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Regge Poles and Finite-Energy Sum Rules for Kaon-Nucleon Scattering

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Generalized finite-energy sum rules (FESR) for kaon-nucleon scattering are evaluated to determine the t dependence and other properties of the relevant Regge-exchange amplitudes: P, P', A_2, ρ, and ω. The FESR's have been evaluated (a) with the available phase-shift analyses for the low-energy KN system as input and (b) in the resonance-saturation approximation with all the appropriate resonances of which J^P is known. For the K^+p system, the phase-shift analysis of Lea *et al.* has been used; for the $\bar{K}N$ system, the multichannel effective-range analysis of Kim, and the resonance-plus-background analysis of Armenteros *et al.* Matching energies of $\sqrt{s}=2$ GeV and $\sqrt{s}=2.15$ GeV have been used for the cases (a) and (b), respectively. In terms of the definite helicity flip amplitudes, A (which is the full forward amplitude at $t=0$) and B, we find that assuming the ρ contribution to be known, the non-spin-flip contributions for the various Regge poles are similar to those deduced from high-energy fits; however, the spin-flip contributions of the high-energy fits are inconsistent with our FESR results. For example, the factorization ratio $(\nu B/A)$ for the A_2, P, P', and ω contributions, where ν is $(s-u)/4M$, is found to have the opposite sign to that used in previous high-energy fits. As far as our results go, the FESR's are consistent with the usual explanation of the crossover phenomenon in terms of a single genuine ω Regge pole, though we cannot regard this conclusion as very strong, because of the poor available input data. We find no evidence of a wrong-signature nonsense zero in α_ω for $-t\lesssim 0.8$ (GeV/c)^2; we find $(\nu B/A)_\omega=+(1\text{–}3)$ for $-t\lesssim 0.6$ (GeV/c)^2. There is some evidence for an exchange degeneracy between the ω and the P' for this ratio, because we also find evidence for $(\nu B/A)_{P,P'}\approx +1$. There is some evidence for the no-compensation mechanism for the P', with $\alpha_{P'}=0$ at $-t\sim 0.5$ (GeV/c)^2, which, however, would make the ω and P' trajectories quite nondegenerate. For the A_2, we find $\nu B/A\sim +10$, which would be expected if the A_2 were degenerate with the ρ. Our determination of the signs of the spin-flip amplitudes B allows us to predict the $K^\pm p$ polarizations semiquantitatively; our results agree with the available K^-p polarization data, while the previous Regge models gave the wrong sign of this polarization. Our new signs for the B amplitudes also improve the agreement of the conventional Regge model with the available K^+n charge-exchange cross section without invoking a ρ' contribution. On the basis of getting good agreement between the FESR results and the Regge expectations, we are able to choose a particular set of low-energy input data as our favored one: We prefer Kim's coupling constants g_Λ^2 and g_Σ^2 for the Born terms, a negligible Y_1^* (1385) coupling (as also found by Kim), and the nonresonant-type solution IV for the K^+p phase-shift analysis. We have also considered FESR's for amplitudes with the wrong crossing properties, generalizing the Schwarz superconvergence relations. A simple model to remove the infinities expected in the case of Schwarz FESR's is seen to be in good agreement with the low-energy data, at least at $t=0$.

1. INTRODUCTION

If an amplitude decreases sufficiently fast with increasing energy, one can use dispersion relations to derive superconvergence relations (SCR) for this amplitude.[1,2] One may use a Regge-pole parametrization for the asymptotic behavior of the scattering amplitude in question. It has been shown recently, however, that one can generalize SCR's to cases in which the amplitude does not superconverge [i.e. behave as (energy)$^{-1-\epsilon}$ where $\epsilon>0$ as energy $\to\infty$]; one essentially subtracts the supposedly known asymptotic part (given, for example, by Regge poles) to write an SCR for the remainder (full amplitude minus Regge part). These Reggeized superconvergence relations or finite-energy sum rules (FESR's) which relate integrals of the full amplitude only over a finite low-energy region to Regge-pole parameters are very important tools in detecting inadequacies of either the low-energy data or the Regge-pole parameters (as determined by fits to high-energy data alone) depending upon which of the two is known better. The FESR's can predict[3] some features of the low- (high-) energy data, given only the high- (low-) energy data. It is this aspect of the FESR's that interests us in this paper.

It is instructive to trace the essential history of the origin of the FESR's. De Alfaro *et al.*[1] pointed out that if an analytic function (for example, a scattering amplitude at a fixed momentum transfer) $f(\nu)$ satisfying a dispersion relation

$$f(\nu)=\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{\operatorname{Im}f(\nu')d\nu'}{\nu'-\nu} \tag{1}$$

[1] See, e.g., V. de Alfaro, S. Fubini, G. Furlan, and G. Rossetti, Phys. Letters **21**, 576 (1966).

[2] G. V. Dass and C. Michael, Phys. Rev. **162**, 1403 (1967), and references therein.

[3] R. Dolen, D. Horn, and C. Schmid, Phys. Rev. **166**, 1768 (1968).

is subject to the asymptotic bound, for energy $\nu \to \infty$,

$$|f(\nu)| < \nu^\gamma, \quad \gamma < -1, \tag{2}$$

it must satisfy the SCR

$$\int_{-\infty}^{\infty} \mathrm{Im} f(\nu)\, d\nu = 0. \tag{3}$$

For amplitudes that are odd under crossing symmetry, this SCR takes the form of an integral over only positive energies:

$$\int_0^{\infty} \mathrm{Im} f(\nu) d\nu = 0. \tag{4}$$

One can, of course, also derive similar SCR's for the amplitudes $\nu^{2n} f(\nu)$, where n is any positive integer, provided the asymptotic behavior permits that derivation. Appealing to Regge-pole theory for the asymptotic behavior, if one chooses the appropriate crossing odd amplitude which corresponds to the exchange of a trajectory that lies sufficiently low to satisfy the condition $\gamma < -1$ [Eq. (2)], the SCR gives a sum rule for the imaginary part of the amplitude. Hopefully, the high-energy part gives a negligible contribution to Eq. (4) which, therefore, gives a condition on the low-energy imaginary parts only. Such sum rules have been studied quite extensively.[2] If the leading Regge term allows only $\gamma > -1$, but is known from high-energy fits, one may study the difference [$f(\nu)$ − the leading Regge term]. For instance, the P' Regge pole was predicted by Igi[4] by a study of the difference of the $\pi N(+)$ amplitude in the forward direction and the P Regge pole contribution to this amplitude. More recently, Igi and Matsuda,[5] Logunov, Soloviev, and Tavkhelidze,[6] and Dolen, Horn, and Schmid[3] have employed this technique further for πN scattering. The FESR's can be derived,[3] for example, by starting with the SCR for the amplitude from which the sum of all the Regge contributions with $\gamma > -1$ has been subtracted. These FESR's hold quite generally for any analytic function that can be expanded at high energies ν ($\geqslant$ a certain number ν_1 at which one believes this asymptotic behavior to have been established) in terms of a Regge-pole parametrization. These FESR's take the form[3]

$$\frac{1}{\nu_1^{n+1}} \int_0^{\nu_1} \nu^n \,\mathrm{Im} f(\nu,t)\, d\nu = \sum_i \frac{\beta_i \nu_1^{\alpha_i}}{(\alpha_i + n + 1)}, \tag{5}$$

where the contribution of a single Regge pole i is given by

$$f_{\mathrm{Regge}}(\nu,t) = \frac{\beta_i(t) \nu^{\alpha_i(t)}}{\sin \pi \alpha_i(t)} (\pm 1 - e^{-i\pi\alpha_i(t)}), \tag{6}$$

the various symbols having their usual meaning. For meson-baryon scattering, we have chosen ν to be $\nu = (s-u)/4M$, where s, t, and u are the usual Mandelstam variables and M is the nucleon mass. It is important[3] to realize that the point $\gamma = -1$ does not have any special role for the FESR's; every Regge-pole contribution, irrespective of whether the trajectory lies high or low, occurs in the same form in the FESR. Also,[3] the various Regge poles contribute to the FESR with the same weight as in the amplitude $f(\nu)$. This makes the FESR's particularly suited to investigate the properties of the Regge poles as they occur at high energies by evaluating only low-energy integrals. One should remember, however, that the derivation of the FESR's assumes that for $\nu > \nu_1$, $f = f_{\mathrm{Regge\ poles}}$ only; if the function f cannot be expanded in terms of a Regge-pole parametrization, one has to reexamine the whole issue all over again. For example, Regge cuts would have to be represented approximately as a superposition of poles, etc.

There is another side of development of the history of the FESR's. So far, we have mentioned finite-energy integrals over only Imf. It is possible to consider FESR's for Ref or for combinations of Ref and Imf also. Gilbert[7] wrote down a similar dispersion relation some years ago. More recently, Liu and Okubo[8] and Olsson[9] and Barger and Phillips[10] have investigated such FESR's for πN scattering. The idea is to consider a function $G(\nu,t)$ which has analyticity properties very similar to those of $f(\nu,t)$ and which also depends on another parameter m. For special values of m, ImG reduces to Imf; otherwise, ImG involves both Ref and Imf in general. For example, for $t=0$, one can use $G = (\mu^2 - \nu^2)^{m/2} f$, where μ is the meson mass and G has the same singularity structure as f. The FESR's for G have exactly the same form as for f above. We evaluate these generalized FESR's involving both the real and imaginary parts of the scattering amplitude for various moments m; the added advantage of these FESR's for $m \neq$ even integral is that they provide information on the phase of the high-energy amplitude also.

Kaon-nucleon scattering, in its various charge and hypercharge modes, is more complicated than pion-nucleon scattering from the theoretical point of view; it has the added disadvantage of poorer experimental information. Regge-pole theory for KN scattering is less well determined than for πN scattering; more types of Regge poles are possible and less information at high energies is available. FESR's are, therefore, of special importance in determining those characteristics of KN Regge poles which cannot be determined otherwise, from high-energy Regge fits alone. The absence of high-

[4] K. Igi, Phys. Rev. Letters **9**, 76 (1962); Phys. Rev. **130**, 820 (1962).

[5] K. Igi and S. Matsuda, Phys. Rev. **163**, 1622 (1967); Phys. Rev. Letters **18**, 625 (1967).

[6] A. A. Logunov, L. D. Soloviev, and A. N. Tavkhelidze, Phys. Letters **24B**, 181 (1967).

[7] W. Gilbert, Phys. Rev. **108**, 1078 (1957).

[8] Y. Liu and S. Okubo, Phys. Rev. Letters **19**, 190 (1967).

[9] M. G. Olsson, Phys. Letters **26B**, 310 (1968).

[10] V. Barger and R. J. N. Phillips, Phys. Letters **26B**, 730 (1968); C. Michael, Phys. Letters **26B**, 392 (1968) gives further evidence on $B^{(+)}$.

energy data on polarization in any channel in elastic KN scattering has led to ambiguities in the phases of the Regge-pole amplitudes (especially the spin-flip amplitude B) because data on high-energy $d\sigma/dt$ alone cannot determine these phases; this has led to predictions of $K^-p \to K^-p$ polarization of the wrong sign as compared to the recent polarization data[11] (though the energy of this experiment is rather low for comparison with a Regge-pole model). Our FESR analysis brings out this inadequacy of the previous Regge fits, predicts the phases of the amplitudes at high energies, and this then leads to the correct sign and magnitude of the expected polarization. The present analysis also confirms some of the notions in current Regge phenomenology and predicts some new ones.

In Sec. 2, we summarize some information on the Regge poles relevant to the KN system. In Sec. 3, we discuss the FESR's that we want to evaluate and also our low-energy data input. In Sec. 4, the results of evaluating the FESR's are discussed along with some relevant information from meson-meson scattering FESR's. Section 5 is devoted to the discussion of some other sum rules related to the Schwarz[12] superconvergence relations. Section 6 gives our predictions for polarization in $K^\pm p$ elastic scattering and for the K^+n charge-exchange cross section, both of which have been regarded as weak points of Regge-pole phenomenology. The conclusions are given in Sec. 7. Some preliminary results have been published elsewhere.[13]

2. REGGE POLES FOR KN SCATTERING

The usual invariant amplitudes[14] A' (which we call A) and B receive contributions from different Regge poles[15] in the t channel ($K\bar{K} \to N\bar{N}$):

$$f(K^-p \to K^-p) = f_P + f_{P'} + f_\omega + f_\rho + f_{A_2}, \quad (7a)$$

$$f(K^+p \to K^+p) = f_P + f_{P'} - f_\omega - f_\rho + f_{A_2}, \quad (7b)$$

$$f(K^-n \to K^-n) = f_P + f_{P'} + f_\omega - f_\rho - f_{A_2}, \quad (7c)$$

$$f(K^+n \to K^+n) = f_P + f_{P'} - f_\omega + f_\rho - f_{A_2}, \quad (7d)$$

$$f(K^-p \to \bar{K}^0 n) = 2f_\rho + 2f_{A_2}, \quad (7e)$$

$$f(K^+n \to K^0 p) = -2f_\rho + 2f_{A_2}, \quad (7f)$$

where f stands for either A or B; the subscripts refer to the contributing Regge pole.[15] We shall use the amplitudes

$$f^{(+)} = \tfrac{1}{2}[f(K^-p \to K^-p) + f(K^+p \to K^+p)] = f_P + f_{P'} + f_{A_2} \quad (8a)$$

and

$$f^{(-)} = \tfrac{1}{2}[f(K^-p \to K^-p) - f(K^+p \to K^+p)] = f_\rho + f_\omega, \quad (8b)$$

which receive contributions from Regge poles of positive and negative signature, respectively. If one had good experimental data on all the amplitudes on the left-hand sign of Eq. (7), one could invert Eq. (7) to extract information on all the four Regge contributions ($P+P'$), ω, ρ, and A_2. One gets

$$4(f_P + f_{P'}) = f_{K^-p} + f_{K^+p} + f_{K^-n} + f_{K^+n}, \quad (9a)$$

$$4f_{A_2} = f_{K^-p} + f_{K^+p} - f_{K^-n} - f_{K^+n}, \quad (9b)$$

$$4f_\rho = f_{K^-p} - f_{K^+p} - f_{K^-n} + f_{K^+n}, \quad (9c)$$

$$4f_\omega = f_{K^-p} - f_{K^+p} + f_{K^-n} - f_{K^+n}, \quad (9d)$$

$$4f_\rho = f_{K^-p \to \bar{K}^0 n} - f_{K^+n \to K^0 p}, \quad (9e)$$

$$4f_{A_2} = f_{K^-p \to \bar{K}^0 n} + f_{K^+n \to K^0 p}. \quad (9f)$$

Since the amplitudes for $K^\pm n$ scattering are not sufficiently well determined, we cannot very reliably separate the Regge contributions beyond that in Eq. (8).

The zeros of the amplitudes A and B as a function of t for a definite Regge pole are very conveniently studied by the FESR approach. These zeros may arise from the trajectory passing through some special α values for which the amplitude develops special properties (for example, the α value may be physically impossible or the amplitude may develop a ghost and so on). The FESR determination of the t dependence of the amplitudes, therefore, could help one to determine the behavior of the trajectory near these zeros: whether it chooses sense or chooses nonsense, what is the ghost-eliminating mechanism, etc. Depending on which amplitude one is considering, one may also determine whether or not the relevant Regge trajectory goes through zero in the t region studied by finding whether or not the appropriate integral over the relevant amplitude passes through zero as a function of t; this is useful only if (a) the zero of $\beta_i(t)$ on the right-hand side of Eq. (5) [if $\beta_i(t)$ be proportional to some positive power of $\alpha_i(t)$] is not cancelled by the factor (α_i+n+1) in the denominator at the t value for which $\alpha_i(t)=0$, and if (b) the contribution of the particular pole for which $\alpha_i(t)$ passes through zero is not masked by that of the other poles in the Regge summation. The various possible modes of behavior of the trajectory couplings have been given by Chiu, Chu, and Wang[16]; we use their notation for the different mechanisms of coupling.

