Pion-Nucleon Elastic Scattering above 1 GeV*

E. Rost and L. Marshall Libby
University of Colorado, Boulder, Colorado 80302
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The diffraction part of the pion-nucleon elastic scattering is described by the Dirac equation, using an absorptive central potential with a hard core. Cross-section and polarization data for pion-proton elastic scattering in the 1.7–18.4-GeV/c region are fitted with three parameters—a potential strength, range, and core radius. The range and core radius are commensurable with values found in other elementary-particle systems.

FOR elastic scattering at GeV energies the number of experimental data is insufficient to determine the large number of phase shifts.[1] Instead, the data may be analyzed in terms of various simple models, for example, by assuming a particle-particle interaction with radial dependence. In the simplest of such models, cross sections for proton-proton interaction have been fitted by assuming the target proton to be a "black" absorbing disk of radius R and mean free absorption length $1/K$, with K large enough that the disk, essentially black initially, is of decreasing blackness with increasing energy.[2] Experimental cross sections at 0.8–2.75 GeV have been fitted with this model.

A more sophisticated model,[3] that of Brown, uses a repulsive (hard) core of 0.45 F with an external absorptive Gaussian potential $\sim\exp(-r^2/R^2)$ with $R=0.86$ F in the Schrödinger equation and fits the elastic and total proton-proton cross sections at 1 GeV and the forward differential elastic scattering. By allowing the hard core to shrink with increasing energy, the model is able to fit differential elastic scattering data at higher energies up to 6 GeV, at least at small angles, still using the Schrödinger equation.

Extending the treatment of the elastic proton-proton scattering to relativistic energies (10–30 GeV), Serber[4] used a purely absorptive Yukawa potential $V(r)=i\eta(1/r)e^{-\Lambda r}$ in the Klein-Gordon equation to obtain an analytic fit, with $\Lambda=(0.45\text{ F})^{-1}$. Real parts were added to Serber's potential by Auerbach and Brown[5] and their effects explored in a computer search, but without yielding significant improvement.

It is apparent that extending analyses of this sort may give useful insight into elementary-particle reactions. In the present paper we have applied it to pion-nucleon scattering at GeV energies. We have found that a single imaginary (absorptive) Yukawa potential is able to fit the elastic πp data, provided that a radial cutoff or hard core is also used. Real terms give no significant improvement.

Differing from the analyses discussed above, we have used the Dirac equation in our computer searches; thus the spin-orbit terms are introduced in a natural way. In order to computerize the Dirac equation we proceed as follows.

We consider a central potential energy $V(r)$ as the fourth component of a 4-vector. The solution to the Dirac equation in polar coordinates is written[6]

$$\psi=r^{-1}\begin{pmatrix} u_1(r)X_\kappa{}^\mu \\ iu_2(r)X_{-\kappa}{}^\mu \end{pmatrix}, \quad (1)$$

where $u_1(r)$ and $u_2(r)$ satisfy coupled first-order equations which may be transformed into a second-order equation for u_1:

$$\frac{d^2u_1}{dr^2}+\frac{dV/dr}{(W+mc^2-V)}\frac{du_1}{dr}+\left(\frac{(W-V)^2-m^2c^4}{\hbar^2c^2}-\frac{\kappa(\kappa+1)}{r^2}+\frac{\kappa}{r}\frac{dV/dr}{(W+mc^2-V)}\right)u_1=0, \quad (2)$$

where κ is the (integral) eigenvalue of $-\beta(\boldsymbol{\sigma}\cdot\mathbf{l}+1)$. The mass m is taken[7] to be the nonrelativistic reduced mass $m=m_\pi m_p/(m_\pi+m_p)$. It is convenient to perform a transformation of variables

$$u_1(r)=[W-V(r)+mc^2]^{1/2}u(r) \quad (3)$$

to obtain a single Schrödinger-like equation with no first derivatives:

$$\frac{d^2u}{dr^2}+\left[\frac{(W-V)^2-m^2c^4}{\hbar^2c^2}+\frac{(\kappa/r)(dV/dr)-\frac{1}{2}d^2V/dr^2}{W-V+mc^2}-\frac{3}{4}\left(\frac{dV/dr}{W-V+mc^2}\right)^2-\frac{\kappa(\kappa+1)}{r^2}\right]u=0. \quad (4)$$

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[1] J. Hamilton and W. S. Woolcock, Rev. Mod. Phys. 35, 737 (1963). The authors state 300 MeV as an upper energy limit in their dispersion-integral analysis. This limit has been exceeded in recent phase-shift analyses [see, e.g., University of California Radiation Laboratory Report No. UCRL-8030, 1968, p. 48 (unpublished)].

