Model of High-Energy Elastic Scattering and Diffractive Excitation Processes in Hadron-Hadron Collisions

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q numbers are introduced to replace c numbers for the density function ρ in a recent model of high-energy elastic scattering. The result is shown to be equivalent to the Glauber theory suitably generalized. The model leads naturally to the distinction between diffractive and nondiffractive excitation processes. The former has finite cross sections at infinite energy. Selection and intensities rules for diffractive excitation processes are discussed.

1. q-NUMBER DENSITY

In a recently proposed model of elastic scattering[1] at infinite energy between two hadrons $AB\to AB$, the S matrix at an impact parameter $\mathbf{b}$ (= two-dimensional vector b_x, b_y in the x, y plane; the z axis being parallel to the incident direction) was written as

$$S(\mathbf{b})=\exp\left(-\int\int\rho_A(x,y,z)\rho_B(x',y',z')\times F(b_x-x'+x,\, b_y-y'+y)d^3xd^3x'\right). \quad (1)$$

In this formula ρ_A (ρ_B) is the "density distribution" of strongly interacting "stuff" inside $A(B)$ at a point (x,y,z) with its center as the origin. In Ref. 1 the function F was taken to be a δ function of its arguments.

We consider two generalizations in this paper: (i) F is not necessarily a δ function, and (ii) we shall consider ρ_A and ρ_B as q numbers, rather than c numbers. The commutation rules for these q numbers will be taken as that of the density $\psi^\dagger\psi$, where ψ is some second quantized fermion field. Actually one should have several fermion fields for each particle A and B, in which case the integral in (1) should be replaced by a sum of integrals:

$$\sum_{i,j}\int\int\rho_A{}^i(x,y,z)\rho_B{}^j(x',y',z')\times F^{ij}(b_x-x'+x,\, b_y-y'+y)d^3xd^3x'. \quad (2)$$

(The necessity of using this more complicated expression derives from the fact that, because of conservation laws, hadrons cannot be envisaged to be made of only one type of fermions and its antiparticles.) However, in the model discussed below we shall ignore this complication and use (1).

To appreciate the meaning of (1), let us apply it to the collision between two nuclei A and B, in which case we take ρ_A and ρ_B to be the q-number nucleon density $\psi^\dagger\psi$ inside A and B, and

$$\exp[-F(b_x,b_y)]=S_1(\mathbf{b})$$

to be the S matrix of nucleon nucleon scattering. In the representation where the coordinates $\mathbf{x}_i$ and $\mathbf{x}_j'$ of the nucleons in A and B are diagonal, $\rho_A(x,y,z)$ is diagonal with diagonal elements $\sum_i\delta^3(x-x_i)$. Thus $S(\mathbf{b})$ of (1) is also diagonal with diagonal elements

$$S_2(\mathbf{b},\mathbf{x}_i,\mathbf{x}_j')=\exp(-\sum_{i,j}F(b_x-x_i+x_j',\, b_y-y_i+y_j'))=\prod_{i,j}S_1(b_x-x_i+x_j',\, b_y-y_i+y_j'). \quad (3)$$

Thus our S matrix is precisely that used in nuclear physics for nucleon nucleus scattering by Glauber,[2] with (a) a generalization to nucleus-nucleus scattering, and (b) the introduction of the physical assumption,[3] suggested by empirical facts, that in high-energy hadron-hadron scattering, each hadron behaves as an extended structure with many degrees of freedom described by ρ_A and ρ_B.

2. ELASTIC SCATTERING

To obtain the elastic scattering amplitude we take the matrix element of (1) between the incoming state $|AB\rangle$ and the outgoing state $\langle AB|$. In this process we consider the q numbers ρ_A as completely independent of (i.e., commute with) the q numbers ρ_B. In other words, the stuff that makes up A is considered totally different from that that makes up B. The empirical basis of this assumption is the fact that at very high energies there is little momentum transfer between the colliding particles. We interpret this fact as meaning

[1] T. T. Chou and C. N. Yang, in *Proceedings of the Second International Conference on High-Energy Physics and Nuclear Structure, Rehovoth, Israel, 1967*, edited by G. Alexander (North-Holland Publishing Co., Amsterdam, 1967), pp. 348–359; Phys. Rev. **170**, 1591 (1968); Phys. Rev. Letters **20**, 1213 (1968). These papers will be referred to as I, II, and III, respectively. Equations in these papers will be referred to as (II.13), etc.