Let us summarize some relevant information on the different Regge poles:

(*a*) ω. At a given lab energy, the experimental $d\sigma/dt$ for ($\bar{p}p \to \bar{p}p$) and ($K^-p \to K^-p$) near the forward direction is bigger and steeper as a function of t than the

[11] C. Daum *et al.*, Nucl. Phys. **B6**, 273 (1968).
[12] J. H. Schwarz, Phys. Rev. **159**, 1269 (1967); **162**, 1671 (1967); Nuovo Cimento **54A**, 529 (1968).
[13] G. V. Dass and C. Michael, Phys. Rev. Letters **20**, 1066 (1968).
[14] W. Rarita, R. J. Riddell, Jr., C. B. Chiu, and R. J. N. Phillips, Phys. Rev. **165**, 1615 (1968).
[15] R. J. N. Phillips and W. Rarita, Phys. Rev. **139**, B1336 (1965); Phys. Rev. Letters **15**, 807 (1965).
[16] C. B. Chiu, S.-Y. Chu, and L.-L. Wang, Phys. Rev. **161**, 1563 (1967).

$d\sigma/dt$ for $(pp\to pp)$ and $(K^+p\to K^+p)$, respectively, and therefore the $\bar{p}p$ and pp (and similarly, K^-p and K^+p) $d\sigma/dt$'s cross over near the forward direction at $t=t_c\sim-0.13$ (we use units GeV$=1$, $\hbar=c=1$). The usual explanation[14,15,17] of this crossover phenomenon in terms of the ω Regge pole can be shown, when combined with the factorization theorem for the Regge-pole residues and the real analyticity for the unfactored residues, to lead to the conclusion that the ω would essentially "decouple from all physics" at $t=t_c$; i.e., the ω-exchange residue functions vanish at $t=t_c$ for every helicity amplitude in every reaction. This explanation has been shown[17] to lead to difficulties in π^0 photoproduction, $\gamma p\to\pi^0 p$, because the data do not show any sign of a dip in the measured $d\sigma/dt$ at $t\simeq t_c$. A recent FESR calculation[18] for $\gamma p\to\pi^0 p$ confirms this difficulty. It would be of interest to see what evidence the FESR's give for KN scattering. The crossover phenomenon only requires that the imaginary part of the helicity-nonflip amplitude A for the ω should vanish at $t=t_c$; the usual explanation, however, puts a much stronger constraint on the ω contribution; it requires the real and imaginary parts of both A_ω and B_ω to vanish at $t=t_c$. It is of interest to study the behavior of the real part of A and both the real and imaginary parts of B for the trajectory corresponding to the ω quantum numbers in the t channel ($I=0$, G negative, and $J^P=1^-$). The FESR calculation, if the input low-energy data were complete and reliable, is capable of giving all this information and also may determine the trajectory function $\alpha_\omega(t)$. If, for example, one finds that at $t=t_c$, only $\mathrm{Im}A_\omega=0$, but $\mathrm{Im}B_\omega\neq0$, $\mathrm{Re}B_\omega\neq0$, and $\mathrm{Re}A_\omega\neq0$, one would tend to believe the conjecture that the usual[14,15] ω Regge pole is an effective mixture of a genuine ω Regge pole and some other contribution $\bar{\omega}$ such that $\mathrm{Im}(A_\omega+A_{\bar{\omega}})=0$ at $t=t_c$; no such restriction being placed on $\mathrm{Re}(A_\omega+A_{\bar{\omega}})$ or $\mathrm{Im}(B_\omega+B_{\bar{\omega}})$ or $\mathrm{Re}(B_\omega+B_{\bar{\omega}})$, all of which may be nonzero at $t=t_c$. If the effective ω contribution in the process $\gamma p\to\pi^0 p$ and in NN and KN scattering were indeed from a genuine single Regge pole, the absence of the zero at $t=t_c$ in the first process and the presence of this zero in the later two processes could be understood if one were to give up the factorization theorem. A recent FESR study[19] of the ω Regge contribution in KN scattering within the resonance saturation approximation shows, as partly expected, that the sum-rule integrals for the amplitudes $\mathrm{Im}A_\omega$ and $\mathrm{Im}\nu B_\omega$ do have this zero at $t\sim t_c$. A possible suggestion of Ref. 19 was to cast doubts on the factorization theorem. It seems unjustified to conclude from this evidence of a zero in only the lowest-moment sum rules for $\mathrm{Im}A_\omega$ and $\mathrm{Im}B_\omega$ at $t=t_c$ that the factorization theorem could be overthrown. The point is that one should study the behavior of $\mathrm{Re}A_\omega$ and $\mathrm{Re}B_\omega$ also (and preferably, the higher moment sum rules also for all the amplitudes $\mathrm{Re}A_\omega$, $\mathrm{Im}A_\omega$, $\mathrm{Re}B_\omega$, and $\mathrm{Im}B_\omega$) near $t=t_c$ in all the relevant processes to arrive at such a conclusion. If one finds that at $t=t_c$, $\mathrm{Re}A_\omega\neq0$ and/or $\mathrm{Re}B_\omega\neq0$ for KN scattering, one already knows that the effective ω is more than a single pole and that one should not expect factorization to hold for such a mixture. Unfortunately, the resonance saturation approximation does not give any reliable information about the real parts of the amplitudes. We return to our evaluation of the FESR's in Sec. 3; we hope to do better than the resonance approximation.

To our knowledge, the only high-energy Regge fit[15] (to KN scattering) that takes the ω contribution into account is unable to determine it very well; actually, $B_\omega=0$ was used.[15] An FESR calculation for KN scattering, therefore, becomes very important for a study of the ω contribution: in particular to find whether B_ω is really very small; also, one might learn something about A_ω.

Contogouris *et al.*[20] have determined the ω trajectory ($\alpha_\omega=0.45+0.9t$) by studying, as a function of t and s, the quantity

$$X(s,t)=\frac{d\sigma}{dt}(\pi^+p\to\rho^+p)+\frac{d\sigma}{dt}(\pi^-p\to\rho^-p)-\frac{d\sigma}{dt}(\pi^-p\to\rho^0n),$$

which receives contributions from ω-like Regge poles. The ω contribution to $X(s,t)$ has a dip at $t=-0.5$ and this mainly determines the position of $\alpha_\omega=0$. While this is the only direct determination of $\alpha_\omega(t)$ (using high-energy data) known to us, the position of $\alpha_\omega=0$ ($t=-0.5$) needs confirmation. A careful analysis of the $\pi N\to\rho N$ data shows that the dip in $X(s,t)$ which could arise due to a peak in $d\sigma/dt(\pi^-p\to\rho^0 n)$ or a dip in $d\sigma/dt(\pi^\pm p\to\rho^\pm p)$ arises mainly from the 4-GeV/c data for $d\sigma/dt(\pi^+p\to\rho^+p)$; one would normally expect it to show up also in the $(\pi^-p\to\rho^-p)$ data which, however, do not go to large enough $|t|$ to allow the conclusion of such a dip; the $(\pi^-p\to\rho^0n)$ data also do not have a peak at about $t=-0.5$. Their analysis[20] also shows this dip at only 4 GeV/c. A detailed Regge-pole analysis of $\pi N\to\rho N$ and $KN\to K^*_{890}N$ by Dass and Froggatt[21] shows that the evidence for this dip is not strong, though the χ^2 does get reduced slightly by using $\alpha_\omega=0.45+0.9t$. One would like to get some confirmation about this type of ω trajectory. Actually, our FESR analysis does not show this zero in α_ω.

Since one cannot very reliably separate the ρ and ω contributions in our FESR calculation because of the

[17] V. Barger and L. Durand, III, Phys. Rev. Letters **19**, 1295 (1967).

[18] P. Di Vecchia, F. Drago, and M. L. Paciello, Nuovo Cimento **55A**, 809 (1968).

[19] P. Di Vecchia, F. Drago, and M. L. Paciello, Phys. Letters **26B**, 530 (1968).

[20] A. P. Contogouris, J. T. T. Van, and H. J. Lubatti, Phys. Rev. Letters **19**, 1352 (1967).

[21] G. V. Dass and C. D. Froggatt (to be published in Nucl. Phys.).

unsatisfactory state of the Kn data, we shall deduce the properties of the ω contribution from the difference $f^{(-)}-f_\rho$, taking the ρ to be well-known from the high-energy KN fits[22] which make use of the $(\pi^- p\rightarrow\pi^0 n)$ and $(\pi^- p\rightarrow\eta^0 n)$ data and factorization constraints. We believe that the ρ Regge pole is well-determined by high-energy fits, assuming that factorization is good for the ρ contribution and the effects of ρ' contribution are not very large.

(*b*) A_2. This is the well-known Regge trajectory which contributes to the process $\pi^- p\rightarrow\eta n$.

Most of the $\pi N\rightarrow\eta N$ and $KN\rightarrow KN$ Regge fits use a curved or a rather flat (slope ≈ 0.4) A_2 trajectory with no zero of α_{A_2} in the region $-t=0\rightarrow 0.8$, while a recent $KN\rightarrow K\Delta$ Regge analysis[23] used a much steeper trajectory (slope ≈ 1). The occurrence of zeros in α_{A_2} causes a ghost in the amplitude A_{A_2} and raises the question of how this ghost is eliminated. Also, one can have many choices for the behavior of the amplitude B_{A_2} near $\alpha_{A_2}=0$. Matsuda and Igi[24] evaluated FESR's for the KN system for the A_2 contribution in the resonance saturation approximation. In the resonance approximation, however, $f_{A_2}=f_\rho$ [see Eqs. (9b) and (9c)] because one does not include any K^+p and K^+n resonances. This approximation $\mathrm{Im}f_\rho=\mathrm{Im}f_{A_2}$ (no $S=+1$ resonances) directly leads to results based on the argument of exchange degeneracy between the ρ and the A_2. However, experimentally, $\mathrm{Im}(f_{K^+p}-f_{K^+n})$ is not identically zero; this makes $\mathrm{Im}f_\rho-\mathrm{Im}f_{A_2}$ nonzero. The differences between the ρ and the A_2 arising because of this will therefore not be reproduced by the resonance approximation. For the ω contribution, on the other hand, the resonance approximation seems more reliable because it only assumes that the background contribution to $\mathrm{Im}f_{K^-n}+\mathrm{Im}f_{K^-p}$ equals $\mathrm{Im}f_{K^+p}+\mathrm{Im}f_{K^+n}$.

The high-energy fits to $d\sigma/dt(\pi N\rightarrow\eta N)$ and $d\sigma/dt(KN\rightarrow KN)$ do not determine the phase of the amplitude B_{A_2}. Because of a lack of polarization data in these reactions, as pointed out in Sec. 1, the FESR's are very useful in predicting the phases of A_{A_2} and B_{A_2}. Indeed, the sign of $(B/A)_{A_2}$ turns out to be opposite to the one used in some high-energy fits. This, combined with the further ill-determined signs of $(B/A)_{P,P'}$ from the high-energy fits led to a negative polarization for $K^-p\rightarrow K^-p$ scattering. With the signs we suggest for B/A for the different contributions, one gets a positive polarization which agrees with the available experimental data.[11]

(*c*) *P and P'*. For the vacuum exchanges again, the high-energy data do not determine the sign of B/A and our FESR analysis is able to pin down the sign of $(B_P+B_{P'})/(A_P+A_{P'})$, though we cannot separate the P and P' contributions unambiguously.

While the P trajectory is always assumed to be rather flat, some recent analyses[10,16] indicate that $\alpha_{P'}$ should go through zero at $t\simeq -0.55$ and the no-compensation mechanism should be followed to parametrize the amplitudes $A_{P'}$ and $B_{P'}$; this gives double dips (zeros) in $\mathrm{Im}A_{P'}$ and $\mathrm{Im}B_{P'}$ at $\alpha=0$. Though our analysis allows one to study only $f^{(+)}$ and not $f_{P'}$ directly (except in the resonance approximation), a dip in $f^{(+)}$ will result (and indeed, it does) if P' did choose the no-compensation mechanism.

(*d*) ρ. We assume that the ρ contribution is correctly given by most of the high-energy fits and we do not take into account any possible ρ' Regge pole.

3. THE FESR's

A. Generalities

For the FESR's, one needs amplitudes having definite crossing symmetry and the $f^{(\pm)}$ are suitable for this purpose. The amplitudes $\nu A^{(+)}$, $B^{(+)}$, $A^{(-)}$, and $\nu B^{(-)}$ are all odd under crossing; we consider the generalized FESR's for the amplitude

$$a(\nu,t,m)=(M/4\pi^2)(\nu_0^2-\nu^2)^{m/2}F(\nu,t)\,, \qquad (10)$$

where m is a variable which could be nonintegral and $\nu_0=\mu+t/4M$. The energy dependence of $F(\nu,t)$ at high energies is parametrized in the form

$$F(\nu,t)=\textstyle\sum_i \nu(\nu_0^2-\nu^2)^{[\alpha_i(t)-\delta]/2}\chi_i(t)\,, \qquad (11)$$

where $F(\nu,t)$ may be one of the four amplitudes $\nu A^{(+)}$, $B^{(+)}$, $A^{(-)}$, and $\nu B^{(-)}$ for which $\delta=0, 2, 1, 1$, respectively, $\chi_i(t)$ is a real function of t, and $\alpha_i(t)$ is the Regge trajectory function; the sum runs over the relevant Regge terms [see Eq. (8)]. At high energies, the parametrization in Eq. (11) resembles the usual Regge expansion,[14,15] so that one can directly use the high-energy Regge parameters in Eq. (11). If one writes a superconvergence relation for the difference of the full amplitude and the Regge contribution and uses analyticity to match the amplitudes evaluated below ν_1 with the Regge parametrization evaluated above ν_1, the set of generalized FESR's takes the form

$$\int_0^{\nu_1} d\nu\,\mathrm{Im}a(\nu,t,m)=\sum_i \frac{\mathrm{Im}a_i(\nu_1,t,m)}{\alpha_i(t)+m+2-\delta}\,\frac{(\nu_1^2-\nu_0^2)}{\nu_1}\,, \qquad (12)$$

where we have chosen the matching energy to be $\nu_1=1.53$ (which corresponds to $\sqrt{s}=2$, $p_{\mathrm{lab}}=1.46$). The derivation of these FESR's in Eq. (12) runs closely parallel to the derivation of the sum rule Eq. (5) in Sec. 1. For computational convenience, we have considered only integral $m=-2$ to 3 at $t=0$ and $m=0$ to 3 for $t\neq 0$. In principle, one could consider nonintegral values of m also. Indeed, we have evaluated some special sum rules for nonintegral moments; we come to these in Sec. 5. For even m, the left-hand side of the FESR (12) requires $(-)^{m/2}\,\mathrm{Im}F$ from the $\Lambda\pi$ threshold to ν_1 together with the Λ and Σ pole terms. For odd m, $(-)^{(m+1)/2}\,\mathrm{Re}F$ is required in the region above the KN threshold and the Λ and Σ pole terms and $\mathrm{Im}F$ below

[22] A. Derem and G. Smadja, Nucl. Phys. **B3**, 628 (1967).
[23] M. Krammer and U. Maor, Nuovo Cimento **52A**, 308 (1967).
[24] S. Matsuda and K. Igi, Phys. Rev. Letters **19**, 928 (1967); **20**, (E) 781 (1968); CERN Topical Conference, 1968 (unpublished).

this threshold. For K^+p scattering, there are no contributions below the physical KN threshold. Indeed, no real parts are needed below the physical threshold; this motivates our particular choice of ν_0 and of the weighting factor in Eq. (10). The above FESR's have the advantage that by varying m one can study the phase of the Regge amplitude; this is not possible in the usual narrow-resonance approximation.

