Static Coulomb Corrections to the Point-Nucleus Fermi Function in Nuclear Beta Decay

L. J. LOGUE* AND B. CHERN†‡
University of Florida, Gainesville, Florida 32601

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In this paper we use a simple model to include the effects of nuclear charge change in β decay. We develop the point-nucleus Fermi function for the daughter nucleus in perturbation theory to $Z^2\alpha^2$ and corrections to it arising from the change in nuclear charge. The model considers only Coulomb interactions between the static parent-daughter nucleus with the leptons and allows for pair creation. This model has been used by Chern *et al.* and by Halpern and Chern to obtain analytical model dependences of the finite nuclear size, screening, and static β-vertex correction to first order in α. The divergences in the Fermi function are shown to have the same source as the divergences occuring in this model due to the change in nuclear charge, so that both types of divergence must be treated in the same manner, including the use of the same cutoff. We find that the $Z\alpha^2$ contribution to the transition probability per unit time calculated from this model is comparable to the screening correction, and thus must be included if one takes the screening into account. An uncertainty has arisen in the literature concerning the incorporation of the first-order radiative correction into the Fermi function. We find from this model that the approximate factorization previously used in the literature to order α is also valid to order α^2 for nuclei with Z less than that of Al^{26}. The validity of this factorization for higher Z is still unresolved.

1. INTRODUCTION

CONSIDERABLE attention has recently been given to the precise determination of the vector coupling constant G_v for nuclear β decay.[1-3] This attention has been focused on the calculation of several small corrections to the theoretical β-decay transition probability obtained by using the point-nucleus Fermi function. These corrections include the electromagnetic radiative corrections[4-7] and the modification of the point-nucleus Fermi function by nuclear size and structure effects,[8-12] and screening of the nuclear Coulomb field by the atomic electrons.[13-15] The vector coupling constant is calculated by applying these corrections to the experimental ft values for a series of pure Fermi decays ($0^+ \rightarrow 0^+$). The precise values of G_v obtained are important in assessing the problem of universality in the weak interactions.[16]

All of these corrections except the radiative correction have in the past been treated by constructing numerical solutions of the Dirac equation for suitably modified static potentials, resulting in the modification of the point-nucleus Fermi function. The radiative correction, however, cannot be generated by a static potential, so it has been treated in perturbation theory, using the techniques of quantum electrodynamics. A simple model has recently been discussed by Chern *et al.*[3] which uses a perturbation calculation and is capable of treating analytically, and in a consistent manner, finite nuclear size, screening, and the Coulomb interaction between the electron and the static parent-daughter nucleus (which gives rise to the bulk of the radiative correction to order α) as corrections to the point-nucleus Fermi function. Halpern and Chern[12] have extended this model to calculate the nuclear model dependences of these effects. The results obtained are found to be in good agreement with those obtained by other methods for those simple static nuclear charge distributions that have been considered previously in the literature. In addition, they have considered a more realistic nuclear model based on the nuclear charge distribution of Hofstadter[17] and the single-particle simple-harmonic-oscillator model for the nuclear matrix elements. It is found that the model dependence of the finite-nuclear-size correction is comparable to the screening correction, making it imperative that one use the most realistic nuclear model available. Since the screening is such a small effect ($\leq 0.3\%$ of f_0t), it was realized that the model used generates logarithmically divergent terms of order $Z\alpha^2$ that could be comparable to the screening and that have their origin in the change in nuclear

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‡ Present Address: National Science Foundation, Washington, D. C. 20550.

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charge during the β decay. In the present work, we use the same basic field-theoretic model, which includes the effects of nuclear charge change in β decay. We develop the point-nucleus Fermi function to $Z^2\alpha^2$ and corrections to it arising from the change in nuclear charge. The model considers only Coulomb interactions between the static parent-daughter nucleus with the leptons and allows for pair creation. We show that these $Z\alpha^2$ contributions are indeed comparable to the screening correction so that consistency requires that they be included in applications where the screening is significant. These $Z\alpha^2$ terms form part of the radiative correction to the β decay as is discussed below.

