High-Energy Polarized-Photon Reactions*

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A general formalism is presented for the analysis of production and decay distributions of meson-baryon final states resulting from interactions of polarized photons and nucleons. Particular emphasis is given to high-energy reactions and Regge-pole exchange models. Circular polarization is shown to provide a test for the presence of two or more exchanges. Linear polarization parallel or perpendicular to the production plane allows the contributions of opposite J-parity exchanges to be separated. This formalism enables one to make use of data at all angles, rather than limit consideration to discrete points. Applications include the separation of diffractive and pion-exchange contributions to vector-meson production, and tests for non-pole-type exchanges in baryon-resonance production. Examination of pion photoproduction reveals that no simple test exists for the discrimination between pion conspiracy and evasion models.

I. INTRODUCTION

THE successes of the Regge-pole model for elastic scattering and quasi-two-body processes in meson-nucleon and nucleon-nucleon interactions have led to its application in high-energy photoproduction reactions.[1–6] Soon experiments with high-energy polarized photons will be possible.[7,8] It has been known for some time that in pion photoproduction by linearly polarized photons the J parity of the crossed-channel exchanges is closely related to the azimuthal angular dependence of the cross section.[9] Recent papers[10–14] have suggested that this mechanism may be useful in examining ambiguous features of the Regge-pole model, such as the importance of cuts, conspiracy and evasion, and relative strength of diffractive versus pion-exchange mechanisms. The purpose of this paper is to examine the general case of the two-body inelastic reaction $\gamma+N\rightarrow V+N^*$, where the photon is linearly or circularly polarized, the nucleon is unpolarized, and V and N^* are mesons and baryons with arbitrary spins. In Sec. II the formalism for the differential cross section, individual decay distributions, and joint decay correlations is presented, and expressions are given for production by unpolarized, circularly polarized, and linearly polarized photons. Applications to specific reactions are given in Sec. III, and Sec. IV contains general conclusions and discussion.

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1 R. W. Childers and W. G. Holladay, Phys. Rev. **132**, 1809 (1963).

2 G. Zweig, Nuovo Cimento **32**, 689 (1963).

3 J. S. Ball, W. R. Frazer, and M. Jacob, Phys. Rev. Letters **20**, 518 (1968).

4 D. Amati, G. Cohen-Tannoudji, R. Jengo, and Ph. Salin, Phys. Letters **26B**, 510 (1968).

5 M. P. Locher and H. Rollnik, Phys. Letters **22**, 696 (1966).

6 A. Ahmadzadeh and R. J. Jacob, Phys. Rev. **160**, 1359 (1967).

7 Stanford Linear Accelerator Center (SLAC) Conference on Polarized Photon Beams (unpublished).

8 R. H. Milburn, invited talk at Chicago APS meeting, 1968 (unpublished).

9 P. Stichel, Z. Physik **180**, 170 (1964).

10 A. V. Berkov and Yu. P. Nitkin, Yadern. Fiz. **5**, 650 (1967) [English transl.: Soviet J. Nucl. Phys. **5**, 461 (1967)].

11 J. Froyland and M. Vaughn, M.I.T. report, 1967 (unpublished).

12 A. Donnachie, Phys. Letters **25B**, 477 (1967); F. Cooper, Phys. Rev. **167**, 1314 (1968).

13 A. S. Krass, Phys. Rev. **165**, 1570 (1968); F. Cooper, *ibid.* **170**, 1602 (1968).

14 F. Cooper, Phys. Rev. Letters **20**, 643 (1968); **20** 1550(E) (1968).

II. FORMALISM

A. Joint Decay Distribution

Consider the s-channel reaction

$$\gamma+N\rightarrow V+N^*, \tag{1}$$

where N is a nucleon, V is a meson of spin J_1, and N^* is a baryon of spin J_2. For simplicity, only the case in which V decays into two or three spinless particles and N^* decays into one spin-0 and one spin-$\frac{1}{2}$ particle is treated. Then parity conservation reduces the number of independent decay amplitudes to one for each process, which may be absorbed into the normalization.[15] Let the direction of the decay particles from V be specified by $\Omega_1=(\theta_1,\varphi_1)$ with respect to some axis in the rest frame of V (in the case of three-particle decay, Ω_1 is the direction of the normal to the decay plane), and the direction of the decay particles from N^* in its rest frame by $\Omega_2=(\theta_2,\varphi_2)$. The angular distribution of the decay products may be written[16]

$$W(\theta_1\varphi_1,\theta_2\varphi_2)=\text{const}\times\sum_{mm'nn'}\rho_{mm',nn'}\,, \\ \times e^{i(m-m')\varphi_1}e^{i(n-n')\varphi_2}d_{m0}{}^{J_1}(\theta_1)d_{m'0}{}^{J_1}(\theta_1)f_{nn'}{}^{J_2}(\theta_2)\,, \tag{2}$$

where the d^J's are the usual rotation coefficients, and

$$f_{nn'}{}^{J}(\theta)\equiv\sum_{\lambda=\pm1/2}d_{n\lambda}{}^{J}(\theta)d_{n'\lambda}{}^{J}(\theta)\,. \tag{3}$$

The joint correlation coefficients $\rho_{mm',nn'}$ depend only on the production mechanism, and have a particularly simple form in the t-channel frame of Gottfried and

15 For decays into higher-spin particles, the same type of analysis may be carried out, but the subsequent decay of the intermediate decay particles must also be studied in some cases in order to extract all of the information.

16 K. Gottfried and J. D. Jackson, Nuovo Cimento **33** 309, (1964); H. Pilkuhn and B. E. Y. Svensson, *ibid.* **38**, 518 (1965).

Jackson.[16] The result for unpolarized target and beam is

$$\rho_{mm',nn'}=\sum_{\text{initial spins}} F_{\lambda n,\mu m}F_{\lambda n',\mu m'}^*/\sum_{\text{all spins}}|F_{\lambda n,\mu m}|^2, \quad (4)$$

where $F_{\lambda m,\mu n}$ is the c.m. helicity amplitude for the crossed t-channel reaction

$$\gamma+\bar{V}\rightarrow \bar{N}+N^*, \quad (5)$$

with helicities μ, m, λ, and n, respectively.

