Structure of Three-Point Functions from $SU(3)\otimes SU(3)$ Algebra*

IRA S. GERSTEIN
Laboratory for Nuclear Science and Physics Department, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

AND

HOWARD J. SCHNITZER†
Department of Physics, Brandeis University, Waltham, Massachusetts 02154

AND

STEVEN WEINBERG‡
Laboratory for Nuclear Science and Physics Department, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

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We construct three-point functions of vector and axial-vector currents consistent with the constraints of chiral $SU(3)\otimes SU(3)$ algebra. We make no single-particle approximations, nor do we assume that the strangeness-changing vector currents are conserved. We do, however, assume that the effects of symmetry breaking may be represented by a set of scalar and pseudoscalar fields transforming as a representation of the algebra. The results are presented in a form in which any approximations will be guaranteed to maintain the constraints of current algebra.

I. INTRODUCTION

THE application of Ward-identity techniques[1,2] to current algebra has made it possible to extend chiral symmetry and the hypothesis of partially conserved axial-vector current (PCAC) to situations involving pions that are not soft. An essential feature of the method is to convert Ward identities for n-point functions of currents to similar identities for hadron proper amplitudes. These identities can then be recast entirely in terms of "primitive" proper functions[2] which are essentially undetermined by current algebra. The formulation thus exhibits the entire content of the current algebra in terms of the primitive functions which are arbitrary, subject only to a constraint on the longitudinal parts if there is a conserved current. As a matter of convenience, this initial work was limited to chiral $SU(2)\otimes SU(2)$ even though the method is more generally applicable. We now extend the development[3] to chiral $SU(3)\otimes SU(3)$ in a way sufficiently general to allow the study of chiral symmetry breaking without any assumption being made concerning the magnitude of the symmetry-breaking term. The approximation of spin-zero propagators by single-particle intermediate states, which was made in our previous work, is also avoided throughout this paper. Here we present results appropriate to a general broken-symmetry group G, restricted to the three-point function for illustrative purposes, while in a companion article we apply the results to $SU(3)\otimes SU(3)$ to discuss strong, electromagnetic, and weak two-body meson decay amplitudes. Further assumptions and approximations which are made for the purpose of practical applications are discussed in the following paper.

We begin with a Lagrangian assumed to be of the form[4,5]

$$\mathfrak{L}=\mathfrak{L}_0+\epsilon_i\phi_i \qquad (1)$$

where $\mathfrak{L}_0$ is invariant under G, and the ϕ_i are local fields which are the basis for a real representation R of G. It follows that the currents constructed from Noether's theorem satisfy the partial conservation law

$$\partial_\mu J_\alpha{}^\mu(x)=\epsilon_i(T_\alpha)_{ij}\phi_j(x)\,, \qquad (2)$$

and the commutation rules

$$\left[\int d^3x\, J_\alpha{}^0(\mathbf{x},0),\phi_j(0)\right]=-i(T_\alpha)_{jk}\phi_k(0)\,. \qquad (3)$$

We also assume the local commutation relations

$$\delta(x_0)[J_\alpha{}^0(x),J_\beta{}^\nu(0)]=iC_{\alpha\beta\gamma}J_\gamma{}^\nu(x)\delta^4(x)+\text{S.T.}\,, \qquad (4)$$

and

$$\delta(x_0)[J_\alpha{}^0(x),\phi_j(0)]=-i\delta^4(x)(T_\alpha)_{jk}\phi_k(x)\,, \qquad (5)$$

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[1] H. J. Schnitzer and S. Weinberg, Phys. Rev. **164**, 1828 (1967).
[2] I. S. Gerstein and H. J. Schnitzer, Phys. Rev. **170**, 1638 (1968).
[3] See S. L. Glashow and S. Weinberg, Phys. Rev. Letters **20**, 224 (1968), where the single-particle approximation and other approximations were made early in the calculation. Further, not all Ward identities for three-point functions were discussed. See, also, S. L. Glashow, in *Proceedings of the International School of Physics, Ettore Majorana (1967)*, edited by E. R. Caianiello (Academic Press Inc., New York, 1968).
[4] The assumptions we make concerning symmetry breaking are the same as those of Ref. 3, where Eqs. (1)–(9) are derived. In particular, only the continuous, connected part of G enters our discussion.
[5] The notation is as given in Refs. 1 and 3 unless otherwise specified. The italic indices $i, j, k, \cdots$ run over the representation space R of the mesons ϕ_i. The Greek indices of $\alpha, \beta, \gamma, \cdots$ refer to the adjoint representation of G, while $\mu, \nu, \lambda, \cdots$ are space-time indices.