In treating the electromagnetic interaction in perturbation theory, one generates the effect of the usual static-Coulomb interaction as well as the electrodynamic effects associated with transverse photons. The Coulomb interaction between the emitted β particle and the static point-charge daughter nucleus has of course been treated by solving the Dirac equation for an electron in a Coulomb field, and results in the point-charge Fermi function $F_0(Z,E)$. Thus, in the perturbation treatment one separates out that part of the transition probability to a given order in α that can be identified with F_0 to the same order; the remainder is then regarded as a radiative correction to $F_0(Z,E)$. This has previously been done in the literature only to first order in α, to which order no divergences appear in $F_0(Z,E)$. This so-called radiative correction can be further divided into two parts. The first of these parts, which we call the β-vertex correction, arises from those processes that involve the interchange of virtual photons only between the nucleus and the leptons. The second part arises from processes that involve interchange of virtual photons among the leptons in addition to possible interchanges with the nucleus. These include processes normally associated with the charge and mass renormalization of the electron.

The model used in this paper treats only the static part of the β-vertex correction. Berman[6] and Chern *et al.*[3] have shown that this static β-vertex correction accounts for $\sim\frac{2}{3}$ of the radiative correction to first order in α, and gives rise to the bulk of the ultraviolet divergence. The major part of the divergence to first order in α is thus shown to arise because of the sudden change in the charge of the nucleus associated with the β decay. It is this divergence, which is directly associated with the divergences in the Fermi function for nuclear β decay, with which we are concerned in this paper, since it has been our purpose to calculate $F_0(Z,E)$ and corrections to it in a consistent manner. The approximation of keeping only the static β-vertex correction is consistent with the previous treatments of Chern *et al.*[3] and Halpern and Chern,[12] which obtain the Fermi function and analytical-model-dependent corrections to it using the physical particle masses and charges, ignoring purely electrodynamic-type corrections. Thus, using the same basic model to calculate the static β-vertex part of the radiative correction we are able to incorporate it consistently as one of the several corrections to the point-nucleus Fermi function.

It should be pointed out that even if one were to treat all the divergences that appear in the radiative correction in some consistent manner, one would then have to recalculate the Fermi function using this new "cutoff" since the point-nucleus Fermi function explicitly contains a configuration-space cutoff and provides the major part of the electromagnetic interaction between the nucleus and the electron. Our approach treats those divergences in the radiative correction that clearly arise in the same manner as the divergences in $F_0(Z,E)$. We, of course, do not claim that our model solves all of the problems associated with the radiative corrections, but it does contain the major part of the physics and makes clear the way of treating some of the divergences in a physically meaningful way, which is forced upon us if we take the point-nucleus Fermi function as our starting point. This is the point of view taken in our series of papers previously referred to and that has implicitly been taken in all previous calculations of the vector coupling constant based on a universal Fermi interaction.

It has been pointed out[18] that the divergent contribution to $F_0(Z,E)$ to order $Z^2\alpha^2$ is comparable to and of the same sign as the ultraviolet divergent part of the first-order static β-vertex correction. The model used here produces both of these divergent terms and allows us to show that they both arise from the same source and have the same form so that the cutoff procedure used must be the same in both cases.

Furthermore, an ambiguity has arisen in the literature concerning the incorporation of the radiative correction into the Fermi function.[3,16] In a first-order perturbation calculation using a plane-wave expansion of the electron field operators, the transition probability per unit time for a negatron decay is found to be proportional to

$$\omega=1+\pi Z\alpha E/p+(\mathrm{R.C.})\alpha. \tag{1.1}$$

The first two terms are recognized as the expansion of $F_0(Z,E)$ to order $Z\alpha$, so that we write

$$\omega=(F_0)_{Z\alpha}+(\mathrm{R.C.})\alpha, \tag{1.2}$$

where $(\mathrm{R.C.})\alpha$ can now be interpreted as a radiative correction added to $F_0(Z,E)$. Since we know $F_0(Z,E)$ to all orders in $Z\alpha$, it is generally assumed that this portion of the perturbation series can be summed to give

$$\omega=F_0(Z,E)+(\mathrm{R.C.})\alpha. \tag{1.3}$$

Further, to this order in α, we can factor Eq. (1.2) in the form

$$\omega(F_0)_{Z\alpha}[1+(\mathrm{R.C.})\alpha]. \tag{1.4}$$

[18] B. Chern, Ph.D. thesis, University of North Carolina (unpublished).