For polarized reactions, Eq. (4) would in general need some modification. However, since the photon has zero mass, the crossing matrix from reaction (1) to (5) is diagonal in photon helicity.[17] The summation over initial spins can be dropped and the index μ set to the values appropriate for the initial polarization state in the s channel. Hence for right- (left-) circular polarization we use only $\mu=+1(-1)$, and for linear polarization the appropriate combination of $\mu=\pm1$. Since the rotation coefficients have the symmetry properties

$$d_{\mu\lambda}{}^J=d_{-\lambda-\mu}{}^J=(-1)^{\mu-\lambda}d_{\lambda\mu}{}^J, \quad (6a)$$

which imply

$$f_{nn'}{}^J=f_{n'n}{}^J=(-1)^{n-n'}f_{-n-n'}{}^J, \quad (6b)$$

only certain linear combinations of the joint correlation coefficients are measurable. One can express the angular distribution in terms of these combinations, making use of (6a), (6b), and the Hermiticity property

$$\rho_{m'm,n'n}=\rho_{mm',nn'}^*. \quad (7)$$

If the normalization constant is adjusted so that the integrated distribution is unity, one obtains the result

$$W(\theta_1\varphi_1,\theta_2\varphi_2)=\frac{(2J_1+1)(2J_2+1)}{\pi^2}\sum_{m\geq|m'|}^{J_1}\epsilon_m\epsilon_{m-|m'|}d_{m0}{}^{J_1}(\theta_1)d_{m'0}{}^{J_1}(\theta_1)\sum_{n\geq|n'|}^{J_2}\epsilon_{n-|n'|}f_{nn'}{}^{J_2}(\theta_2)$$
$$\times[\cos(m-m')\varphi_1\cos(n-n')\varphi_2\operatorname{Re}(Z_{mm'}{}^{nn'}+Z_{m'm}{}^{nn'})-\sin(m-m')\varphi_1\sin(n-n')\varphi_2\operatorname{Re}(Z_{mm'}{}^{nn'}-Z_{m'm}{}^{nn'})$$
$$-\cos(m-m')\varphi_1\sin(n-n')\varphi_2\operatorname{Im}(Z_{mm'}{}^{nn'}+Z_{m'm}{}^{nn'})-\sin(m-m')\varphi_1\cos(n-n')\varphi_2\operatorname{Im}(Z_{mm'}{}^{nn'}-Z_{m'm}{}^{nn'})], \quad (8)$$

where

$$Z_{mm'}{}^{nn'}=\rho_{mm',nn'}+(-1)^{m-m'}\rho_{-m'-m,nn'}$$
$$+(-1)^{n-n'}[\rho_{mm',-n'-n}+(-1)^{m-m'}\rho_{-m'-m,-n'-n}], \quad (9)$$

and

$$\epsilon_m=1, \quad \text{for } m\neq0$$
$$=\tfrac{1}{2}, \quad \text{for } m=0. \quad (10)$$

In the following expressions, the state of photon polarization will be denoted by superscripts on the joint correlation coefficients ρ and the angular distribution W. The notation is 0 for unpolarized, $\pm$ for right- or left-circular polarization, and L for linear polarization.

For unpolarized reactions, one may use the parity relations for the t-channel helicity amplitudes[18]

$$F_{-\lambda-n,-\mu-m}=(\text{phase factor})$$
$$\times(-1)^{\mu-m-(\lambda-n)}F_{\lambda n,\mu m}, \quad (11)$$

along with (7) to show that

$$\rho_{mm',nn'}{}^0=(-1)^{m-m'+n-n'}\rho_{-m-m',-n-n'}{}^0. \quad (12)$$

It follows that

$$\operatorname{Im}Z_{mm'}{}^{nn'}=0 \quad (13)$$

and

$$\operatorname{Re}Z_{mm'}{}^{nn'}$$
$$=2\operatorname{Re}[\rho_{mm',nn'}{}^0+(-1)^{m-m'}\rho_{-m'-m,nn'}{}^0]. \quad (14)$$

The joint decay angular distribution for unpolarized reactions is

$$W^0(\theta_1\varphi_1,\theta_2\varphi_2)=\frac{2(2J_1+1)(2J_2+1)}{\pi^2}\sum_{m\geq|m'|}^{J_1}\epsilon_m\epsilon_{m-|m'|}d_{m0}{}^{J_1}(\theta_1)d_{m'0}{}^{J_1}(\theta_1)\sum_{n\geq|n'|}^{J_2}\epsilon_{n-|n'|}f_{nn'}{}^{J_2}(\theta_2)$$
$$\times\{\cos(m-m')\varphi_1\cos(n-n')\varphi_2\operatorname{Re}[\rho_{mm',nn'}{}^0+\rho_{m'm,nn'}{}^0+(-1)^{m-m'}(\rho_{-m'-m,nn'}{}^0+\rho_{-m-m',nn'}{}^0)]$$
$$-\sin(m-m')\varphi_1\sin(n-n')\varphi_2\operatorname{Re}[\rho_{mm',nn'}{}^0-\rho_{m'm,nn'}{}^0+(-1)^{m-m'}(\rho_{-m'-m,nn'}{}^0-\rho_{-m-m',nn'}{}^0)]\}, \quad (15)$$

and the measurable elements are the combinations given in (14).

For circular polarization one sets $\mu=\pm1$, so that (4) becomes

$$\rho_{mm',nn'}{}^{\pm}=\sum_{\lambda}F_{\lambda n,\pm1m}F_{\lambda n',\pm1m'}^*/\sum_{\lambda nm}|F_{\lambda n,\pm1m}|^2. \quad (16)$$

From (11), we find

$$\sum_{\lambda nm}|F_{\lambda n,1m}|^2=\sum_{\lambda nm}|F_{\lambda n,-1m}|^2, \quad (17)$$

which expresses the equality of the spin-averaged cross section for right- or left-circularly polarized photons on unpolarized targets. Use of the same relations in

[17] T. L. Trueman and G. C. Wick, Ann. Phys. (N. Y.) **26**, 322 (1964).

[18] M. Jacob and G. C. Wick, Ann. Phys. (N. Y.) **7**, 404 (1959).

