Dipole ϱ Meson, Nucleon Form Factors, and Charge-Exchange Scattering*

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On the basis of a dipole model of the ρ meson, we explain the $1/q^4$ behavior of the nucleon form factor. In terms of an extension of Regge-pole ideas to a dipole ρ trajectory, fits are obtained to the charge-exchange cross sections, and a nonzero polarization is predicted.

THE possibility that resonances may have a dipole character has been considered by several people.[1] For a small background scattering, the S matrix has the form

$$S=\left(\frac{E_0-E+i(\frac{1}{2}\Gamma)}{E_0-E-i(\frac{1}{2}\Gamma)}\right)^2. \qquad (1)$$

If we adopt the point of view that the ρ meson is a diploe resonance[2] and assume that the electromagnetic form factors of the nucleon are dominated by this resonance, then these form factors will take the form

$$G_E{}^V(q^2)=\frac{G_M{}^V(q^2)}{(\mu_p-\mu_n)}=\frac{1}{(1+q^2/m_\rho{}^2)^2}, \qquad (2)$$

where μ_p and μ_n are the proton and neutron magnetic moments, respectively. This form factor has the correct asymptotic behavior suggested by the recent nucleon form factor measurements at SLAC.[3]

In order to study the implications of a dipole ρ meson, we must extend the ideas of the conventional Regge-pole theory. The charge-exchange amplitude is defined by

$$A_{\text{ch-ex}}=f+i\frac{\boldsymbol{\sigma}\cdot(\mathbf{q}'\times\mathbf{q})}{q^2}\tilde{f}, \qquad (3)$$

where f and $\tilde{f}$ are the non-spin-flip amplitudes, respectively, and $\mathbf{q}$ and $\mathbf{q}'$ are the center-of-mass momenta of the initial and final states.

For the exchange of a single dipole ρ meson the asymptotic Regge amplitudes f_D and $\tilde{f}_D$ are given by

$$f_D=(\sqrt{2}\gamma_\rho/\sqrt{s})(\nu/\nu_0)^{\alpha_\rho(t)}\{\ln(\nu/\nu_0)\,[i+\tan\tfrac{1}{2}\pi\alpha_\rho(t)]+\tfrac{1}{2}\pi\sec^2[\tfrac{1}{2}\pi\alpha_\rho(t)]\} \qquad (4)$$

and

$$\tilde{f}_D=(1/2\sqrt{2}M)(\nu/\nu_0)^{\alpha_\rho(t)}\{[-\gamma_\rho+\alpha_\rho(t)\tilde{\gamma}_\rho]\{\ln(\nu/\nu_0)\times[i+\tan\tfrac{1}{2}\pi\alpha_\rho(t)]+\tfrac{1}{2}\pi\sec^2[\tfrac{1}{2}\pi\alpha_\rho(t)]\}+\tilde{\gamma}_\rho[i+\tan\tfrac{1}{2}\pi\alpha_\rho(t)]\}. \qquad (5)$$

These forms result from assuming a double pole in the complex l plane, which implies that we differentiate the usual[4] Regge ρ expression for f and $\tilde{f}$ with respect to α. We also assume that the residues are independent of α and t. We effectively recover the usual forms in the limit of large energy. $\alpha_\rho(t)$ is the ρ trajectory and γ_ρ, $\tilde{\gamma}_\rho$ are the non-spin-flip and spin-flip residues, respectively.

In terms of Eqs. (4) and (5) we find for the polarization parameter

$$P=\frac{\pi\sin\theta\,\gamma\tilde{\gamma}(\nu/\nu_0)^{2\alpha_\rho}\sec^2(\tfrac{1}{2}\pi\alpha_\rho)}{2MW[|f|^2-(4t/s)|\tilde{f}|^2]}, \qquad (6)$$

where θ is the scattering angle and $W=\sqrt{s}$. Thus, the dipole Regge ρ exchange predicts a *nonzero* polarization, in contrast to the simple ρ-meson exchange for which the polarization is identically zero.