Replacing $(F_0)_{Z\alpha}$ by $F_0(Z,E)$ would now give

$$\omega=F_0(Z,E)[1+(\text{R.C.})\alpha]. \tag{1.5}$$

However, as pointed out by Chern *et al.*,[3] a proper factorization of Eq. (1.3) is given by

$$\omega=F_0(Z,E)[1+(\text{R.C.})\alpha/F_0(Z,E)]. \tag{1.6}$$

In fact, calculation of Eq. (1.3) to all orders would imply that the complete radiative correction should replace (R.C.)α in Eq. (1.6). Equations (1.5) and (1.6) differ by terms of order $Z\alpha^2$ and higher, which are especially significant for heavy nuclei. In a recent paper, Brene, Roos, and Sirlin[16] have discussed this last point and have emphasized the importance of calculating such $Z\alpha^2$ terms which contribute to the second-order radiative correction. We discuss this point in detail in Sec. 4.

2. GENERAL FORMALISM

It is well known that the major electromagnetic effect in β decay is the Coulomb interaction between the emitted β particle and the daughter nucleus. This effect was taken into account by Fermi[19] by expanding the electron field operators in the set of angular momentum states for an electron in the Coulomb field of a point nucleus with charge Ze. The transition probability per unit time is proportional to

$$\sum_{\text{final spins}} \psi(x)\psi^\dagger(x), \tag{2.1}$$

where $\psi(x)$ is to be evaluated at the point where the decay occurs. If the nucleus is regarded as a point, then $\psi(x)$ must be evaluated at the origin. Only the $j=\frac{1}{2}$ wave functions are nonzero at the origin, and in fact they diverge weakly there. This divergence is avoided by assuming that the decay is most likely to occur near the nuclear surface, so that we set $x=R$, the nuclear radius. The resulting expression, the point-charge Fermi function, is given by

$$F_0(Z,E)=[2!/\Gamma(1+2S_0)]^2 \exp(\pi Z\alpha E/p) \times(2pR)^{2(S_0-1)}|\Gamma(S_0+iZ\alpha E/p)|^2(1+S_0)/2, \tag{2.2}$$

where,

$$S_0=(1-Z^2\alpha^2)^{1/2},$$
$$E^2=p^2+\mu^2.$$

Expanding this expression to second order in $Z\alpha$, we have

$$F_0^{(\mp)}(Z,E)=1\pm\pi Z\alpha E/p +[\tfrac{2}{3}\pi^2E^2/p^2+11/4-\gamma-\ln(2pR)]Z^2\alpha^2, \tag{2.3}$$

where $\gamma=0.57721$ is Euler's constant. The upper and lower signs refer to β^- and β^+ decays, respectively. This form explicitly displays the small distance cutoff to lowest order; the cutoff will of course appear in all higher orders.

[19] E. Fermi, Z. Physik **88**, 161 (1934).

The model that we use in the perturbation calculation is the simplest model that is capable of generating the Fermi function, including the finite-nuclear size and screening, along with the essential features of the radiative correction. As discussed in the Introduction, the physical origin of the major part of the ultraviolet divergence that occurs in the radiative correction to order α is due to the sudden change in nuclear charge due to the β decay. This feature is explicitly displayed in this model by the use of a different potential according to whether the interaction precedes or follows the β decays. The bulk of this contribution comes from the static β-vertex correction, which we calculate in this paper.

We carry out the calculation using ordinary noncovariant perturbation theory, in which only the lepton fields are quantized. For an allowed pure Fermi decay we take the β interaction Hamiltonian

$$H_\beta=g'\int d^3x\,\psi_p{}^\dagger\psi_N\psi_e{}^\dagger(1+\gamma_5)\psi_\nu, \tag{2.4}$$

and the Coulomb interaction Hamiltonian

$$H_c=-Z\alpha\int d^3x'\,\frac{\psi_e{}^\dagger(\mathbf{x}')\psi_e(\mathbf{x}')}{|\mathbf{x}'|}. \tag{2.5}$$

We employ the usual plane-wave expansion for the lepton field operators.[20] The β-decay process, with the static vertex corrections to second order, can then be described by the diagrams in Fig. 1. The time ordering of the vertices is explicitly displayed in these diagrams, with the time increasing upward. As discussed in Sec. 1, these diagrams give only the static β-vertex corrections. Some typical electrodynamic-type corrections, which also have a static limit, are shown in Fig. 2. These may be interpreted as electromagnetic corrections to the first-order β-vertex correction, and along with the corrections to the electron lines, they are not treated in this paper for the reasons discussed in the Introduction. The transition matrix for an allowed pure Fermi decay can then be written in the form

$$\langle f|H_I|i\rangle=g'M_Fu^\dagger(\mathbf{p}) \times\{[a+(b+c)Z\alpha+(d+e+f+g+h+i)Z^2\alpha^2] +[-c\alpha-(f+g+2(h+i))Z\alpha^2+(h+i)\alpha^2]\} \times(1+\gamma_5)v_-(-\mathbf{q}), \tag{2.6}$$

where $u(\mathbf{p})$ is a positive-energy free-particle electron spinor, $v_-(-\mathbf{q})$ is a negative-energy neutrino spinor, M_F is the Fermi-matrix element, and a, b, c, etc., are the contributions from the respective diagrams in Fig. 1. The explicit integrals for these diagrams are given in Sec. 3.