[2] R. J. Glauber, in *Lectures in Theoretical Physics*, edited by W. E. Brittin *et al.* (Interscience Publishers, Inc., New York, 1959), Vol. 1. The ideas of Glauber are quite similar to ideas advanced and developed by S. I. Drozdov, Zh. Eksperim. i Teor. Fiz. **28**, 734 (1955); **28**, 736 (1955); E. V. Inopin, *ibid.* **31**, 901 (1956) [English transl.: Soviet Phys.—JETP **1**, 591 (1955); **1**, 588 (1955); **4**, 764 (1957)]; J. S. Blair, Phys. Rev. **115**, 928 (1959); V. Franco (unpublished).

[3] This idea was first discussed in T. T. Wu and C. N. Yang, Phys. Rev. **137**, B708 (1965); N. Byers and C. N. Yang, *ibid.* **142**, 976 (1966). Subsequent discussion in terms of quark models have been given by L. Van Hove, in *Particle Interactions at High Energies*, edited by T. W. Preist and L. L. J. Vick (Plenum Press, Inc., New York, 1967), p. 96; D. R. Harrington and A. Pagnamenta, Phys. Rev. Letters **18**, 1147 (1967).

no exchange of stuff between A and B. (The collision process may, however, lead to a rearrangement of the stuff in A, and/or in B, resulting in a "diffractive excitation" process to be discussed in Sec. 3.)

The amplitude for elastic scattering at infinite energy is, as in (II.13),

$$-a=\text{two-dimensional Fourier transform of } \langle AB|S-1|AB\rangle,$$

with the elastic differential cross section given by

$$\lim d\sigma/dt=\pi|a|^2. \quad (4)$$

We shall adopt[4] the notation $[\cdots]$ for two-dimensional Fourier transforms so that

$$-a=[\langle AB|S-1|AB\rangle]. \quad (5)$$

To discuss the meaning of this expression, consider the case of nucleus-nucleus scattering for which we may use (3). By definition,

$$\langle AB|S|AB\rangle=\text{arithmetic mean of } S_2(\mathbf{b},\mathbf{x}_i,\mathbf{x}_j') \quad (6)$$

where the weight used in the mean is the absolute value squared of the wave functions for A and B: $\psi_A^\dagger\psi_A\psi_B^\dagger\psi_B \geq 0$. Now we define the geometrical mean of (3) with the same weight:

$$(S_2)_{\text{g.m.}}=\exp\left(-\int\psi_A^\dagger\psi_A\psi_B^\dagger\psi_B d^{3N}xd^{3N'}x'\sum_{i,j}F(\cdots)\right)$$
$$=\text{right-hand side of (1) with } \rho_A \text{ and } \rho_B \text{ replaced by their expectation values.} \quad (7)$$

But this last expression is exactly that for $S(\mathbf{b})$ if we take a c-number theory of the densities ρ_A and ρ_B, as in Ref. 1. Thus

$$S(\mathbf{b}) \text{ in Ref. } 1=(S_2)_{\text{g.m.}} =\text{geometrical mean of } S_2(\mathbf{b},\mathbf{x}_i,\mathbf{x}_j'). \quad (8)$$

Equations (6) and (8) exhibit the relationship between the present model and the model of Ref. 1.

How different are the arithmetic and geometrical means? We make the following comments:

(i) The two means are approximately the same if and only if the blackness of each nucleus, as viewed from the other, does not fluctuate very much as the nucleons in it move around each other.

(ii) These fluctuations are quite small[5] for a tight nucleus. Thus the two means are approximately the same if both nuclei have $\geq$four nucleons. They are, however, quite different for deuteron-nuclei collisions.

(iii) If we apply (1) to a hadron-hadron collision, it is expected that each hadron is composed of many constituent pieces, or of a few tightly bound pieces, much like an α particle is made of nucleons. We expect in either case the arithmetic and geometrical means to be approximately the same.