The nonvanishing polarization in charge-exchange πN scattering has also been explained by introducing a ρ' meson with $J^P=1^-$.[4] However, this model contains eight free parameters, one parameter being constrained by a finite-energy sum rule. In the present model based on a dipole ρ trajectory, we only have four parameters. This is the same number of parameters that occurs in the simple ρ trajectory model. The difference of (π^-p) and (π^+p) cross sections in our model is given by

$$\Delta\sigma_T=\frac{8\pi\gamma_0}{kM}(\nu/\nu_0)^{\alpha_\rho}\ln(\nu/\nu_0). \qquad (7)$$

The superconvergence relations[5,6] for the non-spin-flip amplitude take the form

$$\left(\frac{\gamma_\rho}{1+\alpha_\rho}\right)\left(\frac{\bar{\nu}}{\nu_0}\right)^{\alpha_\rho}\left(\ln(\bar{\nu}/\nu_0)-\frac{1}{1+\alpha_\rho}\right)=0.72\pm0.08 \qquad (8)$$

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[1] M. L. Goldberger and K. M. Watson, Phys. Rev. **136**, B1472 (1964); J. S. Bell and C. J. Goebel, *ibid.* **138**, B1198 (1965); J. S. Bell, CERN Report No. 66/524/5-Th. 658, 1966 (to be published); H. Osborn, Phys. Rev. **145**, 1272 (1966); K. E. Lassila and V. Ruuskanen, Phys. Rev. Letters **17**, 490 (1960); **19**, 762 (1967); T. Sawada, Nuovo Cimento **48**, 534 (1967).

[2] M. Roos, Nuclear Physics **B2**, 615 (1967). In this report the world $\rho\to2\pi$ is analyzed and the ρ masses are found to be t-dependent. A possible explanation of the detailed structure of the ρ meson could be found in the dipole meson which would correspond to a double-peaked structure. Such a structure has been discussed in Ref. 4 in connection with the A_2 meson.

[3] R. E. Taylor, Stanford Linear Accelerator Center Report No. SLAC-PUB-372, 1967 [in Proceedings of the International Symposium on Electron and Photon Interaction at High Energy, 1967 (to be published)]; W. K. H. Panofsky, in *Proceedings of the Heidelberg International Conference on Elementary Particles, 1967*, edited by H. Filthuth (North-Holland Publishing Co., Amsterdam, 1968), p. 371.

[4] T. J. Gajdicar, R. K. Logan, and J. W. Moffat, Phys. Rev. **170**, 1599 (1968).

[5] M. G. Olsson, Phys. Rev. Letters **19**, 550 (1967).

[6] For a review of this subject, see J. W. Moffat, in Proceedings of the IV Boulder Conference on Particle and High Energy Physics (Gordon and Breach Science Publishers, Inc., New York, to be published).

and

$$\left(\frac{\gamma_\rho}{\pi\alpha_\rho}\right)\left(\frac{\bar{\nu}}{\nu_0}\right)^{\alpha_\rho}\left[\tfrac{1}{2}\pi\sec^2(\tfrac{1}{2}\pi\alpha_\rho)\right.$$
$$\left.+\tan\tfrac{1}{2}\pi\alpha_\rho\left(\ln(\bar{\nu}/\nu_0)-(1/\alpha_\rho)\right)\right]=0.73\pm0.15. \quad (9)$$

The scale factor $\nu_0=s_0/2M$ and we shall choose $s_0=2M\mu$,[7] so that $\bar{\nu}/\nu_0=\bar{\nu}/\mu$, where $\bar{\nu}$ denotes the asymptotic laboratory energy.