For $m=-2$, $t=0$, our sum rule for the amplitude $A^{(-)}$ is the usual forward dispersion relation[25]; the sum rule for $B^{(+)}$ is the spin-flip dispersion relation considered, in some way, by Igi[4] (for πN scattering) along with his dispersion relation for the $A^{(+)}$ amplitude to predict the P'. Note that these dispersion relations require the amplitude evaluated at threshold and involve the evaluation of a principal-value integral. Some of our $m=0$, $t=0$ sum rules have already been considered.[26] For example, Lusignoli *et al.* have considered the separate amplitudes $\nu(A_P+A_{P'})$, νA_{A_2}, A_ρ, and A_ω at $t=0$; Razmi and Ueda have evaluated the sum rules for $m=0$, $t=0$ for the $\nu A^{(+)}$ and $A^{(-)}$ amplitudes and also for the amplitudes $A_{K^+n}\pm A_{K^-n}$ and related them to the Regge parameters of Phillips and Rarita[15]; Chan and Yen considered the $t=0$ sum rule for A_ρ, used $SU(3)$ symmetry to relate the $\rho K\bar{K}$ vertex to the $\rho\pi\pi$ vertex, and made use of the πN charge-exchange data in addition to the KN data to predict finally the coupling constants $g_{\Lambda KN}^2$ and $g_{\Sigma KN}^2$; Yoshimura also considered the forward amplitude to investigate the symmetry of the factorized residue functions for the ρ and the (ω,φ) trajectories. To our knowledge, FESR's for the other m values have not been investigated for KN scattering so far.

For $m=0$ and $t\neq 0$, however, the FESR analyses for KN scattering have been used only in the resonance saturation approximation wherein one can explicitly separate the ω,[19] ρ,[24] and A_2 [24] contributions. In addition to the other difficulties mentioned in Sec. 2 about the resonance approximation, one would always prefer to use a more exact form of the low-energy amplitudes A and B on the left-hand side of Eq. (12).

B. Input Data

We use phase-shift analyses for $(K^-p\to K^-p)$ and $(K^+p\to K^+p)$ scattering up to the matching energy $\sqrt{s}=2$. For K^+p scattering, Lea *et al.*[27] have found several solutions in this energy region; we used a solution of type I which suggests an inelastic P_{11} resonance and also a nonresonant solution of type IV. Solutions of type II gave amplitudes A and B very similar to those for type I, while solutions of type III are not favored by the authors.[27] The forward-dispersion relation for the amplitude A for K^+p scattering is already built into their analysis because they used $\mathrm{Re}A(t=0)$ calculated from this dispersion relation as part of the data in their χ^2 minimization. For K^-p scattering, the situation is complicated by the presence of the Born terms (Λ and Σ) and inelastic channels ($\pi\Lambda$ and $\pi\Sigma$) below the $\bar{K}N$ threshold. Fortunately, Kim[28] has done a multichannel analysis of $\bar{K}N$, $\pi\Lambda$, and $\pi\Sigma$ data using a K-matrix–effective-range parametrization for the partial-wave amplitudes; he used data from threshold up to $P_{\mathrm{lab}}=550$ MeV/c. In the unphysical region, we use the direct extrapolation of his parametrization though we allow the $Y_1^*(1385)$ coupling to have its broken $SU(3)$ value[29] as well as the almost negligible value found by Kim. This gives us an idea of what sort of errors to expect from uncertainties in the parametrization of the unphysical region.[30] For the Λ and Σ pole terms, we use Kim's values[25]

$$g_\Lambda^2/4\pi=13.5\,,\quad g_\Sigma^2/4\pi=0.2\,, \tag{13}$$

or alternatively, Zovko's values[25] of 5.7 and 1.7, respectively, for these couplings. We take Zovko's values to be typical of some[31] of the KN forward dispersion relation results.[32–34] For the region 780–1220 MeV/c, Armenteros *et al.*[35] have a preliminary phase-shift analysis using simple backgrounds plus (finite-width) resonances. This still leaves the gaps (550–780 MeV/c) and (1220–1460 MeV/c). Lacking any better procedure, we extrapolated the energy-dependent fits of Armenteros *et al.*[35] to the region 550–1460 MeV/c and confirmed that they still reproduced the experimental K^-p total cross sections.[36] This extrapolation is wrong for each partial wave separately because some of the background amplitudes exceed the unitarity limit; for the full amplitudes A and B which are resonance dominated, however, we

[25] J. K. Kim, Phys. Rev. Letters **19**, 1079 (1967); N. Zovko, Phys. Letters **23**, 143 (1966).

[26] M. S. K. Razmi and Y. Ueda, Phys. Rev. **162**, 1738 (1967); Nuovo Cimento **52A**, 948 (1967); C. H. Chan and Y. L. Yen, Phys. Rev. **165**, 1565 (1968); M. Lusignoli, M. Restignoli, G. Violini, A. Borgese, and M. Colocci, Nuovo Cimento **51A**, 1136 (1967); M. Yoshimura, Tokyo Report, 1967 (unpublished).

[27] A. T. Lea, B. R. Martin, and G. C. Oades, Phys. Rev. **165**, 1770 (1968), and private communication.

[28] J. K. Kim, Phys. Rev. Letters **19**, 1074 (1967).

[29] R. L. Warnock and G. Frye, Phys. Rev. **138**, B947 (1965).

[30] Also, we do not include any D waves below the $\bar{K}N$ threshold. Otherwise, we construct the partial-wave amplitudes and the full amplitudes A and B directly using his tabulated parameters (Ref. 28).

[31] See, e.g., M. Lusignoli, M. Restignoli, G. A. Snow, and G. Violini, Nuovo Cimento **45A**, 792 (1966).

[32] In principle, one could extrapolate the older constant-scattering-length parametrization of Kim (Ref. 33) into the unphysical region instead of using the more recent effective-range parametrization (Ref. 28). It has been shown, however, (Ref. 34) that by using the stability of the couplings g_Λ^2 and g_Σ^2 as a criterion for the compatibility of the extrapolated amplitude in the unphysical region with the known total cross sections in the physical region, one can reject the older parametrization (Ref. 33) in favor of the new one (Ref. 28) which we use.

[33] J. K. Kim, Phys. Rev. Letters **14**, 29 (1965).

[34] C. H. Chan and F. T. Meiere, Phys. Rev. Letters **20**, 568 (1968).

[35] R. Armenteros, M. Ferro-Luzzi, D. W. G. Leith, R. Levi-Setti, A. Minten, R. D. Tripp, H. Filthuth, V. Hepp, E. Kluge, H. Schneider, R. Barloutaud, P. Granout, J. Meyer, and J. P. Porte, Nucl. Phys. **B3**, 592 (1967).

[36] In order to study how crucially our results depend on this extrapolation, we changed the matching energy to $\sqrt{s}=1.9$ which coincides with the higher-energy end of the experiment of Armenteros *et al.* (Ref. 35) and also we changed the point where the Armenteros *et al.* parametrization takes over the Kim parametrization to 670 MeV/c. Making both the changes does not introduce any significant changes in our results.

believe this extrapolation to be a fair approximation. This is better than the narrow-resonance saturation approximation which does not even reproduce the experimental total cross sections. The parametrization of Armenteros *et al.* agrees approximately with the experimental K^-p polarization of Cox *et al.*[37] from 1100 to 1350 MeV/c, which already involves an extrapolation. This shows that our extrapolation may not be very bad for the total amplitudes A and B.

The K^-n analysis of Armenteros *et al.*[33] is less reliable since it does not reproduce the total cross sections as well. Also, the only K^+n analysis is 0–813 MeV/c so that, as pointed out above, we cannot appeal to the neutron data to separate the isospin contributions beyond those in Eq. (8). We do evaluate FESR's for moments $m=0$, 2 for all the four contributions $(P+P')$, A_2, ρ, and ω in the narrow-width resonance saturation approximation taking all the known resonances (of which J^P is known) from Rosenfeld *et al.*[38] This takes one up to an effective cutoff energy $\sqrt{s}=2.15$.[39] If one believes in the conjecture[40] that the Pomeranchuk trajectory is mostly built from the nonresonating background at low energies, one can regard the $P+P'$ results in the resonance approximation to be a representation of the P'.

Let us remind ourselves of the various possibilities in the input data: (a) coupling strength of $Y_1^*(1385)$ [the Kim value or the broken $SU(3)$ value]; (b) solution I or solution IV for K^+p scattering; and (c) Zovko's or Kim's values [the latter are very similar to $SU(3)$ values] for the pole-term couplings. Actually, we use the $SU(3)$ values for $f=0.36$ for the resonance-approximation results instead of the Kim values.

These variations in the input data set mean eight different evaluations of each FESR at each t value. We use the difference, if any, of the results for different data sets to estimate the errors shown in the figures, the central value shown being the one for our favored data set, to be discussed later.

C. Choice of ν_1

One would, in principle, like a high value of ν_1, corresponding to say $P_{\text{lab}}\sim 5$ GeV/c, for the assumption of the Regge behavior having become established at ν_1 to be valid. We have no alternative to choosing ν_1 corresponding to $\sqrt{s}\sim 2$ GeV because the low-energy data do not allow one to do so either with the phase-shift analysis or in the resonance approximation. Though it is certainly desirable to choose a comparatively high ν_1, one need not wait until phase-shift analysis for kaon nucleon scattering extends to momenta as high as 5 GeV/c. Our value of $\nu_1=1.53$ is not too unreasonable if one were to keep in mind the fact that the extrapolation of the Regge amplitude down to ν_1 should represent only a local average of the amplitude and the wiggles (resonances) start appearing only below $\sqrt{s}\sim 2$. Some other calculations[19,24,41] have also used low matching energies like $\sqrt{s}\sim 2$. A high ν_1 would ensure that the right-hand side of the FESR [Eq. (12)] did represent *the* amplitude at that ν_1. The choice of a low ν_1 would give one only the extrapolation (to ν_1) of the high-energy Regge amplitude; this may or may not be *the* amplitude at ν_1 because the low-energy wiggles might be important at a low ν_1. Furthermore, lower-lying Regge trajectories could be important for a low ν_1. In our case, it seems that both these effects are rather small because the agreement of the sum-rule results with the extrapolations of the high-energy Regge amplitudes is quite good in general [see Sec. 4 (A)], at least so far as the question of the over-all normalization is concerned.

D. Sum Rules with Different Moments

The FESR's of Eq. (12) are not equally useful in the form given above for all values of m. In principle, the ones with large m are also equally valid, but their usefulness decreases with increasing m because the higher the m value, the greater is the weight given to the input data immediately below ν_1 in the evaluation of the FESR integral. One cannot make m too small either. With m very small (negative and large), the low-energy data around $\nu=\nu_0$ would be weighted heavily and the sum-rule integrals would be far more sensitive to the low-energy data than to Regge poles; such ones, for example, might call for an accurate knowledge of high partial waves near threshold and in the nearby unphysical region. Since one has to resort to an extrapolation procedure to know the amplitudes in the unphysical region, it is better not to try to learn something about Regge poles from the high-inverse-moment sum rules. From this point of view, the choice $\nu_0=\mu+t/4M$ is better than $\nu_0=0$ because with the latter accurate information of the amplitude in the unphysical region (e.g., analog of scattering length) would be required down to $\nu_0=0$ as an input datum for the low-m sum rules; with the former, this information at only the physical threshold is needed. One needs to know the amplitude at threshold for the $m=-2$ sum rules. Since one does not have this information except for $t=0$ ($t\neq 0$ is unphysical at threshold), we considered $m=-2$ only for $t=0$. Summarizing, therefore, one should consider sum rules with $m\geqslant 0$ as appropriate for giving information on Regge poles. Also, in general, the $m=$even integral sum rules should be more reliable than the

[37] C. R. Cox *et al.*, in *Proceedings of the Heidelberg International Conference on Elementary Particles 1967*, edited by H. Filthuth (Interscience Publishers, Inc., New York, 1968) Contribution No. 253.

[38] A. H. Rosenfeld, N. Barash-Schmidt, A. Barbaro-Galtieri, L. R. Price, P. Soding, C. G. Wohl, M. Roos, and W. J. Willis, Rev. Mod. Phys. **40**, 77 (1968).

[39] This is different from $\sqrt{s}=2$ which is our matching energy in the case of the complete data input. The reason is that we have included some resonances higher in mass than $\sqrt{s}=2$. At the point $\sqrt{s}=2.15$, the higher known resonance next to the highest (in mass) one that we have included should take over.

[40] H. Harari, Phys. Rev. Letters **20**, 1395 (1968).

[41] S.-Y. Chu and D. P. Roy, Phys. Rev. Letters **20**, 958 (1968).

m=odd integral ones because the latter (former) require real (imaginary) parts of the amplitudes in the physical region.

E. Different t Values

In principle, fixed-t dispersion relations are valid for all physical t and because the FESR's are derived from these, one should expect the FESR's to be valid for all the physical t values for which the high-energy Regge expansion holds. Since the Regge poles in question are the mesonic ones in the t channel, one expects the range of validity to be approximately $t=0$ to -1. This immediately requires one to know the input amplitudes out to $-t\sim1$. While for comparatively high s, $-t\sim1$ is in the physical scattering region where experiments can give information on the scattering amplitude, one has to resort to extrapolations in the case of the low-s amplitudes that are input data for the FESR integrals. At the physical threshold, for example, $-t=1$ is well away from the only physical point $t=0$. Similarly, in the unphysical region (below the KN threshold), one has to resort to extrapolation in order to get the $-t=1$ amplitudes. While the $t=0$ extrapolation into the unphysical region can be put to some test[34] by forward dispersion relations, no such reliable test exists for the $t\neq0$ extrapolations. As $-t$ increases, the range of s values for which this extrapolation to unphysical t values becomes necessary increases. This range is small for $t\sim0$. Lacking any subtler method of analytic continuation in $\cos\theta_s$, we have used the common Legendre expansion of the scattering amplitudes to extrapolate to unphysical t values the low-s amplitudes which are input data for the sum-rule integrals.

There is a further complication, however. Suppose one were to use $\nu_0=0$. It so happens that with the physical masses, the effective ν value for some terms can vanish for quite low $-t$ values. The Λ pole term has $\nu_\Lambda\gtrless0$ for $-t\lessgtr0.23$ approximately; the Σ pole term has $\nu_\Sigma\gtrless0$ for $-t\lessgtr0.59$ approximately; at $-t=2$, ν_Λ, ν_Σ, $\nu_{Y_1^*(1385)}$, and $\nu_{Y_0^*(1405)}$ are all negative. This leads to apparent difficulties for the nonintegral and the inverse-moment sum rules. Actually they can be circumvented in the case of genuine FESR's by a proper treatment of the relevant contributions, but they do weight the input data in a rather sensitive manner. For the sum rules to be described in the Sec. 5, however, one cannot avoid this problem and one has to have some prescription to stay away from the t region where such things happen. The choice $\nu_0=\mu+t/4M$ (and not $\nu_0=0$) is very welcome in this respect because now the difficulty due, for example, to $\nu_\Lambda=0$ does not arise; ν_Λ cannot become equal to ν_0 for any real t. The $\nu_0=0$ troubles creep in again at $t=-4M\mu$, but this is at $-t\approx1.9$ and quite far away from the region of interest to us. This point needs special treatment in πN scattering because of the small pion mass. There is another possibility, however. $\nu_\Lambda^2-\nu_0^2$ can vanish when $\nu_\Lambda=-\nu_0$; this happens at $-t\sim1.05$ and calls for a special treatment of the pole term for this t value; we have not gone beyond $-t\lesssim1$ in practice. Not all sum rules (for any m), therefore, are equally simple and valid in their innocent form of Eq. (12) for all t. For the genuine FESR's which we have dealt with so far, the trouble due to $\nu_\Lambda=-\nu_0$ can be circumvented by a proper analytic treatment of the pole; this can be seen by looking at the contour along which the original FESR is evaluated, and deforming it in a harmless manner. For some other sum rules (see Sec. 5), one has to stay away from this point and also use proper definitions of the sum rules; luckily, this does not happen for $-t<1$. One should note that the Λ pole term is the lowest-s contribution and creates difficulties at the lowest $-t$ value. If one can take care of these difficulties or if one does not go as far as the point at which the Λ pole term starts calling for a sensitive treatment, one has automatically guarded oneself against the similar troubles from the other (higher s) contributions.

