where

$$\tilde{D}(q) \equiv D(q)\Delta^{-1}(q)D(q) - \bar{D}(q) \quad (35)$$

and

$$\{T_\alpha, T_\beta\}_+ = T_\alpha T_\beta + T_\beta T_\alpha .$$

IV. CONCLUDING REMARKS

We have extended our previous work to an arbitrary current algebra making as few assumptions as possible. Aside from assuming a local algebra with c-number Schwinger terms, our main assumption has been the specific model of symmetry breaking expressed in Eq. (1). This model is general enough to encompass any set of nonconserved currents. In particular, we have not assumed any propagators to be saturated by single-particle states. As well as making it possible to implement the requirements of unitarity in some future, dynamical approach to current algebra, we note the existence of certain terms in Eqs. (32)–(34) which simply vanish in the single-particle approximation, and hence do not appear in the analogous equations of Refs. 1 and 2.

In the following paper,[7] we shall apply these results to $SU(3)\otimes SU(3)$ to discuss various meson decays. Although our present knowledge forces us to a single-particle-dominance approximation there, such an approximation and its success (or lack thereof) in no way compromises the generality of this paper.

[7] I. S. Gerstein and H. J. Schnitzer, following paper, Phys. Rev. **175**, 1876 (1968).

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Chiral $SU(3)\otimes SU(3)$ Three-Point Functions: Single-Particle Approximation*

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We approximate the three-point functions constructed previously by using a single-particle approximation to all propagators and assuming that the primitive three-point functions are slowly varying functions of the momenta. We use the available data on strong and weak decays of spin-one and spin-zero mesons to compute the values of the parameters introduced in our model, with special reference to K_{l3} decay.

I. INTRODUCTION

In the preceding paper[1] we extended the Ward identity techniques for the three-point functions to chiral $SU(3)\otimes SU(3)$ without making single-particle approximations or special assumptions about symmetry breaking. We now use single-particle dominance for all propagators, assume that the primitive three-point functions are slowly varying functions of the momenta, and assume that the symmetry-breaking term in the Lagrangian transforms as $(3,\bar{3})\oplus(\bar{3},3)$.[2] These approximations lead to predictions for meson decays in terms of a number of arbitrary parameters, which are too numerous to be determined by experiment, so that no specific numerical predictions can be made without further assumptions. We shall discuss these assumptions as we make them. Finally, we determine the remaining parameters from experiment, these are found to be consistent with small $SU(3)$ and chiral symmetry breaking.

II. SINGLE-PARTICLE APPROXIMATION

The spectral representations for the spin-zero mesons are given in Eqs. (I.11), (I.14), and (I.17).[3] We define the single-particle approximation by assuming for the matrix propagators[4]

$$\Delta(q) = \frac{1}{M^2 + Z^{-1}q^2} = Z^{1/2}\frac{1}{\mu^2+q^2}Z^{1/2}, \quad (1)$$

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[1] I. S. Gerstein, H. J. Schnitzer, and S. Weinberg, preceding paper, Phys. Rev. **175**, 1873 (1968). Equations from this paper are denoted by I.

[2] See S. L. Glashow and S. Weinberg, Phys. Rev. Letters **20**, 224 (1968). S. L. Glashow in *Proceedings of the International School of Physics, Ettore Majorana (1967)*. Edited by E. R. Caianiello (Academic Press Inc., New York, 1968).

[3] The notation, unless otherwise specified, is as in I.

[4] These approximations, their relation to the meson mass spectrum and to mixing models are discussed in S. Coleman and H. J. Schnitzer, Phys. Rev. **134**, B863 (1964).

where $\mu^2=Z^{1/2}M^2Z^{1/2}$ is the *physical* mass matrix. This then implies that

$$D(q)=Z^{1/2}\frac{1}{\mu^2}\frac{1}{\mu^2+q^2}Z^{1/2}$$

$$=Z^{1/2}\frac{1}{\mu^2+q^2}\frac{1}{\mu^2}Z^{1/2}, \qquad (2)$$

$$\bar{D}(q)=Z^{1/2}\frac{1}{\mu^2}\frac{1}{\mu^2+q^2}\frac{1}{\mu^2}Z^{1/2}, \qquad (3)$$

and

$$\tilde{D}(q)\equiv D(q)\Delta^{-1}(q)D(q)-\bar{D}(q)=0. \qquad (4)$$

The Ward identities appropriate to these propagators are obtained by substituting Eqs. (1)–(4) into Eqs. (I.32)–(I.34).

From the structure of the resulting equations, one finds that it is useful to renormalize the wave functions of the spin-zero mesons. To this end, we define[2]

$$\tilde{\phi}_j(0)=(Z^{-1/2})_{ji}\phi_i(0), \qquad (5)$$

where $\tilde{\phi}_j$ is the renormalized meson field. If we define the vector (F^α) in the representation space R by

$$F^\alpha=(Z^{-1/2}T_\alpha\lambda), \qquad (6)$$

then (I.21) becomes

$$(T_\alpha\epsilon)=(Z^{-1/2}\mu^2F^\alpha) \qquad (7)$$

in the single-particle approximation. From (I.2), we find

$$\langle 0|J_\mu{}^\alpha(0)|j\rangle=\frac{-iq^\mu}{(2\pi)^{3/2}(2\omega_j)^{1/2}}(F^\alpha)_j, \qquad (8)$$

so that $(F^\alpha)_j$ is the decay amplitude of meson j, via the current $J_\mu{}^\alpha$. The corresponding definitions for the renormalized verticles are

$$\tilde{\Gamma}_{i,j,k}(q,p)=(Z^{1/2})_{ii'}(Z^{1/2})_{jj'}(Z^{1/2})_{kk'}\Gamma_{i',j',k'}(q,p), \qquad (9a)$$

$$\tilde{\Gamma}_{i,j,\gamma\lambda}(q,p)=(Z^{1/2})_{ii'}(Z^{1/2})_{jj'}\Gamma_{i',j',\gamma\lambda}(q,p), \qquad (9b)$$

and

$$\tilde{\Gamma}_{i,\beta\nu,\gamma\lambda}(q,p)=(Z^{1/2})_{ii'}\Gamma_{i',\beta\gamma,\gamma\lambda}(q,p). \qquad (9c)$$

