Study of the $Na^{23}(He^3,d)Mg^{24}$ Reaction*

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The $Na^{23}(He^3,d)Mg^{24}$ reaction induced by 35-MeV He^3 ions has been studied for the first seven states in Mg^{24}. Spectroscopic factors have been extracted and are compared with theoretical predictions. The simple picture of the $K=2^+$ band being built on the one-phonon γ-vibrational level appears to be contradicted by this evidence. A simple Nilsson argument predicts spectroscopic factors in fair agreement with the data.

I. INTRODUCTION

FOR some time now the γ-ray decay properties of the excited states of Mg^{24} have been assumed to arise from some sort of collective nuclear behavior. Figure 1 shows the relation of the first few levels[1] in Mg^{24}. The levels from the ground state through the second excited state are accepted to be members of the $K=0^+$ band. The third through fifth excited states are members of a $K=2^+$ band. The exact explanation of these states has been the subject of much discussion, but three models are most prominent. The first is the simple rotational model[2] with no band mixing. The second is the rotational-vibrational model[3,4] which assumes that the $K=2$ band is built upon the one-phonon vibrational level. The third is the asymmetric-rotor model of Davydov and coworkers.[5,6]

Recently, Robinson and Bent[7] have measured the lifetimes of several γ-ray transitions in Mg^{24} and have compared all the known γ-ray decay data with the predictions of the three models mentioned above. Their conclusions are best summarized by stating that the deformation parameter is $\eta=3$ and that none of the models meet with complete success.

Although the decay of the 2^+ and 4^+ members of the $K=0$ band are fitted quite well by all of the theories, the explanations break down for the $K=2$ band. The best evidence of this breakdown is found in the decay of the $K=2^+$, $J=2^+$ third excited state. The measured branching ratio of the decays to the first excited state and ground state is 1/7 of the ratio predicted from the rotational-vibrational model. For the asymmetric-rotor model, with the asymmetry parameter $\gamma=22°$ (which gives a good fit to the observed level scheme), the predicted value is too large by a factor of 4. For $\gamma=12°$ the ratios are in good agreement, but the $J=2^+$, $K=2^+$ state is then predicted to be at 15 MeV. The decay of the 3^+ state is explained by all of the models and the decay of the 4^+ state by none. The best fit, though not necessarily a good fit, to all the data seems to be that of the rotational model with unmixed bands.

A study of stripping to the states of Mg^{24} will also give some indication of which of these models is most promising—or at least which are wrong. The only previous study of the $Na^{23}(He^3,d)Mg^{24}$ reaction determined only the relative positions of the levels. This work of Hinds and Middleton[8] will be used later in the present study to estimate the fraction of the $l=2$ strength in each member of the pair of states at 4.1 and 4.2 MeV, which we fail to resolve. Unfortunately, the ground state and first excited states are not included in their work.

II. EXPERIMENTAL TECHNIQUE

A beam of 35-MeV He^3 ions from the Argonne cyclotron was used to bombard evaporated metallic sodium targets that were transferred via vacuum locks to the

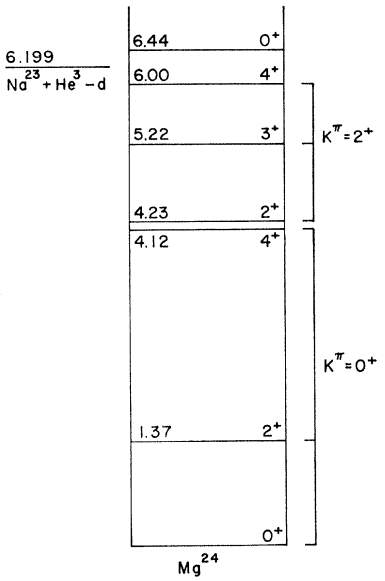

FIG. 1. A level diagram for the first seven states of Mg^{24}. After Ref. 1.

