Momentum-Dependent K_{e4}-Decay Form Factors

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K_{e4} decays are considered within the framework of $SU(3)\times SU(3)$ algebra. An effort has been made to obtain the momentum dependence of the axial-vector form factors involved in these decays. To achieve this, we work with zero-mass external pions (p^2, $q^2\to 0$, where p and q are the four-momenta of the two pions), unlike the soft pions (p, $q\to 0$) incorporated in earlier current-algebra calculations of K_{e4} decays. Further, we use the recently developed on-mass-shell three-point functions throughout our analysis. We also make an estimate of the weak-amplitude term involving K- and Q-meson poles and the scalar term involving the σ meson. In this way, we obtain momentum-dependent K_{e4}-decay form factors. These are used to calculate the dipion energy spectrum, decay rates, and the vector form factor. The fair agreement obtained with the experimental data is indirect evidence of small s-wave final-state interactions. Our calculation neatly brings out the fact that the K_{e4}-decay form factors have significant momentum dependence.

1. INTRODUCTION

In recent years, K_{e4} decays have attracted wide attention within the context of algebra of currents and s-wave π-π phase shifts.[1-7] Most of the calculations deal with the axial-vector form factors only, and further, in almost all of these, the form factors are assumed to be constants. It is our endeavor, in this paper, to bring out the momentum-dependent structure of the K_{e4}-decay axial-vector form factors and show thereby that the momentum dependence is not insignificant. Our starting point is the recent calculation of Weinberg[4] employing $SU(3)\times SU(3)$ chiral algebra. As is usually the case with current-algebra applications, he works with soft pions. The form factors F_1, F_2, and F_3 are, in general, functions of $k\cdot p$, $k\cdot q$, and $p\cdot q$, where k, q, and p are the four-momenta of the K meson and the pions, respectively. In the soft-pion limit, i.e., q, $p\to 0$, the form factors are evaluated essentially at the point $k\cdot p=k\cdot q=p\cdot q=0$. In the present calculation, we work with zero-mass external pions (q^2, $p^2\to 0$), so that the form factors are evaluated as explicit functions of $k\cdot p$, $k\cdot q$, and $p\cdot q$. In our analysis, we extensively use the on-mass-shell three-point functions evaluated in Ref. 8. In that particular calculation, utilizing Ward identities for proper vertices and one-meson dominance, full structure for proper AAV, AVP, and VPP vertices was obtained. Here, A, V, and P stand for axial-vector, vector, and pseudoscalar mesons, respectively. In obtaining these, we used the spectral-function sum rules and introduced a parameter δ which was fixed to be -1 to give mutually consistent good numbers for $K^*\to K+\pi$, $Q\to K^*+\pi$, $Q\to K+\rho$, and $\phi\to K^++K^-$ decay widths. In the present analysis, we will not go into the details of the vertex-function calculation but use the results obtained therein directly. Further, in evaluating some matrix elements (see Sec. 2), we will use the pole model. Proceeding thus, we bring out the essential improvement in the present calculation over that of Weinberg and, also, show how the momentum dependence of the form factors is obtained. In our numerical analysis, we neglect the off-mass-shell corrections arising from extrapolation in q^2 and p^2 from 0 to $-m_\pi^2$. This point is further discussed in the next section.

With our form factors, we compute both the dipion energy spectrum and the decay rates of the CP-conserving K_{e4} decays. The energy spectrum obtained (with the omission of final-state interactions and vector form factors) is in fair agreement with the experimental spectrum and, also, with a recent calculation[5] assuming vector-meson pole dominance. The decay rates obtained are also in good agreement with experiment.

In the final section, we have tried to give a rough

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[1] Nguyen van Hieu, Zh. Eksperim. i Teor. Fiz. **44**, 162 (1963) [English transl.: Soviet Phys.—JETP **17**, 113 (1963)]; B. A. Arbuzov, N. van Hieu, and R. N. Faustov, Zh. Eksperim. i Teor. Fiz. **44**, 329 (1963) [English transl.: Soviet Phys.—JETP **17**, 225 (1963)]; L. M. Brown and M. Faier, Phys. Rev. Letters **12**, 514 (1964); C. Kacser, P. Singer, and T. N. Truong, Phys. Rev. **137**, B1605 (1965); J. Iliopoulos, Nuovo Cimento **38**, 907 (1965); **39**, 413(E) (1965).

[2] N. Cabibbo and A. Maksymowicz, Phys. Rev. **137**, B438 (1965); **168**, 1926(E) (1968); see, also, earlier papers on K_{e4} decays quoted therein. For phase-space integrations, we will follow this paper.

[3] C. Callan and S. Treiman, Phys. Rev. Letters **16**, 153 (1966).

[4] S. Weinberg, Phys. Rev. Letters **17**, 336 (1966); **18**, 1178(E) (1967).

[5] L. J. Clavelli, Phys. Rev. **154**, 1509 (1967).

[6] For the experimental information, we have used the following: R. W. Birge, R. P. Ely, G. Gidal, V. Hagopian, G. E. Kalmus, W. M. Powell, K. Billing, F. W. Bullock, M. J. Esten, M. Govan, C. Henderson, W. J. Knight, D. J. Miller, F. R. Stannard, S. Tovey, O. Treutler, U. Camerini, D. Cline, W. F. Fry, H. Haggerty, R. H. March, and W. J. Singleton (Berkeley-UCR-Wisconsin Collaboration), University of California Radiation Laboratory Report No. 17088, 1965 (unpublished); R. W. Birge, R. P. Ely, G. Gidal, G. E. Kalmus, A. Kernan, W. M. Powell, U. Camerini, W. F. Fry, J. Gaidos, R. H. March, and S. Natali, Phys. Rev. Letters **11**, 35 (1963); R. Birge *et al.*, Phys. Rev. **139**, B1600 (1965).

[7] A. Pais and S. B. Treiman, Phys. Rev. **168**, 1858 (1968); S. Matsuda and S. Oneda, *ibid.* **165**, 1749 (1968); S. M. Berman and P. Roy, Stanford Report, 1968 (unpublished).