The renormalized Ward identities corresponding to (I.32)–(I.34) are, in the single-particle approximation,

$$-(F^\alpha)_iC_{\beta\beta'}C_{\gamma\gamma'}\tilde{\Gamma}_{i',\beta'\nu,\gamma'\lambda}(q,p)=-C_{\alpha\alpha'}C_{\beta\beta'}C_{\gamma\gamma'}q^\mu\Gamma_{\alpha'\mu,\beta'\nu,\gamma'\lambda}(q,p)$$
$$+C_{\alpha\beta'\gamma'}[\Delta^{-1}{}_{\beta'\beta''}{}^{\nu\lambda}(p)C_{\beta''\beta}C_{\gamma'\gamma}-\Delta^{-1}{}_{\gamma'\gamma''}{}^{\nu\lambda}(r)C_{\beta'\beta}C_{\gamma''\gamma}], \qquad (10)$$

$$-(F^\alpha)_i(F^\beta)_jC_{\gamma\gamma'}\tilde{\Gamma}_{i,j,\gamma'\lambda}(q,p)=-C_{\alpha\alpha'}C_{\beta\beta'}C_{\gamma\gamma'}q^\mu p^\nu\Gamma_{\alpha'\mu,\beta'\nu,\gamma'\lambda}(q,p)-q^{\lambda'}C_{\alpha'\beta\gamma'}C_{\gamma\gamma''}\Delta^{-1}{}_{\gamma''\gamma}{}^{\lambda'\lambda}(r)\,\mathrm{tr}(F^{\alpha'}F^\alpha)$$
$$+p^{\lambda'}C_{\alpha\beta'\gamma'}C_{\gamma\gamma''}\Delta^{-1}{}_{\gamma''\gamma}{}^{\lambda'\lambda}(r)\,\mathrm{tr}(F^{\beta'}F^\beta)+\tfrac{1}{2}r^\lambda\,\mathrm{tr}[F^\gamma Z^{-1/2}\{T_\alpha,T_\beta\}_+\lambda]$$
$$-\tfrac{1}{2}(q-p)^\lambda C_{\alpha\beta c'}C_{\gamma'\gamma}+\tfrac{1}{2}(q-p)^\nu C_{\alpha\beta\gamma'}\Delta^{-1}{}_{\gamma'\gamma''}{}^{\nu\lambda}(r)C_{\gamma''\gamma}S, \qquad (11)$$

and

$$-(F^\alpha)_i(F^\beta)_j(F^\gamma)_k\tilde{\Gamma}_{i,j,k}(q,p)=-C_{\alpha\alpha'}C_{\beta\beta'}C_{\gamma\gamma'}q^\mu p^\nu r^\lambda\Gamma_{\alpha'\mu,\nu'\gamma'\lambda}(q,p)-\tfrac{1}{2}(p^2-r^2)C_{\alpha'\beta\gamma}\,\mathrm{tr}(F^{\alpha'}F^\alpha)$$
$$-\tfrac{1}{2}(r^2-q^2)C_{\alpha\beta'\gamma}\,\mathrm{tr}(F^{\beta'}F^\beta)-\tfrac{1}{2}(q^2-p^2)C_{\alpha\beta\gamma'}\,\mathrm{tr}(F^{\gamma'}F^\gamma)+\tfrac{1}{2}\,\mathrm{tr}[F^\alpha(\mu^2+q^2)Z^{-1/2}\{T_\beta,T_\gamma\}_+\lambda]$$
$$+\tfrac{1}{2}\,\mathrm{tr}[F^\beta(\mu^2+p^2)Z^{-1/2}\{T_\gamma,T_\alpha\}_+\lambda]+\tfrac{1}{2}\,\mathrm{tr}[F^\gamma(\mu^2+r^2)Z^{-1/2}\{T_\alpha,T_\beta\}_+\lambda]-\tfrac{1}{6}\,\mathrm{tr}(F^\alpha\mu^2Z^{-1/2}\{T_\beta,T_\gamma\}_+\lambda)$$
$$-\tfrac{1}{6}\,\mathrm{tr}(F^\beta\mu^2Z^{-1/2}\{T_\gamma,T_\alpha\}_+\lambda)+\tfrac{1}{6}\,\mathrm{tr}(F^\gamma\mu^2Z^{-1/2}\{T_\alpha,T_\beta\}_+\lambda), \qquad (12)$$

where the Schwinger term S is

$$S\delta_{\alpha\beta}=C_{\alpha\beta}+\mathrm{tr}(F^\alpha F^\beta). \qquad (13)$$

III. CHIRAL SYMMETRY-BREAKING MODEL: $(3,\bar{3})\oplus(\bar{3},3)$

In order to make definite predictions, one must assume a symmetry-breaking model by choosing the representation (or representations) R for which the fields ϕ_i form a basis. Here we investigate the consequences of assuming that R is the representation $(3,\bar{3})\oplus(\bar{3},3)$ of chiral $SU(3)\times SU(3)$. Recall that

$$\partial_\mu J_\alpha{}^\mu(x)=\epsilon_i(T_\alpha)_{ij}\phi_j(x). \qquad (\mathrm{I}.2)$$

With our particular choice of symmetry breaking, we find, for the axial-vector and vector currents,

$$\partial_\mu A_\alpha{}^\mu(x)=d_{\alpha\beta\gamma}(\lambda_\beta)_b{}^a\epsilon_a{}^b\phi_\gamma(x) \qquad (14)$$

and

$$\partial_\mu V_\alpha{}^\mu(x)=f_{\alpha\beta\gamma}(\lambda_\beta)_b{}^a\epsilon_a{}^b\sigma_\gamma(x), \qquad (15)$$

where $(\lambda_\beta)_b{}^a$ is the 3×3 matrix defined by Gell-Mann, $\phi_\gamma(x)$ is the pseudoscalar meson field with $SU(3)$ index γ, $\sigma_\gamma(x)$ is a scalar meson field, and $d_{\alpha\beta\gamma}$ and $f_{\alpha\beta\gamma}$ are the $SU(3)$ coupling parameters for $8\otimes8$. As a consequence of parity conservation for the strong interactions $\epsilon_b{}^a=\epsilon_{\bar{b}}{}^{\bar{a}}=\epsilon_b{}^{\bar{a}}$, where the barred (unbarred) indices refer to the left (right) chiral component of the $(3,\bar{3})\oplus(\bar{3},3)$ meson representation. It is convenient to label the ϵ by the name of the meson appearing in the right-hand side of Eqs. (14) and (15). Thus, for example,

$$\partial_\mu A_3{}^\mu(x)=\epsilon_\pi\phi_3(x), \quad \text{etc.}$$

Since only the strangeness-changing vector current is not conserved, we have

$$_i(\epsilon)\underset{R}{\longrightarrow}{}_{\bar{b}}{}^a(\epsilon)={}_a{}^{\bar{b}}(\epsilon), \quad R=(3,\bar{3})\oplus(\bar{3},3)$$
$$=-[\tfrac{1}{2}\epsilon_\pi\delta_b{}^a+(\epsilon_\pi-\epsilon_K)\delta_3{}^a\delta_3{}^b], \qquad (16)$$

$$\epsilon_\kappa=\epsilon_\pi-\epsilon_K, \quad \epsilon_K=\tfrac{3}{4}\epsilon_{\eta8}+\tfrac{1}{4}\epsilon_\pi, \qquad (17)$$

and

$$\epsilon_{\eta0}=\tfrac{2}{3}\sqrt{2}(\epsilon_\pi-\epsilon_K). \qquad (18)$$

One can find the same results by defining

$$_i(\epsilon_\alpha) = {}_i(T_\alpha \epsilon),$$

as in Glashow and Weinberg.[2] One can perform a similar analysis for $(T_\alpha \lambda)$, which satisfy equations essentially identical to (16)–(18). Therefore, these results, together with Eqs. (6) and (7), imply

$$-Z_\kappa^{-1/2}\mu_\kappa^2 F_\kappa = Z_\pi^{-1/2}\mu_\pi^2 F_\pi - Z_K^{-1/2}\mu_K^2 F_K, \quad (19)$$

and

$$-Z_\kappa^{1/2} F_\kappa = Z_\pi^{1/2} F_\pi - Z_K^{1/2} F_K, \quad (20)$$

where the renormalization constants and decay rates are given the particle labels[5] as in (16)–(18). Equations (16)–(20), are particular to our choice of symmetry-breaking model. Combining the last two equations gives

$$F_\kappa^2 = \frac{1}{\mu_\kappa^2}\left[\mu_\pi^2 F_\pi^2 + \mu_K^2 F_K^2 - \left(\frac{Z_\pi}{Z_K}\right)^{1/2} \mu_K^2 F_\pi F_K - \left(\frac{Z_K}{Z_\pi}\right)^{1/2} \mu_\pi^2 F_\pi F_K\right]. \quad (21)$$