where $C_{\alpha\beta\gamma}$ are the structure constants of G. Noncovariant or Schwinger terms (S.T.) appropriate to (4) are not explicitly indicated, since we assume that they are c numbers. The matrices (T_α) satisfy

$$[T_{\alpha'}T_\beta]=C_{\alpha\beta\gamma}T_\gamma \tag{6}$$

and

$$(T_\alpha)_{ij}=-(T_\alpha)_{ji}. \tag{7}$$

Since the symmetry is broken, some of the fields ϕ_i may have nonvanishing vacuum expectation values, which we define:

$$\lambda_i\equiv\langle\phi_i\rangle_0. \tag{8}$$

It follows from the vacuum expectation value of Eq. (2) that

$$0=\epsilon_i(T_\alpha)_{ij}\lambda_j. \tag{9}$$

II. TWO-POINT FUNCTIONS

The propagators of the ϕ_i fields,

$$\int d^4x\, e^{-iqx}\langle T(\phi_i(x)\phi_j(0))\rangle_0=-i\Delta_{ij}(q), \tag{10}$$

are assumed to satisfy the spectral representation

$$\Delta_{ij}(q)=\int\frac{dm^2}{m^2+q^2}\rho_{ij}(m^2). \tag{11}$$

From the Ward identity

$$q^\mu\int d^4x\, e^{-iqx}\langle T(J_\alpha{}^\mu(x)\phi_i(0))\rangle_0 =\lambda_k(T_\alpha)_{ki}-\epsilon_j(T_\alpha)_{jk}\Delta_{ki}(q), \tag{12}$$

we learn that

$$\int d^4x\, e^{-iqx}\langle T(J_\alpha{}^\mu(x)\phi_i(0))\rangle_0=q^\mu\epsilon_j(T_\alpha)_{jk}D_{ki}(q), \tag{13}$$

where

$$D_{ki}(q)=\int\frac{dm^2}{m^2(m^2+q^2)}\rho_{ki}{}^{(0)}(m^2). \tag{14}$$

Similarly, from

$$q^\mu\int d^4x\, e^{-iqx}\langle T(J_\alpha{}^\mu(x)J_\beta{}^\nu(0))\rangle_0 =-iq^\nu\,\mathrm{tr}[\epsilon T_\beta D(q)T_\alpha\epsilon] \tag{15}$$

we find that the propagator of currents can be written

$$\int e^{-iqx}\langle T(J_\alpha{}^\mu(x)J_\beta{}^\nu(0))\rangle_0 =-i\Delta_{\alpha\beta}(q)^{\mu\nu}+iq^\mu q^\nu\,\mathrm{tr}[\epsilon T_\beta\bar{D}(q)T_\alpha\epsilon]+i\eta^\mu\eta^\nu S_{\alpha\beta}, \tag{16}$$

where in (15) and (16) the trace is taken in the representation space R of the spin-zero mesons. In Eq. (16) still another spectral representation for the spin-zero mesons is introduced:

$$\bar{D}_{ij}(q)=\int\frac{dm^2}{m^4(m^2+q^2)}\rho_{ij}(m^2), \tag{17}$$

as well as a spectral representation for the spin-1 propagators,

$$\Delta_{\alpha\beta}(q)^{\mu\nu}=\int\frac{dm^2}{m^2+q^2}\rho_{\alpha\beta}{}^{(1)}(m^2)\left(g^{\mu\nu}+\frac{q^\mu q^\nu}{m^2}\right). \tag{18}$$