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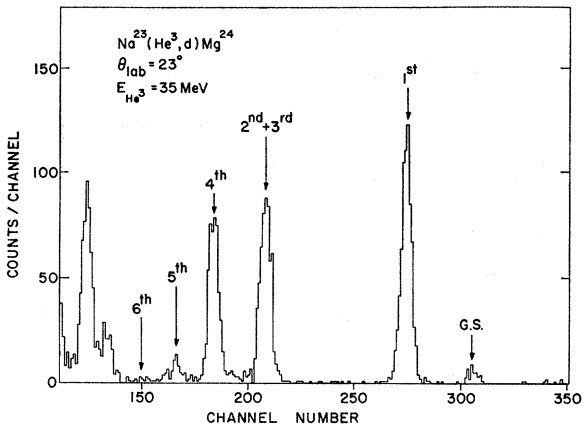

FIG. 2. A particle spectrum taken at 23° (lab). The zero of the abscissa has been suppressed. The resolution width is about 120 keV and transitions are observed through the sixth excited state. States above the sixth are not resolved sufficiently well to allow identification.

60-in. scattering chamber.[9] The thickness of the target was estimated to be $\sim 150\ \mu g/cm^2$ by comparing the He^3 elastic scattering angular distributions with the predictions of code JULIE[10] with the optical-model potentials described in Sec. IV. To stop the approximately 40-MeV deuterons produced in the $Na^{23}(He^3,d)Mg^{24}$ reaction, it was necessary to stack detectors to a sufficient thickness. The detector telescope consisted of a dE detector with a 2-mm depletion depth and an E detector system in which a 1- and a 2-mm detector, selected to have the same depletion voltage, were connected in parallel and their combined output was fed into the preamplifier. Although a standard $(dE/dx)\times E$ multiplier was used, a coincidence between the dE detector and the following detectors was sufficient to guarantee that the event was due to a deuteron without further analysis; the ranges of the He^3, α, and t particles from other reactions were smaller than the thickness of the dE detector, and the energy loss of the ground-state group is 8 MeV less for $Na^{23}(He^3,p)Mg^{25}$ than for $Na^{23}(He^3,d)Mg^{24}$.

Figure 2 shows a typical deuteron spectrum taken at 23°. The positions of the groups that were analyzed in the present work are indicated by arrows. The resolution (full width at half-maximum ≈ 120 keV) was insufficient to resolve the second and third excited states and states higher than the sixth excited state. The sixth excited state is very weak, and it was not possible to extract cross sections at each angle. However, it is possible to say that the distribution is uncharacteristic of a direct reaction and is not more than half as intense as the ground-state group at each angle.

Data were taken from 11° to 35° (lab) in 2° steps for the (He^3,d) reaction. The elastic scattering was studied from 19° to 49° in 2° steps. After the background had been subtracted, the counts in each group were determined and the results were corrected for solid angle. The corrected angular distributions for the (He^3,d) reaction are plotted in Fig. 3. The sum of the second and third excited states is shown. The solid lines are the fits for $l=2$ obtained from the code JULIE with the parameters discussed below. As stated above, the absolute cross sections were estimated by comparing the elastic scattering distributions with the predictions of JULIE, the He^3 optical-model potentials being the ones used in fitting the reaction data.

III. SPECTROSCOPIC FACTORS

The definition of the spectroscopic factor in terms of the measured cross section in a (He^3,d) reaction is

$$\frac{d\sigma}{d\Omega}(\theta)=[(2J_f+1)/(2J_i+1)]N\sum S(l,j)\sigma_{lj}(\theta),$$

where J_i and J_f are the target and final-state spins, respectively, $\sigma_{lj}(\theta)$ is the reduced cross section for proton stripping [evaluated by appropriate distorted-wave Born-approximation (DWBA) techniques], and