We see that the quantity in the first square bracket of Eq. (2.6) is exactly the contribution that would be

[20] See, for example, G. Källén, *Elementary Particle Physics* (Addison-Wesley Publishing Co., Inc., Reading, Mass., 1964).

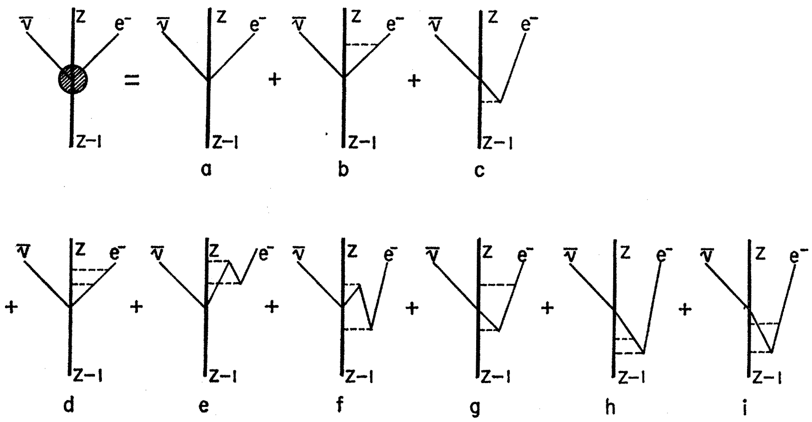

FIG. 1. Diagrams for β^- decay with static-vertex correction to order α^2.

obtained by setting $(Z-1)$ equal to Z in all of the diagrams. This quantity then gives the effect of the Coulomb interaction between the emitted β particle and a fixed nucleus of charge Z; that is, it represents the wave function of the electron in the Coulomb field of the daughter nucleus to this order. It is just this wave function that appears in the Fermi function, so that it is this part of the amplitude that generates the Fermi function. The second square bracket, which comes only from diagrams c, f, g, h, and i, arises because of the change in charge and will give rise to the static β-vertex correction to the Fermi function.

3. CALCULATION OF THE TRANSITION PROBABILITY

The explicit integrals that appear in the transition amplitude are

$$a(\mathbf{p})=1, \tag{3.1a}$$

$$b(\mathbf{p})=\frac{-4\pi}{(2\pi)^3}\int d^3k\,\frac{(E_k+\boldsymbol{\alpha}\cdot\mathbf{k}+\beta\mu)\exp(-i\mathbf{k}\cdot\mathbf{x})}{(|\mathbf{p}-\mathbf{k}|^2+\Delta^2)2E_k(E-E_k)}, \tag{3.1b}$$

$$c(\mathbf{p})=-\frac{4\pi}{(2\pi)^3}\int d^3k\,\frac{(E_k-\boldsymbol{\alpha}\cdot\mathbf{k}-\beta\mu)\exp(-i\mathbf{k}\cdot\mathbf{x})}{(|\mathbf{p}-\mathbf{k}|^2+\Delta^2)2E_k(E+E_k)}, \tag{3.1c}$$

$$d(\mathbf{p})=\frac{(4\pi)^2}{(2\pi)^6}\int d^3k''\int d^3k'\frac{(E_{k''}+\boldsymbol{\alpha}\cdot\mathbf{k}''+\beta\mu)(E_{k'}+\boldsymbol{\alpha}\cdot\mathbf{k}'+\beta\mu)\exp(-i\mathbf{k}'\cdot\mathbf{x})}{(|\mathbf{p}-\mathbf{k}''|^2+\Delta^2)2E_{k''}(E-E_{k''})(|\mathbf{k}''-\mathbf{k}'|^2+\Delta^2)2E_{k'}(E-E_{k'})}, \tag{3.1d}$$