[8] K. C. Gupta and J. S. Vaishya, Phys. Rev. **170**, 1530 (1968). The procedure adopted closely follows the treatment of H. J. Schnitzer and S. Weinberg [Phys. Rev. **164**, 1828 (1967)] for $SU(2)\times SU(2)$ chiral algebra. See, also, R. Arnowitt, M. H. Friedman, and P. Nath, Phys. Rev. Letters **19**, 1085 (1967).

estimate of the vector form factor (F_4). Writing down general structure of the matrix element, assuming F_4 to be a constant, and taking the experimental decay rate for $K^+ \to \pi^+ + \pi^- + e^+ + \nu$ as input, the missing contribution is found and thus the vector form factor is determined. A similar procedure had been adopted by Berends *et al.*[9] with the axial-vector form factors taken from Weinberg.[4] Using our F_1, F_2, and F_3, we obtain F_4 much different from that of Berends *et al.*[9] Finally, we would like to point out that our numerical estimates for F_1, F_2, and F_3 are in good agreement with those obtained from a detailed analysis of K_{e4} decay carried out recently by Berends, Donnachie, and Oades.[10]

2. AXIAL-VECTOR FORM FACTORS IN K_{e4} DECAYS

We shall consider the CP-conserving decays, namely,

$$\begin{aligned} &K^+ \to \pi^+ + \pi^- + e^+ + \nu\,, \\ &K^+ \to \pi^0 + \pi^0 + e^+ + \nu\,, \\ &K_2{}^0 \to \pi^- + \pi^0 + e^+ + \nu\,. \end{aligned} \tag{1}$$

Firstly, we shall fix our notations and definitions.[11] We consider the general process

$$K^m \to \pi^a + \pi^b + e^+ + \nu\,, \tag{2}$$

where m, a, and b are the $SU(3)$ indices. Taking the Cabibbo picture[12] for the hadronic current, the total matrix element is given by

$$(G/\sqrt{2})\langle \pi^a \pi^b | \sin\theta_C [A_\mu{}^n + V_\mu{}^n] | K^m \rangle \bar{u}_e \gamma_\mu (1+\gamma_5) u_\nu\,, \tag{3}$$

where $A_\mu{}^n$ and $V_\mu{}^n$ are the $\Delta S = \Delta Q = 1$, $\Delta I = \frac{1}{2}$ axial-vector and vector currents,[13] respectively, θ_C is the Cabibbo angle, and G is the universal weak-coupling constant. In the present analysis, we shall concentrate mainly on the axial-vector part. For this purpose, the axial-vector form factors are defined as

$$\begin{aligned} &(\sin\theta_C)\langle \pi^a(q), \pi^b(p) | A_\lambda{}^n(0) | K^m(k) \rangle \\ &\quad = i(2\pi)^{-9/2}(8p_0 q_0 k_0)^{-1/2}\left(\frac{1}{m_K}\right) \\ &\qquad \times [(q+p)_\lambda F_1 + (q-p)_\lambda F_2 + (k-q-p) F_3]\,. \end{aligned} \tag{4}$$

The form factors thus defined are dimensionless and functions of $k\cdot q$, $k\cdot p$, and $p\cdot q$, where q, p, and k are the four-momenta of the pions and the kaon, respectively.

Following Weinberg,[4] we maintain that in order to use the partially conserved axial-vector current (PCAC) and the current commutation relations (CCR) systematically, both the pions should be taken off the mass shell simultaneously. Dispersing the pions and using the PCAC relation[14]

$$\partial_\mu A_\mu{}^a(x) = F_\pi m_\pi{}^2 \phi^a(x)\,, \quad a = 1, 2, 3 \tag{5}$$

Eq. (4) becomes

$$\begin{aligned} &(F_\pi m_\pi{}^2)^{-2}(q^2+m_\pi{}^2)(p^2+m_\pi{}^2)\int d^4x d^4y \\ &\quad \times e^{-iq\cdot x - ip\cdot y}\langle 0 | T\{\partial_\mu A_\mu{}^a(x), \partial_\nu A_\nu{}^b(y), A_\lambda{}^n(0)\} | K^m(k)\rangle \\ &= i(2\pi)^{-3/2}(2k_0)^{-1/2}\left(\frac{1}{m_K \sin\theta_C}\right) \\ &\qquad \times [(q+p)_\lambda F_1 + (q-p)_\lambda F_2 + (k-q-p)_\lambda F_3]\,. \end{aligned} \tag{6}$$

Note that now the F_i's are functions of q^2 and p^2 as well. If the two pions are on the mass shell, i.e., q^2, $p^2 = -m_\pi{}^2$, these F_i become identical with the physical form factors defined earlier in Eq. (4). The time-ordered product on the left-hand side in Eq. (6) is analyzed easily, and with the use of the Jacobi identity reduces to the following form:

$$\begin{aligned} &(F_\pi m_\pi{}^2)^{-2}(q^2+m_\pi{}^2)(p^2+m_\pi{}^2)\Big[q_\mu p_\nu \int d^4x d^4y\, e^{-iq\cdot x - ip\cdot y}\langle 0 | T\{A_\mu{}^a(x), A_\nu{}^b(y), A_\lambda{}^n(0)\} | K^m(k)\rangle \\ &\quad - i\delta^{ab}\int d^4x\, e^{-i(p+q)\cdot x}\langle 0 | T\{\sigma(x), A_\lambda{}^n(0)\} | K^m(k)\rangle - \tfrac{1}{2}i(p-q)_\nu (2if^{abc})\int d^4x\, e^{-i(p+q)\cdot x}\langle 0 | T\{V_\nu{}^c(x), A_\lambda{}^n(0)\} | K^m(k)\rangle \\ &\quad - \tfrac{1}{2}(2if^{and})(2if^{bde})\langle 0 | A_\lambda{}^e(0) | K^m(k)\rangle - \tfrac{1}{2}(2if^{bnd})(2if^{ade})\langle 0 | A_\lambda{}^e(0) | K^m(k)\rangle \\ &\quad - (2if^{bnc})\int d^4x\, e^{-iq\cdot x}\langle 0 | T\{V_\lambda{}^c(0), \partial_\mu A_\mu{}^a(x)\} | K^m(k)\rangle - (2if^{anc})\int d^4x\, e^{-ip\cdot x}\langle 0 | T\{V_\lambda{}^c(0), \partial_\mu A_\mu{}^b(x)\} | K^m(k)\rangle \Big]. \end{aligned} \tag{7}$$

[9] F. A. Berends, A. Donnachie, and G. C. Oades, Phys. Letters **26B**, 109 (1967).

[10] F. A. Berends, A. Donnachie, and G. C. Oades, Phys. Rev. **171**, 1457 (1968).