Since $F_\kappa^2 > 0$, one obtains the inequalities:

Case a:

$$(Z_\pi/Z_K)^{1/2} > 0, \quad F_\pi F_K > 0, \quad (22a)$$

$$\mu_\kappa \le \left| \mu_\pi |F_\pi| - \mu_K |F_K| \right| / |F_\kappa| .$$

Case b:

$$(Z_\pi/Z_K)^{1/2} > 0, \quad F_\pi F_K < 0, \quad (22b)$$

$$\mu_\kappa > \left| \mu_\pi |F_\pi| + \mu_K |F_K| \right| / |F_\kappa| .$$

Case c:

$$(Z_\pi/Z_K)^{1/2} < 0, \quad F_\pi F_K < 0, \quad (22c)$$

Same as Case a.

Case d:

$$(Z_\pi/Z_K)^{1/2} < 0, \quad F_\pi F_K > 0, \quad (22d)$$

Same as Case b.

The first two of these were discussed in Ref. 2.

IV. APPLICATIONS

We now apply our results to the meson decays for the unmixed channels. To complete our model we assume that the vector and axial-vector propagators are single-particle dominated, and that the primitive functions[6] are functions[7] of momenta as slowly varying as possible, consistent with the Ward identities. In the Appendix, we give the Ward identities appropriate to each channel, together with the primitive functions for each case.

For our analysis, we take

$$m_\rho^2 = \tfrac{1}{2} m_{A_1}^2 = \tfrac{3}{4} m_{K^*}^2 = \tfrac{3}{8} m_{K_A}^2 = 0.6\ \text{BeV}^2$$

and

$$F_\pi = 94\ \text{MeV}$$

as determined from the pion decay rate. We further define

$$g_{K^*}^2 = \xi_V g_\rho^2, \quad (23a)$$

$$g_{K_A}^2 = \xi_A g_\rho^2, \quad (23b)$$

which, upon using Eq. (13), in the single-particle approximation, and the KSRF relation[8]

$$g_\rho^2 / m_\rho^2 = 2F_\pi^2, \quad (24)$$

lead to

$$F_K^2 = 2F_\pi^2(1 - \tfrac{3}{8}\xi_A), \quad (25a)$$

$$F_\kappa^2 = 2F_\pi^2(1 - \tfrac{3}{4}\xi_V). \quad (25b)$$

Clearly, $0 \le \xi_V \le \frac{4}{3}$ and $0 \le \xi_A \le 8/3$.

In order to consider processes such as K_{l3} decay we have to use the primitive function $\Gamma_{\mu,\nu,\lambda}{}^{K_A A_1 K^*}(q_1,q_2)$ which contains six parameters, $g_1, \cdots, g_6$ not specified by considerations of current algebra. It has been suggested[2,9] that there are reasonable arguments to support the hypothesis that the κ, π, K vertex is no more than quadratic in the momenta, i.e.,

$$\kappa_\lambda q_{1\mu} q_{2\nu} \Gamma_{\mu,\nu,\lambda}{}^{K_A A_1 K^*}(q_1,q_2) = 0 .$$

This requires

$$g_1 = g_2, \quad (26a)$$

$$g_4 + g_5 = g_5 + g_6 = 0, \quad (26b)$$

and we shall assume that these conditions are satisfied. For comparison, note that if $SU(3)$ were an exact symmetry we would have

$$g_1 = g_2 = 1, \quad (27a)$$

$$g_4 = g_5 = g_6 = 0, \quad (27b)$$

$$g_3 = \delta, \quad (27c)$$

where the parameter δ is that introduced previously[7] in a consideration of the primitive function $\Gamma_{\mu,\nu,\lambda}{}^{A_1 A_1 \rho}(q_1,q_2)$. In terms of the $SU(3)$ symmetry-breaking parameter we note that Eq. (27a) is violated only in second order while Eq. (27b) is violated by first-order terms.

The K_{l3} amplitudes, defined in the usual way by

$$\langle \pi(q) | S_\mu(0) | K^0(p) \rangle = (p+q)_\mu f_+(t) + (p-q)_\mu f_-(t) .$$

[5] We have found it more convenient to define F_κ with opposite sign to that in Ref. 2.

[6] I. S. Gerstein and H. J. Schnitzer, Phys. Rev. **170**, 1638 (1968).

[7] H. J. Schnitzer and S. Weinberg, Phys. Rev. **164**, 1828 (1967).

[8] K. Kawarabayashi and M. Suzuki, Phys. Rev. Letters **16**, 225 (1966); Riazuddin and Fayyazuddin, Phys. Rev. **147**, 1071 (1966).

[9] L. N. Chang and Y. C. Leung, Phys. Rev. Letters **21**, 122 (1968).

are found from Eqs. (A19) and (A22) to be

$$2F_\pi F_K f_+(t)=\frac{g_{K^*}^2}{m_{K^*}^2-t}\left(1+\frac{g_{K_A}g_{A_1}}{m_{K_A}^2 m_{A_1}^2}\frac{m_{K^*}^4}{g_{K^*}^2}(-g_2-g_3)\right)+F_\pi^2+F_K^2-F_\kappa^2-\frac{g_{K^*}^2}{m_{K^*}^2}+\frac{g_{A_1}g_{K_A}}{m_{A_1}^2 m_{K_A}^2}m_{K^*}^2(g_2+g_3)\,, \quad (28)$$

$$2F_\pi F_\kappa f_-(t)=\frac{g_{K^*}^2}{m_{K^*}^2}\frac{m_\pi^2-m_K^2}{m_{K^*}^2-t}\times\left(1+\frac{g_{K_A}g_{A_1}}{m_{K_A}^2 m_{A_1}^2}\frac{m_{K^*}^4}{g_{K^*}^2}(-g_2-g_3)\right)+\frac{1}{m_\kappa^2-t}\Bigg[(m_K^2-m_\pi^2)(F_\pi^2+F_K^2-F_\kappa^2)+m_\kappa^2\left(F_K^2-F_\pi^2+F_\kappa^2-2F_KF_\kappa\left(\frac{Z_K}{Z_\kappa}\right)^{1/2}\right)\Bigg]. \quad (29)$$