$$e(\mathbf{p})=\frac{(4\pi)^2}{(2\pi)^6}\int d^3k''\int d^3k'\frac{(E_{k''}-\boldsymbol{\alpha}\cdot\mathbf{k}''-\beta\mu)(E_{k'}+\boldsymbol{\alpha}\cdot\mathbf{k}'+\beta\mu)\exp(-i\mathbf{k}'\cdot\mathbf{x})}{(|\mathbf{p}-\mathbf{k}''|^2+\Delta^2)2E_{k''}(E_{k'}+E_{k''})(|\mathbf{k}''-\mathbf{k}'|^2+\Delta^2)2E_{k'}(E-E_{k'})}, \tag{3.1e}$$

$$f(\mathbf{p})=-\frac{(4\pi)^2}{(2\pi)^6}\int d^3k''\int d^3k'\frac{(E_{k''}-\boldsymbol{\alpha}\cdot\mathbf{k}''-\beta\mu)(E_{k'}+\boldsymbol{\alpha}\cdot\mathbf{k}'+\beta\mu)\exp(-i\mathbf{k}'\cdot\mathbf{x})}{(|\mathbf{p}-\mathbf{k}''|^2+\Delta^2)2E_{k''}(E+E_{k''})(|\mathbf{k}''-\mathbf{k}'|^2+\Delta^2)2E_{k'}(E_{k'}+E_{k''})}, \tag{3.1f}$$

$$g(\mathbf{p})=\frac{(4\pi)^2}{(2\pi)^6}\int d^3k''\int d^3k'\frac{(E_{k''}+\boldsymbol{\alpha}\cdot\mathbf{k}''+\beta\mu)(E_{k'}-\boldsymbol{\alpha}\cdot\mathbf{k}'-\beta\mu)\exp(-i\mathbf{k}'\cdot\mathbf{x})}{(|\mathbf{p}-\mathbf{k}''|^2+\Delta^2)2E_{k''}(E-E_{k''})(|\mathbf{k}''-\mathbf{k}'|^2+\Delta^2)2E_{k'}(E_{k'}+E_{k''})}, \tag{3.1g}$$

$$h(\mathbf{p})=\frac{(4\pi)^2}{(2\pi)^6}\int d^3k''\int d^3k'\frac{(E_{k''}-\boldsymbol{\alpha}\cdot\mathbf{k}''-\beta\mu)(E_{k'}-\boldsymbol{\alpha}\cdot\mathbf{k}'-\beta\mu)\exp(-i\mathbf{k}'\cdot\mathbf{x})}{(|\mathbf{p}-\mathbf{k}''|^2+\Delta^2)2E_{k''}(E+E_{k''})(|\mathbf{k}''-\mathbf{k}'|^2+\Delta^2)2E_{k'}(E+E_{k'})}, \tag{3.1h}$$

$$i(\mathbf{p})=-\frac{(4\pi)^2}{(2\pi)^6}\int d^3k''\int d^3k'\frac{(E_{k''}+\boldsymbol{\alpha}\cdot\mathbf{k}''+\beta\mu)(E_{k'}-\boldsymbol{\alpha}\cdot\mathbf{k}'-\beta\mu)\exp(-i\mathbf{k}'\cdot\mathbf{x})}{(|\mathbf{p}-\mathbf{k}''|^2+\Delta^2)2E_{k''}(E_{k'}+E_{k''})(|\mathbf{k}''-\mathbf{k}'|^2+\Delta^2)2E_{k'}(E+E_{k'})}. \tag{3.1i}$$

The parameter Δ is introduced to account for the screening by the atomic electrons.[12] We have treated the Coulomb potential as the limit of a Yukawa potential. In the following we set $\Delta=0$ except where it would result in an infrared divergence. This procedure is formally equivalent to assigning a small mass to the photon and then allowing the mass to go to zero.[21] The coordinate $\mathbf{x}$ is a nuclear coordinate. Since we are not concerned with finite-nuclear-size corrections here, we will put $x=0$ in what follows except where this would result in an ultraviolet divergence, in which case we put $x=R$, a configuration-space cutoff of the order of the nuclear radius. In Eqs. (3.1d) through (3.1i), $\mathbf{k}'$ represents the momentum of the intermediate electron state that participates in the β vertex.