(iv) In case F is real, which is likely, (6) and (8) represent means taken over positive weights of positive numbers. Thus the arithmetic mean is always larger.

(v) For small fluctuations, we can calculate the ratio of the arithmetic mean divided by the geometrical mean. This ratio is the arithmetic mean of

$$\exp\left(-\int\int F(\cdots)d^3xd^3x'(\rho_A\rho_B-\langle\rho_A\rho_B\rangle)\right) \quad (9)$$

which can be expanded as a power series in $\rho_A\rho_B-\langle\rho_A\rho_B\rangle$.

3. DIFFRACTIVE EXCITATION

At very high energies, the distribution of processes $AB\to CD$ has, empirically, also a very small angular width, in analogy with that for $AB\to AB$. The general description is clearly that the stuff in A suffers a rearrangement and becomes C. This description can be naturally accommodated in the present model. The operator S in (1) has elements between $|AB\rangle$ and $\langle CD|$. The limiting cross section at infinite energy for $AB\to CD$, will be postulated to be again given by (4), with[4]

$$-a=\text{limiting amplitude} =[\langle CD|S-1|AB\rangle]=[\langle CD|S|AB\rangle]. \quad (10)$$

This is the same procedure as that used[2] in nuclear physics by Drozdov, Inopin, Glauber, and Blair. We shall call those processes $AB\to CD$ for which (10) gives a nonvanishing result "diffractive excitation" processes. These processes have finite cross sections at infinite energy in our model. Other processes $AB\to CD$ will be called nondiffractive excitation processes.

The existence of processes $AB\to CD$ at higher and higher energies with nondecreasing cross sections was first[6] discovered by Anderson *et al.* This discovery was subsequently[7] confirmed by Foley *et al.* Theoretical discussions[8] in terms of the "diffraction dissociation" process of Good and Walker[9] have been given. It is clear that our present discussion bears considerable resemblance to the underlying ideas of Good and Walker. However, the present model is in our opinion more completely formulated.

A number of interesting selection and intensity rules can be obtained from our model, for infinite energy elastic and diffractive excitation processes.

[4] Notice the difference of notation from II.

[5] This point was also independently realized by W. Czyż and L. C. Maximon (to be published).

[6] E. W. Anderson, E. J. Bleser, G. B. Collins, T. Fujii, J. Menes, F. Turkot, R. A. Carrigan, Jr., R. M. Edelstein, N. C. Hien, T. J. McMahon, and I. Nadelhaft, Phys. Rev. Letters, **16**, 855 (1966).

[7] K. J. Foley, R. S. Jones, S. J. Lindenbaum, W. A. Love, S. Ozaki, E. D. Platner, C. A. Quarles, and E. H. Willen, Phys. Rev. Letters **19**, 397 (1967).

[8] D. R. O. Morrison, Phys. Letters **22**, 226 (1966).

[9] M. L. Good and W. D. Walker, Phys. Rev. **120**, 1857 (1960); D. Amati and J. Prentki (unpublished); S. D. Drell and K. Hiida, Phys. Rev. Letters **7**, 199 (1961); see also E. L. Feinberg and I. Ia. Pomerancuk, Nuovo Cimento Suppl. **3**, 652 (1956).

(i) Since the S in (1) and (3) is dependent only on space coordinates, one obtains:

For a diffractive excitation process $AB \to CD$, A and C must have the same charge, G, $\mathbf{I}^2$, I_z, strangeness, and nucleon number. So must B and D. (11)

(ii) Spin-parity selection rule:

For a diffractive excitation process $AB \to CD$, A and C cannot both be spinless and have opposite parity. Nor can B and D. (12)

(iii) Forward dip:

If the product of the parities of A, B, C, and D is odd, and $AB \to CD$ is a diffractive excitation process, then $d\sigma/dt=0$ at $t=0$. (13)

To prove this we need only to observe that in (3), S_2 is an even function of $\mathbf{b}$, $\mathbf{x}_i$ and $\mathbf{x}_j'$. Integration over the variables $\mathbf{x}_i$ and $\mathbf{x}_j'$ therefore yields for process (13), a $\langle CD|S|AB\rangle$ which is an odd function of $\mathbf{b}$. Therefore $d\sigma/dt=0$ at $t=0$. We suggest that this selection rule is the reason that in Refs. 6 and 7 the $D_{13}(1.25)$ resonance was observed in $pp \to pp^*$ and $\pi p \to \pi p^*$ only at large $-t$ values, while the $P_{11}(1.40)$ and $F_{15}(1.69)$ resonances were observed at small $-t$ values.