[2] W. B. Fowler, R. P. Shutt, A. M. Thorndike, W. L. Whittemore, V. T. Cocconi, E. Mart, M. M. Block, E. M. Harth, E. C. Fowler, J. D. Garrison, and T. W. Morris, Phys. Rev. 103, 1489 (1956).

[3] G. E. Brown, Phys. Rev. 111, 1178 (1958).

[4] R. Serber, Phys. Rev. Letters 10, 357 (1963).

[5] E. K. Auerbach and G. E. Brown, Phys. Letters 6, 95 (1963).

[6] M. E. Rose, *Relativistic Electron Scattering* (John Wiley & Sons, Inc., New York, 1961).

[7] Alternate choices of m would lead to slightly different values of the optical potential parameters.

If the potential vanishes rapidly [we ignore the Coulomb part of $V(r)$], u_1 and u are asymptotically

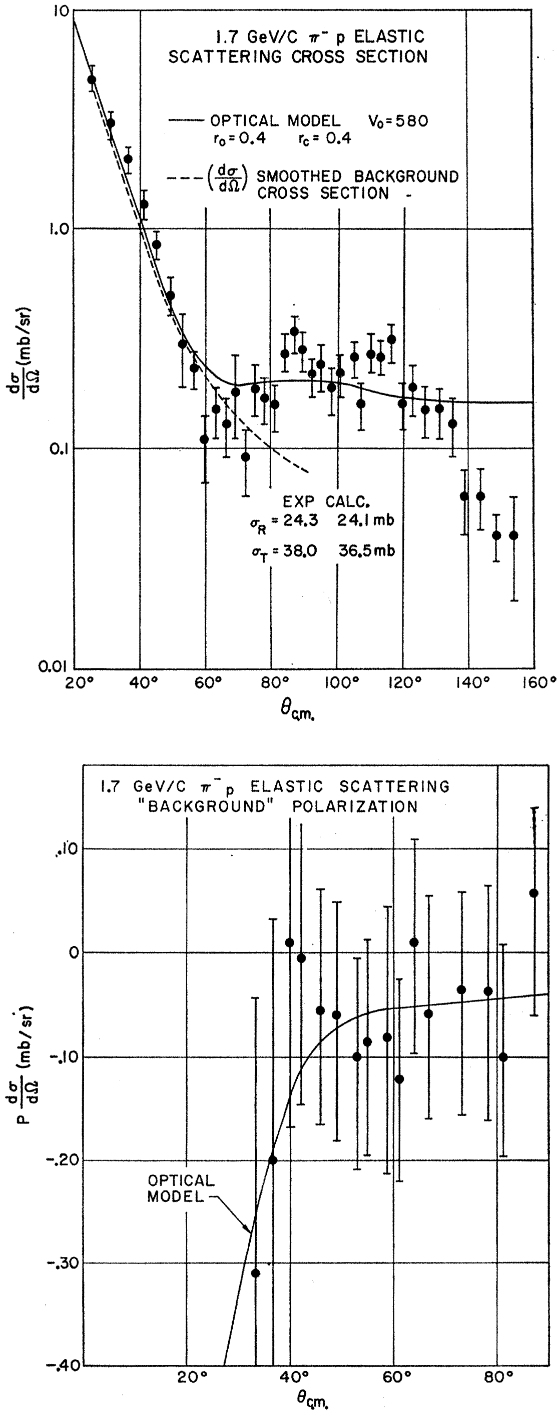

FIG. 1. (a) Optical-model computation (solid curve) fitted to 1.7 GeV/c π^-p experimental differential elastic cross section. The dashed curve is the smoothed "background" contribution extracted by Hoff (Ref. 14). (b) Comparison of computed optical-model prediction of $(Pd\sigma/d\Omega)$ with the extracted "background" value of $(Pd\sigma/d\Omega)_b$.