In order to calculate the transition probability per unit time, we need to construct the quantity

$$T=\sum_{\text{final spins}}\int d\Omega_e d\Omega_\nu |\langle f|H_I|i\rangle|^2, \tag{3.2}$$

where we are summing over the final lepton spins and directions since these are not observed in the particular experiments of interest. Substituting Eq. (2.6) into Eq. (3.2) and performing the indicated sums gives

$$\begin{aligned}T=2g'^2(4\pi)^2|M_F|^2\,\mathrm{Tr}(1+\gamma_5)[[\{a^\dagger\lambda_+(\mathbf{p})a\}+\{a^\dagger\lambda_+(\mathbf{p})(b+c)+(b+c)^\dagger\lambda_+(\mathbf{p})a\}Z\alpha\\ +\{a^\dagger\lambda_+(\mathbf{p})(d+e+f+g+h+i)+(b+c)^\dagger\lambda_+(\mathbf{p})(b+c)+(d+e+f+g+h+i)^\dagger\lambda_+(\mathbf{p})a\}Z^2\alpha^2]\\ +[-\{a^\dagger\lambda_+(\mathbf{p})c+c^\dagger\lambda_+(\mathbf{p})a\}\alpha-\{a^\dagger\lambda_+(\mathbf{p})(f+g+2h+2i)+(f+g+2h+2i)^\dagger\lambda_+(\mathbf{p})a\\ +(b+c)^\dagger\lambda_+(\mathbf{p})c+c^\dagger\lambda_+(\mathbf{p})(b+c)\}Z\alpha^2+\{a^\dagger\lambda_+(\mathbf{p})(h+i)+c^\dagger\lambda_+(\mathbf{p})c+(h+i)^\dagger\lambda_+(\mathbf{p})a\}\alpha^2]]\lambda_-(-\mathbf{q}),\end{aligned} \tag{3.3}$$

where

$$\lambda_+(\mathbf{p})=(E+\boldsymbol{\alpha}\cdot\mathbf{p}+\beta\mu)/2E \tag{3.3$'$}$$

is the positive energy projection operator for the electron, and

$$\lambda_-(-\mathbf{q})=(q+\boldsymbol{\alpha}\cdot\mathbf{q})/2q \tag{3.3$''$}$$

is the negative energy projection operator for the neutrino.

The combination $(b+c)$ has been calculated by Dalitz,[21] in connection with the Coulomb scattering problem, and subsequently by Chern[18] as the zeroth-order term in an expansion in powers of px, with the result

$$\begin{aligned}(b+c)&=-\frac{4\pi}{(2\pi)^3}\int d^3k\,\frac{(E+\boldsymbol{\alpha}\cdot\mathbf{k}+\beta\mu)}{(|\mathbf{p}-\mathbf{k}|^2+\Delta^2)(E^2-E_k{}^2)_F}\\ &=-\frac{4\pi}{(2\pi)^3}\left\{\frac{i\pi^2(E+\beta\mu)}{p}\ln\left(\frac{i\Delta}{2p}\right)\right.\\ &\qquad\left.+\frac{\pi^2\boldsymbol{\alpha}\cdot\mathbf{p}}{p}\left[i+i\ln\left(\frac{i\Delta}{2p}\right)\right]\right\},\end{aligned} \tag{3.4}$$

where we have set $\Delta=0$ where possible in the last line. The integral c itself has been calculated by Halpern[12]

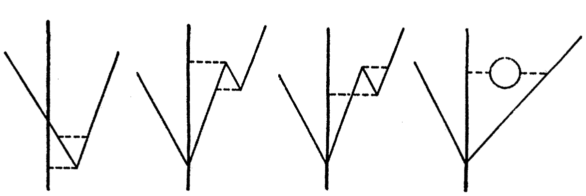

FIG. 2. Some typical static contributions to the radiative correction which have been neglected in this paper.

[21] R. H. Dalitz, Proc. Roy. Soc. (London) **A206**, 509 (1951).

giving

$$(c)\approx-\frac{\pi}{8}\frac{(E+\beta\mu)}{p}+\boldsymbol{\alpha}\cdot\mathbf{p}\left[\frac{\pi}{8p}-\frac{1}{2\pi}\left(\frac{E}{p^2}\right)-i\frac{1}{4p}\right]-\frac{1}{\pi}[2-\gamma-\ln(pR)]. \tag{3.5}$$