We have written these so that the residues of the K^* and κ poles are constants, i.e., they take on their dispersion-theroetic values. Equations (28) and (29) suggest that $f_-(t)$ is unsubtracted (or pole-dominated),[10] while $f_+(t)$ needs one subtraction. Writing

$$f_\pm(t)=f_\pm(0)\left(1+\lambda_\pm\frac{t}{m_\pi^2}\right), \quad (30)$$

Eq. (28) yields

$$f_+(0)=\frac{1}{2F_\pi F_K}(F_\pi^2+F_K^2-F_\kappa^2)\,, \quad (31)$$

$$f_+(0)\lambda_+\frac{m_{K^*}^2}{m_\pi^2}=\frac{1}{2F_\pi F_K}\frac{g_{K^*}^2}{m_{K^*}^2}\times\left(1+\frac{g_{K_A}g_{A_1}}{m_{K_A}^2 m_{A_1}^2}\frac{m_{K^*}^4}{g_{K^*}^2}(-g_2-g_3)\right). \quad (32)$$

Finally, the total width for the decay $K^*\to K+\pi$ is found from Eq. (A19) to be

$$\Gamma=\frac{1}{8\pi}\frac{q^3}{m_{K^*}^2}\frac{g_{K^*}^2}{4F_\pi^2F_K^2}\left(1+\frac{g_{K_A}g_{A_1}}{m_{K_A}^2 m_{A_1}^2}\frac{m_{K^*}^4}{q_{K^*}^2}(-g_2-g_3)\right)^2. \quad (33)$$

We see that Eqs. (31)–(33) depend on only three unknown parameters, ξ_V, ξ_A, and g_2+g_3 so that we may use the experimental information about these decays to deduce the values of the unknowns. It should be noted that the combination g_2+g_3 may be eliminated from Eq. (32) in favor of the K^* width. The fact that the residue of the K^* pole in $f_\pm(t)$ is the physical $K^*K\pi$ vertex is a direct consequence of the definition of the $K^*K\pi$ proper vertex on the mass shell of all three mesons. This leads to

$$\Gamma(K^*\to K\pi)=\frac{1}{8\pi}q^3\frac{m_{K^*}^3}{g_{K^*}^2}\left(f_+(0)\lambda_+\frac{m_{K^*}^2}{m_\pi^2}\right)^2. \quad (34)$$

Taking[11]

$$\Gamma(K^*\to K\pi)=49.2\text{ MeV}\,, \quad (35a)$$

$$F_K/F_\pi f_+(0)=1.28\,, \quad (35b)$$

$$\lambda_+=m_\pi^2/m_{K^*}^2=0.0238\,, \quad (35c)$$

we find

$$\xi_V=1.11\,, \quad (36a)$$

$$\xi_A=1.09\,, \quad (36b)$$

so that

$$g_{K^*}^2\approx g_{K_A}^2\approx 1.1g_\rho^2\,, \quad (37a)$$

$$F_K^2=1.18F_\pi^2\,, \quad (37b)$$

$$F_\kappa^2=0.34F_\pi^2\,, \quad (37c)$$

$$f_+(0)=0.85\,. \quad (37d)$$

Thus the experimental information indicates that the second spectral-function sum rule,[12] requiring the equality of all vector and axial-vector coupling constants, is satisfied for chiral partners as suggested by the algebra of fields.[13] Furthermore, we see that the $SU(3)$ symmetry breaking is small.

It must be noted, however, that the interpretation of present experimental data is not unambiguous. In particular, ξ_A depends extremely sensitively on the value of λ_+. Taking $\lambda_+=0.023$ yields

$$\xi_V=1.18\,,\quad \xi_A=0.85\,,\quad f_+(0)=0.91\,.$$

At present it seems that it is best to use the second sum rule, for chiral partners, as a guide since the experimental error on λ_+ is too large to enable us to determine ξ_A and ξ_V conclusively.

We now turn to K^* decay, which fixes the value of

$$\lambda_{K_A}=g_2+g_3\,. \quad (38)$$

Since we wish to interpret Eq. (37a) as indicating small $SU(3)$ symmetry breaking, we must assume that g_{A_1} and g_{K_A} have the same sign. Then the K^* width of 49.2 MeV predicts $\lambda_{K^*}=-\frac{1}{3}$ or 6.2. The former value is more reasonable and we note that it is of the same order

[10] This is true only if $g_4+2g_5+g_6=0$.

[11] The K^* decay width is known quite accurately; see A. H. Rosenfeld *et al.* Rev. Mod. Phys. **40**, 77 (1968). The quoted error for Eq. (35b) is about 5%; see N. Cabibbo, in *Thirteenth Annual International Conference on High-Energy Physics, Berkeley, 1966* (University of California Press, Berkeley, 1967), p. 29. The quantity λ_+ is not known as well. We have $\lambda_+=0.023\pm0.008$ from K^+ decay and $\lambda=0.013\pm0.009$ from K^0 decay; see W. Willis, *Proceedings of the International Conference on High Energy Physics, Heidelberg, 1967*, edited by H. Filthuth (North-Holland Publishing Co., Amsterdam, 1968).

[12] S. Weinberg, Phys. Rev. Letters **18**, 507 (1967).

[13] T. D. Lee, S. Weinberg, and B. Zumino, Phys. Rev. Letters **18**, 1029 (1967).

of magnitude, but opposite in sign[14] to the analogous parameter $\lambda_{A_1}=1+\delta$ found from ρ and A_1 decay

The total width for the decay $K_A \to K^*+\pi$ is found from Eq. (A16) to be

$$\Gamma(K_A \to K^*\pi)=7.95\xi_V^{-1}[\tfrac{1}{4}(g_3-g_5)^2-3(g_3-g_5)\times(g_2+\tfrac{1}{2}g_3+\tfrac{1}{2}g_5)+25(g_2+\tfrac{1}{2}g_3+\tfrac{1}{2}g_5)^2]\text{ MeV}. \quad (39)$$

Using the previously determined values $\xi_V=1.11$, $g_2+g_3=-\frac{1}{3}$, and taking the K_A width[15] to be 60 MeV, we find

$$g_3-g_5=\begin{Bmatrix}+0.4\\-1.6\end{Bmatrix}.$$

In order further to distinguish these solutions, we need more information than is presently available on K_A decay. In particular, the ratio of transversely polarized to longitudinally polarized K^* mesons arising from the decay would enable us to choose between the two values.

Turning finally to the $f_-(t)$ form factor in K_{l3} decay, we see that Eq. (29) involves two constants not yet determined, m_κ^2 and $(Z_K/Z_\kappa)^{1/2}\equiv y$. There is a relation between these parameters similar to Eq. (21) obtained from Eqs. (19) and (20):

$$ym_\kappa^2=\frac{F_KF_\kappa m_K^2-y(F_K^2m_K^2-F_\pi^2m_\pi^2)}{-F_KF_\kappa y+F_\kappa^2}. \quad (40)$$

From Eqs. (29) and (40), we have

$$f_-(0)=(m_K^2-m_\pi^2)f_+(0)\left(\frac{1}{m_\kappa^2}-\lambda_+\frac{m_{K^*}^2}{m_\pi^2}\frac{1}{m_{K^*}^2}\right)+\frac{1}{2F_\pi F_K}(F_K^2-F_\pi^2+F_\kappa^2-2F_KF_\kappa y). \quad (41)$$