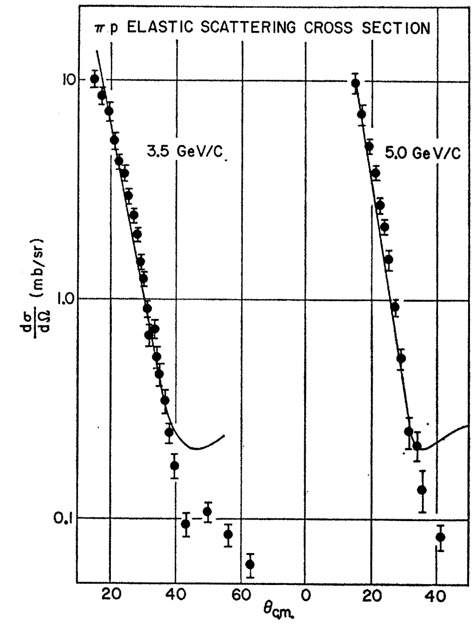

FIG. 2. Forward angle optical-model predictions at 3.5 and 5.0 GeV/c, compared with experimental π^-p differential elastic scattering at these energies (Ref. 12).

proportional and satisfy

$$
\begin{aligned}
u(r) &\underset{r\to\infty}{\longrightarrow} H_l^*(kr)-\eta_\kappa H_l(kr)\,, \\
k^2 &= (W^2-m^2c^4)/\hbar^2c^2\,, \\
l &= \kappa, \qquad \text{if } \kappa>0 \\
&= (-\kappa-1), \quad \text{if } \kappa<0
\end{aligned}
\tag{5}
$$

with $H_l(kr)$ denoting Hankel functions.[8] The scattering amplitudes are then

$$
\begin{aligned}
a_b &= \frac{i}{2k}\sum_\kappa (\iota+\tfrac{1}{2}-|\kappa|\eta_k)P_l(\cos\theta)\,, \\
b_b &= \frac{i}{2k}\sum_\kappa \frac{\kappa}{|\kappa|}\eta_k P_l^1(\cos\theta)\,,
\end{aligned}
\tag{6}
$$

from which one obtains the cross section and polarization:

$$
\begin{aligned}
(d\sigma/d\Omega)_b &= |a_b|^2+|b_b|^2\,, \\
(Pd\sigma/d\Omega)_b &= 2\,\mathrm{Im}(a_b b_b^*)\,,
\end{aligned}
$$

where the spin-nonflip term is a_b and the spin-flip term is b_b.

The central potential was chosen to be pure imaginary with a cutoff Yukawa radial dependence,

$$
\begin{aligned}
V(r) &= -iV_0(r/r_0)^{-1}\exp(-r/r_0)\,, \quad r>r_c \\
u(r) &= 0\,, \quad r\le r_c
\end{aligned}
\tag{7}
$$

[8] $H_l=G_l+iF_l$, where F_l and G_l are defined in *Handbook of Mathematical Functions*, edited by M. Abramowitz and I. A. Stegun (U. S. Department of Commerce, National Bureau of Standards, Appl. Math. Ser. 55, Washington, D. C., 1965), p. 538.