4. RESULTS AND DISCUSSION

Before coming to results of our evaluation of the FESR's of Eq. (12), we record some formulas which we have used. The spin-flip amplitude B and the usual non-spin-flip amplitude A are given in terms of the partial-wave amplitudes $f_{l\pm}$ referring to $J=l\pm\frac{1}{2}$ for a given s-channel isospin state as[42]

$$\begin{bmatrix}B(s,t)\\A(s,t)\end{bmatrix}=\frac{4\pi}{k^2}\sum_{l=1}^{\infty}P_l'(\cos\theta)\Bigg\{(E+M)(f_{l-}(s)-f_{l+}(s))\times\begin{bmatrix}1\\M-W\end{bmatrix}+(E-M)(f_{(l-1)+}(s)-f_{(l+1)-}(s))\begin{bmatrix}1\\M+W\end{bmatrix}\Bigg\}, \quad (14)$$

where k is the c.m. momentum, θ is the c.m. scattering angle, $s=W^2$, and $E=(k^2+M^2)^{1/2}$. The amplitude A' (which we have called A throughout) is related to the amplitudes A and B by the equation[14]

$$A\equiv A'=A+\nu B/(1-x)\,,\quad x=t/4M^2. \quad (15)$$

In terms of A and B, experimental quantities are given by[14]

$$\sigma_{\text{tot}}(s)=(1/p)\,\text{Im}A(s,t=0)\,, \quad (16\text{a})$$

$$\frac{d\sigma}{dt}(s,t)=\frac{1}{\pi s}\left(\frac{M}{4k}\right)^2\left[(1-x)|A|^2-x\left(\frac{p^2+sx}{1-x}\right)|B|^2\right], \quad (16\text{b})$$

$$P(s,t)=-\frac{\sin\theta}{16\pi\sqrt{s}}\frac{\text{Im}(AB^*)}{d\sigma/dt}\,, \quad (16\text{c})$$

[42] See, e.g., Ref. 2.

where p is the pion lab momentum and $P(s,t)$ is the polarization defined relative to the normal $(\mathbf{q}_i \times \mathbf{q}_f)$ to the scattering plane, $\mathbf{q}_i$ and $\mathbf{q}_f$ being the initial and final pion momenta.

Coming to the contributions of the various Born diagrams [Λ, Σ, $Y_0^*(1405)$, and $Y_1^*(1385)$] to the amplitudes A and B, one gets[43]

$$A_\Lambda{}^0=\frac{g_{\Lambda KN}{}^2(M-m_\Lambda)}{m_\Lambda{}^2-s},\quad B_\Lambda{}^0=\frac{g_{\Lambda KN}{}^2}{m_\Lambda{}^2-s},\tag{17a}$$

$$A_\Sigma{}^1=\frac{g_{\Sigma KN}{}^2(M-m_\Sigma)}{m_\Sigma{}^2-s},\quad B_\Sigma{}^1=\frac{g_{\Sigma KN}{}^2}{m_\Sigma{}^2-s},\tag{17b}$$

$$A_{Y_0^*(1405)}{}^0=\frac{g_{Y_0^*KN}{}^2(M+m_{Y_0^*})}{m_{Y_0^*}{}^2-s},\quad B_{Y_0^*(1405)}{}^0=\frac{g_{Y_0^*KN}{}^2}{m_{Y_0^*}{}^2-s},\tag{17c}$$

where the normalization of the coupling constants is $g_{N\pi N}{}^2/4\pi=14.6$ and the superscripts refer to the total isospin in the s channel. The coupling constants $g_{\Sigma KN}{}^2$ and $g_{\Lambda KN}{}^2$ are for a pseudoscalar-type meson-baryon vertex and their $SU(3)$ symmetry values are given in terms of $g_{\pi N}{}^2$ and the F-D mixing parameter f by

$$g_\Lambda{}^2\equiv g_{\Lambda KN}{}^2=\tfrac{1}{3}(1+2f)^2g_{\pi N}{}^2,\tag{18a}$$

$$g_\Sigma{}^2\equiv g_{\Sigma KN}{}^2=(1-2f)^2g_{\pi N}{}^2,\tag{18b}$$

where the currently quoted value of f is ~ 0.36. The effective Lagrangian for the Y_0^*KN vertex is

$$\mathcal{L}=g_{Y_0^*KN}\bar{Y}_0{}^*N\bar{K}+\text{H.c.},$$

where the constant $g_{Y_0^*KN}{}^2/4\pi$ can be determined by relating the Lagrangian calculation with the dispersion-theoretic calculation. In the latter, one introduces a δ function in the appropriate absorptive parts appearing in the dispersion relation and can compare the pole residue obtained in this way with the coupling constant. In the case of Y_0^*, however, one has to extrapolate below the physical $\bar{K}N$ threshold. Warnock and Frye[29] did this with the Dalitz-Tuan model and we use their value $g_{Y_0^*}{}^2/4\pi=0.32$. For the $Y_1^*(1385)$, we use their broken $SU(3)$ value, $g_{Y_1^*KN}{}^2/4\pi=1.9/M^2$, where the relevant Lagrangian is

$$g_{Y_1^*KN}(\bar{Y}_1{}^*)_\mu N\partial_\mu\bar{K}+\text{H.c.}.$$

Again, the structure of the Y_1^* Born term could be determined by means of the dispersion-theoretic calculation. One gets[44]

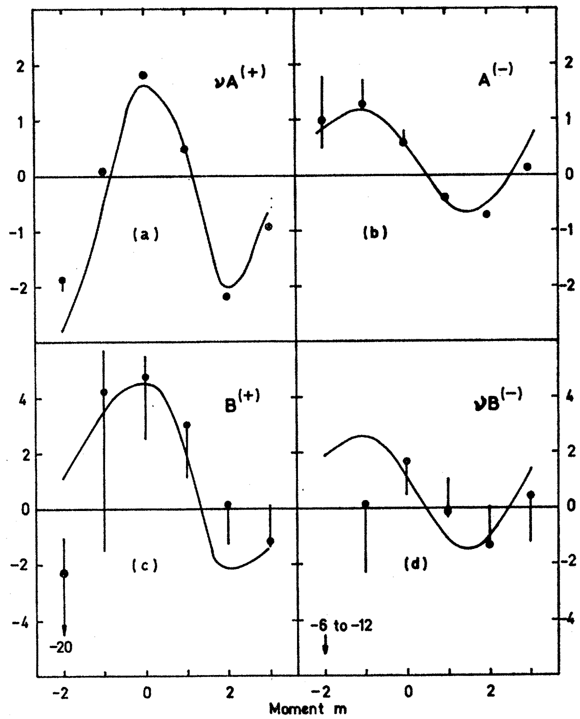

FIG. 1. Evaluation of Eq. (12) (units, GeV=1) at $t=0$ for different moments m and amplitudes F. This is the same as Fig. 1 of Ref. 13. The points are the sum-rule results for our favored data set [Kim's corrected (Ref. 43) coupling constants and unphysical region and a nonresonant K^+p solution], with the error bars showing the extent of the values obtained using the other choices discussed in the text. The continuous curves represent the extrapolations (to our matching energy $\sqrt{s}=2$) of high-energy Regge fits of solution 2 of Phillips and Rarita (Ref. 15) for $A^{(+)}$, $A^{(-)}$, and $B^{(-)}$ ($B\omega\equiv 0$). For $B^{(+)}$, we use the FESR result of Barger and Phillips (Ref. 10) and the Regge fit of Derem and Smadja (Ref. 22).

$$B_{Y_1^*}{}^1=\frac{g_{Y_1^*KN}{}^2}{(m_{Y_1^*}{}^2-s)}\left(\frac{1}{2}t+\frac{[(m_{Y_1^*}+M)^2-\mu^2][(m_{Y_1^*}-M)^2-\mu^2-2Mm_{Y_1^*}]}{6m_{Y_1^*}{}^2}\right),\tag{19a}$$

$$A_{Y_1^*}{}^1=\frac{g_{Y_1^*KN}{}^2}{(m_{Y_1^*}{}^2-s)}\left(\frac{1}{2}t(M+m_{Y_1^*})+\frac{[(M+m_{Y_1^*})^2-\mu^2]}{6m_{Y_1^*}{}^2}\times\{(M+m_{Y_1^*})[(m_{Y_1^*}-M)^2-\mu^2]-m_{Y_1^*}(M^2+\mu^2-m_{Y_1^*}{}^2)\}\right).\tag{19b}$$

[43] The conventional Born terms used in many calculations on KN forward dispersion relations have a factor M_p/M_Y for the Λ and Σ pole terms, relative to the usual definition of the coupling constants. This has been noted, among others, by Chan and Meiere (Ref. 34).

[44] The $Y_1^*(1385)$ contribution to A_{A_2} and A_ρ in Ref. 24 has the wrong t dependence. See S. Matsuda and K. Igi, Phys. Rev. Letters 20, 781 (E) (1968). On reevaluation, their A_{A_2} sum rule results change.

The values of the $Y_0^*(1405)$ and $Y_1^*(1385)$ coupling constants that we use may not be extremely accurate, but they are sufficiently good for our purpose: (a) We have used both of them in the resonance approximation calculation (of which the results are only qualitative anyway), and (b) we have used the $Y_1^*(1385)$ contribution also to provide estimates of errors due to parametrization of the unphysical region in the calculation with the phase-shift analyses input.

In order to evaluate the contributions of the resonances decaying physically into the $\bar{K}N$ channel, one has only to use Eq. (14) above along with the following:

$$f_{l\pm}=\frac{e^{i\delta_{l\pm}}\sin\delta_{l\pm}}{k}, \tag{20}$$

$$\mathrm{Im}f_{l\pm}(s)=\mathrm{Im}\frac{\Gamma_{\mathrm{el}}/2k}{M-W-\frac{1}{2}i\Gamma_{\mathrm{tot}}}\simeq\frac{\pi\Gamma_{\mathrm{el}}}{2k}\delta(M-\sqrt{s}), \tag{21}$$

$\delta_{l\pm}$ being the elastic-scattering phase shift for the appropriate partial wave.

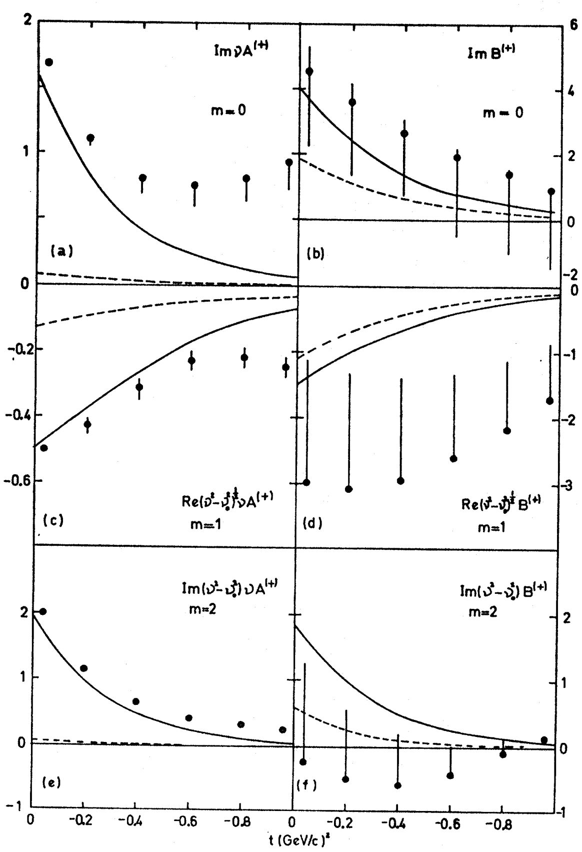

FIG. 2. Evaluation of Eq. (12) (units, GeV=1) for $m=0$, 1, and 2 for the amplitudes $A^{(+)}$ and $B^{(+)}$ for $0<-t<1$. The points and error bars have the same meaning as for Fig. 1. The dashed curve is the A_2 contribution as deduced by an extrapolation (to $\sqrt{s}=2$) of solution 1 of Ref. 22. The full-line curves represent the expected Regge contributions to the $(+)$ amplitudes at $\sqrt{s}=2$, our matching energy. For A_P, $A_{P'}$ we used solution 1 of Phillips and Rarita as such; for B_P, $B_{P'}$, we used $(\nu B/A)_{P,P'}=1$.

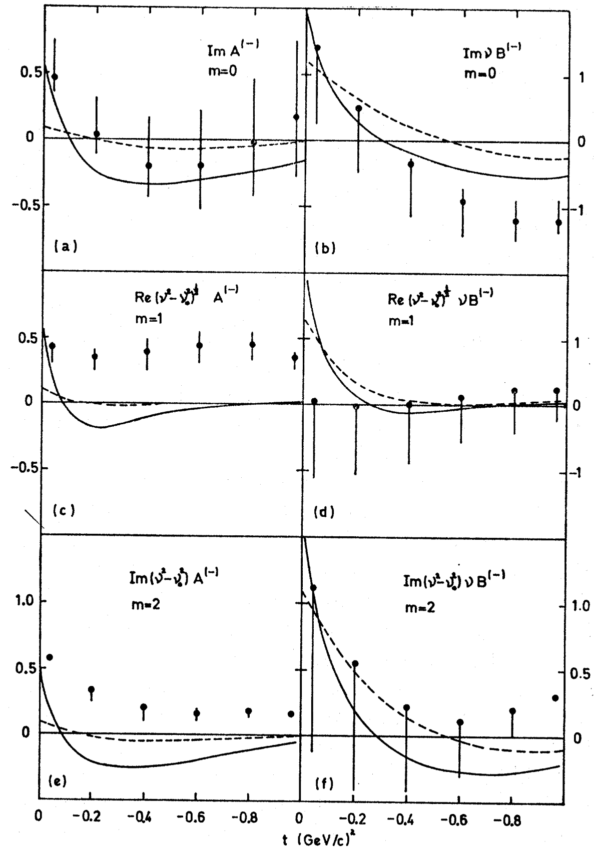

FIG. 3. Evaluation of Eq. (12) (units, GeV=1) for $m=0$, 1, and 2 for the amplitudes $A^{(-)}$ and $B^{(-)}$ for $0<-t<1$. The points and error bars have the same meaning as for Fig. 1. The dashed curve is the ρ contribution as deduced by an extrapolation (to $\sqrt{s}=2$) of solution 1 of Ref. 22. The full-line curves represent the expected Regge contributions to the $(-)$ amplitudes at $\sqrt{s}=2$, our matching energy. For A_ω, we used solution 1 of Phillips and Rarita as such; for B_ω, we used $(\nu B/A)_\omega=1.5$.

Now we turn to our results. We first discuss the case of the data input involving the phase shift and then, the resonance saturation results.

