Lorentz-Pole Model and Polarization in High-Energy Charge-Exchange Pion-Nucleon Scattering

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In this paper we propose a mechanism for explaining the polarization in high-energy charge-exchange πN scattering, based on $O(3,1)$ partial-wave analysis. We found that the nonvanishing polarization may be understood in terms of one Lorentz pole, $\sigma_\rho=\alpha_\rho+1$, and that it is due to the interference of ρ with another Regge pole ρ' belonging to the family $j_0=0$ and given in terms of σ_ρ by the relation $\alpha_{\rho'}=\sigma_\rho-2$.

It is well known that in order to explain the polarization phenomenon in high-energy meson-nucleon charge-exchange scattering, one has to modify the single Regge-pole model[1] (ρ trajectory) either by introducing other singularities in the J plane (Regge cuts, ρ' Regge pole) or by taking into account direct channel effects, for example the effect of tails of low-energy resonances at high energies.

The aim of this work is to study the high-energy behavior of polarization within a simple model in which only one Lorentz pole $\sigma_\rho(t)=\alpha_\rho(t)+1$, degenerate with respect to various quantum numbers (j_0, Lorentz signature χ), is taken into account.

The expansion of the helicity amplitudes of the process $\pi^- p\to\pi^0 n$ in terms of the homogeneous Lorentz group[2] has been studied by Akyeampong, Boyce, and Rashid.[3] According to their results, the dominance of a Lorentz pole at $\sigma_\rho(t)=\alpha_\rho(t)+1$ may reproduce all the qualitative features of the experimental angular distribution, provided that the ρ trajectory belongs to a mixed representation with $j_0=0$ and $j_0=1$.

In what follows we denote by $f_{++}(s,t)$ and $f_{+-}(s,t)$ the non-spin-flip and the spin-flip helicity amplitudes, respectively, corresponding to the direct channel (s channel) of the process

$$\pi^-(\lambda_1)+p(\lambda_2)\to\pi^0(\lambda_3)+n(\lambda_4)\,, \tag{1}$$

where λ_1, λ_3 are the helicities of the mesons, and λ_2, λ_4 the helicities of the nucleons. The $O(3,1)$ expansion of $f_{++}(s,t)$ and $f_{+-}(s,t)$ is given by the following formulas:

$$f_{++}(s,t)=-\frac{1}{2\pi i}\int_{-i\infty}^{i\infty}\sigma^2 d\sigma\, T_{++}{}^{0}(\sigma,t)d_{000}{}^{0\sigma}(\xi_t)\,,$$

$$f_{+-}(s,t)=-\frac{1}{2\pi i}\frac{\sqrt{2}}{3}|t|^{1/2}\int_{-i\infty}^{i\infty}d\sigma[\sigma^2T_{+-}{}^{0}(\sigma,t)d_{1-11}{}^{0\sigma}(\xi_t)$$
$$-(1-\sigma^2)T_{+-}{}^{1}(\sigma,t)d_{111}{}^{1\sigma}(\xi_t)]\,, \tag{2}$$

where $T_{++}{}^{j_0}(\sigma,t)$, $T_{+-}{}^{j_0}(\sigma,t)$ are the $O(3,1)$ "partial-wave" amplitudes ($j_0=0$, 1) and

$$\cosh\xi_t=\frac{s-u}{(4m_N{}^2-t)^{1/2}(4m_\pi{}^2-t)^{1/2}}\,.$$

Before introducing Lorentz poles in the σ plane, we modify Eqs. (2), defining reduced amplitudes $T_{++}{}^{j_0\chi}(\sigma,t)$ and $T_{+-}{}^{j_0\chi}(\sigma,t)$ having $O(3,1)$ signature χ. Following Akyeampong, Boyce, and Rashid,[4] we may write ($\chi=\pm1$):

$$f_{++}(s,t)=-\frac{1}{2\pi i}\sum_\chi\int_{1/2-i\infty}^{1/2+i\infty}\frac{\sigma^2}{\sin\pi\sigma}T_{++}{}^{0\chi}(\sigma,t)$$
$$\times\frac{1-\chi e^{i\pi\sigma}}{2}d_{000}{}^{0\sigma}(\xi_t)d\sigma\,,$$