A fit to the proton form-factor data[3] is shown in Fig. 1(a) using Eq. (2) with $M_{\rho 0}=778$ MeV. At present it is not known whether the neutron form factor is zero and it is clear that since we are assuming that the dipole ρ meson dominates the isovector form factor $G^V=G^P-G^n$ a nonzero neutron form factor can account for the discrepancy of $\sim$20% at low values of $-t$.[8]

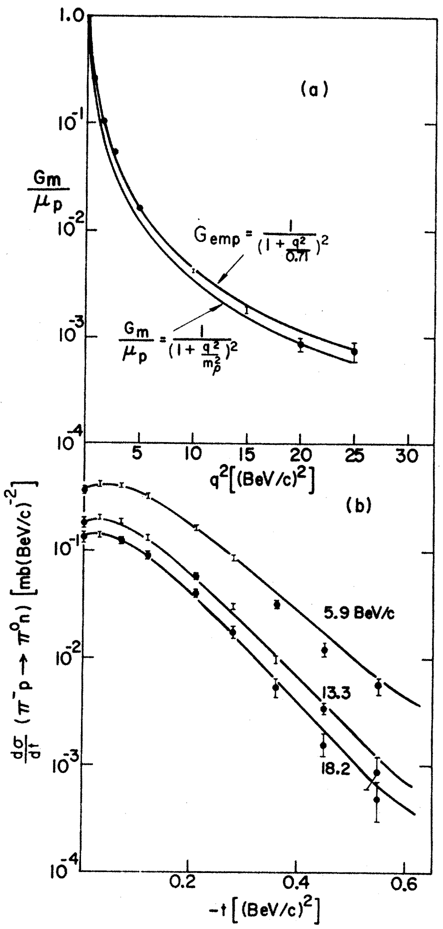

FIG. 1. (a) Comparison of $G_M{}^p(q^2)/\mu_p=(1+q^2/m_\rho{}^2)^{-2}$ with the SLAC data in Ref. 3 using $M_{\rho^0}{}^2=0.61$ BeV^2. The curve corresponding to the empirical dipole $G_{\text{emp}}=(1+q^2/0.71)^{-2}$ is also shown. (b) Fit of $d\sigma/dt$ $(\pi^-+p\to\pi^0+n)$ to the charge-exchange scattering cross-section data in Ref. 10, using the dipole ρ exchange model.

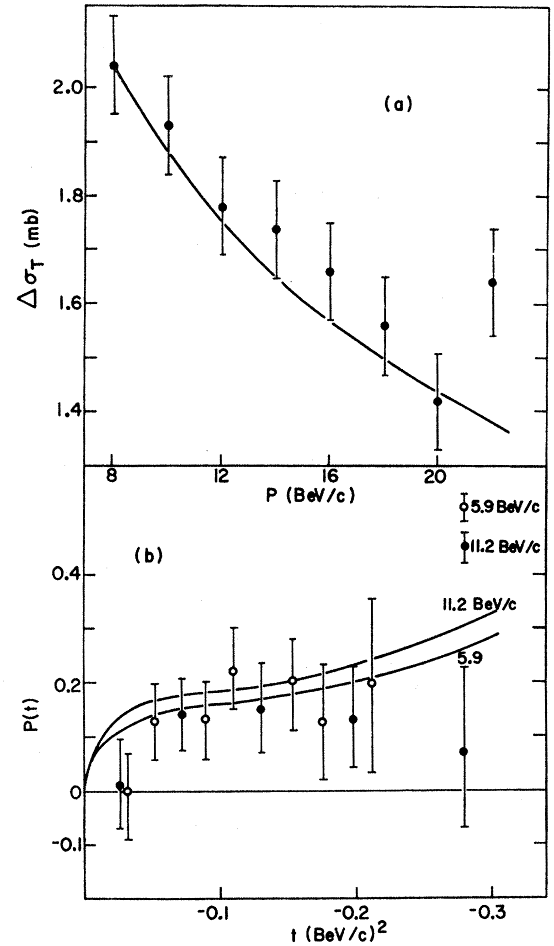

FIG. 2. (a) Prediction of $\Delta\sigma_T=\sigma(\pi^-p)-\sigma(\pi^+p)$ compared with the data of Foley *et al.* in Ref. 11. (b) The predicted polarization P based on the dipole ρ model for two energies 5.9 and 11.2 BeV. The experimental data given in Ref. 13 are shown for both energies.