[11] Our metric is such that $p\cdot q = \mathbf{p}\cdot\mathbf{q} - p_0 q_0$; we work in natural units $\hbar = c = 1$. In our definitions, we follow closely the notation of Ref. 4.

[12] N. Cabibbo, Phys. Rev. Letters **10**, 531 (1963). For the Cabibbo angle, we use the value $\sin\theta_C \simeq 0.26$.

[13] Our currents are twice the usually defined ones.

[14] M. Gell-Mann and M. Lévy, Nuovo Cimento **16**, 705 (1960); Y. Nambu, Phys. Rev. Letters **4**, 380 (1960); S. Adler, Phys. Rev. **137**, B1022 (1965).

Here we have used the conserved-vector-current (CVC) hypothesis[15] to drop one of the eight terms in the T-product expansion, and the CCR's[16] have been used in the form

$$
\begin{aligned}
\delta(x_0-y_0)[A_0{}^a(x),A_\mu{}^b(y)] &= 2if^{abc}V_\mu{}^c(x)\delta^4(x-y)+\cdots\,,\\
\delta(x_0-y_0)[A_0{}^a(x),V_\mu{}^b(y)] &= 2if^{abc}A_\mu{}^c(x)\delta^4(x-y)+\cdots\,, \qquad (8)\\
\delta(x_0-y_0)[A_0{}^a(x),\partial_\mu A_\mu{}^b(y)] &= i\delta^{ab}\sigma(x)\delta^4(x-y)+\cdots\,.
\end{aligned}
$$

Now we shall analyze the structure of all the terms in Eq. (7), one by one, in the off-mass-shell limit q^2, $p^2\to 0$. Since, we have used PCAC we are led to work in this limit. However, we do not work in the much stronger limit q, $p\to 0$ as used by Callan and Treiman,[3] Weinberg,[4] and others.[17] Moreover, we shall evaluate the pole contributions from the first term and also make a plausible estimate of the so-called scalar term. These terms have not been taken into account in earlier papers on K_{l4} decays. Also, for the last two terms in Eq. (7) corresponding to the K_{l3}-decay vector form factors, no momentum dependence was taken. Recently, Gupta and Vaishya[8] with the help of Ward identities (following the Schnitzer-Weinberg[8] approach) have obtained K_{l3}-decay form factors with the K and π mesons on the mass shell. Thus, they have obtained full momentum dependence of the form factors within the framework of $SU(3)\times SU(3)$ chiral algebra. In the same paper, they have obtained expressions for proper VPP and AVP vertices. These expressions involve a parameter δ which has been set equal to -1 (note that this value of δ gave consistently good values for $K^*\to K\pi$, $Q\to K^*\pi$, $Q\to \rho K$, and $\phi\to K\bar{K}$ decay widths). Carrying over those results to Eq. (7), we see that the contribution of the last two terms in Eq. (7) becomes

$$
\begin{aligned}
&i(2\pi)^{-3/2}(2k_0)^{-1/2}\frac{4}{F_\pi}f^{bnc}f^{amc}\\
&\times\Bigg\{(k+q)_\lambda\left[\frac{1}{2}\left(\frac{F_K}{F_\pi}+\frac{F_\pi}{F_K}\right)-\frac{F_\pi}{F_K}\frac{(k-q)^2}{(k-q)^2+m_{K^*}{}^2}\right]\\
&\quad+(k-q)_\lambda\left[\frac{1}{2}\left(\frac{F_K}{F_\pi}-\frac{F_\pi}{F_K}\right)-\frac{F_\pi}{F_K}\frac{m_K{}^2}{(k-q)^2+m_{K^*}{}^2}\right]\Bigg\}\\
&+i(2\pi)^{-3/2}(2k_0)^{-1/2}\frac{4}{F_\pi}f^{anc}f^{bmc}\\
&\times\Bigg\{(k+p)_\lambda\left[\frac{1}{2}\left(\frac{F_K}{F_\pi}+\frac{F_\pi}{F_K}\right)-\frac{F_\pi}{F_K}\frac{(k-p)^2}{(k-p)^2+m_{K^*}{}^2}\right]\\
&\quad+(k-p)_\lambda\left[\frac{1}{2}\left(\frac{F_K}{F_\pi}-\frac{F_\pi}{F_K}\right)-\frac{F_\pi}{F_K}\frac{m_K{}^2}{(k-p)^2+m_{K^*}{}^2}\right]\Bigg\}\,, \qquad (9)
\end{aligned}
$$

where F_K is the kaon decay constant defined by

$$\langle 0|\,\partial_\mu A_\mu{}^n(0)\,|K^m(k)\rangle=F_K m_K{}^2(2\pi)^{-3/2}(2k_0)^{-3/2}\delta^{nm}\,. \qquad (10)$$

With the above definition, the contributions of the fourth and fifth terms in Eq. (7) are readily written down:

$$i(2\pi)^{-3/2}(2k_0)^{-1/2}\frac{2F_K}{F_\pi{}^2}k_\lambda(f^{and}f^{bdm}+f^{bnd}f^{adm})\,. \qquad (11)$$

The third term in Eq. (7) is again readily computed by using the proper AVP vertex obtained in Ref. 8. Here, we would like to point out that in order to calculate this term Weinberg[4] had to invoke the Low model[18] and thus introduced some more parameters in the model. In our case, this term attains the form

$$
\begin{aligned}
&i(2\pi)^{-3/2}(2k_0)^{-1/2}\frac{4f^{abc}f^{mnc}m_\rho{}^2}{F_K[m_\rho{}^2+(p+q)^2]}\Bigg\{(q+p)_\lambda\left[-\frac{m_\rho{}^2k\cdot(p-q)}{m_Q{}^2[m_Q{}^2+(k-q-p)^2]}\right]\\
&\quad+(q-p)_\lambda\left[\frac{m_\rho{}^2m_Q{}^{-2}(p\cdot k+q\cdot k+m_K{}^2)-m_\rho{}^2+m_Q{}^2-2k\cdot(p+q)-m_K{}^2}{m_Q{}^2+(k-q-p)^2}\right]\\
&\quad+(k-q-p)_\lambda\left[\frac{2k\cdot(p-q)p\cdot q}{m_Q{}^2[m_Q{}^2+(k-q-p)^2]}+\frac{2k\cdot(p-q)}{m_K{}^2+(k-q-p)^2}\left(\frac{p\cdot q}{m_\rho{}^2}(F_K{}^2F_\pi{}^{-2}-1)+\frac{1}{2}\frac{F_K{}^2}{F_\pi{}^2}\right)\right]\Bigg\}\,. \qquad (12)
\end{aligned}
$$

As far as the second term in Eq. (7) is concerned, one may adopt the viewpoint, as Weinberg does, that this matrix element, being proportional to $m_\pi{}^2$, is negligible. However, one may, working in a σ model,[16] pick up the meson poles and evaluate them in terms of unknown parameters defining the SPP and SAP couplings, where S

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[15] The CVC hypothesis [R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958)] gives $\partial_\mu V_\mu\equiv 0$.