The Schwinger term is

$$S_{\alpha\beta}=C_{\alpha\beta}-\mathrm{tr}[\epsilon T_\beta D(0)T_\alpha\epsilon], \tag{19}$$

where

$$C_{\alpha\beta}=\int\frac{dm^2}{m^2}\rho_{\alpha\beta}{}^{(1)}(m^2). \tag{20a}$$

In Eq. (16) η^μ is the timelike unit vector (0,0,0,1). Notice the necessity of defining three different representations $\Delta_{ij}(q)$, $D_{ij}(q)$, and $\bar{D}_{ij}(q)$ for the spin-zero mesons in order to satisfy the Ward identities. They are of course connected by the equations

$$q^2D(q)+\Delta(q)=\Delta(0), \tag{20b}$$

and

$$q^2\bar{D}(q)+D(q)=D(0). \tag{20c}$$

The relation

$$\epsilon_i(T_\alpha)_{ij}=\lambda_i[T_\alpha\Delta^{-1}(0)]_{ij}, \tag{21}$$

used in Eqs. (15), (16), and (19), is obtained from Eq. (12) in the limit $q^2\to 0$. Equation (21) interconnects the parameters ϵ_i of the symmetry-breaking Lagrangian and $\langle\phi_i\rangle_0$, which exhibits the asymmetry of the vacuum.

III. THREE-POINT FUNCTIONS

In this section we exploit the Ward identities for the three-point functions of currents and their derivatives. We find the following notation useful:

$$\int d^4xd^4yd^4z\, e^{-iqx}e^{-ipy}e^{-irz}\langle T(\phi_i(x)\phi_j(y)\phi_k(z))\rangle_0 =(2\pi)^4\delta^4(q+p+r)\langle\phi_i(q)\phi_j(p)\phi_k(r)\rangle. \tag{22}$$

There are three Ward identities which must be considered, namely,

$$q_\mu\langle J_\alpha{}^\mu(q)\phi_j(p)\phi_k(r)\rangle =-i\epsilon_i(T_\alpha)_{ii'}\langle\phi_{i'}(q)\phi_j(p)\phi_k(r)\rangle +i[T_\alpha\Delta(r)]_{jk}-i[\Delta(p)T_\alpha]_{jk}, \tag{23}$$

$$q_\mu\langle J_\alpha{}^\mu(q)J_\beta{}^\nu(p)\phi_k(r)\rangle =-i\epsilon_i(T_\alpha)_{ii'}\langle\phi_{i'}(q)J_\beta{}^\nu(p)\phi_k(r)\rangle -C_{\alpha\beta\gamma}r^\nu\epsilon_i[T_\alpha D(r)]_{ik}+p^\nu\epsilon_i[T_\beta D(p)T_\alpha]_{ik}, \tag{24}$$

and

$$q_\mu\langle J_\alpha{}^\mu(q)J_\beta{}^\nu(p)J_\gamma{}^\lambda(r)\rangle = -i\epsilon_i(T_\alpha)_{ii'}\langle\phi_{i'}(q)J_\beta{}^\nu(p)J_\gamma{}^\lambda(r)\rangle - iC_{\alpha\beta\delta}\{\Delta_{\delta\gamma}(r)^{\nu\lambda}-r^\nu r^\lambda\ \mathrm{tr}[\epsilon T_\alpha\tilde{D}(r)T_\delta\epsilon]\} - iC_{\alpha\beta\delta}\{\Delta_{\beta\delta}(p)^{\nu\lambda}-p^\nu p^\lambda\ \mathrm{tr}[\epsilon T_\delta\tilde{D}(p)T_\beta\epsilon]\}. \quad (25)$$

Further, since we have assumed that the Schwinger terms are c numbers, we have

$$p_\nu[q_\mu\langle J_\alpha{}^\mu(q)J_\beta{}^\nu(p)J_\gamma{}^\lambda(r)\rangle]=q_\mu[p_\nu\langle J_\alpha{}^\mu(q)J_\beta{}^\nu(p)J_\gamma{}^\lambda(r)\rangle],$$

which requires

$$C_{\alpha\beta\delta}S_{\delta\gamma}-S_{\alpha\delta}C_{\delta\beta\gamma}=0. \quad (26)$$