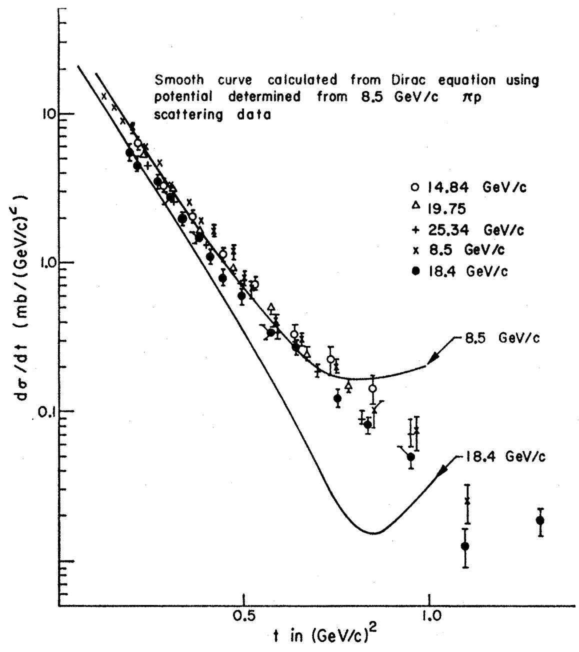

FIG. 3. Differential elastic scattering cross section for negative pions on protons, at 8.5 and 18.4 GeV/c measured by D. O. Caldwell, B. Elsner, D. Harting, A. C. Helmholz, W. C. Middelkoop, B. Zacharov, P. Dalpiaz, S. Focardi, G. Giacomelli, L. Monari, J. A. Beaney, R. A. Donald, P. Mason, and L. W. Jones, Phys. Letters **8**, 288 (1964); Nuovo Cimento **38**, 60 (1965), and at 14.84, 19.75, and 25.34 GeV/c measured by K. J. Foley, R. S. Gilmore, S. J. Lindenbaum, W. A. Love, S. Ozaki, E. H. Willen, R. Yamada, and L. C. L. Yuan, Phys. Rev. Letters **15**, 45 (1965). The solid curves have been computed from the Dirac equation using the potential determined from pion elastic scattering at 8.5 GeV/c and shown to fit elastic scattering from 1.7 to to above 8.5 GeV/c. The curve computed from this potential, for elastic scattering at 18.4 GeV/c, falls below the 18.4-GeV/c data but may be fitted by a slight change of any of the three parameters, well depth, core radius, or Yukawa radius; see Fig. 4 for examples.

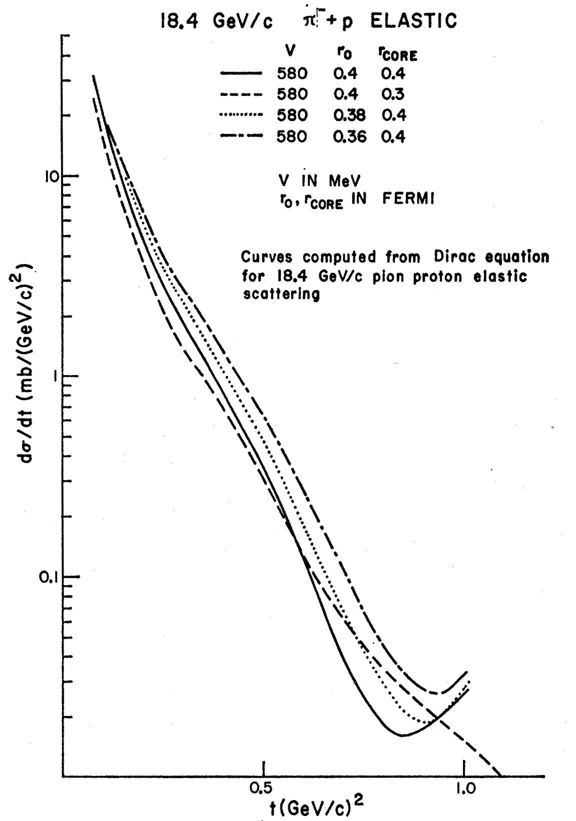

FIG. 4. Curves computed from the Dirac equation for 18.4-GeV/c negative pion proton scattering, using the potential found to fit elastic scattering from 8.5 down to 1.7 GeV/c, and showing the effect of varying the Yukawa radius and the core radius separately. An acceptable fit to the 18.4-GeV/c data is found, for example, by changing only the Yukawa radius by 5.5% and holding well depth and core radius unchanged. The total cross sections at all energies are correctly computed.

so that r_c is essentially a hard-core radius. This radius determines the large-angle scattering while V_0 and r_0 govern the forward exponential scattering. Note that the polarization is determined by $V(r)$ without any scaling parameter, and the corresponding spin-orbit term in Eq. (4) is decidedly complex.[9]

Trials in fitting 1.7-GeV/c elastic differential $\pi^- p$ data[10] yielded the parameters $V_0 = 580$ MeV, $r_0 = 0.4$ F, and $r_c = 0.4$ F. Using these, the total, reaction, and differential cross sections are correctly given. The coincidence of our value of $r_0(\pi p)$ with Serber's value of $r_0(pp)$ and of our value of $r_c(\pi p)$ with Brown's value of $r_c(pp)$ is interesting and probably significant. Also, this small range of the potential, 0.4 F, is found for other elementary-particle systems,[11] e.g., the Λ-nucleon system and the $\bar{p}p$ system.[4]

The potential (7) as determined above was found to fit forward diffractive scattering[12] for $\pi^- p$ data from 1.7–8.4 GeV/c (see Figs. 1–4) with no change in V_0, r_0, and r_c, and to give the total cross sections correctly. No attempt was made to obtain a best fit at large angles because of the lack of information on N^* resonances at high energy. It is hoped that the model may be useful in extracting resonance information in the high-energy region, and alternatively to gain understanding of the energy dependence of the hard core.

For example, the diffraction contribution to the proton polarization computed from this model (see Fig. 5) may be compared with polarizations measured at high energy.[13] Taking into account the polarization in the Regge-pole model computed from interference

[9] However, unitarity is not violated ($|\eta_\kappa| < 1$).