Using the values for F_K^2 and F_κ^2 derived from $f_+(t)$, we have[16]

$$f_-(0)=-0.005+(0.196/m_\kappa^2)-0.58|y|. \quad (42)$$

Thus, if we had an accurate measurement of $f_-(0)$ we would be able to determine m_κ^2 and all of the ratios of the renormalization constants. In the absence of such a measurement at the present we make the ansatz.[17]

$$y=g_{K_A}/g_{K^*}=0.99. \quad (43)$$

This yields

$$m_\kappa=635\text{ MeV}, \quad (44)$$

$$f_-(0)=-0.09. \quad (45)$$

Some comments about these values are in order. As mentioned above, we feel that our values of ξ_A and ξ_V imply small $SU(3)$ symmetry breaking, and hence that the product $F_\pi F_K$ is positive. Then, as Glashow and Weinberg have pointed out, $m_\kappa \leq 670$ MeV follows from Eq. (22a). Thus our small value of m_κ is not surprising. If F_π had the opposite sign, then Eq. (42) would become

$$f_-(0)=-0.005+(0.196/m_\kappa^2)+0.58|y|, \quad (46)$$

and one would expect the sign of $f_-(0)$ to be positive and the magnitude to be somewhat larger than we have found. It is worth observing that 635 MeV is precisely the $K\pi$ threshold.

From Eq. (29), we have

$$f_-(0)\lambda_-\frac{m_{K^*}^2}{m_\pi^2}=(m_K^2-m_\pi^2)f_+(0)\times\left(\frac{1}{m_\kappa^2}\frac{m_{K^*}^2}{m_\kappa^2}-\lambda_+\frac{m_{K^*}^2}{m_\pi^2}\frac{1}{m_{K^*}^2}\right)+\frac{m_{K^*}^2}{m_\kappa^2}\frac{1}{2F_\pi F_\kappa}(F_K^2-F_\pi^2+F_\kappa^2-2F_KF_\kappa y). \quad (47)$$

For $y=0.99$, we obtain

$$f_-(0)\lambda_-m_{K^*}^2/m_\pi^2=0.06. \quad (48)$$

It seems to be a common feature of this kind of analysis that $f_-(0)$ comes out small and negative, with λ_- large. This might suggest an alternative way of analyzing the experiments.

V. DISCUSSION

A. Other Literature

Several papers recently[18] have been concerned with questions of a similar nature as those we have discussed. We shall examine the relationship of our work to some of them.

This paper is most similar in spirit to those of Glashow and Weinberg,[2] and Chang and Leung.[9] Glashow and Weinberg use the Ward-identity method, but work at zero momentum transfer, so that they do not have an expression for λ_+ in K_{l3} decay. To make up for this lack, they assumed $\xi_A=\xi_V$. Since we find that this relation is predicted from λ_+, our values for $g_{K^*}^2$, $g_{K_A}^2$, F_K, F_κ, and $f_+(0)$ agree with theirs.

[14] If g_{A_1} and g_{K_A} had opposite signs, then the signs of λ_{K_A} would change relative to those in the text. $\lambda_{A_1}=\frac{1}{3}$ predicts $\Gamma(\rho\to\pi\pi)=115$ MeV, $\Gamma(A_1\to\rho\pi)=95$ MeV. There is no particular justification for asking that $\lambda_{K_A}=\lambda_{A_1}$.

[15] All candidates for our K_A with mass $\sim\sqrt{2}m_{K^*}$ are given this width in the tables. However, we cannot tell what fraction of this is background $K\pi\pi$ direct decay relative to $K^*\pi$ decay. A similar problem arises in A_1 decay and in Ref. 6 we have argued that this may be a non-negligible effect.

[16] Since our solution has small $SU(3)$ breaking, we assume the sign of F_π and F_κ are the same. Then Eqs. (40) and (37b) imply that the sign of y is the same as the product F_KF_κ. This accounts for the absolute value sign in Eq. (42).

[17] This assumption was first introduced by Chang and Leung in Ref. 9. It states that scalar and vector chiral partners undergo the same renormalization. We want to emphasize that we are only making this assumption because of the confused experimental situation regarding $f_-(0)$.

[18] We have only elected to discuss papers by other authors which we feel are similar in spirit to ours. Our list of references to other papers is not meant to be exhaustive.

Our basic equations agree with those of Chang and Leung. These authors use Eq. (43) but, they add some other hypothetical relationships among the model parameters which express a notion of κ dominance. A technical, although crucial, difference is that we take the parameter F_π as determined from the π_{l2} rate, in keeping with the type of calculation made in the previous section while they use the Goldberger-Treiman relation. It is not surprising, considering the sensitivity of the parameters to the experimental input that we obtain such different results. They find no evidence for the equality of ξ_A and ξ_V. Much of the difference may be characterized by their value of $\lambda_+=0.018$ compared to ours, $\lambda_+=0.0238$. Accurate information about this parameter is the most crucial input at the present time.

Lee[19] has studied K_{l3} decay using a chiral phenomenological Lagrangian. It has previously been observed that this technique yields results essentially similar to ours. However, Lee's method of symmetry breaking is quite different from ours. He rejects partial conservation for the strangeness-changing vector currents, takes its divergence proportional to a bilinear function of π and K fields, and has no κ meson. His Eqs. (2) and (3) for $f_\pm(t)$ are obtained from our Eqs. (28) and (29) if we take

$$m_\kappa^2\to\infty\,, \tag{49}$$

$$F_\kappa=0\,, \tag{50}$$

and identify

$$F_K^2-F_\pi^2+\frac{g_{K_A}g_{A_1}}{m_{K_A}^2m_{A_1}^2}m_{K^*}^2(g_2+g_3)=\beta^2F_K^2(1+\delta)\,. \tag{51}$$

We have some comments about the numerical values for the parameters found by Lee:

(i) The solution of our Eq. (35b) using Eq. (2) of Ref. (19) yields $F_K/F_\pi=1.33$, $f_+(0)=1.04$, rather than $F_K/F_\pi=1.28$, $f_+(0)=1.0$ as quoted there.

(ii) Using either of the above in our Eq. (34) yields $\lambda_+m_{K^*}^2/m_\pi^2=0.9$, a value midway between the one favored by us and that quoted by Lee.

Fenster and Hussain,[20] and Lai and Young[21] use the Ward-identity approach in their work on three-point functions. However, they allow the strangeness-changing vector current to be conserved so they do not examine the type of question we are consерned with. Furthermore, they do not consider K_{l3} decay and hence ignore a powerful constraint on the theory.

Riazuddin and Sarker[22] have used the Ward-identity approach and studied K_{l3} decays. However, they *assume* $F_\kappa=0$ and further, make the questionable demand that $f_+(0)=1$, *including* second-order symmetry breaking.

[19] B. W. Lee, Phys. Rev. Letters **20**, 617 (1968).
[20] S. Fenster and F. Hussain, Phys. Rev. **169**, 1314 (1968).
[21] C. S. Lai and B. L. Young, Phys. Rev. **169**, 1241 (1968).
[22] Riazuddin and A. Q. Sarker, Phys. Rev. **173**, 1752 (1968).

Finally, there have been papers[23] which use dispersion relations for $f_\pm(t)$ along with current-algebraic derivations of the subtraction constants. It is easy to connect our results with theirs, since we have written our expressions for $f_\pm(t)$ in dispersion-theoretic form. There have also been some attempts to compute quantities such as F_π/F_K by using some set of postulated (usually without particularly firm foundations) sum rules.[24] The values obtained by this approach do not particularly agree with ours.