$$f_{+-}=-\frac{1}{2\pi i}\frac{\sqrt{2}}{3}|t|^{1/2}\sum_\chi\int_{3/2-i\infty}^{3/2+i\infty}\frac{d\sigma}{\sin\pi\sigma}\frac{1-\chi e^{i\pi\sigma}}{2}$$
$$\times[\sigma^2T_{+-}{}^{0\chi}(\sigma,t)d_{1-11}{}^{0\sigma}(\xi_t)$$
$$-(1-\sigma^2)T_{+-}{}^{1\chi}(\sigma,t)d_{111}{}^{1\sigma}(\xi_t)]\,. \tag{3}$$

We assume now that the helicity amplitudes (3) are dominated by one Lorentz pole σ_ρ. We also assume that the location of the Lorentz pole is independent of j_0. Therefore all amplitudes $T_{++}{}^{0\chi}$, $T_{+-}{}^{0\chi}$, $T_{+-}{}^{1\chi}$ have a pole at the same point $\sigma=\sigma_\rho(t)$ corresponding to a "degenerate" trajectory in the sense that it accommodates both $j_0=0$ and $j_0=1$ states.[3] If we shift the path of integration in Eqs. (3) to the left, we obtain the following expressions for the contribution of the

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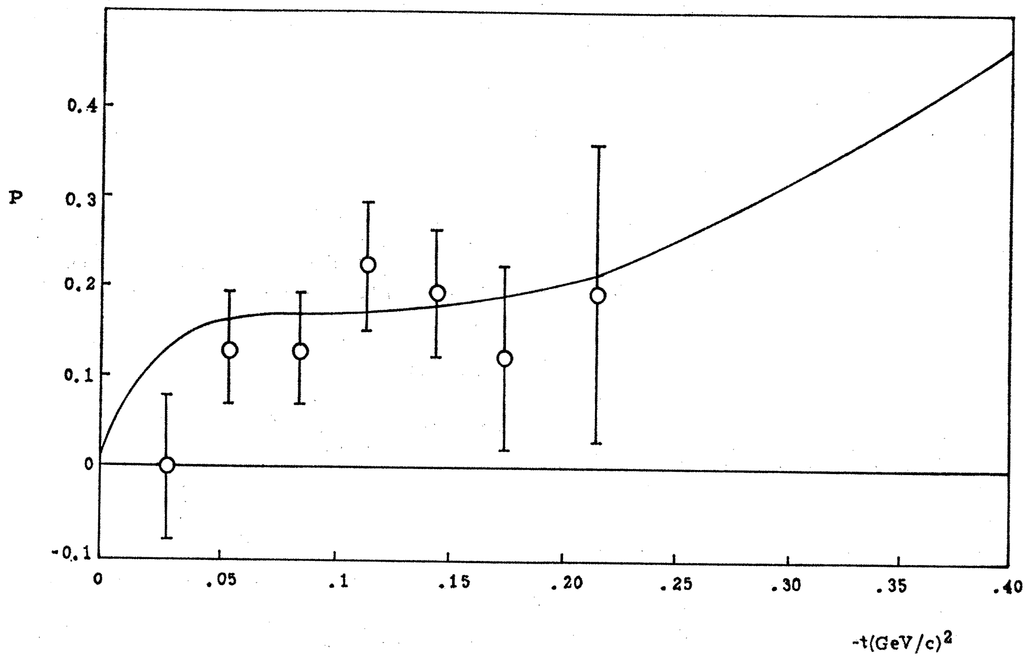

FIG. 1. The polarization of the neutron in the reaction $\pi^- p \to \pi^0 n$ for pion momentum $P_{\rm lab}=5.9$ GeV/c is plotted. The solid line represents our result. Experimental points are also given.