A. $t=0$

For $t=0$, we plot the results of evaluating the left-hand side of Eq. (12) versus m; this gives one a feeling for the relative importance of the different moments m and the phase of the amplitude in question. We recall that FESR's with m odd involve the real part and those with m even involve the imaginary part of the amplitude. For even m, the sum rules for the amplitude A could be evaluated directly by using total cross sections when a higher matching energy might be employed. Some earlier work for the $m=0$ sum rules has been mentioned already in Sec. 3. The vertical error bars in Fig. 1 give an estimate of the difference arising from using different input low-energy data, the central dots are for our favored set which has the Kim couplings for the Λ and Σ pole terms, a negligible Y_1^* coupling (as in

Kim's solution[28]) and a nonresonant K^+p solution. This notation has been followed in Figs. 2 and 3, also. The smooth curves in Fig. 1 represent extrapolations to 1.53 GeV of high-energy Regge fits to the 6–20 GeV data. We used solution 2 of Phillips and Rarita.[15] (Using solution 1 would make very slight differences in the comparison of the Regge curve with our points.) For $\nu A^{(+)}$ and $A^{(-)}$, the Phillips-Rarita parameters show a good agreement with our points remembering that a confrontation of data below 1.53 GeV and above 6 GeV is what is being presented. For $\nu B^{(-)}$, the Phillips-Rarita solution had $\nu B/A=+11$ for the ρ and $B_\omega=0$; these fit with our results approximately, the real part (m odd) results not being in good agreement. As pointed out earlier, the reason for their[15] choice $B_\omega=0$ is that the high-energy data were not good enough to determine it well. Hopefully, the agreement could be improved by taking $B_\omega\neq 0$. Indeed, we do use $(\nu B/A)_\omega=1.5$ as a typical expected sample value in all the subsequent comparisons of the sum-rule results with the extrapolations to ν_1 of the high-energy Regge fits; we were led to this choice of $(\nu B/A)_\omega$ by a comparison of the sum-rule results for $B^{(-)}$ and the known ρ contribution. For $B^{(+)}$, reasonable agreement can be obtained only by using the more recent[22] result of an analysis of the process $K^-p\to \bar{K}^0n$ that $\nu B/A=+8.3$ for the A_2 (rather than ~ -8 as in the Phillips and Rarita[15] analyses) and also the recent FESR result[10] in πN scattering that $\nu B/A=+1$ (rather than negative as in the Phillips-Rarita analyses[15]) for the P and P'. The fact that our results require $\nu B/A$ = positive for all the contributions to the (+) amplitudes is very important (especially for the A_2) and we return to it in Sec. 6. That high-energy data alone cannot determine the sign of $\nu B/A$ is clearly brought out by the four solutions that Reeder and Sarma[45] obtain for the A_2 trajectory couplings. This ambiguity in the determination of the sign of $\nu B/A$ is removed by our analysis and we are able to pick out their solution 3 (which resembles the Derem-Smadja solution,[22] but differs from the Phillips-Rarita solutions[15]) as our favored one. As noted by Reeder and Sarma, this has definite experimental consequences for the polarization in $K^-p\to\bar{K}^0n$ (and actually, also for the $K^\pm p\to K^\pm p$ processes which are more easily accessible for polarization measurements; see Sec. 6); their[45] neutron polarization near the forward direction is about -50% for solution 3 as compared to about -100% for the solutions having the opposite sign of $\nu B/A$.

One sees that the agreement of our results with the extrapolation of the high-energy Regge fits is not very good (especially for the B amplitudes) for $m=-2$ and -1. For $m=-2$, the FESR for $A^{(-)}$ (which is the forward-dispersion relation evaluated, for example, by Kim[25]) is much more sensitive to low-energy data and to coupling constants (g_Λ^2 and g_Σ^2) than it is to the Regge contribution. Indeed, we regard our FESR's for $m=-2$ and $m=-1$ (which we evaluate only for $t=0$) as providing a consistency check on the input data set rather than teaching us something about the relevant Regge poles. Actually, Fig. 1 shows that there is some inconsistency in our data set since one can see that our $m=-2$ results for the B amplitudes cannot be attributed to lower-lying Regge contributions, but imply some error in the P waves near threshold, or in Kim's treatment of the unphysical region which we adopt, etc. It is precisely these B amplitudes which are much more sensitive to the Born-term coupling constants than are the A amplitudes.

B. Favored Data Set

One should consider several independent sum rules in choosing one's "favored data set" and, in particular, in determining the coupling constants g_Λ^2 and g_Σ^2 because of the sensitivity of different sum rules to different aspects of the input data. For example, the forward dispersion relation (the $A^{(-)}$ sum rule for $m=-2$, $t=0$) is nearly as well satisfied by Zovko's coupling constants plus a "broken $SU(3)$" value for $Y_1^*(1385)$ coupling as with our favored set. Our choice of the favored set was based on a study of the whole family of our sum rules.

It is obvious that since total cross-section data are ingredients of the K^+p phase-shift analysis that we use as our data input, one does not expect our results using the K^+p solutions I and IV to be significantly different for the A amplitudes. For $\mathrm{Im}A^{(\pm)}$, especially at $t=0$, this is strictly true because of the optical theorem; for $\mathrm{Re}A^{(+)}$ also, it is true because of the fact that the forward-dispersion relation was built into the K^+p analysis of Lea *et al.*[27] Indeed, we find that the FESR results for the two K^+p solutions are essentially the same for the A amplitude. From our point of view, the main difference between the two K^+p solutions is in the $B^{(\pm)}$ amplitudes and we use this difference to select our favored data set. Also, the difference between the Kim and Zovko-Born couplings is more important for the $B^{(\pm)}$ amplitudes than for the $A^{(\pm)}$ amplitudes. The reason for this becomes clear if we consider, as an example, the Λ contribution in Eq. (17a) which shows that while the coupling constants have their full strength in the B amplitudes, their contribution to the A amplitude is weakened by the factors $M-m_\Lambda$ and ν_Λ (the value of ν for $\sqrt{s}=m_\Lambda$), both ν_Λ (in the region of interest to us) and $M-M_\Lambda$ being much less than 1 in our units. One has

$$A_\Lambda^0=(M-m_\Lambda+\nu_\Lambda/(1-x))B_\Lambda^0. \tag{22}$$

The B amplitudes, therefore, are more important than the A amplitudes as a guide to the choice of the favored set. The third type of variation that we have considered [$Y_1^*(1385)$ contribution] can contribute to both the $A^{(\pm)}$ and $B^{(\pm)}$ amplitudes almost equally

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[45] D. D. Reeder and K. V. L. Sarma, Phys. Rev. **172**, 1566 (1968).

strongly, and the variation it causes depends in detail upon which sum rule one is considering.

For choosing between the different input data sets, we compared our predictions for these different data sets with the expected (extrapolated down to $\nu_1=1.53$) Regge contribution. Let us restrict our attention to $t=0$ because this comparison is more reliable for $t=0$. In fact, the t dependence of the results with different data sets is quite similar and, if anything, the data set which we favor on the basis of our considerations at $t=0$ is also the favored one for our $t\neq 0$ results. For the $B^{(-)}$ amplitude, for example, our FESR results for $m=-1$, 1, 2, and 3 have the same and the opposite signs to that expected from the Regge extrapolation for the K^+p solutions IV and I, respectively; for $m=-2$, our results are quite different from the expected one possibly because of the extreme sensitivity of this sum rule to the very-low-energy data; for $m=0$, the magnitude is in much better agreement with the Regge expectation for solution IV. The $B^{(+)}$ results are not completely in favor of solution IV; solutions I and IV are better for $m=2$ and 3, respectively; $m=0$ and ± 1 results are about equally good for both the solutions; $m=-2$ results are not good for either. On the whole, therefore, the nonresonant solution IV is preferable to the resonant solution I. As for the difference between the Kim and Zovko couplings and the presence or almost absence of the Y_1^* contribution, one has to consider the A sum rules also. For the $A^{(-)}$ results, the results are surely better for the Kim couplings than for the Zovko ones for $m=0$, 1, -2, -1, the results for $m=2$ and 3 being about the same for both the cases. In some cases, the combination (Zovko's couplings + full Y_1^* contribution) is only slightly worse than the choice (Kim couplings + almost no Y_1^* contribution), though the latter is, in some other cases, much better (for example, the $m=0$ result for the $A^{(-)}$ amplitude). When the combination (Kim couplings + full Y_1^* contribution) is significantly different from the combination (Kim couplings + almost no Y_1^* contribution), it is generally worse than the latter (for example, the $m=-2$ case for $A^{(-)}$). On the whole, therefore, the preferred choice is (a) Kim's Born couplings for the Λ and the Σ contributions, (b) almost zero $Y_1^*(1385)$ contribution, as found by Kim,[28] and (c) the nonresonant K^+p solution IV. We hope that when future analyses of low-energy kaon-nucleon scattering become sufficiently well determined, one would not have to rely on a comparison with the extrapolation of the Regge result; under the present circumstances, we found this to be a useful possibility. It may very well happen that only two or one or none of the three choices that we have preferred is the true one and that the combination of the three is only a close approximation to the true representation of the data; this must await better and more complete phase-shift analyses than those available at present. It is unfortunate that, as found by Queen *et al.*,[46] all recent parametrizations of the low-energy $\bar{K}N$ scattering amplitude are inconsistent with the remainder of our knowledge of the KN interactions because they do not satisfy the important constraint that the Born coupling constants g_Λ^2 and g_Σ^2 defined by the dispersion relations be energy-independent. Queen *et al.*[46] have suggested that future parametrizations of the experimental data should incorporate more theoretical constraints (for example, their important consistency requirement of the constancy of g_Λ^2 and g_Σ^2). As mentioned in Sec. 3, Chan and Meiere[34] have shown, in a somewhat related context, that the constant-scattering-length extrapolation (with Kim's[33] parameters) into the unphysical region leads to inconsistencies because the coupling constants g_Λ^2 and g_Σ^2 tend to vary wildly and even become negative for different allowed values of a certain parameter β. Hopefully, things will improve in future and a future FESR analysis of KN scattering will be more informative than it is now.

C. $t\neq 0$

The results carry information in the form of dips and zeros in the various Regge contributions as a function of t. For $t\neq 0$, one should rely upon the results of only those FESR's which agree reasonably well at $t=0$ with the Regge expectation. We have shown the results for the (+) and (−) amplitudes for $m=0$, 1, and 2 in Figs. 2 and 3, respectively. The smooth full-line curves in these figures are the sum of all the relevant Regge contributions (extrapolated down to our matching energy) and the dashed curves are for the ρ (for Fig. 3) contribution in the case of the (−) amplitudes and the A_2 (for Fig. 2) contribution in the case of the (+) amplitudes. The ρ and A_2 contributions are extrapolations from the results of Derem and Smadja.[22] For the A_2, we have used their solution 1. Their other solution is very similar. The A_P, $A_{P'}$ and A_ω contributions are taken from solution I (which is very similar to their solution II which we have used only for our Fig. 1) of Phillips and Rarita.[15] B_P, $B_{P'}$, and B_ω are determined by using our qualitative (for all t) conclusions (confirmed for the P and P' by the πN FESR results of Barger and Phillips[10]) that $\nu B/A=+1$ for P and P' and $\nu B/A=+1.5$ for the ω. The curves do not represent, therefore, true high-energy Regge fits; they are partly motivated by the FESR results. With the high-energy Regge fit[14,15] extrapolations taken as such, even the signs of the spin-flip amplitudes B would not agree with ours. The curves are meant to represent what we believe the extrapolation of high-energy Regge fits should look like. Detailed Regge fits to high-energy data incorporating our FESR "prejudices" about the phases and

[46] N. M. Queen, S. Leeman, and F. E. Yeomans, Birmingham University Report (unpublished).

magnitudes of the various contributions are in progress.[47]

For $A^{(+)}$ and $m=0$, we find [Fig. 2(a)] slight evidence of a dip (presumably due to the P' contribution) at $-t=0.5$; this agrees with the conclusions of Barger and Phillips,[10] who studied the πN sum rules. From high-energy fits, it appears that the P' trajectory is less strongly coupled to the KN system than to the πN system; Phillips and Rarita[15] found that at $t=0$, for example,

$$(A_{KN}/A_{\pi N})_{P'}=(B_{KN}/B_{\pi N})_{P'}=0.29.$$

Indeed, we find a less pronounced dip than in the πN case. This dip is consistent with its interpretation as a double zero in $A_{P'}$ according to the no-compensation mechanism if $\alpha_{P'}=0$ at this t value. Such a dip could not be attributed to the P contribution because the P trajectory is rather flat and is not expected to go through zero at such a low value of $-t$. Also, it could not be due to the A_2 contribution because the latter is much smaller than the P and P' contributions to $A^{(+)}$. The $m=1$ [Fig. 2(c)] results for the $A^{(+)}$ (involving $\text{Re}A^{(+)}$) are also consistent with the presence of this dip in the P' contribution; here, however, the A_2 contribution is not negligible and the conclusion cannot be definite because the same dip could be attributed to the A_2 contribution. The extrapolations of the Regge fits down to our matching energy seems to be systematically below the FESR results for the $A^{(+)}$ amplitude. This can be due to a number of reasons. (a) The extrapolation of the Regge fits to as low an energy as $\sqrt{s}=2$ may not be completely justified; (b) the high-energy fits may be inconsistent with the FESR results, in which case either the low-energy data or the high-energy fits need to be reexamined. [It is worth remarking that our results for the $A^{(+)}$ sum rules are fairly reliable especially for $m=0$ in the sense that the vertical error bars are small and the different input data sets do not lead to very different results. The small error bars for $m=2$ in Fig. 2(e) are *a priori* expected because of the relatively small weight given to the very-low-energy data in the high-m sum rules.] The disagreement at large $-t\sim 1$ is not as bad as it looks because the plotted Regge extrapolation did not take into account the no-compensation mechanism for the P and P'.

Coming to the results for the $B^{(+)}$ amplitude shown in Figs. 2(b), 2(d), and 2(f), we see that, as pointed out in Sec. 4 B, the vertical bars are quite big and a difference in the input data sets causes an appreciable difference in the results. The two K^+p solutions give fairly different results. (For $B^{(-)}$, the variation caused by varying the K^+p solution is much larger than the one caused by varying the Y_1^* or the Λ and Σ couplings for the same K^+p solution.) It is difficult to draw any strong conclusion from the $B^{(+)}$ results partly because of the very big errors. The agreement is not good for our favored data set, though it is reasonable for $m=0$ [Fig. 2(b)] within the rather large error bars. In $A^{(+)}$, the A_2 trajectory contribution is masked by P and P'; not so in $B^{(+)}$ for $m=1$ [Fig. 2(d)] where real parts are involved. In this case, our results, *as they stand*, show no evidence of a zero for $-t<0.8$ approximately so that the Chew mechanism (or the no-compensation mechanism) type nonsense-choosing zero is excluded. Either a Gell-Mann type nonsense-choosing zero or else no zero of α_{A_2} in this range is possible.[48] Unfortunately, this conclusion is in direct contradiction to the results of Chu and Roy,[41] who considered the sum rule corresponding to $\int \text{Im}B d\nu$ and $\int \nu^2 d\nu\, \text{Im}B$ for the A_2 contribution to photoproduction; they conclude that their results strongly favor the Chew or the no-compensation mechanism over that of Gell-Mann; they did find a zero in the zero-moment sum rule and they thought that the behavior of their second-moment sum rule suggests strongly a double-zero behavior. It is true that our data input is not extremely reliable for the real parts (Sec. 4 D) and that our conclusion cannot be regarded as absolutely final, but it appears that their conclusion is not final either. Their second-moment sum rule does have a definite single zero, even though one could perhaps regard it as only a slight displacement of a double-zero–type behavior. We believe that more conclusive evidence is needed to decide the issue one way or the other. In fact, considerations of exchange degeneracy between the ρ and the A_2 would tend to support our conclusion. Also, the $\pi\eta\to\pi\rho$ FESR's support our conclusion.

The results for the $(-)$ amplitudes are shown in Fig. 3 for $m=0$, 1, and 2. We assume the ρ to be given and investigate the ω contribution which is our main interest in the $(-)$ amplitudes. Turning to $m=0$ [Fig. 3(a)] for $A^{(-)}$ which should be dominated by the ω, we do find solutions (using Kim's coupling constants) in which the imaginary part changes sign for $-t\sim 0.2$ as is needed to explain the crossover phenomenon. There appears to be some dilution, however, by lower-lying Regge poles or else the data are not sufficiently reliable since we find no corresponding zero either in $\text{Re}A^{(-)}(m=1)$ [Fig. 3(c)] which is well nigh constant as a function of t, or in the $m=2$ moment for $A^{(-)}$. Actually, the FESR results for the $m=2$ moment of $A^{(-)}$ [Fig. 3(e)] are quite insensitive to the considered variations in the input data and could be reliable except for the fact that all the variations we have allowed (for the A amplitudes) are in the very-low-energy data to which the $m=2$ are insensitive by construction.[49] The present

[47] G. V. Dass, C. Michael, and R. J. N. Phillips (unpublished).