[16] The first two CCR follow easily from the quark model; see, e.g., M. Gell-Mann, Phys. Rev. **125**, 1067 (1962); Physics **1** 63 (1964). The last one follows from the "σ model"; see J. Schwinger, Ann. Phys. (N. Y.) **2**, 407 (1967) and M. Gell-Mann and M. Lévy in Ref. 14. Throughout, we ignore the singular terms arising in the CCR.

[17] In fact, limits p, $q\to 0$ lead to different results for the form factors depending on the order in which these limits are taken; see e.g., Berman and Roy in Ref. 7. In our case, the limits p^2, $q^2\to 0$ lead to an unambiguous result.

[18] F. E. Low, Phys. Rev. **110**, 974 (1958).

stands for a scalar meson. This term can be written as equal to (q^2, $p^2 \to 0$)

$$-i\delta^{ab}F_\pi^{-2}\int d^4x\, e^{-i(p+q)\cdot x}\langle 0|T\{\sigma(x),A_\lambda{}^n(0)\}|K^m(k)\rangle. \tag{13a}$$

We assume that[19]

$$\int d^4x d^4y\, e^{-iq\cdot x+ip\cdot y}\langle T\{\partial_\mu A_\mu{}^a(x),A_\nu{}^b(y),\sigma(0)\}\rangle_0 \equiv (-i\delta^{ab})\frac{F_a m_a{}^2}{q^2+m_a{}^2}\left[\frac{g_{Ab}}{p^2+m_{Ab}{}^2}\left(g_{\nu\eta}+\frac{p_\nu p_\eta}{m_{Ab}{}^2}\right)\right.$$
$$\left.\times\frac{F_\sigma m_\sigma{}^2}{(p-q)^2+m_\sigma{}^2}\Gamma_\eta{}^{A\sigma P}(q,p)+\frac{F_b p^\nu}{p^2+m_b{}^2}\frac{F_\sigma m_\sigma{}^2}{(p-q)^2+m_\sigma{}^2}\Gamma^{P\sigma P}(q,p)\right]. \tag{13b}$$

Further, we take the simplest momentum dependence for the vertices:

$$\Gamma_\eta{}^{A\sigma P}(q,p)\simeq\Gamma_1(p-q)_\eta+\Gamma_2 q_\eta,\quad \Gamma^{P\sigma P}\simeq\Gamma_3(p\cdot q), \tag{13c}$$

where the Γ's are unknown parameters, F_σ is the σ decay constant, and g_{Ab} $(=g_Q)=\sqrt{2}m_\rho F_\pi$ [8]; note also that whereas Γ_1, Γ_2 are dimensionless, Γ_3 has the dimensions of $(\text{mass})^{-1}$. Thus, expression (13a) simplifies to give

$$i(2\pi)^{-3/2}(2k_0)^{-1/2}\frac{F_\sigma}{F_\pi}\delta^{ab}\delta^{mn}\frac{m_\sigma{}^2}{m_\sigma{}^2+(p+q)^2}\left\{(q+p)_\lambda\left[(\Gamma_1+\Gamma_2)\frac{\sqrt{2}m_\rho F_\pi}{m_Q{}^2+(k-q-p)^2}\right]+(k-q-p)_\lambda\right.$$
$$\left.\times\left[\frac{F_K\Gamma_3 p\cdot q}{m_K{}^2+(k-q-p)^2}+\frac{\sqrt{2}m_\rho F_\pi}{m_Q{}^2+(k-q-p)^2}\{\Gamma_2-m_Q{}^{-2}[\Gamma_1((p+q)^2-k\cdot(p+q))+\Gamma_2(m_K{}^2+k\cdot(p+q))]\}\right]\right\}. \tag{14}$$

Finally, we consider the contribution of the first term in Eq. (7). In order to evaluate this matrix element, we write one of the terms in the following form:

$$q_\mu p_\nu\int d^4x d^4y\, e^{-iq\cdot x-ip\cdot y}\langle 0|A_\mu{}^a(x)A_\nu{}^b(y)A_\lambda{}^n(0)|K^m(k)\rangle. \tag{15a}$$

Because of the time ordering, we will have five more similar terms. Now, we introduce a complete set of one-particle intermediate states and obtain

$$q_\mu p_\nu\int d^4x d^4y\, e^{-iq\cdot x-ip\cdot y}\sum_{m,n}\langle 0|A_\mu{}^a(x)|m\rangle\langle m|A_\nu{}^b(y)|n\rangle\langle n|A_\lambda{}^n(0)|K^m(k)\rangle. \tag{15b}$$

For these states, we take vector, axial-vector, and pseudoscalar mesons only.[20] Then, using the three-point functions obtained in Ref. 8, it is easy to calculate the above contribution. Notice that in the q^2, $p^2 \to 0$ limits, only K- and Q-meson poles survive. Summing over all such matrix elements, we finally arrive at the following contribution from the first term in Eq. (7):

$$i(2\pi)^{-3/2}(2k_0)^{-1/2}\frac{2}{F_K}\left\{-f^{amc}f^{bnc}\frac{m_{K^*}{}^2-m_Q{}^2m_{K^*}{}^2m_\rho{}^{-2}+(k-q)^2}{[m_Q{}^2+(k-q-p)^2][m_{K^*}{}^2+(k-q)^2]}\left[(q+p)_\lambda\left(\tfrac{1}{2}p\cdot k+\frac{q\cdot kp\cdot(k-q)}{m_{K^*}{}^2}\right)\right.\right.$$
$$\left.+(q-p)_\lambda\left(\tfrac{1}{2}p\cdot k-p\cdot q-\frac{q\cdot kp\cdot(k-q)}{m_{K^*}{}^2}\right)+(k-q-p)_\lambda\left(\frac{k\cdot qp\cdot(k-q)}{m_Q{}^2}+\frac{k\cdot qp\cdot(k-q)}{m_{K^*}{}^2}+\frac{q\cdot kp\cdot(k-q)(k-q)^2}{m_{K^*}{}^2m_Q{}^2}\right)\right]$$
$$+(a\leftrightarrow b)(p\leftrightarrow q)-\frac{(k-q-p)_\lambda f^{amc}f^{bnc}}{[m_K{}^2+(k-q-p)^2]m_{K^*}{}^2}\left[\frac{1}{m_{K^*}{}^2+(k-q)^2}\left(p\cdot q+\frac{(k-q)\cdot p(k-q)\cdot q}{m_{K^*}{}^2}\right)\right.$$
$$\left.\left.\times\left(m_{K^*}{}^2+(k-q)^2-\frac{m_Q{}^2m_{K^*}{}^2}{m_\rho{}^2}\right)^2\right]+(a\leftrightarrow b)(p\leftrightarrow q)\right\}. \tag{16}$$