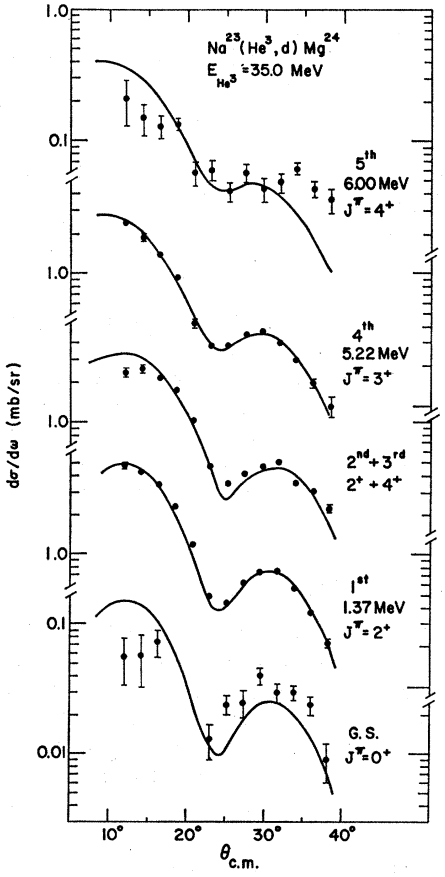

FIG. 3. Angular distributions extracted from the data. The solid lines are the predictions of JULIE for a pure $l=2$ transition.

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[10] We are indebted to Dr. R. M. Drisko for the use of the code JULIE.

TABLE I. Optical-model potentials used in code JULIE.

Reaction channel	V (MeV)	r (fm)	a (fm)	W (MeV)	r_W (fm)	a_W (fm)	W_D (MeV)	r_c (fm)
Incoming								
$Na^{23}+He^3$	153.5	1.150	0.694	30.5	1.455	0.890	...	1.40
Outgoing								
$Mg^{24}+d$	61.2	1.416	0.571	0.0	1.088	0.847	17.4	1.40

N is a normalization factor taking into account the overlap between He^3 and $(d+p)$ as well as the transition strength itself. We have taken the value obtained by Bassel,[11] $N=4.4$.

Spectroscopic factors were extracted by use of the code JULIE and the optical-model potentials shown in Table I. The form of the potential was the usual independent Woods-Saxon plus derivative (option 7). These potentials were chosen since they have been shown[12] to give good fits in this region of the periodic table. Calculations were performed with both a finite value and a zero value of V_{so}, but the latter results were used to extract spectroscopic factors because only the l and not the J^π of the transferred nucleon is known *a priori*.

Since our data do not resolve the second and third excited states, we turned to the data of Hinds and Middleton. With the potentials listed in Table I, JULIE was run for the lower (10.19 MeV) He^3 energy. By comparing these results with their data, we are able to split up our total $l=2$ spectroscopic factors for the sum of the second and third excited states into the individual components. Such a process undoubtedly introduces a larger error into the determination. The results of these manipulations are shown in Table II. The ratios of the spectroscopic factors extracted from the data of Hinds and Middleton for the second through sixth excited states are in excellent agreement with the present results. An error of about 30% should be assigned to the absolute value of the results with the exception of the second and third excited states which are probably good to 50%. The relative spectroscopic factors are, of course, much better known. The determination of the $l=0$ strengths in the two cases in which such transfer is possible (the first and third excited states) is discussed in the next section.

TABLE II. Spectroscopic factors obtained from the data and code JULIE. The values shown for the second and third excited states were extracted by comparing the present data with those of Hinds and Middleton.[a]

Final state	(J^π,K)	Spectroscopic factor $l=0$	Spectroscopic factor $l=2$
Ground	$(0^+, 0)$	...	0.05
1st exc.	$(2^+, 0)$	<0.06	1.1
2nd exc.	$(4^+, 0)$		0.05
3rd exc.	$(2^+, 2)$	0.13	0.30
4th exc.	$(3^+, 2)$		0.24
5th exc.	$(4^+, 2)$		0.03
6th exc.	$(0^+,)$		~0.005

[a] Reference 8.