B. Conclusions

We have presented a consistent picture of several processes involving strange mesons. Our basic assumption is the validity of the local chiral algebra of vector and axial-vector currents. We have supplemented this with a specific model of symmetry breaking, single-particle dominance of propagators, and minimum momentum dependence of primitive functions. Our results indicate a simple pattern of symmetry breaking in which the second spectral-function sum rule is valid for chiral partners and the variation from exact $SU(3)$ results is small.

We used experimental values for $F_K/[F_\pi f_+(0)]$, λ_+, and $\Gamma(K^*\to K\pi)$ to compute g_{K^*}, g_{K_A}, F_K, and F_κ. It is unfortunate that at present we can only say that the pattern of symmetry breaking we found is consistent with the data; it is by no means predicted uniquely. In particular, more accurate measurement of λ_+ is needed to make satisfactorally definite statements.

Our theory involves a κ meson whose existence is certainly in doubt. Our calculation of g_{K^*} and g_{K_A} from $f_+(t)$ and the K^* decay rate does not depend on the existence of a physical scalar-meson state corresponding to the κ field. If we knew λ_+ more accurately and had a measurement of $f_-(0)$, then we would have a unique prediction for the position of the κ and it would be a simpler experimental question to verify that portion of the theory. The value, 635 MeV, which we give is not a firm prediction since it is subject to the uncertainties introduced by λ_+ as well as depending on our use of Eq. (43).

If the κ does not exist, then we believe that an attractive alternative is to let $m_\kappa^2\to\infty$, $F_\kappa\to 0$ as in Lee's theory.[19] However, an accurate value for λ_+ could rule out this possibility since then we can simply compute F_κ. We recall that for $\lambda_+=0.9m_\pi^2/m_{K^*}^2$ we have $F_\kappa=0$. Were λ_+ not given by this value and nevertheless no κ found, we would then try parametrizing the κ propagator by a continuum rather than a pole.

[23] For example, H. T. Nieh, Phys. Rev. Letters **21**, 116 (1968); N. H. Fuchs, Phys. Rev. **170**, 1310 (1968); **172**, 1532 (1968); J. Mackey, J. McKissic, D. Scott, and W. Wada, Phys. Rev. **172**, 1590 (1968).
[24] For example, D. Majumdar, Phys. Rev. Letters **20**, 971 (1968). P. K. Mitter and L. J. Swank, University of Maryland Report (unpublished); P. P. Srivastava, CERN Report (unpublished).

Recall, too, that we have imposed Eqs. (26) on the primitive function $\Gamma^{A_1K_AK^*}$. It is also possible that these are not satisfied although it then appears to be difficult to analyze the data. It may be reasonable to require only

$$g_4+2g_5-g_6=0\,,$$

which follows from demanding that the K_{l3} form factor $f_-(t)$ be unsubtracted.

There are several questions we have not considered here, although most of the machinery for doing so is contained in the paper. First of all, there is the study of channels where mixing is permitted when $SU(3)$ symmetry is broken. Secondly, there are the abnormal-parity vertices (AAA and AVV) such as π^0 decay and ω decay. Some discussion of these have been given by other authors.[25] In any event it appears that sufficient new parameters (such as mixing angles) must be introduced so that consideration of these processes does not put any new constraints on the parameters we have been studying.

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APPENDIX: WARD IDENTITIES FOR PARTICLE CHANNELS

We have used the symbol C_x when single-particle dominance is not implied. In general,

$$C_x=\int dm^2\frac{\rho_x^{(1)}(m^2)}{m^2}\,,$$

and in the single-particle model $C_x=g_x^2/m_x^2$.

$\rho\rho\rho$

Vector Constraint

$$0=-C_\rho q^\mu\Gamma_{\mu,\nu,\lambda}{}^{\rho\rho\rho}(q,p)+[\Delta_\rho{}^{-1}(p)_{\nu\lambda}-\Delta_\rho{}^{-1}{}_{\nu\lambda}(r)]\,,\qquad\text{(A1)}$$

where we have defined

$$\Gamma_{\alpha\mu,\beta\nu,\gamma\lambda}{}^{\rho\rho\rho}(q,p)=f_{\alpha\beta\gamma}\Gamma_{\mu,\nu,\lambda}{}^{\rho\rho\rho}(q,p)\,.$$

Primitive Function:

$$\Gamma_{\mu,\nu,\lambda}{}^{\rho\rho\rho}(q,p)=-(m_\rho{}^2/g_\rho{}^4)[g_{\mu\nu}(q-p)_\lambda+2(g_{\mu\lambda}r_\nu-g_{\nu\lambda}r_\mu)-(g_{\nu\lambda}q_\mu-g_{\mu\lambda}p_\nu)]\,.\qquad\text{(A2)}$$

$K^*K^*\rho$

Vector Constraint

$$0=-C_\rho r^\lambda\Gamma_{\mu,\nu,\lambda}{}^{K^*K^*\rho}(q,p)+[\Delta_{K^*}{}^{-1}{}_{\mu\nu}(q)-\Delta_{K^*}{}^{-1}{}_{\mu\nu}(p)]\,,\qquad\text{(A3)}$$

where

$$\Gamma_{\alpha\mu,\beta\nu,\gamma\lambda}{}^{K^*K^*\rho}(q,p)=f_{\alpha\beta\gamma}\Gamma_{\mu,\nu,\lambda}{}^{K^*K^*\rho}(q,p)\,.$$

Primitive Function:

$$\Gamma_{\mu,\nu,\lambda}{}^{K^*K^*\rho}(q,p)=-(m_\rho{}^2/g_{K^*}{}^2g_\rho{}^2)[g_{\mu\nu}(q-p)_\lambda+2(1+\delta_{K^*})(g_{\mu\lambda}r_\nu-g_{\nu\lambda}r_\mu)-(g_{\nu\lambda}q_\mu-g_{\mu\lambda}p_\nu)]\,.\qquad\text{(A4)}$$

Nonprimitive Functions:

$$F_\kappa\tilde{\Gamma}_{\nu,\lambda}{}^{\kappa K^*\rho}(q,p)=1C_{K^*}q^\mu\Gamma_{\mu,\nu,\lambda}{}^{K^*K^*\rho}(q,p)-[\Delta_{K^*}{}^{-1}{}_{\nu\lambda}(p)-\Delta_{K^*}{}^{-1}{}_{\nu\lambda}(r)]\,,\qquad\text{(A5)}$$

where

$$\tilde{\Gamma}_{\delta,\beta\nu,\gamma\lambda}{}^{\kappa K^*\rho}(q,p)=d_{\delta\beta\gamma}\tilde{\Gamma}_{\nu,\lambda}{}^{\kappa K^*\rho}(q,p)\,,$$

$$-F_\kappa{}^2\tilde{\Gamma}_\lambda{}^{\kappa\kappa\rho}(q,p)=-C_{K^*}{}^2q^\mu p^\nu\Gamma_{\mu,\nu,\lambda}{}^{K^*K^*\rho}(q,p)-\tfrac{1}{2}(q-p)_\lambda+\tfrac{1}{2}(q-p)^\nu\Delta_\rho{}^{-1}{}_{\nu\lambda}(r)(C_\rho-2F_\kappa{}^2)\,,\qquad\text{(A6)}$$

where

$$\tilde{\Gamma}_{\gamma,\delta,\alpha\lambda}{}^{\kappa\kappa\rho}(q,p)=f_{\gamma\delta\alpha}\tilde{\Gamma}_\lambda{}^{\kappa\kappa\rho}(q,p)\,.$$