Schur's lemma implies

$$S_{\alpha\beta}=S\delta_{\alpha\beta}, \quad (27)$$

where S *can* have a different value for *each simple* subalgebra of G.[6]

The next step is to define proper functions which are "diagonalized" according to the singularity structure and transformation properties of the external currents. These are defined by

$$\langle\phi_i(q)\phi_j(p)\phi_k(r)\rangle=\Delta_{ii'}(q)\Delta_{jj'}(p)\Delta_{kk'}(r)\Gamma_{i',j',k'}(q,p), \quad (28)$$

$$\langle J_\alpha{}^\mu(q)\phi_j(p)\phi_k(r)\rangle=i\Delta_{\alpha\alpha'}{}^{\mu\mu'}(q)\Delta_{jj'}(p)\Delta_{kk'}(r)\Gamma_{\alpha'\mu',j',k'}(q,p)+iq^\mu\epsilon_i[T_\alpha D(q)]_{ii'}\Delta_{jj'}(p)\Delta_{kk'}(r)\Gamma_{i',j',k'}(q,p), \quad (29)$$

$$\langle J_\alpha{}^\mu(q)J_\beta{}^\nu(p)\phi_k(r)\rangle=-\Delta_{\alpha\alpha'}{}^{\mu\mu'}(q)\Delta_{\beta\beta'}{}^{\nu\nu'}(p)\Delta_{kk'}(r)\Gamma_{\alpha'\mu',\beta'\nu',k'}(q,p) -\Delta_{\alpha\alpha'}{}^{\mu\mu'}(q)p^\nu\epsilon_j[T_\beta D(p)]_{jj'}\Delta_{kk'}(r)\Gamma_{\alpha'\mu',j',k'}(q,p)-q^\mu\epsilon_i[T_\alpha D(q)]_{ii'}\Delta_{\beta\beta'}{}^{\nu\nu'}(p)\Delta_{kk'}(r)\Gamma_{i',\beta'\nu',k'}(q,p) -q^\mu\epsilon_i[T_\alpha D(q)]_{ii'}p^\nu\epsilon_j[T_\beta D(p)]_{jj'}\Delta_{kk'}(r)\Gamma_{i',j',k'}(q,p), \quad (30)$$

and

$$\langle J_\alpha{}^\mu(q)J_\beta{}^\nu(p)J_\gamma{}^\lambda(r)\rangle=-i\Delta_{\alpha\alpha'}{}^{\mu\mu'}(q)\Delta_{\beta\beta'}{}^{\nu\nu'}(p)\Delta_{\gamma\gamma'}{}^{\lambda\lambda'}(r)\Gamma_{\alpha'\mu',\beta'\nu',\gamma'\lambda'}(q,p) -i\Delta_{\alpha\alpha'}{}^{\mu\mu'}(q)\Delta_{\beta\beta'}{}^{\nu\nu'}(p)r^\lambda\epsilon_k[T_\gamma D(r)]_{kk'}\Gamma_{\alpha'\mu',\beta'\nu',k'}(q,p)-i\Delta_{\alpha\alpha'}{}^{\mu\mu'}(q)p^\nu\epsilon_j[T_\beta D(p)]_{jj'}\Delta_{\gamma\gamma'}{}^{\lambda\lambda'}(r)\Gamma_{\alpha'\mu',j',\gamma'\lambda'}(q,p) -iq^\mu\epsilon_i[T_\alpha D(q)]_{ii'}\Delta_{\beta\beta'}{}^{\nu\nu'}(p)\Delta_{\gamma\gamma'}{}^{\lambda\lambda'}(r)\Gamma_{i',\nu'\beta',\gamma'\lambda'}(q,p)-i\Delta_{\alpha\alpha'}{}^{\mu\mu'}(q)p^\nu\epsilon_j[T_\beta D(p)]_{jj'}r^\lambda\epsilon_k[T_\gamma D(r)]_{kk'}\Gamma_{\alpha'\mu',j',k'}(q,p) -iq^\mu\epsilon_i[T_\alpha D(q)]_{ii'}\Delta_{\beta\beta'}{}^{\nu\nu'}(p)r^\lambda\epsilon_k[T_\gamma D(r)]_{kk'}\Gamma_{i',\nu'\beta',k'}(q,p)-iq^\mu\epsilon_i[T_\alpha D(q)]_{ii'}p^\nu\epsilon_j[T_\beta D(p)]_{jj'}\Delta_{\gamma\gamma'}{}^{\lambda\lambda'}(r)\Gamma_{i',j',\gamma,\lambda'}(q,p) -iq^\mu\epsilon_i[T_\alpha D(q)]_{ii'}p^\nu\epsilon_j[T_\beta D(p)]_{jj'}r^\lambda\epsilon_k[T_\gamma D(r)]_{kk'}\Gamma_{i',j',k'}(q,p). \quad (31)$$