Lorentz pole σ_ρ:

$$f_{++}=-\sum_{\chi}\frac{\sigma_\rho^2}{\sin\pi\sigma_\rho}\frac{1-\chi e^{i\pi\sigma_\rho}}{2}d_{000}{}^{0\sigma_\rho}(\xi_t)\beta_{++}{}^{0\chi}(t)\,,$$

$$f_{+-}=-\tfrac{1}{3}\sqrt{2}|t|^{1/2}\sum_{\chi}\frac{1}{\sin\pi\sigma_\rho}\frac{1-\chi e^{i\pi\sigma_\rho}}{2}\times[\sigma_\rho^2\beta_{+-}{}^{0\chi}(t)d_{1-11}{}^{0\sigma_\rho}(\xi_t)-(1-\sigma_\rho^2)\beta_{+-}{}^{1\chi}(t)d_{111}{}^{1\sigma_\rho}(\xi_t)]\,. \quad (4)$$

It is easy to show that the signatures of the Regge trajectories associated with the Lorentz pole σ_ρ are given by the formula[1]

$$\xi_n=\chi(-1)^n\,, \quad (5)$$

where ξ_n is the signature of the nth member of the family: $\alpha_n=\sigma-n-1$. Since the only Regge poles that are allowed in this process are those with the quantum numbers of the ρ meson, namely with negative signature ($\xi=-1$) and natural parity ($\eta\xi=+1$), we distinguish the following coupling schemes (η is the parity of the exchanged particle):

(a) The leading Regge particle, associated with the amplitudes $T_{++}{}^{0\chi}\sim\beta_{++}{}^{0\chi}d_{000}{}^{0\sigma_\rho}$ and $T_{+-}{}^{1\chi}\sim\beta_{+-}{}^{1\chi}d_{111}{}^{1\sigma_\rho}$, corresponds to the member $n=0$ of the family $\alpha_n=\sigma_\rho-n-1$, and therefore, according to (5), we must have $\chi=-1$ since $\xi=-1$ (ρ meson). Hence, the non-spin-flip amplitude f_{++} and the leading part of the spin-flip amplitude ($\beta_{+-}{}^{1\chi}$) are controlled by the exchange of the $j_0=0$ and $j_0=1$ components of the ρ trajectory.

(b) The leading Regge trajectory, associated with the amplitude $T_{+-}{}^{0\chi}\sim\beta_{+-}{}^{0\chi}d_{1-11}{}^{0\sigma_\rho}$ which contributes to the spin-flip amplitude [Eqs. (4)], corresponds to the member $n=1$ of the family $\alpha_n=\sigma_\rho-n-1$, as can easily be shown by decomposing the $O(3,1)$ d function $d_{1-11}{}^{0\sigma_\rho}$ in terms of $O(2,1)$ matrix elements, following the formalism of Sciarrino and Toller.[5] The contribution of this Regge pole is not zero only if $\xi=-1$, i.e., if $\chi=+1$, according to formula (5).

We identify this Regge particle with the ρ' meson which therefore belongs to the following class: $j_0=0$, $\eta\xi=+1$, $\xi=-1$.

Moreover, the trajectory $\alpha_{\rho'}(t)$ is given in terms of $\alpha_\rho(t)$ by the following simple relation:

$$\alpha_{\rho'}(t)=\alpha_\rho(t)-1\,. \quad (6)$$

Keeping now the appropriate signatures in Eqs. (4) and using the asymptotic expressions of the d functions, we may write the contributions of ρ and ρ' as follows:

$$f_{++}(s,t)=\frac{\beta_{++}{}^{0-1}}{\sin\pi\alpha_\rho}\left(\frac{1-e^{-i\pi\alpha_\rho}}{2}\right)(\alpha_\rho+1)\left(\frac{s}{2m_Nm_\pi}\right)^{\alpha_\rho},$$

$$f_{+-}(s,t)=\sqrt{3}|t|^{1/2}\frac{(\alpha_\rho+1)\beta_{+-}{}^{01}}{(\alpha_\rho+2)\sin\pi\alpha_\rho}\times\left(\frac{1+e^{-i\pi\alpha_\rho}}{2}\right)\left(\frac{s}{2m_Nm_\pi}\right)^{\alpha_\rho-1}+\sqrt{3}|t|^{1/2}\frac{\beta_{+-}{}^{1-1}\alpha_\rho}{\sin\pi\alpha_\rho}\times\left(\frac{1-e^{-i\pi\alpha_\rho}}{2}\right)\left(\frac{s}{2m_Nm_\pi}\right)^{\alpha_\rho} \quad (7)$$

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[5] A. Sciarrino and M. Toller, Nota Interna No. 108, 1966, Istituto di Fisica "G. Marconi," Università di Roma (unpublished).