[10] D. D. Allen, G. P. Fisher, G. Godden, J. B. Kopelman, L. Marshall, and R. Sears, Phys. Letters **21**, 468 (1966).

[11] S. Ali and A. R. Bodmer, Phys. Letters **24B**, 343 (1967); Z. Koba and G. Takeda, Progr. Theoret. Phys. (Kyoto) **19**, 269 (1958).

[12] See compilation, Ned Dikman, Ph.D. thesis, University of Michigan, 1966 (unpublished).

[13] M. Borghini, C. Coignet, L. Dick, K. Kuroda, L. di Lella, P. C. Macq, A. Michalowicz, and J. C. Olivier, Phys. Letters **24B**, 77 (1967).

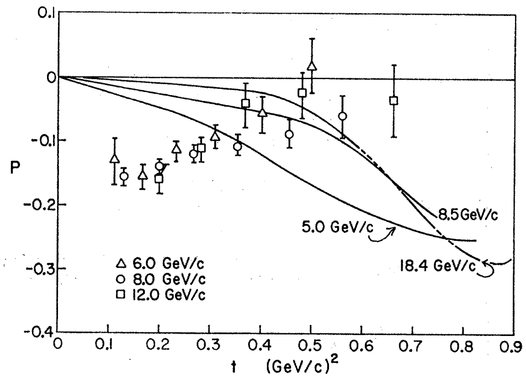

FIG. 5. Curves show polarizations computed from the Dirac equation for 5.0, 8.5, and 18.4 GeV/c negative pion proton elastic scattering using the imaginary Yukawa potential with hard core described in the text. Measured polarizations for 6.0, 8.0, and 12.0 GeV/c are shown, taken from Ref. 13. It is obvious that resonances make a large contribution to the interference terms producing polarization, and that resonance terms like those computed by Hoff (Ref. 14) at lower energy must be taken into account. In Ref. 13 are shown predictions of polarization computed in the Regge framework. It would appear that away from the forward direction, other Regge pole terms will be needed if the diffraction polarization is taken into account.

terms contributed by the ρ meson (as shown in Ref. 13), one tentatively concludes that higher resonances may play a role.

APPENDIX

It is interesting that using the model described above we have been able to compute the background polarization deriving from optical diffraction scattering as recently estimated by Hoff.[14]

Hoff separated the scattering amplitude into two terms: a resonant term and an optical diffraction term called "background." Thus the spin-nonflip term a and the spin-flip term b are written

$$a = a_b + a_r,\quad b = b_b + b_r. \tag{A1}$$

The differential cross section $(d\sigma/d\Omega)$ and the polarization P are given by

$$d\sigma/d\Omega = (d\sigma/d\Omega)_b + (d\sigma/d\Omega)_i + (d\sigma/d\Omega)_r,\quad P\,d\sigma/d\Omega = (Pd\sigma/d\Omega)_b + (Pd\sigma/d\Omega)_i, \tag{A2}$$

where the subscript i denotes a background-resonance interference term. By making reasonable assumptions about phases, Hoff has written fixed-angle relationships connecting the background and resonance components with the data at different energies "near" the resonance. The dominant N^* resonances of mass 2.0 GeV have $J=\frac{7}{2}$; in particular, Hoff, in application to $\pi^- $-$p$ scattering near 2.0 GeV/c, has computed a resonant component for $N^*(2070)$ of $G_{7/2}$.

In the Hoff analysis, the background amplitude $(d\sigma/d\Omega)_b$ is not computed. Instead, it is estimated as the difference between the experimental data and the computed resonance scattering. However, we have computed it from the Dirac equation as described above using the previously mentioned values of V_0, r_0, and r_c and show the computed values in Fig. 3. For comparison $(d\sigma/d\Omega)_b$, estimated by Hoff's procedure,[15] is shown as the smoothed curve.

Since the resonance-interference term in (A2) dominates, the extraction of the background term is inaccurate, as shown by the error bars. Nevertheless, the optical-model computation gives the over-all behavior *with no adjustable parameters*, the parameters V_0, r_0, and r_c having been obtained from the experimental elastic-scattering data. In contrast to the situation for the nuclear optical model, there is needed no large scale factor to account for the polarization.

[14] G. T. Hoff, Phys. Rev. Letters **18**, 816 (1967).

[15] The original data are presented in A. Yokosawa *et al.*, Phys. Rev. Letters **16**, 714 (1967). The extraction of the background contribution is taken from Ref. 14.