(iv) Lack of right-left asymmetry:

For elastic scattering or a diffractive excitation process on a target transversely polarized, there is no right-left asymmetry at infinite energy. (14)

This follows from the fact that if the product of the parities of A, B, C, and D is even (odd), $\langle CD|S|AB\rangle$ is an even (odd) function of $\mathbf{b}$. Therefore there is no right-left asymmetry (To have right-left asymmetry, $\langle CD|S|AB\rangle$ must be neither even nor odd in $\mathbf{b}$.). It should be emphasized, however, that more complicated polarization effects than simple right-left asymmetry can and should obtain.

4. REMARKS

(i) The selection rules and intensities rules above do not change if we introduce several types of densities $\rho_A{}^i$, $\rho_B{}^j$, as in (2), provided no spin, isospin, and strangeness dependence is introduced in F.

(ii) At finite energies, F is presumably dependent on spin, isospin, and strangeness coordinates. It could also depend on variables that cause a nucleon number transfer from A to B. Such dependences are presumably partially responsible for the nondiffractive excitation processes.

(iii) Since $\psi_D{}^\dagger\psi_C{}^\dagger\psi_A\psi_B$ has a fluctuating phase it is expected that the matrix element for a diffractive excitation process is small. Furthermore, for highly excited states, the fluctuation should become more rapid and the cross sections become smaller. Thus the existence of many diffractive excitations does not imply that the total two-body cross section would increase without bound even though more channels are opened when the incoming energy increases.

(iv) The existence of diffractive excitation processes such as $pp \to pp^*$, $pp \to p^*p^*$ implies that at very high energies, these processes become the main source of inelastic high energy outgoing protons. Similarly, in $\pi p \to \pi p^*$, the diffractive excitation processes become the main source of inelastic high energy outgoing pions.

(v) It is tempting to try, in analogy with elastic scattering, to write down for $\langle CD|S|AB\rangle$ the expression (1) with ρ_A and ρ_B replaced by, respectively, $\langle C|\rho_A|A\rangle$ and $\langle D|\rho_B|B\rangle$. One would then have,[10] by expanding in powers of the exponent, a relationship between $pp \to pp^*$ and the form factors for $ep \to ep$ and $ep \to ep^*$, like that for elastic scattering first discussed[3] by Wu and Yang. This procedure is not justifiable in the present model, since the arguments of Sec. 2 comment (iii) above do not apply when the weight used in taking the mean is not necessarily positive. Experimentally, it is interesting to note that while[6,7] in $pp \to pp^*$ and $\pi p \to \pi p^*$, the $P_{11}(1.40)$ is very much excited, in $ep \to ep^*$ it is very little excited.[11]

(vi) Our discussion centers on hadron-hadron collisions. For p-neucleus or π-nucleus collisions, diffractive excitation processes such as p nucleus $\to p$ nucleus*, p nucleus $\to p^*$ nucleus, and p nucleus $\to p^*$ nucleus* have been discussed. Some of the selection rules discussed in this paper for hadron-hadron collisions have been discussed[12] in the literature for these collisions.

(vii) What are ρ_A and ρ_B? Why is the procedure of taking matrix elements of the operator S in (1) a good one at infinite energies? Why is F dependent only on space coordinates? These are questions that cannot be answered within the model itself. In fact they can only be answered when a fundamental theory of hadrons emerges.

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[11] F. W. Brasse, J. Engler, E. Ganssauge, and M. Schweizer, DESY Report (unpublished); also private communication from the SLAC group on electron scattering.

[12] See Refs. 2 and 9; A. S. Goldhaber and M. Goldhaber, in *Preludes in Theoretical Physics*, edited by A. de-Shalit, H. Feshbach, and L. Van Hove (North-Holland Publishing Co., Amsterdam, 1966); L. Stodolsky, Phys. Rev. **144**, 1145 (1966).