(16) yields

$$\rho_{mm',nn'}^{\pm}=(-1)^{m-m'+n-n'}\rho_{-m-m',-n-n'}^{\mp}, \quad (18)$$

which leads to

$$\mathrm{Re}\rho_{mm',nn'}^{\pm}+\mathrm{Re}\rho_{mm',nn'}^{\mp}=2\,\mathrm{Re}\rho_{mm',nn'}^{0} \quad (19)$$

and

$$\mathrm{Im}\rho_{mm',nn'}^{\pm}-\mathrm{Im}\rho_{mm',nn'}^{\mp}=\rho_{mm',nn'}^{C}, \quad (20)$$

where we have defined

$$\rho_{mm',nn'}^{C}=\mathrm{Im}\sum_{\lambda}(F_{\lambda n,1m}F_{\lambda n',1m'}^{*}-F_{\lambda n,-1m}F_{\lambda n',-1m'}^{*})\Big/2\sum_{\lambda nm}|F_{\lambda n,1m}|^{2}. \quad (21)$$

Using this notation, one can write the joint angular distribution in the form

$$W^{\pm}(\Omega_1,\Omega_2)=W^{0}(\Omega_1,\Omega_2)\mp W^{C}(\Omega_1,\Omega_2), \quad (22)$$

where W^0 is the unpolarized distribution given by (15). W^C has the same form as W^0, with the substitutions $\rho^0\to\rho^C$, $\cos(n-n')\varphi_2\to\sin(n-n')\varphi_2$, and $\sin(n-n')\varphi_2\to-\cos(n-n')\varphi_2$. This form is useful for partially polarized beams, where the W^C term is to be multiplied by the degree of circular polarization.

The measurable elements of ρ^C occur in combinations $\rho_{mm',nn'}^{C}+(-1)^{m-m'}\rho_{-m'-m,nn'}^{C}$, for $m>|m'|$ and $n>|n'|$, and multiply terms proportional to

$$\sin(m-m')\,\varphi_1\cos(n-n')\varphi_2$$

or

$$\sin(n-n')\varphi_2\cos(m-m')\varphi_1.$$

A simple test for the presence of the W^C terms is to look for asymmetry in the distribution about φ_1 (or φ_2)$=0$ correlated with a symmetric distribution in φ_2 (or φ_1). To guard against accidental cancellations, one may project out the various mm',nn' components of W^C in the usual manner by examination of the $\theta_1\theta_2$ dependence. In general, there will be $J_1(J_1+1)(J_2+\frac{1}{2})^2$ independent terms in the joint decay distribution. Since ρ^C is made of imaginary parts of products of helicity amplitudes, a nonzero value implies unequal phases, which is an indication of two or more exchanges in the Regge-pole model.

For linear polarization, one uses the relations between states of definite helicity and plane-polarized states. Let the photon momentum be along the z axis and the production plane be the x-z plane. Then the helicity polarization vectors are[18]

$$\epsilon_{\pm1}=\mp(\epsilon_x\pm i\epsilon_y)/\sqrt{2}. \quad (23)$$

This may be inverted to give

$$\epsilon_x=(\epsilon_{-1}-\epsilon_1)/\sqrt{2} \quad (24)$$

and

$$\epsilon_y=i(\epsilon_{-1}+\epsilon_1)/\sqrt{2}. \quad (25)$$

For a photon polarized at an angle Φ with respect to the production plane (in such a sense that a clockwise rotation of the polarization plane through an angle Φ about the z axis brings it into the production plane), the polarization vector is

$$\epsilon(\Phi)=(\epsilon_{-1}e^{-i\Phi}-\epsilon_1e^{i\Phi})/\sqrt{2}. \quad (26)$$

This is the appropriate combination of t-channel helicity amplitudes to use in (4) to get the joint correlation coefficients ρ^L. The result is

$$\sum|F^L|^2\rho_{mm',nn'}^{L}=\tfrac{1}{2}\sum_{\lambda}(F_{\lambda n,-1m}F_{\lambda n,-1m'}^{*}+F_{\lambda n,1m}F_{\lambda n',1m'}^{*}-F_{\lambda n,-1m}F_{\lambda n',1m'}e^{-2i\Phi}-F_{\lambda n,1m}F_{\lambda n',-1m'}^{*}e^{2i\Phi}), \quad (27)$$

where

$$\sum|F^L|^2\equiv\tfrac{1}{2}\sum_{\lambda mn}|F_{\lambda n,-1m}e^{-i\Phi}-F_{\lambda n,1m}e^{i\Phi}|^2. \quad (28)$$

This may be rewritten in the form

$$\sum|F^L|^2\rho_{mm',nn'}^{L}=\tfrac{1}{2}\sum|F^0|^2(\rho_{mm',nn'}^{0}-\cos2\Phi\rho_{mm',nn'}^{1}+i\sin2\Phi\rho_{mm',nn'}^{2}), \quad (29)$$

where we have defined

$$\rho_{mm',nn'}^{1,2}\equiv\sum_{\lambda}(F_{\lambda n,-1m}F_{\lambda n',-1m'}^{*}\pm F_{\lambda n,1m}F_{\lambda n',-1m'}^{*})\Big/\sum|F^0|^2 \quad (30)$$

and

$$\sum|F^0|^2=\sum_{\mu\lambda nm}|F_{\lambda n,\mu m}|^2. \quad (31)$$

Note that $d\sigma^L/dt=K\sum|F^L|^2$ and $d\sigma^0/dt=\frac{1}{2}K\sum|F^0|^2$, with K a kinematic factor, so that the elements ρ^0 and ρ^L always occur weighted by their respective differential cross sections. The joint decay distribution may now be written in the form

$$\frac{d\sigma^L}{dt}W^L(\Omega_1,\Omega_2)=\frac{d\sigma^0}{dt}[W^0(\Omega_1,\Omega_2)-\cos2\Phi\,W^1(\Omega_1,\Omega_2)-\sin2\Phi\,W^2(\Omega_1,\Omega_2)], \quad (32)$$

where W^0 is again the unpolarized distribution (15), $W^{1,2}$ have the same form as W^0 with the replacement of ρ^0 by $\rho^{1,2}$, and the substitutions $\cos(n-n')\varphi_2\to\sin(n-n')\varphi_2$ and $\sin(n-n')\varphi_2\to-\cos(n-n')\varphi_2$ are to be made *in* W^2 *only*. For partially polarized beams, W^1 and W^2 are multiplied by the degree of linear polarization. The measurable elements again occur in combinations like (14).