The expression for $(b+c)$ given in the first line of Eq. (3.4) can also be obtained by an approximation procedure that will be used to evaluate some of the higher-order terms. Integrals b and c can be separately arranged to read

$$\begin{aligned}(b)=&-\frac{4\pi}{(2\pi)^3}\int d^3k\\ &\times\left(\frac{E+E_k}{2E_k}\right)\frac{(E+\boldsymbol{\alpha}\cdot\mathbf{k}+\beta\mu)}{(|\mathbf{p}-\mathbf{k}|^2+\Delta^2)(E^2-E_k{}^2)_F}\\ &+\frac{4\pi}{(2\pi)^3}\int d^3k\,\frac{1}{(|\mathbf{p}-\mathbf{k}|^2+\Delta^2)2E_k},\end{aligned} \tag{3.6}$$

$$\begin{aligned}(c)=&\frac{4\pi}{(2\pi)^3}\int d^3k\\ &\times\left(\frac{E-E_k}{2E_k}\right)\frac{(E+\boldsymbol{\alpha}\cdot\mathbf{k}+\beta\mu)}{(|\mathbf{p}-\mathbf{k}|^2+\Delta^2)(E^2-E_k{}^2)_F}\\ &-\frac{4\pi}{(2\pi)^3}\int d^3k\,\frac{1}{(|\mathbf{p}-\mathbf{k}|^2+\Delta^2)2E_k}.\end{aligned} \tag{3.7}$$

In each case the second term diverges for large k while the first term does not, but in $(b+c)$ the second terms exactly cancel. The denominators in the first term nearly vanish for $k\approx p$ so that we expect these integrands to be sharply peaked in the region $k\approx p$. In this region

$$\left(\frac{E+E_k}{2E_k}\right)\approx 1,\quad \left(\frac{E-E_k}{2E_k}\right)\approx 0. \tag{3.8}$$

Making this approximation then, the first term in b is equal to $(b+c)$ while the corresponding part of c vanishes. That is, the major contribution to $(b+c)$ comes from the diagram that represents the interaction with the daughter nucleus, while the effect of the interaction with the parent nucleus simply serves to cancel the ultraviolet divergence of the first diagram.

It should also be noted that diagram b has a nonrelativistic analog, while diagram c, involving virtual pair production, does not. Since the relativistic and nonrelativistic Fermi functions are identical to first order in $Z\alpha$, it is reasonable that diagram b should provide almost all of the contribution to this order. The major difference between the relativistic and nonrelativistic cases occurs to second and higher orders because of the divergence of the relativistic Coulomb wave functions at the origin. It is an important feature of this approximation that it does not affect the divergent parts of the integrals; in particular, the divergent part of c given in Eq. (3.5) would be unchanged.

With this in mind we are now in a position to evaluate the higher-order contributions to the transition probability. The appropriate combination for the $Z^2\alpha^2$ term can be reduced to

$$(d+e+f+g+h+i)=\frac{(4\pi)^2}{(2\pi)^6}\int d^3k''\int d^3k'\frac{(E+\boldsymbol{\alpha}\cdot\mathbf{k}''+\beta\mu)(E+\boldsymbol{\alpha}\cdot\mathbf{k}'+\beta\mu)\exp(-i\mathbf{k}'\cdot\mathbf{x})}{(|\mathbf{p}-\mathbf{k}''|^2+\Delta^2)(E^2-E_{k''}{}^2)_F(|\mathbf{k}''-\mathbf{k}'|^2+\Delta^2)(E^2-E_{k'}{}^2)_F}. \tag{3.9}$$

This expression is convergent except for the $(\boldsymbol{\alpha}\cdot\mathbf{k}')(\boldsymbol{\alpha}\cdot\mathbf{k}'')$ term. Treating this term separately,

$$J_{\alpha\alpha}=\int d^3k''\frac{\boldsymbol{\alpha}\cdot\mathbf{k}''}{(|\mathbf{p}-\mathbf{k}''|^2+\Delta^2)(E^2-E_{k''}{}^2)_F}\int d^3k'\frac{\boldsymbol{\alpha}\cdot\mathbf{k}'}{(|\mathbf{k}''-\mathbf{k}'|^2+\Delta^2)(E^2-E_{k''}{}^2)_F}$$

$$=\pi^2\int d^3k''\frac{k''}{(|\mathbf{p}-\mathbf{k}''|^2+\Delta^2)(E^2-E_{k''}{}^2)_F}\left[\frac{ip}{k''}+i\frac{p^2+k''^2}{2k''^2}\ln\left(\frac{p-k+i\Delta}{p+k+i\Delta}\right)\right]. \tag{3.10}$$