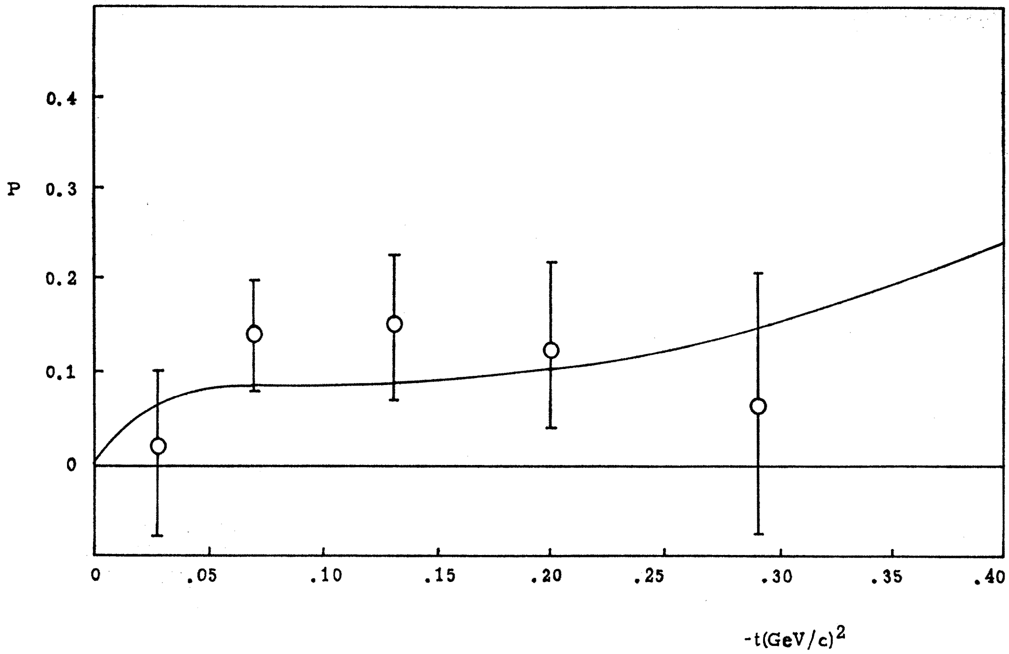

FIG. 2. The polarization of the neutron in the reaction $\pi^- p \to \pi^0 n$ for pion momentum $P_{\rm lab}=11.2$ GeV/c is plotted. The solid line represents our result. Experimental points are also given.

We observe that, although the angular distribution is dominated by the exchange of the ρ meson alone, the nonvanishing polarization at high energies is due to the interference between the ρ and ρ' terms in Eqs. (7). This mechanism coincides with the $\rho+\rho'$ model introduced for the explanation of the polarization within the Regge theory[6,7] so the identification of the amplitude $T_{+-}^{0\chi} \sim \beta_{+-}^{0\chi} d_{1-11}^{0\sigma_\rho}$ with the conventional ρ' meson exchange is therefore justified. The details of the mechanism which explains the polarization data within the present model are as follows.