IV. APPLICABILITY OF ROTATIONAL MODEL

Since it is generally assumed that the nuclei around Mg^{24} are deformed, it becomes possible to examine these nuclear states as arising from some rotational model. This would imply that K is a good quantum number. The observation that the Nilsson model[13] correctly predicts the ground state of Na^{23} to be $\frac{3}{2}^+$ (whereas the simplest undeformed-shell-model assignment would be $\frac{5}{2}^+$) indicates that it has $K=\frac{3}{2}^+$. For transfer reactions there then arises a selection rule which was first pointed out by Satchler.[14] Since the low-lying states of Mg^{24} (through the second excited state) are members of a $K=0^+$ band, any direct reaction linking the Na^{23} ground state to these states requires a transfer of a $K=\frac{3}{2}$ particle. Since K is the projection of the angular momentum onto the symmetry axis of the target nucleus, it follows that the j of this transferred nucleon must be greater than this value of K. Hence j must be greater than $\frac{3}{2}$ for stripping to the first three states of Mg^{24}. In the case of the ground state and second excited state, conservation of angular momentum would also require $j\geq\frac{3}{2}$; but for the first excited state this is not so since $j=\frac{1}{2}$ $(l=0)$ stripping is allowed by angular-momentum conservation laws. Therefore, if the rotational model applies to Na^{23} and Mg^{24}, K is a good quantum number; and unless the Na^{23} ground state contains an admixture of $K=\frac{1}{2}^+$ bands, $l=0$ stripping is forbidden to the Mg^{24} first excited state.

To search for an $l=0$ admixture to the first excited state, the angular distribution was assumed to be of the form $\sigma(\theta)=A[\sigma_2(\theta)+b\sigma_0(\theta)]$, where σ_2 and σ_0 are the angular distributions computed with the code JULIE. A χ^2 determination was then made for varying amounts of b (where for each b, a best A was determined by a least-squares fit).

Since we are not able to state unequivocally that this function is the exact form of the angular distributions (as, for instance, $\sum a_nP_n$ may be assumed to fit γ-ray angular distributions), the actual value of the minimum χ^2 is not an indication of the confidence that may be placed in the data. To get a better feel for the meaning of the χ^2 test in such cases, we therefore performed exactly the same kind of test on the 3^+ state which, from angular-momentum considerations, is not allowed to have $l=0$ components. And to show the effects of actually having an $l=0$ component, a similar χ^2 test was performed on the combined second and third excited states; since $K=2^+$, $J=2^+$ for the third excited state, $J=\frac{1}{2}$ transfer is allowed. The results are shown in

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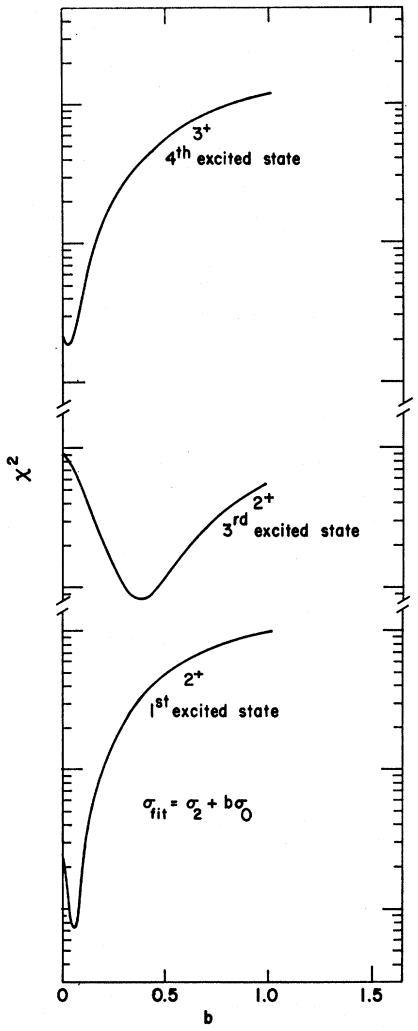

FIG. 4. Result of the χ^2 tests for the first, fourth, and combined second and third excited states. The results for the first and fourth excited states are practically indistinguishable and indicate little, if any, $l=0$ contribution, whereas the results for the third excited state show a large contribution from $l=0$ transfer.