Equations (28)–(31) are inserted into Eqs. (23)–(25) to obtain Ward identities for the proper functions. We do not write these lengthy expressions, but instead go on to the next step, which is to express the proper functions in terms of a "primitive" function. That is to say, from the Ward identities for the proper functions one observes that $\Gamma_{i,j,k}(q,p)$, $\Gamma_{\alpha\mu,j,k}(q,p)$, and $\Gamma_{\alpha\mu,\beta\nu,k}(q,p)$ cannot be chosen arbitrarily. Rather, they are constructed from the Ward identities once $\Gamma_{\alpha\mu,\beta\nu,\gamma\lambda}(q,p)$ is specified. This latter proper amplitude we call the primitive function.[2] It can be specified arbitrarily, subject only to a vector constraint if there is a conserved current, and still satisfy all the Ward identities. We now express the proper functions in terms of the primitive function $\Gamma_{\alpha\mu,\beta\nu,\gamma\lambda}(q,p)$ and the various two-point functions. It is in this sense that we "solve" the Ward identities.

The solutions are

$$\epsilon_i[T_\alpha\Delta(0)]_{ii'}C_{\beta\beta'}C_{\gamma\gamma'}\Gamma_{i',\beta'\nu,\gamma'\lambda'}(q,p) =-C_{\alpha\alpha'}C_{\beta\beta'}C_{\gamma\gamma'}q^\mu\Gamma_{\alpha'\mu,\beta'\nu,\gamma'\lambda}(q,p)-p^\nu p^{\lambda'}C_{\alpha\beta'\gamma'}C_{\gamma\gamma''}\Delta^{-1}{}_{\gamma'\gamma''}{}^{\lambda\lambda'}(r)\ \mathrm{tr}[\epsilon T_{\beta'}\tilde{D}(p)T_\beta\epsilon] +r^{\nu'}r^\lambda C_{\alpha\beta'\gamma'}C_{\beta\beta''}\Delta^{-1}{}_{\beta'\beta''}{}^{\nu'\nu}(p)\ \mathrm{tr}[\epsilon T_{\gamma'}\tilde{D}(r)T_\gamma\epsilon]+C_{\alpha\beta'\gamma'}[\Delta^{-1}{}_{\beta'\beta''}{}^{\nu\lambda}(p)C_{\beta''\beta}C_{\gamma'\gamma}-\Delta^{-1}{}_{\gamma'\gamma''}{}^{\nu\lambda}(r)C_{\beta'\beta}C_{\gamma''\gamma}, \quad (32)$$