The differential cross section $d\sigma/dt$ of the reaction $\pi^- p \to \pi^0 n$ is given by the formula

$$\frac{d\sigma}{dt}=\frac{1}{2\pi s^2}(|f_{++}|^2+|f_{+-}|^2) \quad (s\to\infty) \tag{8}$$

and the polarization P of the neutron by the expression

$$P=-\frac{2\,\mathrm{Im}(f_{++}f_{+-}^*)}{|f_{++}|^2+|f_{+-}|^2}. \tag{9}$$

From Eqs. (7) and (9) we easily find that the energy dependence of the polarization P in the asymptotic region ($s\to\infty$) is $P\sim 1/s$. The experimental results in πN charge-exchange scattering at 5.9 and 11.2 GeV/c give for the mean value of P in a given momentum transfer integral, the value $\langle P_{11.2}\rangle/\langle P_{5.9}\rangle=0.8\pm0.2$. This experimental result[8] indicates a slower decrease of P with energy as compared with our prediction, but nevertheless our result is consistent with the above value within the experimental errors (Figs. 1 and 2). The explicit form of P is found using Eqs. (7) and (9). We obtain

$$P=\frac{2\sqrt{3}\beta_{++}^{0-1}\beta_{+-}^{01}(\alpha_\rho+1)^2|t|^{1/2}\cot\frac{1}{2}\pi\alpha_\rho}{(\alpha_\rho+2)[\frac{1}{4}(\beta_{++}^{0-1})^2(\alpha_\rho+1)^2+3|t|\alpha_\rho^2(\beta_{+-}^{1-1})^2]}\times\left(\frac{2m_\pi m_N}{s}\right). \tag{10}$$

The ρ trajectory is taken to be $\alpha_\rho(t)=0.57+0.91t$.

The residues β_{++}^{0-1}, β_{+-}^{1-1} treated as constants, are determined by fitting the angular distribution. In Eq. (10) the residue β_{+-}^{01} remains a free parameter and it is chosen to fit the polarization[8] data at 5.9 and 11.2 GeV/c in the process $\pi^- p\to\pi^0 n$. The results are shown in Figs. 1 and 2.

To conclude, we have shown that within the $O(3,1)$ partial-wave expansion of the πN charge-exchange scattering amplitudes, two leading Regge poles emerge and they are identified with the ρ and ρ' mesons. The ρ trajectory corresponds to the first member ($n=0$) of the family $\alpha_n=\sigma_\rho-n-1$ generated by the Lorentz pole σ_ρ with quantum numbers $\eta\chi=+1$, $\chi=-1$, ($j_0=0$, 1).

The ρ' trajectory corresponds to the leading Regge pole which contributes to the amplitude $T_{+-}^{0\chi} \sim\beta_{+-}^{0\chi}d_{1-11}^{0\sigma_\rho}$ [Eqs. (4)]. It is generated by a Lorentz pole with quantum numbers $j_0=0$, $\eta\chi=+1$, $\chi=+1$ and lying on the same trajectory $\sigma_\rho(t)$. Moreover, the ρ' trajectory is given in terms of α_ρ by a simple relation $\alpha_{\rho'}=\alpha_\rho-1$. We wish to point out that the ρ' trajectory

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[7] H. Högaasen and W. Fischer, Phys. Letters 22, 516 (1966).

[8] P. Bonamy *et al.*, Phys. Letters 23, 501 (1966).