Fig. 4. Inspection shows immediately that the position and shape of the minima for the first and fourth excited states are the same but that they are greatly different from the combined results for the second and third excited states. So it appears that the rotational model is not violated. The χ^2 tests were also used to determine the values of the $l=0$ spectroscopic factors displayed in Table II.

We conclude then that the results do not violate the restrictions placed on the reaction by a quantum number K, defined to be a constant in the rotational model. Hence we consider further predictions of the model.

V. DISCUSSION

Obviously this reaction should shed light on the extent to which various models are applicable to the

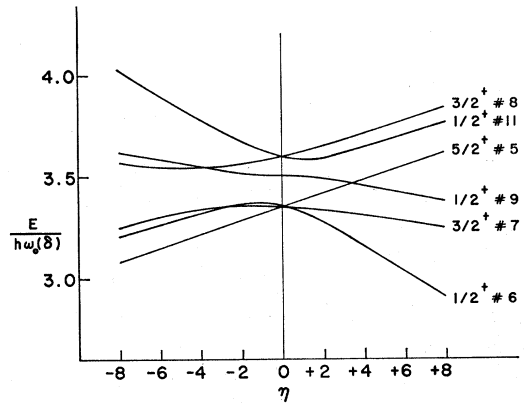

FIG. 5. Nilsson orbitals as a function of the deformation η for the $2s$–$1d$ shell. The zero of the ordinate has been suppressed.

nucleus studied here. However, it must be remembered that the spectroscopic factors are a measure of the overlap between $Na^{23}+p$ and Mg^{24} and information about Mg^{24} can only be extracted on the basis of assumptions about Na^{23}.

It is possible to make some statements about viewing the $K=2$ band in Mg^{24} as being built on the first vibrational state. Satchler[14] has pointed out that direct reactions may only link states of the same vibrational character (because of overlap, i.e., $\langle j\text{-phonon}|i\text{-phonon}\rangle = \delta_{ij}$); and hence if Na^{23} is a zero vibrational state, no strong direct reactions may proceed to states in the $N_\gamma=1$ band. Since we see quite strong transitions to the $K=2$ band in Mg^{24}, either Na^{23} is a mixture of large amounts of vibrational and zero vibrational states or

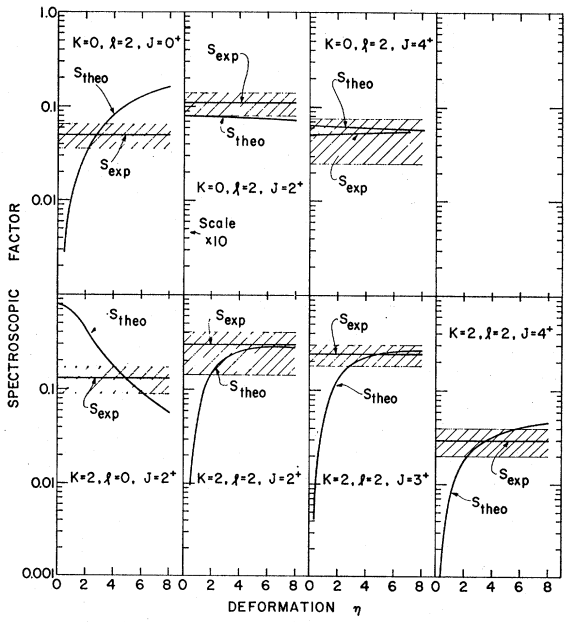

FIG. 6. A comparison between the measured spectroscopic factors and those calculated from the Nilsson model. A value $\eta=3$, in agreement with Ref. 7, gives a good over-all fit.

the $K=2$ band does not arise from a simple vibrational picture.