A preliminary fit to the $d\sigma/dt$ and $\Delta\sigma_T$ data[9–11] at $t=0$ gives the results $\alpha_\rho(0)=0.40$ and $\gamma_\rho=0.076$. A fit to $d\sigma/dt$ for various values of $-t$ and for the three energies 5.9, 13.3, and 18.2 BeV is shown in Fig. 1(b). To fit these data we have assumed a linear relationship for the t dependence of $\alpha_\rho(t)$, viz., $\alpha_\rho(t)=\alpha_\rho(0)+\alpha_\rho{}'t$. We find that $\bar{\gamma}=1.2$ and $\alpha_\rho{}'=0.88$. Again we have

[7] I. R. Gatland and J. W. Moffat, Phys. Rev. **132**, 442 (1963).

[8] If we take our dipole prediction Eq. (2) for the vector form factor and take the empirical dipole fit $G_E{}^P(q^2)=1/(1+q^2/0.71)^2$, then we predict the slope at the origin to be $dG_E{}^n/dq^2=0.018$ F^2, which agrees remarkably well with the experimental value $dG_E{}^n/dq^2=0.0193\pm0.0004$ F^2 [V. E. Krohn and G. R. Ringo, Phys. Rev. **148**, 1303 (1966); E. Melkonian, B. M. Rustad, and W. W. Havens, *ibid.* **114**, 1571 (1959)].

[9] A. V. Stirling. P. Sonderegger, J. Kirz, P. Falk-Vairant, O. Guisan, C. Bruneton, and P. Borgeaud, Phys. Rev. Letters **14**, 763 (1965); P. Sonderegger, J. Kirz, O. Guisan, P. Falk-Vairant, C. Bruneton, P. Borgeaud, A. V. Stirling, C. Caverzasio, J. P. Guillard, M. Yvert, and B. Amblard, Phys. Letters **20**, 75 (1966); I. Mannelli, A. Bigi, R. Canara, M. Wahlig, and L. Sodickson, Phys. Rev. Letters **14**, 408 (1965).

[10] K. J. Foley, R. S. Jones, S. J. Lindenbaum, W. A. Love, S. Ozaki, E. D. Platner, C. A. Quarles, and G. H. Willen, Phys. Rev. Letters **19**, 330 (1967).

[11] W. Galbraith, E. W. Jenkins, T. F. Kycia, B. A. Leontic, R. H. Phillips, A. L. Read, and R. Rubinstein, Phys. Rev. **138**, B913 (1965).

chosen $\nu_0=\mu$. This choice of ν_0 is important because of the logarithmic factors in f_D and $\tilde{f}_D$. A pronounced minimum occurs in the spin-flip amplitude at $-t\sim0.6$ BeV^2.

The prediction of $\Delta\sigma_T$ is shown in Fig. 2(a). With the above obtained values of $\alpha_\rho(t)$, γ_ρ, and $\tilde{\gamma}_\rho$, we can predict the polarization from Eq. (6) and this result is shown at 5.9 and 11.2 BeV, in Fig. 2(b). At $-t=0.12$ BeV^2 the polarization is $P\approx15\%$, in reasonable agreement with the data.[12] We observe that P increases with increasing $-t$ at both 5.9 and 11.2 BeV, and that there is a slight increase in P going from energies of 5.9 to 11.2 BeV.

The ratio of $\mathrm{Re}f_D$ to $\mathrm{Im}f_D$ in the model is given by

$$\frac{\mathrm{Re}f_D}{\mathrm{Im}f_D}=\tan\tfrac{1}{2}\pi\alpha_\rho+\frac{\pi}{2\cos^2(\frac{1}{2}\pi\alpha_\rho)\ln(\nu/\nu_0)}. \qquad (10)$$

We obtain for this ratio at 5.9 and 18.2 BeV the values 1.37 and 1.22, in agreement with the data.[9] This is to be expected since we fit the $\Delta\sigma_T$ and $d\sigma/dt$ data.