[19] In writing this quantity, we are essentially picking up the axial-vector meson and pseudoscalar meson poles from the axial-vector current and the scalar-meson pole from the scalar current. This is in the same spirit as the earlier vertex-function calculations of Ref. 8.

[20] Notice that one can again introduce a scalar-meson intermediate state here but, as such, since this term gives a small contribution on the whole, we do not think that inclusion of σ will produce any change in the final results.

Combining Eqs. (6)–(16), we obtain the momentum-dependent decay form factors

$$
\begin{aligned}
&F_1(q^2=0,\, p^2=0,\, k\cdot q,\, k\cdot p,\, p\cdot q)\\
&=(m_K \sin\theta_C)\left[\frac{2}{F_K}\left\{f^{amc}f^{bnc}\frac{[-m_{K^*}{}^2+m_Q{}^2m_{K^*}{}^2/m_\rho{}^2-(k-q)^2]}{m_Q{}^2+(k-q-p)^2}\,\frac{[\frac{1}{2}p\cdot k+q\cdot kp\cdot(k-q)/m_{K^*}{}^2]}{m_{K^*}{}^2+(k-q)^2}+(a\leftrightarrow b)(p\leftrightarrow q)\right\}\right.\\
&\quad+\frac{F_\sigma}{F_\pi{}^2}\delta^{ab}\delta^{mn}\frac{m_\sigma{}^2}{m_\sigma{}^2+(p+q)^2}(\Gamma_1+\Gamma_2)\frac{\sqrt{2}m_\rho F_\pi}{m_Q{}^2+(k-q-p)^2}-\frac{4f^{abc}f^{mnc}m_\rho{}^4k\cdot(p-q)}{F_Km_Q{}^2[m_\rho{}^2+(p+q)^2][m_Q{}^2+(k-q-p)^2]}\\
&\quad-\frac{2F_K}{F_\pi{}^2}(f^{anc}f^{bmc}+f^{bnc}f^{amc})+\frac{4}{F_\pi}f^{bnc}f^{amc}\left(\frac{F_K}{F_\pi}+\frac{1}{2}\frac{F_\pi}{F_K}-\frac{1}{2}\frac{F_\pi}{F_K}\frac{3(k-q)^2+m_K{}^2}{(k-q)^2+m_{K^*}{}^2}\right)\\
&\qquad\left.+\frac{4}{F_\pi}f^{anc}f^{bmc}\left(\frac{F_K}{F_\pi}+\frac{1}{2}\frac{F_\pi}{F_K}-\frac{1}{2}\frac{F_\pi}{F_K}\frac{3(k-p)^2+m_K{}^2}{(k-p)^2+m_{K^*}{}^2}\right)\right],
\end{aligned}\tag{17a}
$$

$$
\begin{aligned}
&F_2(q^2=0,\, p^2=0,\, k\cdot q,\, k\cdot p,\, p\cdot q)\\
&=(m_K \sin\theta_C)\left\{\frac{2}{F_K}\left[f^{amc}f^{bnc}\frac{[-m_{K^*}{}^2+m_Q{}^2m_{K^*}{}^2/m_\rho{}^2-(k-q)^2]}{m_Q{}^2+(k-q-p)^2}\,\frac{[\frac{1}{2}p\cdot k-p\cdot q-q\cdot kp\cdot(k-q)/m_{K^*}{}^2]}{m_{K^*}{}^2+(k-q)^2}\right.\right.\\
&\quad\left.-(a\leftrightarrow b)(p\leftrightarrow q)\right]+\frac{4m_\rho{}^2f^{abc}f^{mnc}[(m_\rho{}^2/m_Q{}^2)(k\cdot p+k\cdot q+m_K{}^2)-m_\rho{}^2+m_Q{}^2-m_K{}^2-2k\cdot(p+q)]}{F_K[m_\rho{}^2+(p+q)^2][m_Q{}^2+(k-q-p)^2]}\\
&\qquad\left.+\frac{2}{F_K}\left[f^{bnc}f^{amc}\left(1+\frac{m_K{}^2-(k-q)^2}{(k-q)^2+m_{K^*}{}^2}\right)-(a\leftrightarrow b)(p\leftrightarrow q)\right]\right\},
\end{aligned}\tag{17b}
$$