To extract the ρ^i from the decay distribution, one may separate the W^i by weighting each event by a Φ-dependent factor and forming new distributions from (32):

$$W^0=\left(\frac{d\sigma^0}{dt}\right)^{-1}\frac{1}{2\pi}\int_0^{2\pi}\frac{d\sigma^L}{dt}W^L(\Phi)d\Phi, \quad (33)$$

$$W^1=-\left(\frac{d\sigma^0}{dt}\right)^{-1}\frac{1}{\pi}\int_0^{2\pi}\frac{d\sigma^L}{dt}W^L(\Phi)\cos2\Phi\,d\Phi, \quad (34)$$

$$W^2=-\left(\frac{d\sigma^0}{dt}\right)^{-1}\frac{1}{\pi}\int_0^{2\pi}\frac{d\sigma^L}{dt}W^L(\Phi)\sin2\Phi\,d\Phi. \quad (35)$$

These distributions may be analyzed by the usual methods, i.e., maximum likelihood, least squares, or method of moments,[19] to project out the individual ρ^i's. One interesting combination of measurable elements is

$$\begin{aligned}\mathrm{Re}[(\rho_{mm',nn'}{}^0&+(-1)^{m-m'}\rho_{-m'-m,nn'}{}^0)\\&\pm(\rho_{mm',nn'}{}^1+(-1)^{m-m'}\rho_{-m'-m,nn'}{}^1)]\\=\frac{1}{\sum|F^0|^2}&\mathrm{Re}\sum_\lambda[(F_{\lambda n,-1m}\pm F_{\lambda n,1m})\\&\times(F_{\lambda n',-1m'}\pm F_{\lambda n',1m'})^*\\&+(-1)^{m-m'}(F_{\lambda n,-1-m'}\pm F_{\lambda n,1-m'})\\&\times(F_{\lambda n'-1-m}\pm F_{\lambda n',1-m})^*].\quad(36)\end{aligned}$$

Note from (27) that this is equal to a combination of the total joint correlation coefficients for linearly polarized photons at angles $\Phi=0$ or $\Phi=\frac{1}{2}\pi$ from the reaction plane. *This formalism, however, enables one to make use of data at all angles, rather than limit consideration to discrete points.*

It is shown in the Appendix that for the exchange of a definite J parity [J parity=parity$\times$signature, $\sigma=P(-1)^J$], one of the combinations $F_{\lambda n,-10}\pm F_{\lambda n,10}$ is either identically zero (when $\lambda=n$) or is much smaller than the opposite combination at high energy (when $\lambda\neq n$). The examination of expressions such as (36) for m or m' zero and arbitrary n and n' will enable one to separate the contributions of exchanges with different J parity.

Another possibility is to consider the element

$$\begin{aligned}\mathrm{Re}\rho_{mm',nn'}{}^2=&\frac{1}{\sum|F^0|^2}\mathrm{Re}\sum_\lambda[(F_{\lambda n,-1m}+F_{\lambda n,1m})\\&\times(F_{\lambda n',-1m'}-F_{\lambda n',1m'})-(F_{\lambda n,-1m}-F_{\lambda n,1m})\\&\times(F_{\lambda n',-1m'}+F_{\lambda n',1m'})].\quad(37)\end{aligned}$$

This will also vanish (exactly or asymptotically with energy, as before) for definite J-parity exchanges when both m *and* m' are zero, and will be useful to separate opposite J-parity exchanges (except when $n=n'$, in which case the element is identically zero).

B. Meson Decay

The meson decay distribution may be obtained by integrating the joint decay distribution over the baryon decay angles. The result is

$$\begin{aligned}W(\theta_1\varphi_1)=&\frac{2J_1+1}{2\pi}\sum_{m\geq|m'|}^{J_1}\epsilon_m\epsilon_{m-|m'|}d_{m0}{}^{J_1}(\theta_1)d_{m'0}{}^{J_1}(\theta_1)\\&\times\{\cos(m-m')\varphi_1[\mathrm{Re}\rho_{mm'}+(-1)^{m-m'}\mathrm{Re}\rho_{-m-m'}]\\&-\sin(m-m')\varphi_1[\mathrm{Im}\rho_{mm'}-(-1)^{m-m'}\mathrm{Im}\rho_{-m-m'}]\},\quad(38)\end{aligned}$$

where

$$\rho_{mm'}=\sum_n\rho_{mm',nn}.\quad(39)$$

The same type of manipulations as in the previous part yield the following results.

For unpolarized photons,

$$\begin{aligned}W^0(\theta_1\varphi_1)=&\frac{2J_1+1}{\pi}\sum_{m\geq|m'|}^{J_1}\epsilon_m\epsilon_{m-|m'|}d_{m0}{}^{J_1}(\theta_1)d_{m'0}{}^{J_1}(\theta_1)\\&\times\cos(m-m')\varphi_1\,\mathrm{Re}\rho_{mm'}{}^0.\quad(40)\end{aligned}$$

For circular polarization,

$$W^\pm(\Omega_1)=W^0(\Omega_1)\mp W^C(\Omega_1),\quad(41)$$

where W^C has the same form as W^0 except

$$\cos(m-m')\varphi_1\to\sin(m-m')\varphi_1$$

and $\rho_{mm'}{}^0$ is replaced by $\rho_{mm'}{}^C$. Just as in the joint decay distribution, the $\sin(m-m')\varphi_1$ terms detect interference terms between amplitudes with different phases. This is also an indication of two or more exchange contributions, but is less reliable because of the possibility of cancellations in the sum over N^* helicities.

For linear polarization,

$$\begin{aligned}\frac{d\sigma^L}{dt}W^L(\Omega_1)=\frac{d\sigma^0}{dt}[W^0(\Omega_1)&-\cos2\Phi\,W^1(\Omega_1)\\&-\sin2\Phi\,W^2(\Omega_1)],\quad(42)\end{aligned}$$

where W^1 and W^2 have the same form as W^0 with the replacements $\rho^0\to\rho^{1,2}$, and $\cos(m-m')\varphi_1\to\sin(m-m')\varphi_1$ *in* W^2 *only*. One can form the combination of measurable elements

$$\begin{aligned}\mathrm{Re}(\rho_{mm'}{}^0\pm\rho_{mm'}{}^1)=&\frac{1}{\sum|F^0|^2}\mathrm{Re}\sum_{\lambda n}(F_{\lambda n,-1m}\pm F_{\lambda n,1m})\\&\times(F_{\lambda n,-1m'}\pm F_{\lambda n,1m'})^*,\quad(43)\end{aligned}$$

which separate different J-parity exchanges for m or m' zero. There is no useful expression involving $\rho_{mm'}{}^2$ for meson decay, since $\mathrm{Re}\rho_{00}{}^2\equiv0$ from (30) and (39).