The Nilsson model, on the other hand, allows a more detailed comparison with the data since spectroscopic factors may be obtained from the wave functions of the intrinsic states, which are generally expressed as expansions of single-particle components. Figure 5 shows the Nilsson orbits computed for the mass region near $A=24$ by use of the parameters of Siemssen *et al.*[15] The Na^{23} ground state then arises from an odd proton in orbit No. 7 (prolate deformation is accepted for this region), and the $K=0^+$ band in Mg^{24} is produced by stripping a proton into that orbit. A $K=2^+$ band can be constructed by stripping a particle into the $K=\frac{1}{2}$ orbit No. 9. The 0^+ sixth excited state then may be pictured as two nucleons in orbit No. 5 or 9 and hence could not be strongly excited in the (He^3,d) reaction. (Such excitation would require a two-step process of promoting one particle in the target nucleus to orbit No. 5 or 9 before or after stripping into it.)

[15] R. H. Siemssen, L. L. Lee, Jr., and D. Cline, Phys. Rev. **140**, B1258 (1965).

With these assumptions, spectroscopic factors as a function of deformation have been determined by the methods of Siemssen *et al.* The results are shown in Fig. 6.

It can be seen that for $\eta\approx 3$ (the value taken by Robinson and Bent[7] as the best fit to the γ-decay data), all of the experimental spectroscopic factors are in reasonable agreement with the model. A better fit would be obtained if all of the measured spectroscopic factors were made smaller by about 10%.

It would be interesting to compare the asymmetric-rotor picture of, say, Bar-Touv and Kelson[16] with the present results, but unfortunately this is not now feasible.

ACKNOWLEDGMENTS

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Lifetime Measurements from the $K^{39}(p,\gamma)Ca^{40}$ Reaction

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Lifetime measurements with a Ge(Li) detector have been made for several excited states of Ca^{40} by the Doppler-shift attenuation method utilizing the $K^{39}(p,\gamma)Ca^{40}$ reaction. Agreement with results of $(p,p'\gamma)$ lifetime measurements is found for the levels at 3.904, 5.280, and 6.285 MeV. New lifetime results are reported for levels at 5.615 MeV (>0.8 psec), 7.465 MeV ($0.010_{-0.006}{}^{+0.005}$ psec), and 7.562 MeV ($0.26_{-0.07}{}^{+0.14}$ psec. An upper limit of 1% is set for the branching of the 3.904-MeV (2^+) to the 3.354-MeV (0^+) levels. Comparison is made with Gerace and Green's predictions that the 5.280-, 3.904-, and 3.354-MeV levels form a rotational band. Calibration of the resonance spectrum with Co^{56} γ rays has resulted in precise establishment of level energies for Ca^{40}, a better Q value for the reaction of 8.3295±0.0009 MeV, and the association of the lower member of the doublet near 5.615 MeV with the 4^- level in this region. New decay modes are established for the levels at 7.113, 7.562, and 7.811 MeV.

I. INTRODUCTION

A RECENT innovation in nuclear theory has been the prediction of coexistence of single-particle and collective modes of excitation in nuclei near closed shells.[1–5] Initial evidence for the existence of deformed states in addition to the usual spherical states in O^{18} and O^{16} was the observation of anamolously large $E2$ transition probabilities.[6] More conclusive evidence came from the observation that low-lying excited states in the oxygen nuclei could be fitted into rotational energy bands characteristic of deformed nuclei.[7] Since calculations based on this model have had significant success in explaining the experimental features of the oxygen isotopes which close the $1p$ shell,[1–4] it is essential to look at the calcium isotopes which close the $2s$-$1d$ shell and see if the same phenomenon repeats.

Ca^{40} does seem to exhibit nuclear excited states that can be fitted into even- or odd-parity rotational bands

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