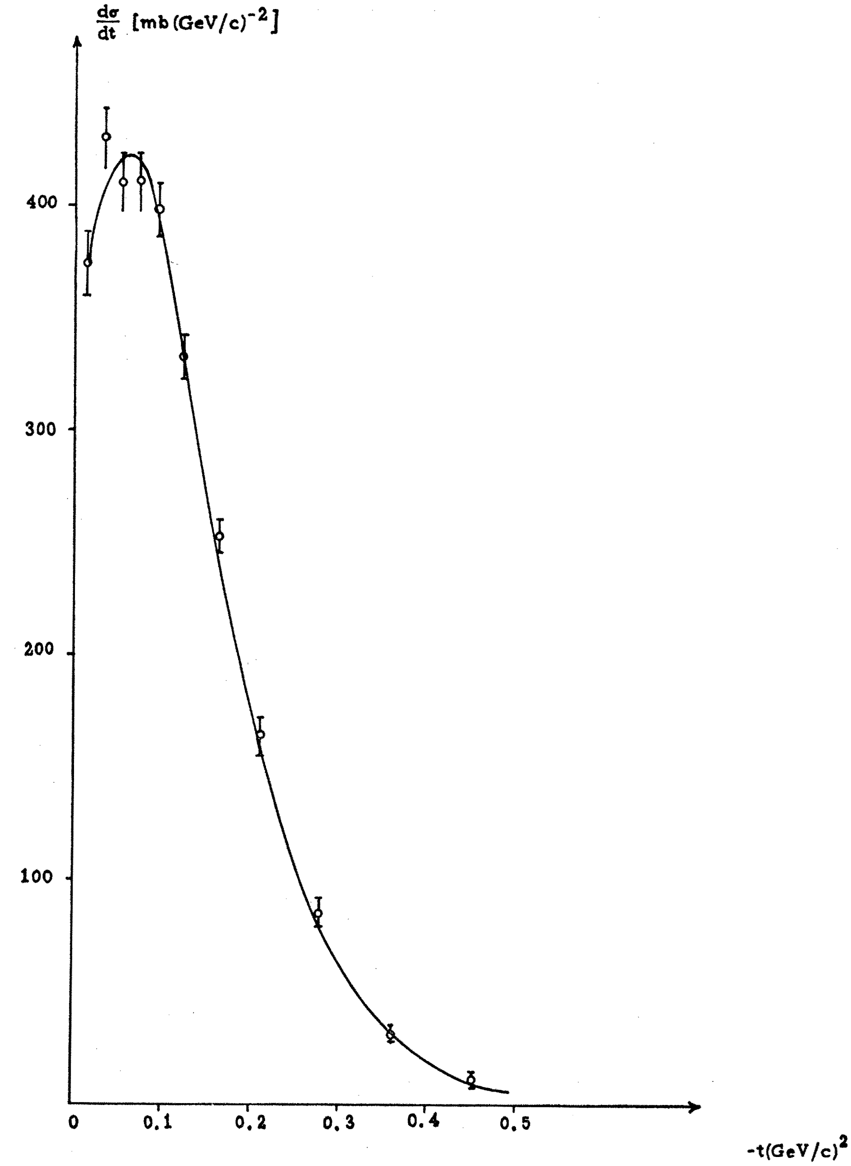

FIG. 3. In this figure we plot the angular distribution of the reaction $\pi^- p \to \pi^0 n$ using the parameters of Table I, for energy of the incident pion, $P_{\rm lab}=5.9$ GeV/c. Experimental points are also given.

must not be interpreted as a daughter of the ρ trajectory, since these Regge poles correspond to different $O(3,1)$ signatures. From the interference of these two trajectories, a nonzero polarization is obtained and a reasonable fitting of the data is given.

TABLE I. The parameters of the Lorentz pole σ_ρ which have been used to fit the angular distribution and the energy dependence of the present charge-exchange data are shown in this table. The residue $\beta_{++}{}^{0-1}$ is given in units mb$^{1/2}$ and the residues $\beta_{+-}{}^{1-1}$, $\beta_{+-}{}^{01}$ in units mb$^{1/2}$/GeV.

$\alpha_\rho(0)$	0.6
$\alpha_\rho'(0)$	0.9
$\beta_{++}{}^{0-1}$	47
$\beta_{+-}{}^{1-1}$	364
$\beta_{+-}{}^{01}$	6.2×10^3

Finally, we compare the present model with the results of Regge theory where the parameters of the ρ' are determined by fitting the experimental data. (See Table I and Fig. 3). In order to satisfy the constraints of both the polarization data and the superconvergence relations, Sertorio and Toller[9] introduced a conspirator ρ' with Lorentz quantum number $j_0=1$ in disagreement with the present model according to which ρ' is a nonconspiring Regge pole with $j_0=0$. More recently, Gajdicar, Logan, and Moffat,[10] using the $\rho+\rho'$ Regge model, have analyzed in detail the πN charge-exchange scattering by fitting the differential cross section, the

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charge-exchange polarization, the πp total cross section, the real forward amplitude $D^{(-)}$, and the non-spin-flip superconvergence relations. They found that it is not necessary to introduce a conspiring ρ' trajectory, but that it is possible to satisfy the superconvergence relations and the polarization data with a nonconspiring ρ' ($j_0=0$). Moreover, one of the two solutions of their model gives $\alpha_{\rho'}(0)=-0.5$, $\alpha_\rho(0)=0.57$, in very good agreement with the prediction $\alpha_{\rho'}=\alpha_\rho-1$.