C. Baryon Decay

Cancellations similar to those in the previous section give the baryon distribution

$$\begin{aligned}W(\theta_2\varphi_2)=&\frac{2J_2+1}{4\pi}\sum_{n\geq|n'|}^{J_2}\epsilon_{n-|n'|}f_{nn'}{}^{J_2}(\theta_2)\\&\times\{\cos(n-n')\varphi_2[\mathrm{Re}\rho_{nn'}+(-1)^{n-n'}\mathrm{Re}\rho_{-n-n'}]\\&-\sin(n-n')\varphi_2[\mathrm{Im}\rho_{nn'}-(-1)^{n-n'}\mathrm{Im}\rho_{-n-n'}]\},\quad(44)\end{aligned}$$

where

$$\rho_{nn'}=\sum_m\rho_{mm,nn'}.\quad(45)$$

[19] N. Schmitz, in Proceedings of the 1965 Easter School for Physicists, CERN 65-24, Vol. I (unpublished).

For unpolarized photons,

$$W^0(\theta_2\varphi_2)=\frac{2J_2+1}{2\pi}\sum_{n\geq|n'|}^{J_2}\epsilon_{n-|n'|}f_{nn'}{}^{J_2}(\theta_2)\times\cos(n-n')\varphi_2\,\mathrm{Re}\rho_{nn'}{}^0. \quad (46)$$

For circular polarization,

$$W^{\pm}(\Omega_2)=W^0(\Omega_2)\mp W^C(\Omega_2), \quad (47)$$

where W^C has the same form as W^0 with the replacements $\rho^0\to\rho^C$ and $\cos(n-n')\varphi_2\to\sin(n-n')\varphi_2$. This also provides a test for interference terms between amplitudes with different phases, but now the possibility of cancellation comes from the sum over meson helicities.

For linear polarization,

$$\frac{d\sigma^L}{dt}W^L(\Omega_2)=\frac{d\sigma}{dt}[W^0(\Omega_2)-\cos2\Phi\,W^1(\Omega_2)-\sin2\Phi\,W^2(\Omega_2)], \quad (48)$$

with the same replacements for W^1 and W^2 as in the meson-decay case. However, in this case the $\mathrm{Re}\rho^{0,1,2}$ involve sums over meson helicities, so that the $m=0$ states cannot be projected out. The baryon decay is useful in selecting J-parity exchanges only when the meson has spin $J_1=0$. In this case the differential cross section may be used directly. If (48) is integrated over all baryon decay angles, one obtains

$$\frac{d\sigma^L}{dt}=\frac{d\sigma^0}{dt}(1-\beta\cos2\Phi), \quad (49)$$

where

$$\beta=\sum_n\rho_{nn}{}^1=\mathrm{Re}\sum_{\lambda n}F_{\lambda n,-10}F_{\lambda n,10}{}^*/\sum_{\lambda n}|F_{\lambda n,10}|^2. \quad (50)$$

The parameter β may be projected out by weighting each event by $\cos2\Phi$ and forming a new differential cross section:

$$\beta=-\left(\frac{d\sigma^0}{dt}\right)^{-1}\frac{1}{\pi}\int_0^{2\pi}\frac{d\sigma^L}{dt}\cos2\Phi\,d\Phi. \quad (51)$$

The combinations $1\pm\beta$ separate opposite J-parity exchange contributions.

III. ANALYSIS OF SPECIFIC REACTIONS

First consider the case in which the N^* is a nucleon. Then there are only two independent couplings at the nucleon vertex for a Regge-pole exchange. The "reduced" spin-flip amplitudes $G_{\frac{1}{2}-\frac{1}{2},\pm10}$ are defined by the relation

$$F_{\frac{1}{2}-\frac{1}{2},\pm10}=\tfrac{1}{2}(1\pm x)G_{\frac{1}{2}-\frac{1}{2},\pm10}, \quad (52)$$

where x is the cosine of the t-channel c.m. scattering angle. Consider the contribution of two opposite J-parity exchanges A and B ($\sigma_A=+1$, $\sigma_B=-1$) to the production of a meson with J parity σ_V. Using the parity relations in the Appendix, one can write

$$\sum|F^0|^2(\rho_{00}{}^0+\sigma_V\rho_{00}{}^1)=8|F_{\frac{1}{2}\frac{1}{2},10}{}^B|^2+2|G_{\frac{1}{2}-\frac{1}{2},10}{}^A+xG_{\frac{1}{2}-\frac{1}{2},10}{}^B|^2, \quad (53)$$

$$\sum|F^0|^2(\rho_{00}{}^0-\sigma_V\rho_{00}{}^1)=8|F_{\frac{1}{2}\frac{1}{2},10}{}^A|^2+2|xG_{\frac{1}{2}-\frac{1}{2},10}{}^A+G_{\frac{1}{2}-\frac{1}{2},10}{}^B|^2. \quad (54)$$

A. Neutral Mesons with Odd C

The exchanges in this case are limited to those associated with diffractive production (P and P') and pion exchange. Consider photoproduction of ρ^0 and ω^0 as examples.[13] Since the pion couples only to nucleons with equal helicity in the t channel, (53) and (54) become

$$\sum|F^0|^2(\rho_{00}{}^0+\rho_{00}{}^1)=8|F_{\frac{1}{2}\frac{1}{2},10}{}^\pi|^2+2|G_{\frac{1}{2}-\frac{1}{2},10}{}^{P,P'}|^2, \quad (55)$$

$$\sum|F^0|^2(\rho_{00}{}^0-\rho_{00}{}^1)=8|F_{\frac{1}{2}\frac{1}{2},10}{}^{P,P'}|^2+2x^2|G_{\frac{1}{2}-\frac{1}{2},10}{}^{P,P'}|^2. \quad (56)$$

There are three features of these equations which offer tests of present theoretical ideas.

(a) The energy dependence of $(\rho_{00}{}^0+\rho_{00}{}^1)/(\rho_{00}{}^0-\rho_{00}{}^1)$ should be $\approx s^{-2}$, since the difference of the Pomeranchuk and pion trajectories is approximately one unit. If the energy dependence is slower, this would indicate a contribution to $\rho_{00}{}^0+\rho_{00}{}^1$ not corresponding to the exchange of a definite parity, e.g., a Regge cut or absorptive corrections to pion exchange.