With $\alpha_\rho(0)=0.40$ and $\gamma_\rho=0.076$, the superconvergence relations, Eqs. (8) and (9), are satisfied giving the values 0.65 and 0.81, respectively.

In summary, we have found that a dipole ρ meson gives a good account of the form-factor data of the proton, and that a Regge dipole ρ provides a good description of charge-exchange data with a nonzero polarization in terms of only four parameters, two of which can be considered to be constrained by the superconvergence relations. In view of the degree of success of the dipole-ρ-meson model in describing nucleon form factors and charge-exchange scattering, it would be interesting to study its consequences in other related problems.

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Note added in proof. A question of much interest with respect to the dipole model is the ρ mass distribution in the pion-pion system. In particular, a recent compilation of experiments [J. Pišut and M. Roos, CERN report (unpublished)] does not reveal a double peaking. One might assume that the dipole peaks should be 60 MeV apart, and that they should have been seen, but neither assumption is necessarily correct. First of all, as Roos says, "An ultimate understanding of the ρ meson can, however, only be reached through analyzing a simultaneous distribution in angles and invariant mass." What Roos does, however, is to integrate over the physical range, instead of going to the unphysical one-pion pole. This introduces factors designed to correct for being off-shell. He also looks at only the invariant mass distribution, which corresponds to looking at the π-π total cross section, instead of looking at the partial-wave cross sections. This will combine the ρ contribution with the other (decidedly nonzero) partial waves. In order to see the ρ in its cleanest form, one should extrapolate to the pion pole and then extract the separate partial-wave cross sections. This has in fact been done by one of Roo's references [Baton *et al.*, Nucl. Phys. **B3**, 349 (1967)]. In order to compare the predictions of any model (and the dipole, in particular) to experiment, one must first fold the predicted cross section with a Gaussian corresponding to the experimental resolution, and then average over each energy bin actually used in compliing the data. For a relatively smooth function this procedure has little effect but, for the dipole it strongly modifies the shape of the curve —if the width is sufficiently small, which in this case it is. In terms of specific numbers, we have taken a resolution of 10 MeV as a reasonable value. The bin size used by Baton *et al.* in the partial-wave analysis is 40 MeV for reasons of statistics. He fits the data with a simple Breit-Wigner shape, quoting a mass of 755 ± 5 MeV and a width of 110 ± 9 MeV. The pure dipole (no averaging at all) with a peak-to-peak separation of 35 MeV yields a width at half-height of 80 MeV. If one now folds in a 10-MeV Gaussian, the width at half-height goes to 100 MeV, and the relative height at the central minimum is 0.6 (instead of zero for the unaveraged dipole). The reason for this is the narrow ($\simeq$15 MeV) half-height separation of the pure peaks. When one now averages over the 40-MeV bins, the half-height width goes to 115 MeV (which is rather near the experimental value, in spite of the original narrow peak separation) and the relative height of the central minimum to 0.92, only 8% lower than the maximum. The curve, individual points of which are the numbers relevant to experiment, has roughly the shape of a molar, and this is certainly consistent with, but by no means implied by, the Baton data. In order to compare it to the Roos data, one must make a model for the other partial waves, (and in principle the nonresonant background also). After adding in their contributions, one must make some model for going off-shell. It would be surprising if very much remained of the p-wave dip in the total cross section after all this, but if one looks at the five highest values in Fig. 2 of Roos, the central point is $\simeq$5% lower than its two neighbors, a fact which is at least suggestive. It is on this basis that we are able to remark that "we regard the data of Roos as not only consistent with the dipole model, but perhaps even supporting it. The "perhaps" is meant seriously; by no stretch of the imagination can the data be called a positive confirmation of a dipole model.

[12] P. Bonamy, P. Borgeaud, C. Bruneton, P. Falk-Vairant, O. Guisan, P. Sonderegger, C. Caverzasio, J. P. Guillard, J. Schneider, M. Yvert, I. Manelli, F. Sergiampietri, and L. Vincelli, Phys. Letters 23, 501 (1966).