$$
\begin{aligned}
&F_3(q^2=0,\, p^2=0,\, k\cdot q,\, k\cdot p,\, p\cdot q)\\
&=(m_K \sin\theta_C)\left\{-\frac{2}{F_K}f^{amc}f^{bnc}\frac{[m_{K^*}{}^2+(k-q)^2-m_Q{}^2m_{K^*}{}^2m_\rho{}^{-2}]}{m_{K^*}{}^2+(k-q)^2}\left[\frac{[m_{K^*}{}^2+(k-q)^2-m_Q{}^2m_{K^*}{}^2m_\rho{}^{-2}]}{m_{K^*}{}^2[m_K{}^2+(k-q-p)^2]}\right.\right.\\
&\quad\left.\times\left(p\cdot q+\frac{q\cdot(k-q)p\cdot(k-q)}{m_{K^*}{}^2}\right)+\frac{[q\cdot kp\cdot(k-q)/m_Q{}^2+q\cdot kp\cdot(k-q)/m_{K^*}{}^2+q\cdot kp\cdot(k-q)(k-q)^2/m_{K^*}{}^2m_Q{}^2]}{m_Q{}^2+(k-q-p)^2}\right]\\
&\quad+(a\leftrightarrow b)(p\leftrightarrow q)+\frac{F_\sigma}{F_\pi{}^2}\delta^{ab}\delta^{mn}\frac{m_\sigma{}^2}{m_\sigma{}^2+(p+q)^2}\left[\frac{F_K\Gamma_3p\cdot q}{m_K{}^2+(k-q-p)^2}+\frac{\sqrt{2}m_\rho F_\pi}{m_Q{}^2+(k-q-p)^2}\right.\\
&\quad\left.\times\{\Gamma_2-m_Q{}^{-2}[\Gamma_1(2p\cdot q-k\cdot(p+q))+\Gamma_2(m_K{}^2+k\cdot(p+q))]\}\right]+\frac{4m_\rho{}^2f^{abc}f^{mnc}k\cdot(p-q)}{F_K[m_\rho{}^2+(p+q)^2]}\\
&\quad\times\left[\frac{2p\cdot q}{m_Q{}^2[m_Q{}^2+(k-q-p)^2]}+\frac{2}{m_K{}^2+(k-q-p)^2}\left(\frac{p\cdot q}{m_\rho{}^2}\left(\frac{F_K{}^2}{F_\pi{}^2}-1\right)+\frac{1}{2}\frac{F_K{}^2}{F_\pi{}^2}\right)\right]-\frac{2F_K}{F_\pi{}^2}(f^{anc}f^{bmc}+f^{amc}f^{bnc})\\
&\qquad\left.+\frac{4}{F_\pi}f^{bnc}f^{amc}\left(\frac{F_K}{F_\pi}-\frac{F_\pi}{F_K}\frac{m_K{}^2+(k-q)^2}{(k-q)^2+m_{K^*}{}^2}\right)+\frac{4}{F_\pi}f^{anc}f^{bmc}\left(\frac{F_K}{F_\pi}-\frac{F_\pi}{F_K}\frac{m_K{}^2+(k-p)^2}{(k-p)^2+m_{K^*}{}^2}\right)\right\}.
\end{aligned}\tag{17c}
$$

Now, we shall assume that the form factors are smooth functions of q^2 and p^2. In other words, we assume that the extrapolation from q^2, $p^2=0$ to q^2, $p^2=-m_\pi{}^2$ is smooth and small.[21] With this assumption, the form factors obtained above become the physical K_{l4} decay axial-vector form factors. In order to calculate the decay rates and dipion energy spectrum, we shall use the full structure[22] of F_i's as displayed in Eqs. (17).

[21] It is suggested that by writing a once-subtracted dispersion relation for the form factors, taking the subtraction constant from the afore-determined values (for q^2, $p^2=0$), and estimating the dispersion integral with the help of known resonances, one can determine the off-mass-shell corrections [S. C. Bhargava, S. N. Biswas, K. C. Gupta, and K. Datta, Phys. Rev. Letters **20**, 558 (1968); see also, S. Okubo, in *Proceedings of the 1967 Internationl Conference on Particles and Fields, Rochester* (Interscience Publishers, Inc., New York, 1967), p. 469].

[22] However, we shall drop the σ term. We notice that it does not contribute to F_2. Also, unlike the assertion made by Berman and Roy (see Ref. 7), in our non-soft-pions limit, its contribution to either F_1 or F_3 is always finite. The good agreement obtained with experiments justifies *a priori* the assumption about the smallness of the σ contribution (at least for F_1). We feel that since the commutator that gives rise to the σ term is proportional to $F_\pi m_\pi{}^2$, the contribution of the scalar term will be important particularly in those cases where the extrapolation in q^2 from 0 to $-m_\pi{}^2$ is appreciable.

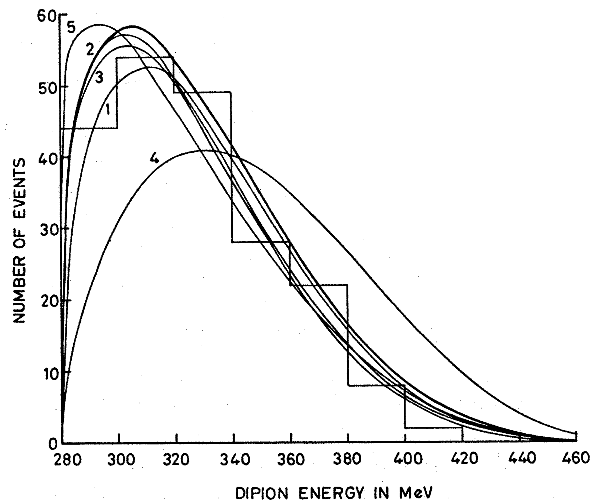

FIG. 1. Dipion energy spectrum for $K^+ \to \pi^+\pi^- e^+\nu$. The thick (unnumbered) curve corresponds to the present calculation (without vector form factor and final-state interactions). All other curves have been taken from Ref. 10. These correspond to (1) Constant form factors; (2) only F_1 enhanced; (3) all form factors enhanced; (4) all form factors enhanced, the $I=0$ s-wave phase being given by a Breit-Wigner resonance at 500 MeV with a width of 100 MeV; (5) all form factors enhanced, the $I=0$ s-wave phase having a scattering length of 1.0 and a resonance at 500 MeV with a width of 100 MeV. The histogram represents experimental mass spectrum for 208 events.

For the sake of comparison and clarity, it is worthwhile to calculate[23] these form factors at certain particular points. These, along with a discussion, are given in the Appendix.

3. DIPION ENERGY SPECTRUM AND DECAY RATES

In order to calculate the dipion energy spectrum and the decay rates, we consider Eqs. (3) and (4), substitute for the form factors from Eqs. (17), square up the resulting expression, and sum over the spins. For the phase-space integrations,[24] we follow exactly the procedure of Cabibbo and Maksymowicz.[2] In doing these integrations we retain the full momentum dependence of the form factors obtained in Eqs. (17) and further do not introduce either the vector form factor or the final-state interactions. Also, in this section, form factor F_3 is completely dropped since it gives a contribution to decay rate proportional to $(m_e/m_K)^2$. Since now we are retaining $k\cdot q$ and $k\cdot p$ dependence of the form factors in addition to the $p\cdot q$ dependence, the expression for the dipion energy spectrum is slightly modified

TABLE I. Predictions for K_{e4} decay rates (10^3 sec^{-1}).