(b) One may separate the pion contribution from the diffractive part exactly, with the use of additional information from Regge-pole fits in elastic scattering. Note that even though the diffractive part $G_{\frac{1}{2}-\frac{1}{2},10}{}^P\propto s^{\alpha_P-1}$ and the pion part $F_{\frac{1}{2}\frac{1}{2},10}{}^\pi\propto s^{\alpha_\pi}$, they may still be of the same order of magnitude over a large energy range, since $\alpha_P-\alpha_\pi\approx1$. However, one can relate $F_{\frac{1}{2}-\frac{1}{2},10}{}^P$ to $F_{\frac{1}{2}\frac{1}{2},10}{}^P$ from the fits to pion-nucleon elastic scattering[20] by using the factorization theorem. The result is expressed in terms of the ratio of pion exchange to the total *helicity-zero* cross section:

$$\frac{(d\sigma/dt)^\pi}{(d\sigma/dt)^0}=\frac{\rho_{00}{}^0+\rho_{00}{}^1-(\rho_{00}{}^0-\rho_{00}{}^1)/x^2(1+\gamma^2)}{2\rho_{00}{}^0}, \quad (57)$$

where γ is defined by

$$F_{\frac{1}{2}\frac{1}{2},10}{}^{P,P'}=\tfrac{1}{2}x\gamma_{P,P'}G_{\frac{1}{2}-\frac{1}{2},10}{}^{P,P'}, \quad (58)$$

and the difference in x values for πN and γN reactions at high energies is neglected. The values of $\gamma_{P,P'}$ are shown in Fig. 1, as obtained from fit 1 of Ref. 20. Note that γ in (57) is actually some linear combination of γ_P and $\gamma_{P'}$, depending on the (unknown) relative couplings of P and P' at the $\gamma\rho^0$ or $\gamma\omega^0$ vertex. Since x^2 is large at high energy (away from the forward direction), it is possible that this uncertainty will not have much effect on the ratio (57). Justification of this procedure must await the experimental values.

[20] W. Rarita, R. J. Riddell, C. B. Chiu, and R. J. N. Phillips, Phys. Rev. 165, 1615 (1968).

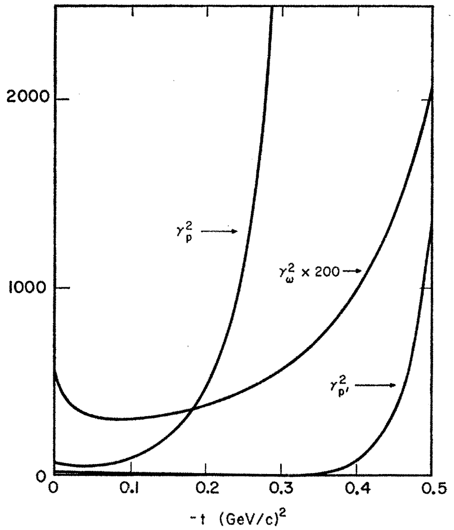

FIG. 1. Ratios of non-spin-flip to spin-flip couplings of Regge trajectories to nucleons from parameters of Ref. 20.

One may also use (57) for both ρ^0 and ω^0 production to determine the ratio of pion coupling constants. Neglecting the ρ-ω mass difference, one can write

$$\frac{(d\sigma/dt)_{\gamma p\to\rho^0 p^\pi}}{(d\sigma/dt)_{\gamma p\to\omega^0 p^\pi}}=\frac{g_{\pi\rho\gamma}{}^2}{g_{\pi\omega\gamma}{}^2}. \tag{59}$$

This ratio is predicted to be $\approx\frac{1}{9}$ from $SU(6)$ and also has an upper limit of $\approx\frac{1}{3}$ from experimental decay widths.

(c) At nonforward angles and high energies where $x^2\gg1$, one can find the relative contribution of diffractive processes. From (55) and (56) one has

$$\frac{\rho_{00}{}^0-\rho_{00}{}^1}{2\rho_{00}{}^0}\underset{x^2\gg1}{\longrightarrow}\frac{(d\sigma/dt)^{P,P'}}{(d\sigma/dt)^0}. \tag{60}$$

One can determine the ratio of diffractive production of ρ^0 and ω^0 and compare with the predictions of $SU(3)$, quark model, vector dominance, etc.

B. Neutral Mesons with Even C

Here the candidates for exchange are the vector mesons ρ, ω, ϕ and also axial vector mesons with odd C. The two opposite J-parity contributions may be separated by using linearly polarized photons just as in the previous case. Consider the case of π^0 photoproduction. Here $\rho_{00}{}^0=1$ and $\rho_{00}{}^1=\beta$ [see Eq. (51)]. The two dominant contributions are assumed to be ω and B exchange, due to small $\rho\pi\gamma$ and $\phi N\bar{N}$ couplings. The analogous result to (57) is

$$\frac{(d\sigma/dt)^B}{(d\sigma/dt)^0}=\frac{1}{2}\left(1-\beta-\frac{1+\beta}{x^2(1+\gamma_\omega{}^2)}\right), \tag{61}$$

where γ_ω is defined as in (58). The values are determined from Regge-pole fits to nucleon-nucleon elastic scattering[20] and are presented in Fig. 1.

One may also write

$$\frac{(d\sigma/dt)^\omega}{(d\sigma/dt)^0}\underset{x^2\gg1}{\longrightarrow}\tfrac{1}{2}(1+\beta). \tag{62}$$

Even when x^2 is not large, however, the combination $1+\beta$ receives contributions only from the ω. It may be used to study the details of ω exchange, such as the nonsense zero and crossover zero.[20]

C. Charged Mesons

For charged mesons the diffractive processes are absent, but both even and odd C exchanges are allowed. The most interesting reaction at present is pion photoproduction, where a sharp forward peak at high energies[21] indicates that the exchange of a single set of definite quantum numbers cannot be the dominant mechanism.[22] Two models have been suggested to fit the data. One is the conspiracy model, in which a pion and its parity doublet partner act in a cooperative manner to produce the forward peak.[3] The other is a pion-exchange contribution interfering destructively with a background term of nondefinite parity coming from a Regge cut, fixed pole, absorption correction, or some other mechanism.[4] Both models fit the high-energy forward-direction data, and knowledge of the individual spin amplitudes is needed to distinguish between them. It would seem reasonable that polarized photon interactions may be able to provide such a test.