$A_1A_1\rho$

Vector Constraint

$$0=-C_\rho r^\lambda\Gamma_{\mu,\nu,\lambda}{}^{A_1A_1\rho}(q,p)+[\Delta_{A_1}{}^{-1}{}_{\mu\nu}(q)-\Delta_{A_1}{}^{-1}{}_{\mu\nu}(p)]\,,\qquad\text{(A7)}$$

where

$$\Gamma_{\alpha\mu,\beta\nu,\gamma\lambda}{}^{A_1A_1\rho}(q,p)=f_{\alpha\beta\gamma}\Gamma_{\mu,\nu,\lambda}{}^{A_1A_1\rho}(q,p)\,.$$

Primitive Function:

$$\Gamma_{\mu,\nu,\lambda}{}^{A_1A_1\rho}(q,p)=(-m_\rho{}^2/g_{A_1}{}^2g_\rho{}^2)[g_{\mu\nu}(q-p)_\lambda+2(1+\delta_{A_1})\times(g_{\mu\lambda}r_\nu-g_{\nu\lambda}r_\mu)-(g_{\nu\lambda}q_\mu-g_{\mu\lambda}p_\nu)]\,.\qquad\text{(A8)}$$

Nonprimitive Functions:

$$F_\pi\tilde{\Gamma}_{\nu,\lambda}{}^{\pi A_1\rho}(q,p)=-C_{A_1}q^\mu\Gamma_{\mu,\nu,\lambda}{}^{A_1A_1\rho}(q,p)+[\Delta_{A_1}{}^{-1}{}_{\nu\lambda}(p)-\Delta_{A_1}{}^{-1}{}_{\nu\lambda}(r)]\,,\qquad\text{(A9)}$$

where

$$\tilde{\Gamma}_{\alpha,\beta\nu,\gamma\lambda}{}^{\pi A_1\rho}(q,p)=f_{\alpha\beta\gamma}\tilde{\Gamma}_{\nu,\lambda}{}^{\pi A_1\rho}(q,p);$$

$$-F_\pi{}^2\tilde{\Gamma}_\lambda{}^{\pi\pi\rho}(q,p)=-C_{A_1}{}^2q^\mu p^\nu\Gamma_{\mu,\nu,\lambda}{}^{A_1A_1\rho}(q,p)-\tfrac{1}{2}(q-p)_\lambda+\tfrac{1}{2}(q-p)^\nu\Delta_\rho{}^{-1}{}_{\nu\lambda}(r)(C_\rho-2F_\pi{}^2)\,,\qquad\text{(A10)}$$

where

$$\tilde{\Gamma}_{\alpha,\beta,\gamma\lambda}{}^{\pi\pi\rho}(q,p)=f_{\alpha\beta\gamma}\tilde{\Gamma}_\lambda{}^{\pi\pi\rho}(q,p)\,.$$

$K_AK_A\rho$

Vector Constraint

$$0=-C_\rho r^\lambda\Gamma_{\mu,\nu,\lambda}{}^{K_AK_A\rho}(q,p)+[\Delta_{K_A}{}^{-1}{}_{\mu\nu}(q)-\Delta_{K_A}{}^{-1}{}_{\mu\nu}(p)]\,,\qquad\text{(A11)}$$

where

$$\Gamma_{\alpha\mu,\beta\nu,\gamma\lambda}{}^{K_AK_A\rho}(q,p)=f_{\alpha\beta\gamma}\Gamma_{\mu,\nu,\lambda}{}^{K_AK_A\rho}(q,p)\,.$$

Primitive Function:

$$\Gamma_{\mu,\nu,\lambda}{}^{K_AK_A\rho}(q,p)=(-m_\rho{}^2/g_{K_A}{}^2g_\rho{}^2)[g_{\mu\nu}(q-p)_\lambda+2(1+\delta_{K_A})(g_{\mu\lambda}r_\nu-g_{\nu\lambda}r_\mu)-(g_{\nu\lambda}q_\mu-g_{\mu\lambda}p_\nu)]\,.\qquad\text{(A12)}$$

Nonprimitive Functions:

$$F_K\tilde{\Gamma}_{\nu,\lambda}{}^{KAK_A\rho}(q,p)=-C_{K_A}q^\mu\Gamma_{\mu,\nu,\lambda}{}^{K_AK_A\rho}(q,p)+[\Delta_{K_A}{}^{-1}{}_{\nu\lambda}(p)-\Delta_\rho{}^{-1}{}_{\nu\lambda}(r)]\,,\qquad\text{(A13)}$$

[25] Riazuddin and A. Q. Sarker, Phys. Rev. Letters **20**, 1455 (1968). See also Refs. 20 and 21.

where

$$\tilde{\Gamma}_{\alpha,\beta\nu,\gamma\lambda}{}^{K_AK_A\rho}(q,p)=f_{\alpha\beta\gamma}\tilde{\Gamma}_{\beta,\gamma}{}^{K_AK_A\rho}(q,p);$$

$$-F_K{}^2\tilde{\Gamma}_\lambda{}^{KK\rho}(q,p)=-C_{K_A}{}^2q^\mu p^\nu\Gamma_{\mu,\nu,\lambda}{}^{K_AK_A\rho}(q,p)-\tfrac{1}{2}(q-p)_\lambda+\tfrac{1}{2}(q-p)^\nu\Delta_\rho{}^{-1}{}_{\nu\lambda}(r)(C_\rho-2F_K{}^2),\quad (A14)$$

where

$$\tilde{\Gamma}_{\alpha,\beta,\gamma\lambda}{}^{KK\rho}(q,p)=f_{\alpha\beta\gamma}\tilde{\Gamma}_\lambda{}^{KK\rho}(q,p).$$

$A_1K_AK^*$

No Vector Constraint

Primitive Function:

$$\Gamma_{\mu,\nu,\lambda}{}^{A_1K_AK^*}(q,p)=-(m_{K^*}{}^2/g_{K^*}{}^2g_{A_1}g_{K_A})\{g_1g_{\mu\nu}(q-p)_\lambda+g_2[g_{\mu\lambda}(r-q)_\nu+g_{\nu\lambda}(p-r)_\mu]+g_3(g_{\mu\lambda}r_\nu-g_{\nu\lambda}r_\mu)+g_4g_{\mu\nu}r_\lambda+g_5[g_{\mu\lambda}(r-q)_\nu-g_{\nu\lambda}(p-r)_\mu]+g_6(g_{\mu\lambda}r_\nu+g_{\nu\lambda}r_\mu)\},\quad (A15)$$

where

$$\Gamma_{\alpha\mu,\beta\nu,\gamma\lambda}{}^{A_1K_AK^*}(q,p)=f_{\alpha\beta\gamma}\Gamma_{\mu,\nu,\lambda}{}^{A_1K_AK^*}(q,p).$$