Finally, Högaasen and Fischer,[7] in their attempt to fit the experimental data on nucleon-nucleon charge-exchange scattering, found for the intercept of the ρ' trajectory the value $\alpha_{\rho'}(0)=-0.63$, again in good agreement with the results of our model.

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Intrinsic Invariance Groups, Mass Operators, and Linear Wave Equations*

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Relativity theory permits motions of a free-spinning particle in which the instantaneous velocity and linear momentum are not collinear. Therefore, it is necessary to specify two invariance groups in order to completely describe the space-time symmetry properties of a particle with intrinsic spin. If the quantal description of such a particle is given by a covariant linear field equation, then the *external* space-time symmetry is specified by the 10-element Poincaré group generated by the linear four-momentum and total angular momentum operators. However, invariance of the field equation under external inhomogeneous Lorentz transformations does not complete the algebra of the 10-element group generated by the instantaneous four-velocity and spin angular-momentum operators. Several formulations of linear field equations admitting a mass-spin spectrum are based on different choices for this latter, *intrinsic*, space-time symmetry group. We begin with the simplest choice, namely, intrinsic Poincaré invariance, and establish the formal connection between these several formulations by constructing a unitary transformation which generates an infinite sequence of linear wave equations describing ascendingly more complex intrinsic space-time symmetry, but linked by a common mass operator. Of special interest is the infinitesimal transformation which generates the familiar intrinsic DeSitter group. Finally, some kinematical properties of this transformation are discussed for various proposed mass operators.

I. INTRODUCTION

Theoretical attempts to explain the proliferation of elementary particles have, of late, led to extensive investigation of the class of Lorentz-covariant wave equations of the form,[1]

$$(i\Gamma_\mu P_\mu+Mc)\psi=0. \tag{1}$$

If $x_\mu\equiv(\mathbf{x},ict)$ is the instantaneous position four-vector, then the linear four-momentum is given by $P_\mu=-i\hbar\partial_\mu$, so that

$$(x_\mu,P_\nu)=i\hbar\delta_{\mu\nu} \tag{2}$$

specifies the conjugate relation between the position and momentum. Unlike the momentum, however, the remaining two operators appearing in (1), Mc and Γ_μ, do not possess unique representations although their physical interpretation is understood as follows.

First, the Lorentz frame defined by $P_\mu\equiv(0,iE/c)$ is the "momentum rest" frame, and is usually referred to as the rest frame. In this frame, Eq. (1) becomes

$$\Gamma_4 E\psi=Mc^2\psi, \tag{3}$$

so that Mc is a mass operator, whose specification should lead, via the eigenvalue equation (3), to the spectrum of rest energies admitted by (1). Explicit representation of the Lorentz-invariant mass operator Mc depends on the dynamical model studied, if one exists, and on the desired algebraic properties. It is most generally represented by a finite- or infinite-dimensional matrix, which may possess more structure than constant multiple of the unit matrix.[1]

Second, the covariant wave equation, (1), defines the proper-time Hamiltonian operator,[2]

$$H=i\Gamma_\mu P_\mu+Mc, \tag{4}$$

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[2] The role of proper time in quantum mechanics, and its application to the temporal evolution of the wave packet, is rigorously treated in the literature. See, for example, S. Shanmugadhasan, Can. J. Phys. **29**, 593 (1951); G. Szamosi, Nuovo Cimento, **20**, 1090 (1961); R. Schiller, Phys. Rev. **125**, 1116; **128**, 1402 (1962); G. N. Fleming, J. Math. Phys. **7**, 1959 (1966).