The pion-conspiracy model predicts the following form for the measurable quantities:

$$\sum|F^0|^2(1-\beta)=8|F_{\frac{1}{2}\frac{1}{2},10}{}^\pi|^2+2|G_{\frac{1}{2}-\frac{1}{2},10}{}^{\pi'}|^2, \tag{63}$$

$$\sum|F^0|^2(1+\beta)=8|F_{\frac{1}{2}\frac{1}{2},10}{}^{\pi'}|^2+2x^2|G_{\frac{1}{2}-\frac{1}{2},10}{}^{\pi'}|^2. \tag{64}$$

The nonconspiring-pion model predicts a change in (63):

$$\sum|F^0|^2(1-\beta)=8|F_{\frac{1}{2}\frac{1}{2},10}{}^\pi+\tfrac{1}{2}(F_{\frac{1}{2}\frac{1}{2},10}{}^B-F_{-\frac{1}{2}-\frac{1}{2},10}{}^B)|^2 +2|F_{\frac{1}{2}-\frac{1}{2},10}{}^B-F_{-\frac{1}{2}\frac{1}{2},10}{}^B|^2. \tag{65}$$

The essential difference is that the leading-order pion-conspirator contribution to $1-\beta$ vanishes, leaving only a term $\propto s^{2\alpha-2}$, while the background term contributes full strength, since it has no definite parity. Cooper[14] has suggested that the energy dependence of a quantity like $1-\beta$ could distinguish between these models. In practice, however, this difference in energy behavior would be measurable only outside the forward peak, for momentum transfers in the range -0.5 to -1.0 (GeV/c)^2. In this region the restrictions on the models are not as stringent. For example, there may be nonconspiring negative J-parity contributions to $F_{\frac{1}{2}-\frac{1}{2},10}$

[21] A. M. Boyarski *et al.*, Phys. Rev. Letters **20**, 300 (1968).
[22] S. D. Drell and J. D. Sullivan, Phys. Rev. Letters **19**, 268 (1967).

which become large at large momentum transfers and are approximately energy-independent (an A_1 with a flat trajectory). Alternatively, the combination of background terms $F_{\frac{1}{2}\frac{1}{2},10}{}^B-F_{-\frac{1}{2}-\frac{1}{2},10}{}^B$ may be very small at large momentum transfers, with the main energy-independent contribution coming from the opposite combination in $1+\beta$. These possibilities make the interpretation of large-momentum-transfer data somewhat ambiguous. Conversely, if one considers only small momentum transfers it is fairly certain that the dominent contributions are the pion-exchange term in combination with some additional term to form the forward peak.

One essential feature of the conspiracy model is a zero in the pion residue function at $t\approx -0.03$ $(\mathrm{GeV}/c)^2$. An obvious test is to look for a dip in $1-\beta$, since the leading-order contribution from the conspirator is absent. Using the amplitudes of Ref. 3, one can write

$$\tfrac{1}{2}(1-\beta)=\frac{[1-\lambda(1+y)]^2+(Os^{-2})}{[1-\lambda(1+y)]^2+[(1-\lambda)(1+y)]^2}, \quad (66)$$

where $y\equiv -t/\mu^2$, μ is the pion mass, and λ is an adjustable parameter which determines the position of the pion residue zero. The result for $\lambda=0.4$ is shown in Fig. 2, and indeed shows a pronounced dip at $t=-1.5\mu^2$.

One might expect that the interference model would not exhibit such a dip, since the pion residue function does not need a zero, and also the background term contributes full strength. However, this is not true. An examination of the fit of Ref. 4 shows that a cancellation between the pion and background terms occurs at approximately the same place as the zero in the conspiracy model. Values for two energies are shown for comparison in Fig. 2. Although the position of the minimum moves with energy, the curves are qualitatively the same as in the conspiracy model, so that no clear distinction is possible. In the notation of Ref. 4, the curves are values of the expression

$$\tfrac{1}{2}(1-\beta)=\frac{\{A_1{}^b+[t/(t-\mu^2)]a_2{}^\pi\xi^\pi\}^2}{\{A_1{}^b+[t/(t-\mu^2)]a_2{}^\pi\xi^\pi\}^2+(A_1{}^b)^2}. \quad (67)$$

It has also been shown by Donohue[23] in an absorption-model calculation that the corresponding term also has a minimum at about the same point. In general, it is evident that any model which fits the sharp peak in the differential cross section and includes pion exchange will predict such a structure. The pion amplitude alone is too large for momentum transfer greater than $(3\text{–}4)\mu^2$, so that there must either be a zero in the residue function, or else a cancellation with some other contribution.

One other possibility is to examine the energy dependence of $1-\beta$ at the position of the dip. The conspiracy model predicts a s^{-2} dependence while the

[23] J. T. Donohue (private communication).

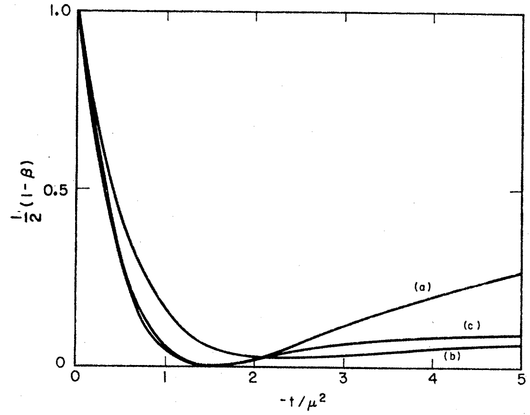

FIG. 2. Predicted ratio of pion-photoproduction cross section for photons polarized in the reaction plane to that for unpolarized photons. (a) Pion conspiracy model of Rev. 3. (b) Interference model of Ref. 4 at 5 GeV/c. (c) Interference model of Ref. 4 at 16 GeV/c.

other models predict essentially no energy dependence. However, the same uncertainties present themselves here as in the large-momentum-transfer case. The addition of some small term which would not affect the fits to the cross section could greatly modify the predictions at the dip. It seems that the use of polarized photons alone cannot clearly distinguish between conspiracy versus interference models of pion photoproduction.

D. Baryon Resonances

The most common baryon resonance production is the $\Delta(1238)$. Since it has $I=\frac{3}{2}$, the exchange contributions with $I=0$ are ruled out. However, since it has $J=\frac{3}{2}$, there are twice as many independent amplitudes for each exchange as in the nucleon case. In addition, since there are no G-parity restrictions in the t channel for the $\bar{N}\Delta$ state, the exchanges with π or B type quantum numbers will couple to all four independent helicity combinations, rather than just to the equal helicities as in $\bar{N}N$. Consequently there will always be amplitudes with nonzero spin flip, so that J-parity separation will be only approximate for all exchanges.