Nonprimitive Functions:

$$F_\pi\tilde{\Gamma}_{\nu,\lambda}{}^{\pi K_AK^*}(q,p)=-C_{A_1}q^\mu\Gamma_{\mu,\nu,\lambda}{}^{A_1K_AK^*}(q,p)+[\Delta_{K_A}{}^{-1}{}_{\nu\lambda}(p)-\Delta_{K^*}{}^{-1}{}_{\nu\lambda}(r)],\quad (A16)$$

where

$$\tilde{\Gamma}_{\alpha,\beta\nu,\gamma\lambda}{}^{\pi K_AK^*}(q,p)=f_{\alpha\beta\gamma}\tilde{\Gamma}_{\nu,\lambda}{}^{\pi K_AK^*}(q,p);$$

$$F_K\tilde{\Gamma}_{\mu,\lambda}{}^{A_1KK^*}(q,p)=-C_{K_A}p^\nu\Gamma_{\mu,\nu,\lambda}{}^{A_1K_AK^*}(q,p)+[\Delta_{K^*}{}^{-1}{}_{\mu\lambda}(r)-\Delta_{A_1}{}^{-1}{}_{\mu\lambda}(q)],\quad (A17)$$

where

$$\tilde{\Gamma}_{\alpha\mu,\beta,\gamma\lambda}{}^{A_1KK^*}(q,p)=f_{\alpha\beta\gamma}\tilde{\Gamma}_{\mu,\lambda}{}^{A_1KK^*}(q,p);$$

$$F_\kappa\tilde{\Gamma}_{\mu,\nu}{}^{A_1K_A\kappa}(q,p)=-C_{K^*}r^\lambda\Gamma_{\mu,\nu,\lambda}{}^{A_1K_AK^*}(q,p)+[\Delta_{A_1}{}^{-1}{}_{\mu\nu}(q)-\Delta_{K_A}{}^{-1}{}_{\mu\nu}(p)],\quad (A18)$$

where

$$\tilde{\Gamma}_{\alpha\mu,\beta\nu,\gamma}{}^{A_1K_A\kappa}(q,p)=d_{\alpha\beta\gamma}\tilde{\Gamma}_{\mu,\nu}{}^{A_1K_A\kappa}(q,p);$$

$$-F_\pi F_K\tilde{\Gamma}_\lambda{}^{\pi KK^*}(q,p)=-C_{A_1}C_{K_A}q^\mu p^\nu\Gamma_{\mu,\nu,\lambda}{}^{A_1K_AK^*}(q,p)-\tfrac{1}{2}(q-p)_\lambda+\tfrac{1}{2}(q-p)^\nu\Delta_{K^*}{}^{-1}{}_{\nu\lambda}(r)(C_\rho-F_\pi{}^2-F_K{}^2)-(1/2C_{K^*})r_\lambda\{(F_K{}^2-F_\pi{}^2)+(1/\mu_\kappa{}^2)[\mu_\pi{}^2F_\pi{}^2-\mu_K{}^2F_K{}^2+\mu_\pi{}^2F_\pi F_K(Z_\pi/Z_K)^{-1/2}-\mu_K{}^2F_\pi F_K(Z_\pi/Z_K)^{1/2}]\},\quad (A19)$$

where

$$\tilde{\Gamma}_{\alpha,\beta,\gamma\lambda}{}^{\pi KK^*}(q,p)=f_{\alpha\beta\gamma}\tilde{\Gamma}_\lambda{}^{\pi KK^*}(q,p);$$

$$-F_\kappa F_K\tilde{\Gamma}_\mu{}^{A_1K\kappa}(q,p)=-C_{K^*}C_{K_A}p^\nu r^\lambda\Gamma_{\mu,\nu,\lambda}{}^{A_1K_AK^*}-\tfrac{1}{2}(p-r)_\mu+\tfrac{1}{2}(p-r)^\nu\Delta_{K_A}{}^{-1}{}_{\mu\nu}(q)(C_\rho-F_\kappa{}^2-F_K{}^2)-(1/2C_{A_1})q_\mu[F_\kappa{}^2+F_\pi{}^2-F_K{}^2-2F_\pi F_K(Z_\pi/Z_K)^{-1/2}],\quad (A20)$$

where

$$\tilde{\Gamma}_{\alpha\mu,\beta,\gamma}{}^{A_1K\kappa}(q,p)=d_{\alpha\beta\gamma}\tilde{\Gamma}_\mu{}^{A_1K\kappa}(q,p);$$

$$-F_\kappa F_\pi\tilde{\Gamma}_\nu{}^{\pi K_A\kappa}(q,p)=-C_{K^*}C_{A_1}q^\mu r^\lambda\Gamma_{\mu,\nu,\lambda}{}^{A_1K_AK^*}(q,p)-\tfrac{1}{2}(r-q)_\nu+\tfrac{1}{2}(r-q)^\mu\Delta_{K_A}{}^{-1}{}_{\mu\nu}(p)(C_\rho-F_\kappa{}^2-F_\pi{}^2)+(1/2C_{K_A})p_\nu[F_\kappa{}^2-F_\pi{}^2-F_K{}^2+2F_\pi F_K(Z_\pi/Z_K)^{1/2}],\quad (A21)$$

where

$$\Gamma_{\alpha,\beta\nu,\gamma}{}^{\pi K_A\kappa}(q,p)=d_{\alpha\beta\gamma}\tilde{\Gamma}_\nu{}^{\pi K_A\kappa}(q,p);$$

$$F_\pi F_K F_\kappa\tilde{\Gamma}^{\pi K\kappa}(q,p)=-C_{A_1}C_{K_A}C_{K^*}q^\mu p^\nu r^\lambda\Gamma_{\mu,\nu,\lambda}{}^{A_1K_AK^*}(q,p)-\tfrac{1}{2}(p^2-r^2)F_\pi{}^2-\tfrac{1}{2}(r^2-q^2)F_K{}^2-\tfrac{1}{2}(q^2-p^2)F_\kappa{}^2+\tfrac{1}{2}(\mu_\pi{}^2+q^2)[F_\pi{}^2-2F_\pi F_K(Z_\pi/Z_K)^{-1/2}]+\tfrac{1}{2}(\mu_K{}^2+p^2)[-F_K{}^2+2F_\pi F_K(Z_\pi/Z_K)^{1/2}]+\tfrac{1}{2}[(\mu_\kappa{}^2+r^2)/\mu_\kappa{}^2]\times[\mu_K{}^2F_K{}^2-\mu_\pi{}^2F_\pi{}^2+F_\pi F_K\mu_K{}^2(Z_\pi/Z_K)^{1/2}-F_\pi F_K\mu_\pi{}^2(Z_K/Z_\pi)^{1/2}]+\tfrac{1}{2}F_\pi F_K[\mu_\pi{}^2(Z_K/Z_\pi)^{1/2}-\mu_K{}^2(Z_\pi/Z_K)^{1/2}],\quad (A22)$$

where

$$\tilde{\Gamma}_{\alpha,\beta,\gamma}{}^{\pi K\kappa}(q,p)=d_{\alpha\beta\gamma}\tilde{\Gamma}^{\pi K\kappa}(q,p).$$