[48] Exchange degeneracy of the A_2 with the ρ would suggest that the A_2 trajectory passed through zero at $-t\sim 0.5$, while high-energy Regge fits to $\pi N\to\eta N$ and to the $\bar{K}N$ charge-exchange data tend to favor a flatter (or curved) trajectory with no such zero.

[49] For the B amplitudes, however, we have allowed big variations at all energies because the two K^+p solutions give quite different spin-flip amplitudes at even high energies. At low energies, the different choices for the couplings g_Λ^2, g_Σ^2, and $g_{Y_1^*(1385)}^2$ provide this variation.

experimental situation does not allow one to consider any variations (for the A amplitudes) in the higher-energy end of our input data—the region which is weighted heavily for high positive m values; it is possible that allowing for such variations would reproduce results consistent with the usual zero[17] in the effective ω contribution to the $(-)$ amplitudes. We do not regard the absence of a crossover in the $m=1$ results as a serious difficulty because of the extent to which the real parts of the input data are reliable. We return to the question of the reliability of our sum rules involving the real parts in Sec. 4 D.

The case of the $B^{(-)}$ amplitudes is shown in Fig. 3(b), 3(d), and 3(f). The error bars here are bigger, in general, than for $A^{(-)}$. Overlooking again the $m=1$ [Fig. 3(d)] results which involve real parts and which are very sensitive to the input data variations (particularly, the K^+p solution), we find that after subtracting the known ρ contribution, the $m=0$ results [Fig. 3(b)] for the effective ω contribution to $B^{(-)}$ show a behavior similar to the one for $A^{(-)}$ for $m=0$. Within the error bars shown, the $m=2$ results [Fig. 3(f)] are also consistent with the occurrence of the usual crossover at $-t\sim 0.2$.

We find no evidence in the $B^{(-)}$ sum rules of any additional zero in B_ω (a sense-nonsense zero at the wrong-signature unphysical point $\alpha_\omega=0$) for $-t<1$ which is inconsistent with $\alpha_\omega=0.45+0.9t$ found by Contogouris *et al.*[20] (see also Sec. 2), from a $\pi N\to\rho N$ analysis; our results tend to favor a flatter trajectory (if the zero at $t=t_c$ is not to be associated with $\alpha_\omega=0$). Also, we find $(\nu B/A)_\omega=+1$ to $+3$ for $0<-t<0.7$. This shows that the amplitude B_ω could be appreciably more important than A_ω in this t region and it is not a good approximation to set $B_\omega=0$[15] in a Regge fit. One can get a qualitative estimate of $(\nu B/A)_\omega$ at $t=m_\omega^2$ by assuming the ω and ρ dominance of the isoscalar and isovector nucleon form factors, respectively. It has been shown by Rarita *et al.*[14] that

$$\left(\frac{\omega_{\text{lab}}B}{A}\right)_\rho\equiv\frac{R_-}{R_+}=\frac{34.8}{13.7}\simeq 2.54 \tag{23}$$

at $t=m_\rho^2$, assuming factorization for the ρ residues and using the results in their Sec. V (xii). Using the known proton and neutron magnetic moments and charges and the fact that the isovector part is the ρ contribution and the isoscalar part is the ω contribution, one can deduce from the ratio of the isoscalar and isovector total magnetic moments that

$$\frac{B_\omega}{B_\rho}=\frac{(\gamma_1+2M\gamma_2)_\omega}{(\gamma_1+2M\gamma_2)_\rho}\approx 0.2\,, \tag{24}$$

where the residues γ_1 and γ_2 are defined by Ball and Wong.[50] Similarly, since the isovector and isoscalar charges are equal, $A_\omega/A_\rho\simeq(\gamma_1)_\omega/(\gamma_1)_\rho=1$. Hence $\omega_{\text{lab}}B_\omega/A_\omega\simeq 0.5$ at $t=m_\omega^2$, assuming $m_\omega^2\simeq m_\rho^2$. Keeping in mind the various approximations that we have made on the way, the comparison of this number $(\nu B/A)_\omega$ of $\sim +0.5$ with our value ($+1$ to $+3$), though it involves an extrapolation in t, is qualitatively satisfactory. On the other hand, in their fit to NN data, Rarita *et al.*[14] used $\nu B/A\approx -6$ for the ω, which they took as the predominant spin-flip contribution to fit the pp polarization data. Their model is in conflict with the recent $\bar{p}p$ polarization data of Daum *et al.*[11] at 2–3 GeV/c. This lends support to the conclusion that additional important spin-flip contributions (for example, ρ, A_2, and ω correctly) should be included in a Regge fit to NN scattering to achieve agreement with experiment. We have seen that the sign of $\nu B/A$ used by Rarita *et al.*[14] is negative for P, P', and ω; our results want this sign to be positive for all these three contributions. It may very well be that if the Regge parameters are constrained to be consistent with the FESR results, one would get agreement with experimental data on NN and $N\bar{N}$ scattering.

D. How Good Are Our Input Data? Implications for Different Moments

A word about the extent to which our results for the FESR's with different moments are reliable is now in order. Here we want to consider this question in the light of our input data. These remarks are supplementary to those of Sec. 3D. To get some idea of how well our input data determined the low-energy amplitudes A and B which went into our FESR integrals, we compared our amplitudes A with some forward-dispersion-relation calculations. Unfortunately, there is not very much else to compare with, especially at $t\neq 0$. ImA at $t=0$ is, of course, all right because all our input data reproduce the observed total cross sections. However, one still has not confirmed that the $t\neq 0$ amplitudes $A^{(\pm)}$ are correct. Our extrapolation of the parametrization of Armenteros *et al.*[35] below their region introduces very small discontinuities in ImA (and similarly, in ImB) at the point ($p_{\text{lab}}=550$ MeV/c) where the parametrization of Armenteros *et al.* takes over the Kim[28] parametrization. For Re$A(K^+p\to K^+p)$, the agreement is very good. The situation about Re$A(K^-p\to K^-p)$ is not very good. We (a) overestimate it in the region 550–780 MeV/c (the region between the analyses of Kim and Armenteros *et al.*), (b) more or less agree with the results of a very recent forward-dispersion-relation calculation by Carter[51] in the region of the analysis of Armenteros *et al.*, and (c) underestimate it in the region (1220–1460 MeV/c); all this, in such a manner that the area under the plot of Re$A(K^-p\to K^-p)$ versus ν is about the same as for Carter's numbers. Our $m=$odd (real parts of $A^{(\pm)}$ involved) sum rules always have factors of $(\nu^2-\nu_0^2)^{m/2}$ multiplying the amplitude; this

[50] J. S. Ball and D. Y. Wong, Phys. Rev. **133**, B179 (1964).

[51] A. A. Carter, Cavendish Laboratory Report No. HEP 68-10 (unpublished).

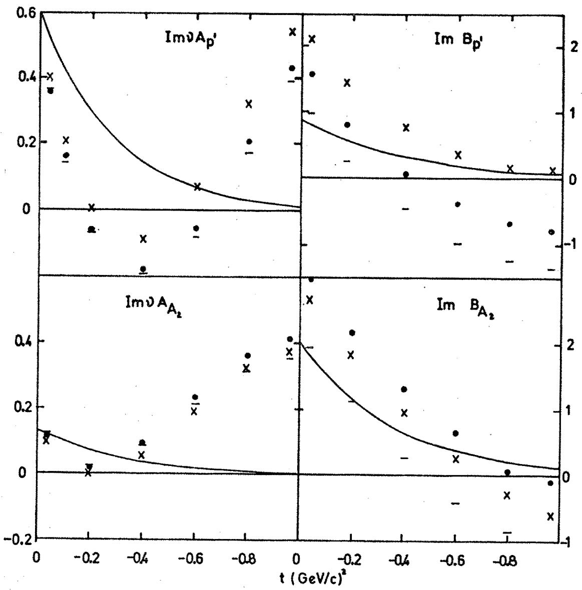

FIG. 4. Evaluation of Eq. (12) (units, GeV=1) for $m=0$, $\nu_0=0$ in the resonance saturation approximation for the contributions $f_{P+P'}$ (identified as $f_{P'}$), f_{A_2} of Eq. (9) with all the bound states and resonances (Ref. 38) (of which J^P is known) included. The full-line curves are the Regge expectation as for Figs. 2 and 3 evaluated at $\sqrt{s}=2.15$ (Ref. 39). The crosses (×) are for corrected Kim coupling constants (Ref. 43) g_Λ^2 and g_Σ^2 and a negligible $Y_1^*(1385)$ contribution, the resonance analog of our favored data set; the points are for $SU(3)$ couplings g_Λ^2 and g_Σ^2 ($g_{\pi N}^2 \sim 14.5$ and $f=0.36$) and $g_{Y_1^*(1385)}{}^2=1.9/M^2$; the lines are for uncorrected Zovko's couplings (Ref. 43) g_Λ^2 and g_Σ^2, and $g_{Y_1^*}{}^2=1.9/M^2$.

places a greater or smaller weighting on the high-energy region of our input data than on the low-energy region, depending upon whether m is positive or negative. If this weighting is not too strong, our results for m odd may be reliable. Otherwise, this makes our results for m odd sum rules less reliable than those for the m even (imaginary parts involved) sum rules because of this different weighting of the two regions in which our real parts of the $A^{(\pm)}$ amplitudes do not reproduce very well the results of Carter. It is obvious that one would be very lucky if the t dependence of the m odd sum rules came out correctly, given the fact that the input data set might be bad enough to make them wrong at even $t=0$. Our $m=3$ results have not been shown because these would grossly over estimate the higher-energy end of the input data. The $m=\pm 1$ results are not bad at $t=0$ (see Fig. 1); the t dependence may not be correct (see Figs. 2 and 3).

We have no way to check our B amplitudes and these are particularly sensitive to variations in the input data; one can believe only those conclusions (we have mentioned only such ones) which stand inspite of these variations. Again, we rely on our sum-rule results for ImB more than on ReB. Summarizing, therefore, we regard our m even sum rules as more reliable than the m odd ones.

We wish to emphasize that our data input is a good representation of all the known experimental data, though it may not be the only possible unique representation. For example, it reproduces the known total cross section (both elastic and reaction ones) and the known differential cross sections from threshold up to our matching energy; these indeed are the ingredients of the data on which the phase-shift analyses which we have used were based. Also correctly reproduced are the available data on polarization in K^-p scattering even in the region 1100–1350 MeV/c,[37] in which some extrapolation beyond the region of Armenteros *et al.* is already involved. The forward-dispersion relation for K^+p scattering, being built into the analysis of Lea *et al.*,[27] guarantees our Re$A\,(K^+p \to K^+p)$ at $t=0$ being correct. The only obvious imperfections in our input are (a) Re$A\,(K^-p \to K^-p)$ is not well reproduced. This weakens our m odd results somewhat. (b) There is a slight discontinuity in ImB and Re$A\,(K^-p \to K^-p)$ at the point of changeover from the Kim parametrization to the parametrization of Armenteros *et al.*; the discontinuities in ImB are not very crucial because these are very small compared to the other more important ambiguities like g_Λ^2, g_Σ^2, and $g_{Y_1^*}{}^2(1385)$ for the B amplitude; the discontinuities in ImA are negligibly small anyway; the one in Re$A\,(K^-p \to K^-p)$ is not negligible. Nonetheless, an over-all average of Re$A\,(K^-p)$ as a function of energy is nearly correctly given, when compared with Carter's numbers.

E. Resonance Saturation Approximation

We have evaluated (with $\nu_0=0$) the $m=0$ and $m=2$ results for the separate contributions $(P+P')$, A_2, ω, and ρ in the resonance approximation. For $m=0$, the results are shown in Fig. 4 for the $P+P'$ and A_2 and in Fig. 5 for the ρ and the ω. As mentioned in Sec. 2, we expect the resonance-saturation-approximation results to be quite reliable for the ω. Keeping in mind the rather large uncertainties in the known resonance parameters, we thought it good enough to work within the approximation of narrow-width resonances and therefore not to worry about the energy dependence of the relevant width, etc. The full-line curves in Figs. 4 and 5 are the expected Regge contributions extrapolated down to $\sqrt{s}=2.15$,[39] evaluated as for Figs. 2 and 3. The crosses are for Kim coupling constants for the Λ and Σ terms (corrected[43] for the factors M_p/M_Y) and a negligible $Y_1^*(1385)$ contribution; the points are for $SU(3)$ couplings for the Λ and Σ terms ($g_{\pi N}^2=14.5$ and $f=0.36$) and a broken $SU(3)$ value[29] for the Y_1^* coupling; the broken lines are for Zovko couplings (not corrected[43] for the factors M_p/M_Y) for the Λ and Σ terms and a broken $SU(3)$ value[29] for the Y_1^* coupling; these cases were chosen as indicative of the errors and uncertainties involved, within the resonance approximation. Assuming[40] that the $P+P'$ contribution of Eq. (9a) is only P' contribution in the resonance approximation, the results for all the four contributions are summarized below. One must remember[41] that the resonance approximation can give only crude results and that slight shifts in the positions of the expected zeros (as a function of t) or double zeros becoming broken into two nearby zeros (and vice versa) or other similar things are likely to happen in the resonance-saturation results. We have taken the resonance parameters from Rosenfeld *et al.*[38] and have included all the relevant resonances of which J^P is known.

P'. A single zero in the B sum rule and an almost double zero in the A sum rule (a similar behavior for the $m=2$ results) confirm our more exact FESR result that the P' trajectory seems to choose the no-compensation mechanism type coupling ($\alpha_{P'}=0$ at $-t\sim0.5$). The agreement with the Regge curves is not good, possibly because the relevant Regge fits did not incorporate this feature of the P' coupling.

ω. The results for both $m=0$ and $m=2$ indicate zeros in the A_ω and B_ω at $-t\sim0.1$ which is expected on the basis of the usual explanation[17] of the crossover phenomenon. The agreement with the Regge curves is fairly good. We recall that one can believe the resonance-approximation results for the ω to be reasonably correct. Again, we do not see evidence of a second zero (at $-t\sim0.5$) in B_ω as expected due to a wrong-signature sense-nonsense zero if one accepts $\alpha_\omega=0.45+0.9t$.[20] The results for the ω also, therefore, confirm our more exact FESR results given already. While this paper was being written up, we saw a paper by Di Vecchia *et al.*[19] who evaluated the $m=0$ case with the older (1967) resonance parameters of Rosenfeld *et al.*,[52] neglecting the $Y_1^*(1385)$ contribution, as suggested by Kim.[28]

ρ. We do not rely very much on our resonance results for the ρ and A_2, both of which have been evaluated for $m=0$ by Matsuda and Igi.[24,44] For the ρ, the sense-nonsense zero in B_ρ is at about the right place, the $m=2$ result behaving similarly. The Regge A_ρ is very small and is not very well given by the resonance approximation, though the Kim case with no $Y_1^*(1385)$ is not far out from the Regge curve.

A_2. The $m=0$ results have been evaluated by Mitsuda and Igi[24,44] with the older 1967 resonance parameters of Rosenfeld *et al.*[52] The results are consistent with a double zero in A_{A_2} at $-t\sim0.2$ and a single zero in B_{A_2} at $t\approx-0.6$, both of which, if at the same t value, could result from a no-compensation mechanism for the A_2 (as for P'), though A_{A_2} is numerically somewhat small in magnitude and also sensitive to the inclusion or omission of certain resonances. Also, the $m=2$ result for A_{A_2} shows two zeros (at $-t\approx0.13$ and 0.5; similarly for A_ρ) and not a double zero as for $m=0$. The $m=2$ result for B_{A_2} is similar to that for $m=0$; it has a single zero at $-t\approx0.4$.

On the whole, therefore, our resonance approximation results give support to our more exact results obtained with phase-shift analyses as input data.

[52] A. H. Rosenfeld, A. Barbaro-Galtieri, W. T. Podolsky, L. R. Price, Matts Roos, P. Soding, W. J. Willis, and C. G. Wohl, Rev. Mod. Phys. 39, 1 (1967).