Decay mode	Current-algebra predictions: Present calculation	Current-algebra predictions: Weinberg[a]	From detailed analysis of Berends *et al.*[b]	Expt.[c]
$K^+ \to \pi^+\pi^- e^+\nu$	2.83	1.87	2.9 $\pm$0.6	2.9$\pm$0.6
$K^+ \to \pi^0\pi^0 e^+\nu$	0.96	0.78	1.13$\pm$0.25	···
$K_2{}^0 \to \pi^-\pi^0 e^+\nu$	0.62	0.30	0.64$\pm$0.25	···

[a] Reference 4.
[b] References 9 and 10.
[c] Reference 6.

(from that given in Ref. 2) and given by

$$\frac{d\Gamma(x^2)}{dx^2}=\frac{G^2m_K}{16(2\pi)^5}\left[\tfrac{2}{3}\beta_1\int P^3\{|F_1(x^2,R_0)|^2+\tfrac{1}{3}\beta_1^2 \times|F_2(x^2,R_0)|^2+\beta_1^2x^2K^2P^{-2}|F_2(x^2,R_0)|^2\}dR_0\right], \quad (18)$$

where

$$\begin{aligned}
&R^2=(p+q)^2=W^2=\text{total c.m. energy of pions},\\
&x^2=R^2/m_K^2,\\
&k\cdot q=\tfrac{1}{2}m_KR_0,\quad k\cdot p=\tfrac{1}{2}m_KR_0,\\
&\qquad p\cdot q=\tfrac{1}{2}(m_K^2x^2-2m_\pi^2),\\
&\beta_1=\left(1-\frac{4m_\pi^2}{m_K^2x^2}\right)^{1/2}, \qquad (19)\\
&K^2=m_K^2(1+x^2)-2m_KR_0=(k-R)^2,\\
&P=(R_0^2-m_K^2x^2)^{1/2},\\
&(R_0)_{\max}=\tfrac{1}{2}m_K(1+x^2),\quad (R_0)_{\min}=xm_K.
\end{aligned}$$

The mass spectrum obtained for $K^+\to\pi^++\pi^-+e^++\nu$ along with the experimental histogram and results of Ref. 10 are plotted in Fig. 1. We observe good agreement. In order to calculate the decay rates, we integrate over x^2 from $4m_\pi^2/m_K^2$ to unity. The numbers obtained are in good agreement with experiments. These are listed in Table I. It is easy to calculate, in a similar manner, total decay rates for $K_{\mu 4}$ decays. However, in these, one must include the form factor F_3.

4. VECTOR FORM FACTOR

From invariance considerations, we can write, in particular for the decay $K^+\to\pi^++\pi^-+e^++\nu$,

$$\langle\pi^+(q),\pi^-(p)|V_\lambda{}^+(0)|K^+(k)\rangle =\frac{iF_4}{m_K{}^3}\epsilon_{\lambda\mu\nu\sigma}(k-q-p)_\mu(q+p)_\nu(q-p)_\sigma. \quad (20)$$

Again, the form factor F_4 is, in general, a function of $k\cdot q$, $k\cdot p$, and $p\cdot q$. If we assume it to be a constant, its contribution (because of parity considerations it does

[23] For numerical purposes, we take $F_K/F_\pi=1.17$, $0.22\ \pi F_\pi^2=m_\pi^2$; see Ref. 8 and the literature quoted therein. All the masses have been taken from A. H. Rosenfeld *et al.*, Rev. Mod. Phys. **39**, 1 (1967).

[24] The metric used in Ref. 2 is $p\cdot q=p_0q_0-\mathbf{p}\cdot\mathbf{q}$. Since the form factors obtained in Eqs. (17) are functions of invariant quantities and only their moduli squared appear in the final integrations, our results are easily carried over to this metric.

not interfere with the axial-vector form factors) to the total decay rate is easily calculated. It comes out to be equal to[9]

$$3.31\,|F_4|^2\ \text{sec}^{-1}.$$

The total contribution of the axial-vector form factors has been found to be equal to

$$2.83\times 10^3\ \text{sec}^{-1}.$$

Taking the experimental rate[6] to be the sum of these two, we obtain

$$2.9\pm 0.6 = 2.83 + 3.31\times 10^{-3}|F_4|^2; \tag{21}$$

thereby,

$$|F_4| \simeq 4.6_{-4.6}^{+9.6}. \tag{22}$$

5. CONCLUDING REMARKS

Working within the framework of $SU(3)\times SU(3)$ chiral algebra, we have obtained momentum-dependent K_{l4}-decay axial-vector form factors. The dipion energy spectrum obtained is in good agreement with experiment and this supports the fact that the final-state interactions are indeed small. Comparing with Weinberg's calculation, we find that our form factors (at zero momentum transfer) are different from his, and the total decay rates obtained are in better agreement with experiment. We would like to point out that in carrying out the phase-space integrations for the decay rates, we have retained full momentum dependence of the form factors. Further, since our calculation is a non-soft-pions calculation, the form factors obtained are free from the ambiguities (related with the limiting procedure) arising in earlier current-algebra calculations of K_{l4} decays. We close with the remark that, within a current algebra and PCAC framework, it is better to work with off-mass-shell pions than with soft pions.

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APPENDIX

Here, we calculate the form factors at three sets of points and show thereby that they vary appreciably from point to point. For the first set, we choose $k\cdot q = k\cdot p = p\cdot q = 0$. From Eqs. (17),

$$F_1(0,0,0) = (m_K \sin\theta_C)\left\{\delta^{ab}\delta^{mn}\frac{F_\sigma}{F_\pi{}^2}(\Gamma_1+\Gamma_2)\frac{\sqrt{2}m_\rho}{m_Q{}^2-m_K{}^2}+\frac{2}{F_\pi}(f^{anc}f^{bmc}+f^{bnc}f^{amc})\left[\frac{F_K}{F_\pi}+\frac{F_\pi}{F_K}\left(1+\frac{2m_K{}^2}{m_{K^*}{}^2-m_K{}^2}\right)\right]\right\}, \tag{A1}$$

$$F_2(0,0,0) = (m_K \sin\theta_C)\left[\frac{4}{F_K}(1-m_\rho{}^2 m_Q{}^{-2})f^{abc}f^{mnc}+\frac{2}{F_K}\left(\frac{m_{K^*}{}^2+m_K{}^2}{m_{K^*}{}^2-m_K{}^2}\right)(f^{amc}f^{bnc}-f^{anc}f^{bmc})\right], \tag{A2}$$

$$F_3(0,0,0) = (m_K \sin\theta_C)\left[\frac{1}{F_K}(f^{amc}f^{bnc}+f^{anc}f^{bmc})\left\{\frac{2F_K{}^2}{F_\pi{}^2}+\frac{(m_{K^*}{}^2-m_K{}^2-m_Q{}^2m_{K^*}{}^2m_\rho{}^{-2})^2}{m_{K^*}{}^2(m_{K^*}{}^2-m_K{}^2)}\alpha\right\}\right.$$
$$\left.+\delta^{ab}\delta^{mn}\frac{F_\sigma}{F_\pi}\left(-\frac{F_K\Gamma_3}{2F_\pi}\alpha+\frac{\sqrt{2}m_\rho\Gamma_2}{m_Q}\right)+\frac{2F_K}{F_\pi}f^{abc}f^{mnc}\beta\right], \tag{A3}$$

TABLE II. Predictions for "constant" K_{e4}-decay axial-vector form factors.