$$-\epsilon_i[T_\alpha\Delta(0)]_{ii'}\epsilon_j[T_\beta\Delta(0)]_{jj'}C_{\gamma\gamma'}\Gamma_{i',j',\gamma'\lambda}(q,p) =-C_{\alpha\alpha'}C_{\beta\beta'}C_{\gamma\gamma'}q^\mu p^\nu\Gamma_{\alpha'\mu,\beta'\nu,\gamma'\lambda}(q,p)+q^{\lambda'}C_{\alpha'\beta\gamma'}C_{\gamma\gamma''}\Delta^{-1}{}_{\gamma''\gamma'}{}^{\lambda'\lambda}(r)\ \mathrm{tr}[\epsilon T_{\alpha'}\Delta(0)\Delta^{-1}(q)D(q)T_\alpha\epsilon] -p^{\lambda'}C_{\alpha\beta'\gamma'}C_{\gamma\gamma''}\Delta^{-1}{}_{\gamma''\gamma'}{}^{\lambda'\lambda}(r)\ \mathrm{tr}[\epsilon T_{\beta'}\Delta(0)\Delta^{-1}(p)D(p)T_\beta\epsilon] -\tfrac{1}{2}r^\lambda\ \mathrm{tr}[\epsilon T_\gamma D(r)\Delta^{-1}(r)\{T_\alpha,T_\beta\}_+\lambda]-\tfrac{1}{2}(q-p)^\lambda C_{\alpha\beta\gamma'}C_{\gamma'\gamma} +\tfrac{1}{2}(q-p)^\nu C_{\alpha\beta\gamma'}\Delta^{-1}{}_{\gamma'\gamma''}{}^{\nu\lambda}(r)C_{\gamma''\gamma}S+\tfrac{1}{2}(q^2-p^2)r^\lambda C_{\alpha\beta\gamma'}\ \mathrm{tr}[\epsilon T_{\gamma'}\tilde{D}(r)T_\gamma\epsilon], \quad (33)$$

$$\epsilon_i[T_\alpha\Delta(0)]_{ii'}\epsilon_j[T_\beta\Delta(0)]_{jj'}\epsilon_k[T_\gamma\Delta(0)]_{kk'}\Gamma_{i',j',k'}(q,p) =-C_{\alpha\alpha'}C_{\beta\beta'}C_{\gamma\gamma'}q^\mu p^\nu r^\lambda\Gamma_{\alpha'\mu,\beta'\nu,\gamma'\lambda}(q,p)+\tfrac{1}{2}(p^2-r^2)C_{\alpha'\beta\gamma}\ \mathrm{tr}[\epsilon T_{\alpha'}\Delta(0)\Delta^{-1}(q)D(q)T_\alpha\epsilon] +\tfrac{1}{2}(r^2-q^2)C_{\alpha\beta'\gamma}\ \mathrm{tr}[\epsilon T_{\beta'}\Delta(0)\Delta^{-1}(p)D(p)T_\beta\epsilon]+\tfrac{1}{2}(q^2-p^2)C_{\alpha\beta\gamma'}\ \mathrm{tr}[\epsilon T_{\gamma'}\Delta(0)\Delta^{-1}(r)D(r)T_\gamma\epsilon] -\tfrac{1}{2}\ \mathrm{tr}[\epsilon T_\alpha\Delta(0)\Delta^{-1}(q)\{T_\beta,T_\gamma\}_+\lambda]-\tfrac{1}{2}\ \mathrm{tr}[\epsilon T_\beta\Delta(0)\Delta^{-1}(p)\{T_\gamma,T_\alpha\}_+\lambda]-\tfrac{1}{2}\ \mathrm{tr}[\epsilon T_\gamma\Delta(0)\Delta^{-1}(r)\{T_\alpha,T_\beta\}_+\lambda] +\tfrac{1}{6}\ \mathrm{tr}[\epsilon T_\alpha\{T_\beta,T_\gamma\}_+\lambda]+\tfrac{1}{6}\ \mathrm{tr}[\epsilon T_\beta\{T_\gamma,T_\alpha\}_+\lambda]+\tfrac{1}{6}\ \mathrm{tr}[\epsilon T_\gamma\{T_\alpha,T_\beta\}_+\lambda], \quad (34)$$

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[6] Equation (27) is the generalization of the first spectral-function sum rule to G. See S. Weinberg, Phys. Rev. Letters **18**, 507 (1967). The generalization to $SU(3)\times SU(3)$ was given in S. L. Glashow, H. J. Schnitzer, and S. Weinberg, Phys. Rev. Letters **19**, 139 (1967); and T. Das, V. Mathur, and S. Okubo, Phys. Rev. Letters **18**, 761 (1967). Recall that the application of Schur's lemma requires $C_{\alpha\beta\gamma}$ to be irreducible. If not, then S can take a different value for each *simple* subalgebra of G. This means that for a chiral subalgebra, the algebra for $V+A$ and $V-A$ could have separate Schwinger terms. However, because of parity conservation of the strong interactions, these Schwinger terms must in fact be equal.