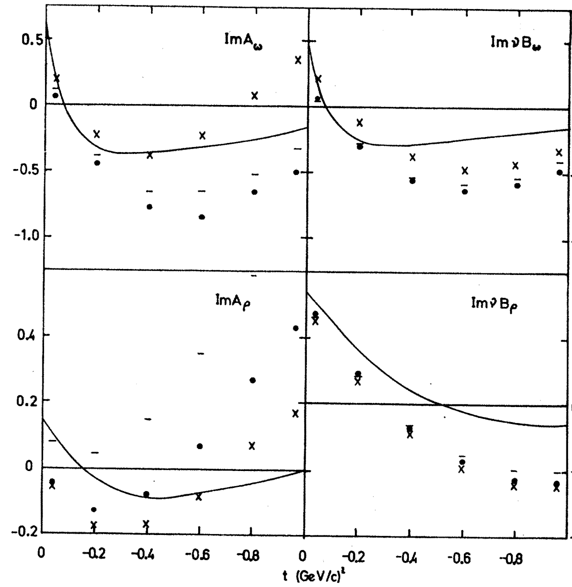

FIG. 5. Evaluation of Eq. (12) (units, GeV=1) for $m=0$, $\nu_0=0$ in the resonance saturation approximation for the contributions f_ρ, and f_ω of Eq. (9) with all the bound states and resonances (Ref. 38) (of which J^P is known) included. The full-line curves are the Regge expectation as for Figs. 2 and 3 evaluated at $\sqrt{s}=2.15$ (Ref. 39). The crosses (**X**) are for corrected Kim coupling constants (Ref. 43) g_Λ^2 and g_Σ^2 and a negligible $Y_1^*(1385)$ contribution, the resonance analog of our favored data set; the points are for $SU(3)$ couplings g_Λ^2 and g_Σ^2 ($g_{\pi N}^2\sim14.5$ and $f=0.36$) and $g_{Y_1^*(1385)}^2=1.9/M^2$; the lines are for uncorrected Zovko's couplings (Ref. 41) g_Λ^2 and g_Σ^2, and $g_{Y_1^*}{}^2=1.9/M^2$.

F. Relevant Meson-Meson Scattering FESR Analyses in Resonance Approximation

By considering the process $KK\to KK$, one can show that the zero in the residue of the helicity-nonflip amplitude for the ω contribution has a zero nearer to $t=0$ than the zero in the ρ residue which in, for example, the $\pi\pi\to\pi\pi$ calculation of Schmid[53] is at $-t\simeq0.3$. This is encouraging for the usual[17] ω-crossover explanation which would indeed want it at $-t\simeq0.15$. For the other trajectories, the $KK\to KK$ calculation does not give very unambiguous results.

One could consider the process $\pi\rho\to\pi\eta$ to get information on B_{A_2}. If one uses Eq. (12c) of Ademollo *et al.*[54] in their Eq. (12b), one can deduce that B_{A_2}/α_{A_2} has no zero for $-t<1.4$ approximately. This could mean the Gell-Mann nonsense-choosing mechanism $\beta\sim\alpha$ near $\alpha=0$ if α_{A_2} goes through zero for $-t<1.4$; otherwise,[55] it could mean the Chew mechanism or the no-compensation type coupling ($\beta\sim\alpha^2$ near $\alpha=0$). Also, the $SU(3)$ symmetry limit results of Ademollo *et al.*[54] include exchange degeneracy between the ρ and A_2; this implies $\beta_{A_2}\sim\alpha_{A_2}$ near $\alpha_{A_2}=0$.

[53] C. Schmid, Phys. Rev. Letters 20, 628 (1968).

[54] M. Ademollo, H. R. Rubinstein, G. Veneziano, and M. A. Virasoro, Phys. Rev. Letters 19, 1502 (1967).

[55] The results of Ademollo *et al.* (Ref. 54) favor the possibility that α_{A_2} does go through zero for $-t<1.4$.

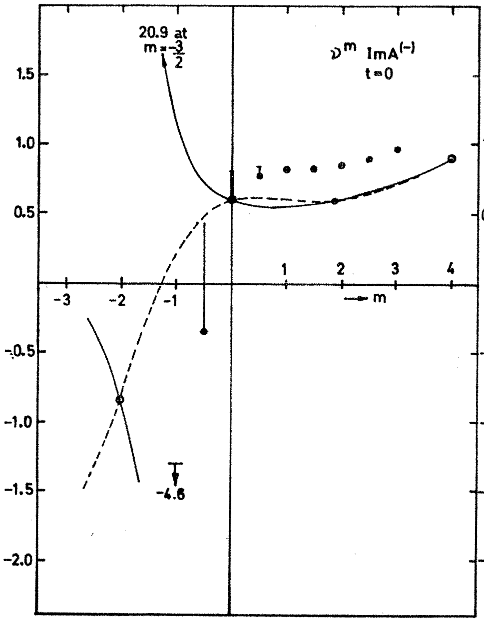

FIG. 6. Evaluation of Eq. (33) at $t=0$, $\lambda=m_K$ in the normalization of Eq. (12) (units, GeV=1) for various m values. For m even, the whole thing is a genuine FESR, shown by circles where the naive Regge expectation (evaluated as a sum of genuine ω and ρ Regge poles as for Figs. 2 and 3) and the modified Regge expectation coincide. The full line is the naive Regge expectation having two infinite discontinuities at $m=-1.52(=-1-\alpha_\omega)$ and $m=-1.57(=-1-\alpha_\rho)$; the dashed curve is the modified version incorporating the m plane pole plus background model and has no infinities at the points $m=-1.52$, -1.57. The integral in Eq. (33) is shown by points and error bars in the same notation as for the other figures. For $m=-1.5$, the integral varies from -8.6 to -21.7 for the various input data sets; this agrees better with the dashed curve. An ideal agreement with the dashed curve would mean $\tilde{f}(m)=0$.

5. OTHER SUM RULES

The finite-energy sum rules that we have considered so far depend on analyticity and a Regge-like parameterization of the high-energy data. However, one may evaluate relations which depend in a more sensitive fashion on the Regge-pole approximation. Effectively what one does is to evaluate the Froissart-Gribov representation of the l-plane amplitude $a(l)$ using the finite-energy trick to continue the representation to smaller l values. We shall discuss this procedure in terms of the Khuri plane[56] which is much simpler for practical evaluation, while the correspondence between leading Regge poles and leading Khuri poles is maintained. Let us take ν_0 to be zero to simplify the discussion further and consider a typical amplitude $A^{(-)}(\nu,t)$ which is an odd function of ν. Then at fixed t, say, $t=0$, we consider the following relations:

$$0=\int_0^\infty \mathrm{Im}\left[\left(-\frac{\nu^2}{\lambda^2}\right)^{m/2} A^{(-)}(\nu)\right]d\nu, \qquad (25)$$

[56] N. N. Khuri, Phys. Rev. Letters **10**, 420 (1963); Phys. Rev. **132**, 914 (1963); D. Z. Freedman and J.-M. Wang, *ibid.* **153**, 1596 (1967).

$$b(m)=\int_0^\infty \mathrm{Re}\left[\left(-\frac{\nu^2}{\lambda^2}\right)^{m/2} A^{(-)}(\nu)\right]d\nu, \qquad (26)$$

$$f(m)=\int_0^\infty \left(\frac{\nu}{\lambda}\right)^m \mathrm{Im}A^{(-)}(\nu)d\nu. \qquad (27)$$

The expression on the right-hand side of Eq. (25), which is the usual generalized SCR we have discussed previously, is zero from crossing and analyticity. Equation (26) is not zero, and defines an analytic function of m, $b(m)$. There is a relation between Eqs. (26) and (27): $f(m)=b(m)\sin\frac{1}{2}m\pi$. Equation (27) similarly defines an analytic function of m, $f(m)$, which is the same function as was discussed by Khuri[56] (noting that ν is proportional to $\cos\theta_t$ for our kinematics). Thus one expects $f(m)$ to have poles at $m=-\alpha-1$, $-\alpha+1$, $-\alpha+3$, $\cdots$ if there is a pole in $a(l)$ at $l=\alpha$. Assuming that the highest l-plane singularity is at $l=\alpha$, one must seek a method of analytic continuation to discuss Eqs. (25)–(27) for $m>-\alpha-1$ and this is provided by the finite-energy trick. We evaluate the integrals from ν_1 to ∞ using a Regge parametrization of the high-energy data and obtain, if $A^{(-)}(\nu)\sim\sum\beta e^{-i(\pi/2)(\alpha-1)}(\nu/\lambda)^\alpha$,

$$0=\int_0^{\nu_1} \mathrm{Im}\left[\left(-\frac{\nu^2}{\lambda^2}\right)^{m/2} A^{(-)}(\nu)\right]d\nu - \sum\lambda\beta\frac{\sin[\frac{1}{2}\pi(\alpha+m+1)](\nu_1/\lambda)^{\alpha+m+1}}{\alpha+m+1}, \qquad (28)$$

$$b(m)=\int_0^{\nu_1} \mathrm{Re}\left[\left(-\frac{\nu^2}{\lambda^2}\right)^{m/2} A^{(-)}(\nu)\right]d\nu + \sum\lambda\beta\frac{\cos[\frac{1}{2}\pi(\alpha+m+1)](\nu_1/\lambda)^{\alpha+m+1}}{\alpha+m+1}, \qquad (29)$$

$$f(m)=\int_0^{\gamma_1} \left(\frac{\nu}{\lambda}\right)^m \mathrm{Im}A^{(-)}d\nu - \sum\lambda\beta\frac{\sin[\frac{1}{2}\pi(\alpha+1)](\nu_1/\lambda)^{\alpha+m+1}}{\alpha+m+1}. \qquad (30)$$

In these equations, the summation is over the relevant Regge poles (ρ and ω for the $A^{(-)}$ amplitude). Equation (28) can clearly be continued in m beyond $-\alpha-1$ and this is the technique we have been employing in the previous sections. Equations (29) and (30), however, have poles in $b(m)$ and $f(m)$, as we have noted, and to continue these one must make a model for the m-plane amplitude as a pole plus background:

$$b(m)=\sum\frac{\lambda\beta}{(\alpha+m+1)}+\tilde{b}(m). \qquad (31)$$

Then, Eqs. (29) and (30) may be written as

$$\tilde{b}(m)=\int_0^{\nu_1}\mathrm{Re}\left[\left(-\frac{\nu^2}{\lambda^2}\right)^{m/2}A^{(-)}(\nu)\right]d\nu+\sum\frac{\lambda\beta}{\alpha+m+1}\times\left[\cos[\tfrac{1}{2}\pi(\alpha+m+1)]\left(\frac{\nu_1}{\lambda}\right)^{\alpha+m+1}-1\right],\quad(32)$$

$$\tilde{f}(m)\equiv\sin(\tfrac{1}{2}\pi m)\,\tilde{b}(m)=\int_0^{\nu_1}\left(\frac{\nu}{\lambda}\right)^m\mathrm{Im}A^{(-)}(\nu)d\nu-\sum\frac{\lambda\beta}{\alpha+m+1}\{\sin[\tfrac{1}{2}\pi(\alpha+1)]\times(\nu_1/\lambda)^{\alpha+m+1}+\sin(\tfrac{1}{2}\pi m)\}\quad(33)$$

and the latter equations (32) and (33) are in a form suitable to continue to values of m larger than $-\alpha-1$. We note that Schwarz[12] has considered expressions in Eqs. (27) and (30) for m odd (for m even this is the form of the FESR's used by Dolen *et al.*[3]) and has argued that $f(m)$ is zero at these nonsense, wrong-signature values if third double-spectral-function effects are negligible. However, $f(m)$ may even be infinite, as is exhibited in Eq. (30).

Before evaluating relations such as Eq. (32), one should discuss the choice of ν_0 and λ and possible t-dependent singularities. As they stand, $b(m)$ and $f(m)$ are singular for $t\simeq-0.23$ when $\nu_\Lambda=0$; such left-hand cuts in t are well known in the physical partial-wave amplitudes $a(l)$, of course. Thus we see that $\tilde{b}(m)$ must contain these cuts and we may learn rather little from evaluating the relations numerically—what we seek, of course, is some representation which will minimize the background term in the region of interest.

Equation (33) with $\nu_0=0$ has the advantage that only imaginary parts of the data are required; thus at $t=0$ for $A^{(\pm)}$ the data are essentially the total cross sections and are reliable. Choosing $A^{(-)}$ since this is expected to have effectively only one contribution with $\alpha\simeq0.5$ at $t=0$, we plot in Fig. 6, with error bars as usual, the integral in Eq. (33) and compare with the Regge terms. The normalization used is similar to that for the usual FESR's [Eq. (12)] considered previously, rather than as in Eq. (33). The m-plane pole model using $\lambda=0.494$ (since $\cos\theta_t=\nu/\mu$ at $t=0$) is the more plausible and is shown dashed, together with the naive result of setting $f(m)=0$. Though the normalization of the high-energy parameters is not very reliable (because the agreement for $m>0$ is not good even for m values corresponding to genuine FESR's) in this context, we see that the Schwarz condition $f(m)=0$ for $m=-1$ is not satisfied, while $\tilde{f}(m)=0$ (shown dashed) is a more plausible assumption. For even m, $f(m)=0$ and one has the FESR's evaluated by Dolen, Horn, and Schmid.[3]

In Fig. 7, we show the results of evaluating the analogs of Eq. (32) at $t=0$, for the four amplitudes

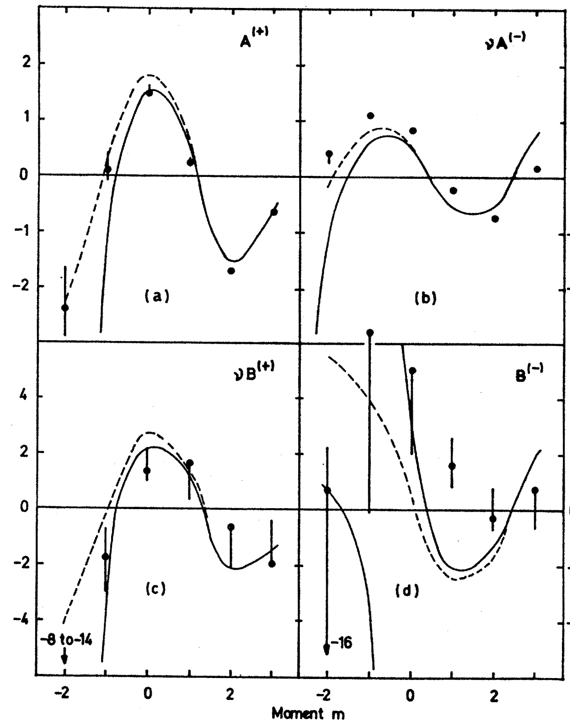

FIG. 7. Evaluation of the analogs of Eq. (32) at $t=0$ for $-2\leqslant m$ (integral only) $\leqslant3$ for the four amplitudes $A^{(+)}$, $\nu A^{(-)}$, $\nu B^{(+)}$, and $B^{(-)}$. The normalization is the same as for Eq. (12) (units, GeV=1). Also, $\nu_0=\mu+t/4M$ was used. The points and error bars have the same meaning as for the other figures; they are for the left-hand side of Eq. (12) evaluated for the wrong crossing-symmetry amplitudes $A^{(+)}$, $\nu A^{(-)}$, $B^{(-)}$, and $\nu B^{(+)}$ and correspond to the integral in Eq. (32). The naive (full line) and the modified (dashed line) Regge expectations are calculated with the same high-energy parameters as for Figs. 2 and 3 and for $\lambda=m_K$. The infinities at $m=-1-\alpha_i$, $-2-\alpha_i$, $-1-\alpha_i$, and α_i for the amplitudes $A^{(+)}$, $\nu A^{(-)}$, $\nu B^{(+)}$, and $B^{(-)}$, respectively, (where α_i refers to any contributing Regge pole) occur for the unmodified (full line) case [$b(m)=0$], but not in the modified model [$\tilde{b}(m)=0$] which agrees better with the low-energy integrals.