Decay mode	Form factor	Current-algebra predictions: Present calculation Set 1	Set 2	Set 3	Weinberg[a]	Detailed analysis of Berends *et al.*[b]
$K^+\to\pi^+\pi^-e^+\nu$	F_1	2.18	1.33	1.52	0.97	1.19±0.03
	F_2	2.12	2.76	2.44	0.97	1.34±0.30
	F_3	$(0.91+2.6\alpha+0.91\beta)$	1.88	2.27	$1.41[1-k\cdot(p-q)/k\cdot(p+q)]$	···
$K^+\to\pi^0\pi^0e^+\nu$	F_1	2.18	1.33	1.52	0.97	···
	F_2	0	0.05	0.06	0	···
	F_3	$(0.91+2.6\alpha)$	1.88	2.27	1.41	···
$K_2{}^0\to\pi^-\pi^0e^+\nu$	F_1	0	0	0	0	···
	F_2	−2.12	−2.71	−2.37	−0.97	···
	F_3	-0.91β	0	0	$-1.41[k\cdot(p-q)/k\cdot(p+q)]$	···

[a] Reference 4.
[b] Reference 10.

where

$$\alpha=\frac{p\cdot q}{k\cdot q+k\cdot p-p\cdot q},\quad \beta=\frac{k\cdot(q-p)}{k\cdot q+k\cdot p-p\cdot q}. \tag{A4}$$

Alternatively, one may say that these form factors are functions of $(k-q)^2$, $(k-p)^2$, and $(k-q-p)^2$, and thus, one wants to evaluate them at zero values of these momentum transfer squares. Hence we define a second set such that $k\cdot p=k\cdot p=p\cdot q=-\frac{1}{2}m_K^2$. Also, from kinematical considerations, these are the maximum. Now, the form factors are given by

$$\begin{aligned}F_1(-\tfrac{1}{2}m_K^2,\,-\tfrac{1}{2}m_K^2,\,-\tfrac{1}{2}m_K^2)
&=(m_K\sin\theta_C)\Bigg[\frac{1}{2F_K}(f^{amc}f^{bnc}+f^{anc}f^{bmc})\left\{m_K^2(m_Q^{-2}-m_\rho^{-2})+4\left(1-\frac{m_K^2}{m_{K^*}^2}\right)+\frac{4F_K^2}{F_\pi^2}\right\}\\
&\qquad+\frac{F_\sigma}{F_\pi}\delta^{ab}\delta^{mn}\left(\frac{m_\sigma^2}{m_\sigma^2-m_K^2}\right)(\Gamma_1+\Gamma_2)\frac{\sqrt{2}m_\rho}{m_Q^2}\Bigg].\end{aligned} \tag{A5}$$

$$\begin{aligned}F_2(-\tfrac{1}{2}m_K^2,\,-\tfrac{1}{2}m_K^2,\,-\tfrac{1}{2}m_K^2)
&=(m_K\sin\theta_C)\Bigg[-\frac{1}{2F_K}(f^{amc}f^{bnc}+f^{anc}f^{bmc})m_K^2(m_Q^{-2}-m_\rho^{-2})+\frac{4f^{abc}f^{mnc}}{F_K}\left(\frac{m_\rho^2}{m_\rho^2-m_K^2}\right)\left(\frac{m_Q^2+m_K^2-m_\rho^2}{m_Q^2}\right)\\
&\qquad+\frac{2}{F_K}(f^{bnc}f^{amc}-f^{anc}f^{bmc})\left(\frac{m_{K^*}^2+m_K^2}{m_{K^*}^2}\right)\Bigg].\end{aligned} \tag{A6}$$

$$\begin{aligned}F_3(-\tfrac{1}{2}m_K^2,\,-\tfrac{1}{2}m_K^2,\,-\tfrac{1}{2}m_K^2)
&=(m_K\sin\theta_C)\Bigg[\frac{1}{F_K}(f^{amc}f^{bnc}+f^{anc}f^{bmc})\left\{\left(1-\frac{m_Q^2}{m_\rho^2}\right)^2+\frac{2F_K^2}{F_\pi^2}-\frac{4m_K^2}{m_{K^*}^2}\right\}\\
&\qquad+\frac{F_\sigma}{F_\pi}\delta^{ab}\delta^{mn}\left(\frac{m_\sigma^2}{m_\sigma^2-m_K^2}\right)\left(-\frac{F_K}{2F_\pi}\Gamma_3+\frac{\sqrt{2}m_\rho}{m_Q^2}\Gamma_2\right)\Bigg].\end{aligned} \tag{A7}$$

However, from phase-space considerations (see Sec. 3), we see that

$$(k\cdot q)_{\max}=(k\cdot p)_{\max}=(p\cdot q)_{\max}\simeq-0.5m_K^2,$$
$$(k\cdot q)_{\min}=(k\cdot p)_{\min}=(p\cdot q)_{\min}\simeq-0.155m_K^2.$$

So, if the decay rates are evaluated by treating the form factors as constants, one must evaluate these at the point

$$(k\cdot q)=(k\cdot p)=(p\cdot q)=-0.327m_K^2.$$

This defines a third set of constant form factors. It is evident that these three sets receive different contributions from Eqs. (17). For numerical purposes, we drop the scalar-term contributions for reasons mentioned in Ref. 22. The values of the form factors obtained from the three sets defined above along with the values obtained by Weinberg[4] and Berends *et al.*[20] are listed in Table II. We see that the form factors vary appreciably from point to point (compare sets 2 and 3). Thus, we are led to believe that the momentum dependence of the form factors cannot be ignored.