where

$$\bar{D}(q) \equiv D(q)\Delta^{-1}(q)D(q) - \bar{D}(q) \quad (35)$$

and

$$\{T_\alpha, T_\beta\}_+ = T_\alpha T_\beta + T_\beta T_\alpha .$$

IV. CONCLUDING REMARKS

We have extended our previous work to an arbitrary current algebra making as few assumptions as possible. Aside from assuming a local algebra with c-number Schwinger terms, our main assumption has been the specific model of symmetry breaking expressed in Eq. (1). This model is general enough to encompass any set of nonconserved currents. In particular, we have not assumed any propagators to be saturated by single-particle states. As well as making it possible to implement the requirements of unitarity in some future, dynamical approach to current algebra, we note the existence of certain terms in Eqs. (32)–(34) which simply vanish in the single-particle approximation, and hence do not appear in the analogous equations of Refs. 1 and 2.

In the following paper,[7] we shall apply these results to $SU(3)\otimes SU(3)$ to discuss various meson decays. Although our present knowledge forces us to a single-particle-dominance approximation there, such an approximation and its success (or lack thereof) in no way compromises the generality of this paper.

[7] I. S. Gerstein and H. J. Schnitzer, following paper, Phys. Rev. **175**, 1876 (1968).

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Chiral $SU(3)\otimes SU(3)$ Three-Point Functions: Single-Particle Approximation*

IRA S. GERSTEIN

Laboratory for Nuclear Science and Physics Department, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

AND

HOWARD J. SCHNITZER†

Department of Physics, Brandeis University, Waltham, Massachusetts 02154

(Received 15 July 1968)

We approximate the three-point functions constructed previously by using a single-particle approximation to all propagators and assuming that the primitive three-point functions are slowly varying functions of the momenta. We use the available data on strong and weak decays of spin-one and spin-zero mesons to compute the values of the parameters introduced in our model, with special reference to K_{l3} decay.

I. INTRODUCTION

IN the preceding paper[1] we extended the Ward identity techniques for the three-point functions to chiral $SU(3)\otimes SU(3)$ without making single-particle approximations or special assumptions about symmetry breaking. We now use single-particle dominance for all propagators, assume that the primitive three-point functions are slowly varying functions of the momenta, and assume that the symmetry-breaking term in the Lagrangian transforms as $(3,\bar{3})\oplus(\bar{3},3)$.[2] These approximations lead to predictions for meson decays in terms of a number of arbitrary parameters, which are too numerous to be determined by experiment, so that no specific numerical predictions can be made without further assumptions. We shall discuss these assumptions as we make them. Finally, we determine the remaining parameters from experiment, these are found to be consistent with small $SU(3)$ and chiral symmetry breaking.

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[1] I. S. Gerstein, H. J. Schnitzer, and S. Weinberg, preceding paper, Phys. Rev. **175**, 1873 (1968). Equations from this paper are denoted by I.

[2] See S. L. Glashow and S. Weinberg, Phys. Rev. Letters **20**, 224 (1968). S. L. Glashow in *Proceedings of the International School of Physics, Ettore Majorana* (*1967*). Edited by E. R. Caianiello (Academic Press Inc., New York, 1968).

II. SINGLE-PARTICLE APPROXIMATION

The spectral representations for the spin-zero mesons are given in Eqs. (I.11), (I.14), and (I.17).[3] We define the single-particle approximation by assuming for the matrix propagators[4]

$$\Delta(q) = \frac{1}{M^2 + Z^{-1}q^2} = Z^{1/2}\frac{1}{\mu^2+q^2}Z^{1/2}, \quad (1)$$

[3] The notation, unless otherwise specified, is as in I.

[4] These approximations, their relation to the meson mass spectrum and to mixing models are discussed in S. Coleman and H. J. Schnitzer, Phys. Rev. **134**, B863 (1964).