$A^{(+)}$, $\nu A^{(-)}$, $\nu B^{(+)}$, and $B^{(-)}$. In this case, to avoid the necessity for real-part data below the physical KN threshold, we used $\nu_0=\mu+t/4M$ as for the FESR's. Indeed the integrals evaluated are formally the same as for the FESR's of Eq. (12) except for an interchange of the (+) and (−) labels. Again the high-energy contribution (within a 10% error due to neglecting ν_0^2 relative to ν_1^2) is in good agreement if $\tilde{b}(m)=0$ (dashed line) and not if $b(m)=0$ (continuous line).

Using this value of ν_0, there is no t singularity until $\nu_0=-\nu_\Lambda$ at $t\simeq-1.05$, so that the t dependence of the expression should be of value if the background is really negligible. We find the P' dip again in $A^{(+)}$ but no ω crossover at all for $A^{(-)}$. In view of the extra assumptions involved in carrying the model in Eq. (31) to $t\neq0$ for these sum rules, we are not able to make a definite conclusion about this apparent lack of ω crossover. Introduction of adjustable parameters (to fix up, for example, the ω crossover for $A^{(-)}$) would reduce the chances of learning more from these sum rules.

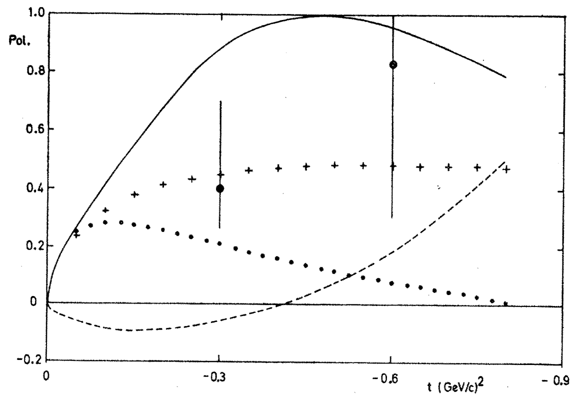

FIG. 8. $K^{\pm}p$ polarizations at $P_{lab}=1.46$ GeV/c versus t. Continuous and dashed curves are for K^-p scattering and are the predictions for choices (I) and (II), respectively, as described in the text. The points with error bars are from Ref. 11; the error bars represent the range of values in this energy region. The K^+p predictions are shown with crosses and dots for choices (I) and (II), respectively.

In conclusion, we have shown that the "off l-shell" Regge-pole contributions can be calculated by using the wrong-crossing generalized moment sum rules. These depend on the m-plane background amplitude which seems to be negligible for our value of ν_1 and m sufficiently large.

6. PREDICTIONS

In order to predict any specific feature of KN scattering, it is necessary to extract the invariant amplitudes and their energy dependence from our FESR results. Having obtained the invariant amplitudes, one can calculate any observable like $d\sigma/dt$ and polarization for KN scattering at high energies or a local energy-averaged value at lower energies. For instance, the modulus and phase of the $K_2{}^0p \to K_1{}^0p$ regeneration amplitude can be evaluated; it is dominated by the ω-exchange contribution.

If the FESR evaluations had negligibly small errors, the extraction of the invariant amplitudes could be accomplished by fitting the left-hand side of Eq. (12) to a sum of effective poles and determining the $\alpha_i(t)$ and $\beta_i(t)$ for each amplitude at each t value from the results for the different m values. In practice, however, these errors are not small, and we choose a less ambitious procedure (which we call I): We employ effective trajectories[57] $\alpha(t)$ deduced qualitatively from previous high-energy fits, in order to extract the effective residues $\beta(t)$ from our $m=0$ and $m=1$, FESR results with our favored data set. This is somewhat inconsistent since the phase of the amplitude may not correspond with the trajectory $\alpha(t)$. An alternative (which we call II) is to use (as mentioned previously in Sec. 4C and employed for Figs. 2–7) the high-energy fits of Phillips and Rarita[15] and of Derem and Smadja[22] with the modifications that $\nu B/A=+1$, $+1$, $+1.5$ for P, P', and ω respectively—although this representation of our FESR results is quantitatively rather poor, especially for Re$A^{(-)}$ and Re$B^{(+)}$ [Figs. 3(c) and 2(d), respectively]. We believe that these choices (I) and (II) represent qualitatively the correct amplitudes; the quantitatively exact answer may perhaps lie in between the two if and when the predictions with (I) and (II) differ appreciably.[58]

In Fig. 8, we show the $K^{\pm}p$ polarizations resulting from the above two choices for $p_{lab}=1.46$ GeV/c, our matching energy. Also shown is an energy-averaged representation of the K^-p polarization data of Daum *et al.*[11] from 1.4 to 2.3 GeV/c, the error bars showing the range of values encountered in their energy region. The experimental values lie between our predictions for the two choices, (I, the FESR results) and (II, the expected extrapolation of the modified high-energy Regge fit), at least up to $-t\sim0.6$ beyond which the reliability of these predictions decreases. In our preliminary report[13] on the sum-rule evaluations, we had used a modification of choice (I): we did not use directly the FESR results for Re$A^{(-)}$ and Re$B^{(-)}$, but determined them from the FESR results for the corresponding imaginary parts (Im$A^{(-)}$ and Im$B^{(-)}$, respectively), assuming some effective α values; this leads to predictions, for K^-p polarization, of about 40% at $-t\sim0.3$ and about 100% at $-t\sim0.6$ (as is indeed observed[11] experimentally), with the K^+p (K^-p) polarization being larger than the K^-p (K^+p) polarization for $-t$ less (greater) than approximately 0.3.

We hope that our results will be qualitatively valid also at higher energies[58] with, of course, a reduction in normalization. Previous high-energy fits with $B_\omega=0$ and the opposite sign of B_{A_2} had predicted a large negative K^-p polarization and a small K^+p polarization. One should note that in an exchange-degeneracy limit for the residues (at all t values) where $\nu B/A$ is the same for ρ and A_2, the polarization will be zero for K^+n and K^-p charge-exchange scattering. Further, if P' and ω (as well as the ρ and A_2) were also each degenerate in their trajectories and residue functions, the polarization and the real part of the forward-scattering amplitude would both be zero in the K^-p case and both positive in the K^+p case. Also, the two charge-exchange cross sections would be identical, $K^-p\to\bar{K}^0n$ having a purely imaginary amplitude and $K^+n\to K^0p$, a purely real one.

Another source of difficulty for the Regge-pole model in the intermediate-energy region has been the K^+n charge-exchange data at 2.3 GeV/c, as discussed by

[57] The effective α values we used for our $m=0$ and $m=1$ results are $(0.9+0.3t)$ for Im$A^{(+)}$; $(0.6+0.4t)$ for Im$B^{(+)}$; $(0.5+0t)$ for Re$A^{(+)}$ and Re$B^{(+)}$; $(0.5+0.6t)$ for Im$A^{(-)}$ and Re$A^{(-)}$, and $(0.5+0.8t)$ for Im$B^{(-)}$ and Re$B^{(-)}$. Only the Im$B^{(+)}$ sum rule for $m=0$ is sensitive to the chosen α values because the corresponding denominator is just α.

[58] Future high-energy fits (Ref. 47) incorporating the information that the FESR's give can help to make these predictions quantitative.

Rarita and Schwarzchild,[59] who found that the conventional Regge fits gave only half the differential cross section needed in the peak region ($-t\sim0.2$). This process is spin-flip-dominated with the ρ and A_2 trajectory exchanges contributing; the sign change of the spin-flip amplitude B_{A_2} (retaining everything else unchanged as in Ref. 59) is enough to increase the predictions by up to 50% for $-t\sim0.2$ without the need to introduce a ρ' contribution. Hopefully, a researching[47] of the parameters after one takes into account this sign change will make theory agree with the experimental cross section even better.

7. CONCLUSIONS

We have seen that FESR's can provide a very useful tool to determine several features of Regge-pole parameters. If one had a complete and well-determined phase-shift analysis, one could hopefully learn something also about the lower-lying Regge poles which would not be very important to the high-energy fits, but could be important at the low matching energy that we have to use. Even with the present state of the low-energy KN phase-shift analysis, we have learnt some useful things. We now summarize these.

ω. Our sum-rule results, as far as they go, are consistent with the usual explanation of the crossover phenomenon. These results are based mainly on the sum rules involving the imaginary parts of the amplitudes A and B. Our sum-rule results involving the real parts are not as reliable as the ones for imaginary parts. If it were not for the lack of a zero in $\mathrm{Re}A_\omega$ as determined by our sum rules for $\mathrm{Re}A^{(-)}$, we should be unreserved about our confirmation of the usual ω-crossover mechanism of only a single pole with all residues passing through zero at $t=t_0$ because of factorization.

We find no evidence of a wrong-signature nonsense zero in B_ω for $-t\lesssim0.8$. This is in contradiction with what is expected for a trajectory function $\alpha_\omega=0.45+0.9t$ found by Contogouris *et al.*[20] from an analysis of the ω contribution in the reaction $\pi N\to\rho N$. Our FESR results would prefer a flatter trajectory for the effective ω contribution.

We find $(\nu B/A)_\omega=+1$ to $+3$ for $-t\lesssim0.6$ which, again, is in contradiction to what has been assumed in high-energy fits which have taken this ratio to be either zero[15] or negative.[14]

P and P′. We find $(\nu B/A)_{P,P'}\sim+1$ which is, again, of opposite sign to that in high-energy fits. This agrees with the πN FESR results of Barger and Phillips.[10] Also, we find some evidence of the no-compensation mechanism type coupling for the P', $\alpha_{P'}$ passing through zero at $-t\sim0.5$. Our results support exchange degeneracy of P' and ω for the ratio of the residues $\nu B/A$, though $\alpha_{P'}$ and α_ω are not found to be degenerate. For example, we suggest that $\alpha_{P'}$ has a zero at $-t\sim0.5$, while α_ω has no zero for $-t\lesssim0.8$.

ρ. We have not really learnt anything about the ρ-trajectory contribution from the present analysis. We have taken the ρ Regge pole to be well known and used it to teach us something about the ω pole.

A_2. Our results would want $(\nu B/A)_{A_2}=+10$ (nearly the same as for ρ) which, again, is of opposite sign to that previously used in some high-energy fits.[15]

The situation is somewhat confused about the type of mechanism of coupling that the A_2 chooses. Our sum-rule results (and also the $\pi\eta\to\pi\rho$ FESR's in the resonance approximation) would prefer either the Gell-Mann mechanism, or else no zero in α_{A_2} for $-t<1$ approximately, while the resonance-approximation FESR's for the KN system could be consistent with the no-compensation mechanism and the photoproduction sum rules of Chu and Roy[41] would perhaps like either the Chew mechanism or the no-compensation mechanism. A really convincing FESR analysis in this context would be very welcome.[60] Again, our results support exchange degeneracy of ρ with A_2 for the ratio of the residues $\nu B/A$ for $t\sim0$. We cannot really say very much about the A_2 trajectory; our input data are not accurate enough to allow one to calculate α_{A_2} explicitly.

We have not considered the possibility of more than one t-channel pole having the quantum numbers of the ρ and A_2. Our A_2 and ω contributions, therefore, are only effective ones.

Favored data set. Though the input data are not very well determined, a choice of the favored data set which leads to the best agreement with extrapolations down to our matching energy of the high-energy Regge fits is possible. This may not be *the* correct and final representation of the data at low energies (up to $\sqrt{s}=2$ for our purpose). We would prefer Kim's couplings g_Λ^2 and g_Σ^2 for the Born diagrams, a negligible $Y_1^*(1385)$ coupling (as found by Kim) and the nonresonant (type IV) K^+p phase-shift solution of Lea *et al.*[27]

Predictions. Having been able to determine the signs and the t dependence of the Regge-pole spin-flip contributions in KN scattering, we see that the older signs of $\nu B/A$ for P, P', ω, and A_2 are not consistent with our results. If we take them as our sum rules prefer, we are able to predict quite confidently the expected polarization in K^+p and K^-p elastic scattering and our prediction agrees with the available[11] experimental data on K^-p polarization while the previous fits gave the wrong sign of the polarization. Also, we are able to remove, at least partially, the other difficulty that the Regge-pole theory meets in the KN system: The $K^+n\to K^0p\,d\sigma/dt$ comes out in better agreement with the experimental

[59] W. Rarita and B. M. Schwarzschild, Phys. Rev. **162**, 1378 (1967).

[60] Note that our result (either the Gell-Mann mechanism or $\alpha_{A_2}\neq0$ for $-t\lesssim0.8$) is based mainly on our $\mathrm{Re}B^{(+)}$ sum rule which, apart from involving real parts, gets non-negligible contributions from the P'. If one had $|\mathrm{Re}B_{P'}|$ for $-t\sim1$ much larger than $|\mathrm{Re}B_{P'}|$ for $-t\sim0$ one could perhaps allow a zero in the $\mathrm{Re}B_{A_2}$ sum rule and, therefore, a Chew or a no-compensation type mechanism which corresponds to $B\sim\alpha^2$ near $\alpha=0$.

data than the previous older prediction because of the change of sign of the ratio $(\nu B/A)_{A_2}$.

Other sum rules. We have considered generalized Schwarz sum rules which evaluate the "off l-shell" amplitudes in the Khuri plane. We find the background to be small in general, so that these relations are satisfied with Regge-pole parameters alone. The t dependence of these relations implies that the background amplitude has cuts, however, and this limits the applications, since further parameters to describe the background will then be needed.

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Analyticity and Broken $SL(2,C)$ Symmetry for Regge Families*

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Explicit constraints on the mass dependence of daughter Regge trajectories, near zero mass, are obtained for fermion trajectories contributing to πN scattering. Both the analyticity and the group-theoretic approaches are investigated. We find agreement between these two methods, but disagreement between our constraints and those previously published. For the dependence on the mass W of the kth daughter trajectory with parity designation $\pm$, we find that $\alpha_k^{(\pm)}(W)=\sigma-k\pm A(\sigma-k+\frac{1}{2})W+[B_1+B_2(\sigma-k)(\sigma-k+1)+A^2(\sigma-k+\frac{1}{2})]W^2\pm\cdots$, where σ, A, B_1, and B_2 are constants over the family. For each of the two methods, we stress the assumptions leading to the MacDowell symmetry evident above.

I. INTRODUCTION

In a recent paper[1] it has been pointed out that two different approaches to daughter Regge trajectories, analyticity and group-theoretic, lead to the same results for the scattering of spinless particles. Mathematically the equivalence of these two approaches has been established.[2] Namely, in order to make the analyticity requirement for scattering amplitudes compatible with Lorentz invariance and Regge behavior, it is necessary and sufficient to classify singularities according to the irreducible representations of the homogeneous Lorentz group $SL(2,C)$. However, at the practical level, the ways by which these approaches lead to a given result differ considerably. At present their relationship is by no means trivial.[3] In this paper we compare these approaches for fermion trajectories, with particular emphasis on the mass formula that they yield. Even though the two methods agree, we find that each of the methods seems to have some advantages over the other. We reserve a more detailed discussion of this point for later. The mass formula that we obtain does not agree completely with that obtained previously by Domokos and Surányi,[4] hereafter referred to as DS, using their group-theoretic method. In order to facilitate comparison, our group-theoretic approach closely parallels that of DS. In our approach this disagreement is resolved by recognizing some subtleties associated with the use of wave functions having nonphysical angular momentum values.

In Sec. II we examine the implication of analyticity on the πN scattering amplitude near $u=0$ (u is the square of the momentum transfer for exchange scattering) in some detail, using the method of Ref. 1. In Sec. III we use our apparently modified version of the perturbation theory developed in DS to reproduce the results of Sec. II. Section IV contains some discussion concerning the relative merit of the two approaches and the degree to which the daughters are determined by experiment.

II. ANALYTICITY APPROACH TO πN SCATTERING AMPLITUDE

The πN scattering is dominated in the backward region by the exchange of fermion trajectories. For this reason we go to the u channel and define the invariant

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