,iE^{\rm op}/c)$, Eq. (1) becomes

$$\beta_4W\psi=[1+2a(J^2+K^2)]\psi, \tag{6}$$

where $a=m'/m$ and $W=E^{\rm op}/mc^2$. It is the solution of the eigenvalue problem (6) we wish to present here.

With $A=1+2aJ^2$, squaring (6) yields

$$(A+2aK^2)\beta_4W\psi=(A^2+4aAK^2+4a^4K^4)\psi, \tag{7}$$

where we have used $(J^2,K^2)=0$. After some lengthy, but straightforward, algebra, we see that

$$K^2\beta_4+\beta_4K^2=3\beta_4 \tag{8}$$

and

$$K^4=3K^2-J^2. \tag{9}$$

With these last two algebraic identities, and the linear equation (6) in the form

$$2aK^2\psi=(\beta_4W-A)\psi, \tag{10}$$

we obtain, from (7),

$$W^2\beta_4^2\psi=(A^2+4aJ^2+6aA)\psi. \tag{11}$$