The reaction $\gamma+p\rightarrow\pi^0+\Delta^+$ is of particular interest, since only B exchange is expected to be important. Its leading-order term may be eliminated by using linearly polarized photons, so that the remainder must come from either lower-lying poles or non-pole-type contributions. It has been suggested that the energy dependence of this reaction is a good test for the presence of Regge cuts.[12] It is not likely that this will be done in the near future, due to experimental difficulties in identifying two neutrals in the final state. An easier reaction to examine is $\gamma+p\rightarrow\pi^-+\Delta^{++}$. The exchange of π, π', ρ, A_1, and A_2 are all allowed. However, if the A_1 term is assumed small, the pion-exchange term may be approximately separated from the others due to its different J parity. An experiment to look for the pion-exchange term has been done at low energies,[24] and essentially a null result obtained. It will be interesting to see if this result persists at high energies.

The reactions $\gamma+p\to(\rho^0\omega^0)+\Delta^{++}$ differ from the ordinary vector-meson photoproduction reactions, since the diffractive terms are not present. The only important exchange is thought to be the pion. This term may be isolated as usual with the study of $\rho_{00}d\sigma/dt$ for linearly polarized photons, and again a test for the presence of non-pole-type terms is possible.

E. Strange-Particle Production

For reactions which produce a strange meson, and hence also a strange baryon, the only possible exchanges are K and K^* types. The reactions $\gamma+p\to K^++(\Sigma^0,\Lambda^0)$ have been fitted with a K parity doublet conspiracy and K^* exchange. Linearly polarized photons can separate the K exchange to leading order, and possibly provide a check on the relative magnitudes of the $K\bar{N}\Sigma$ and $K\bar{N}\Lambda$ coupling constants. Similar analyses are possible for reactions such as $\gamma+p\to K^*+Y^*$.

IV. CONCLUSION

The analysis of quasi-two-body final states for polarized photons has been shown to provide two main tests of theoretical models. For circular polarization, an analysis of the azimuthal asymmetry in individual or joint decay angular distributions for any two-body reaction can give information on the presence of two or more exchange terms with different phases. The interference terms of the amplitude may be determined explicitly from an extraction of the $\sin m\varphi$ dependence of the decay distribution.

For linear polarization, a leading-order separation (exact if there is no spin flip) of opposite J-parity exchange contributions is possible. This occurs in the differential cross section for the production of spin zero mesons, or in meson decay angular distributions via the elements $\mathrm{Re}\rho_{m0}$ and in joint decay distributions via the elements $\mathrm{Re}\rho_{m0,nn'}$. The main applications are the separation of pion exchange and the diffractive mechanism in vector meson production, tests for the presence of non-pole-type contributions in baryon resonance production, and separation of kaon exchange from positive J-parity exchanges in strange-particle photoproduction. An application to pion photoproduction reveals that there exist no simple tests for the presence or absence of pion parity doublet conspiracy.

ACKNOWLEDGMENT

I wish to thank Dr. Gordon Ringland for a careful reading of the manuscript.

[24] R. W. Morrison, D. J. Drickey, and R. F. Mozley, Stanford Report No. HEPL 545, 1967 (unpublished).

APPENDIX

The t-channel c.m. helicity amplitudes for Regge-pole exchange in the process $\gamma+\bar{C}\to\bar{B}+D$ may be written

$$F_{\lambda n,\mu m}(t,x)=R_{\lambda n,\mu m}(t)d_{\mu-m,\lambda-n}{}^{\alpha}(x)\,,\tag{A1}$$

where λ, n, μ, m are the helicities of $\bar{B}$, D, γ, and $\bar{C}$, respectively, R is the residue function, α is the trajectory value, t is the square of the total energy, and x is the cosine of the scattering angle. From parity relations for helicity amplitudes,[18] it may be shown that

$$R_{-\lambda-n,\mu m}(t)=\sigma_B\sigma_D\sigma_E R_{\lambda n,\mu m}(t)\tag{A2}$$

and

$$R_{\lambda n,-\mu-m}(t)=\sigma_C\sigma_E R_{\lambda n,\mu m}(t)\,,\tag{A3}$$

where $\sigma_i=P_i(-1)^{J_i\ (J_i-1/2\text{ for fermions})}$ is the J parity and E refers to the exchanged trajectory. We use the properties[18]

$$d_{-\lambda-\mu}{}^{J}(x)=(-1)^{\lambda-\mu}d_{\lambda\mu}{}^{J}(x)\tag{A4}$$

and

$$d_{m,\pm1}{}^{J}(x)=C(J,m)\left(\frac{1\pm x}{2}\right)\left(\frac{1-x^2}{4}\right)^{(m-1)/2}\times s^{J-m}[1+O(1/s)],\tag{A5}$$

for $m\geqq1$, where $C(J,m)$ is independent of x and s is the square of the c.m. energy for the s-channel reaction $\gamma=B\to C+D$.[25] For no spin flip at the baryon vertex ($\lambda=n$), we can combine (A1), (A3), and (A4) and write

$$F_{\lambda\lambda,-\mu-m}=\sigma_C\sigma_E(-1)^{m-\mu}F_{\lambda\lambda,\mu m}\,,\tag{A6}$$

so that when $m=0$, the combination

$$F_{\lambda\lambda,-10}+\sigma_C\sigma_E F_{\lambda\lambda,10}=0\,.\tag{A7}$$

When $\lambda\neq n$, we use (A5) and write

$$\frac{F_{\lambda n,-10}+\sigma_C\sigma_E F_{\lambda n,10}}{F_{\lambda n,-10}-\sigma_C\sigma_E F_{\lambda n,10}}=\frac{1+O(1/s)}{x}\,.\tag{A8}$$

Thus the leading-order term is missing in one combination. Note that the parameter is x, not s. Away from the forward direction $x\approx s$, so that at high energies the ratio (A8) is very small. In the forward direction the ratio approaches unity for all energies, but for linearly polarized photons this region is not useful, since the angle between reaction and polarization planes is not well defined. In practice this region is very small, typically less than 0.01 $(\mathrm{GeV}/c)^2$ in the multi-GeV range of energies.

[25] The expansion parameter in the terms of dynamical origin has been changed from x to s to satisfy the analyticity requirements of inelastic scattering amplitudes. See D. Z. Freedman and J.-M. Wang, Phys. Rev. Letters **17**, 569 (1966).