The only ultraviolet divergence comes from the last term. Performing the remaining angular integration and examining the asymptotic form of the integrand,

$$(J_{\alpha\alpha})_\infty=-\frac{i\pi^3}{2p}\int_0^\infty dk''\frac{k''^2}{(E^2-E_{k''}{}^2)_F}\ln\left(\frac{p-k''+i\Delta}{p+k''+i\Delta}\right)\ln\left(\frac{k''^2+p^2+2pk''+\Delta^2}{k''^2+p^2-2pk''+\Delta^2}\right)\underset{k''\to\infty}{\longrightarrow}2\pi^4\int_\Lambda^\infty\frac{dk''}{k''}. \tag{3.11}$$

By retaining the factor $\exp(-i\mathbf{k}'\cdot\mathbf{x})$ and applying a Feynman identity to the original integral we would obtain a convergence factor $\exp(-\mathbf{k}''\cdot\mathbf{x})$ in Eq. (3.11). This integral is then of exactly the same from as the divergent part of c. We must then use the same cutoff procedure to evaluate both of these divergences. Thus Eq. (3.11) gives a contribution of the form $\ln(pR)$, and carrying out the trace indicated in Eq. (3.3) shows that it has exactly the same coefficient as the logarithmic term in the expansion of the Fermi function.

The remaining parts of Eq. (3.9) can then be approximated by noting, as in Eq. (3.8), that the k'' integrand is peaked about $k''\approx p$. Setting $\mathbf{k}''\approx\mathbf{p}$ in the second momentum transfer denominator has the effect of decoupling the two integrals, giving

$$(d+e+f+g+h+i)\approx(b+c)^2+(J_{\alpha\alpha})_\infty. \tag{3.12}$$

Proceeding to the second order static β-vertex corrections, we next calculate the $Z\alpha^2$ contribution from the second diagrams.

$$(f+g+2h+2i)=\frac{(4\pi)^2}{(2\pi)^6}\Bigg\{\int d^3k''\frac{(E+\boldsymbol{\alpha}\cdot\mathbf{k}''+\beta\mu)}{(|\mathbf{p}-\mathbf{k}''|^2+\Delta^2)(E^2-E_{k''}{}^2)_F}\Bigg[\int d^3k'\left(\frac{E_{k'}+E_{k''}}{E+E_{k'}}\right)$$

$$\times\frac{(E_{k'}-\boldsymbol{\alpha}\cdot\mathbf{k}'-\beta\mu)}{(|k''-k'|^2+\Delta^2)2E_{k'}(E_{k'}+E_{k''})}+\frac{(E_{k''}-E)}{E_{k''}}\int d^3k'\left(\frac{E_{k'}}{E+E_{k'}}\right)\frac{(E_{k''}-\boldsymbol{\alpha}\cdot\mathbf{k}'-\beta\mu)}{(|\mathbf{k}''-\mathbf{k}'|^2+\Delta^2)2E_{k'}(E_{k'}+E_{k''})}\Bigg]$$

$$-\int d^3k''\frac{1}{(|\mathbf{p}-\mathbf{k}''|^2+\Delta^2)E_{k''}}\Bigg[\int d^3k'\frac{(\boldsymbol{\alpha}\cdot\mathbf{k}'+\beta\mu)}{(|\mathbf{k}''-\mathbf{k}'|^2+\Delta^2)2E_{k'}(E_{k'}+E_{k''})}$$

$$+(E+\boldsymbol{\alpha}\cdot\mathbf{k}''+\beta\mu)\int d^3k'\left(\frac{1}{E+E_{k'}}\right)\frac{(\boldsymbol{\alpha}\cdot\mathbf{k}'+\beta\mu)}{(|\mathbf{k}''-\mathbf{k}'|^2+\Delta^2)2E_{k'}(E_{k'}+E_{k''})}\Bigg]\Bigg\}. \tag{3.13}$$

Of these four terms, only the first contains an ultraviolet divergence that can survive the final spin sums and integrations. Other combinations of ultraviolet, and infrared, divergences occur, but with imaginary coefficients so that they do not contribute to the transition probability. Applying the same reasoning used in Eq. (3.8), we see that the first term dominates over the second. The remaining two terms are seen to be of the same order as the second. Thus, as an approximation to Eq. (3.13) we take

$$(f+g+2h+2i)\approx(b+c)(c). \tag{3.14}$$

We again emphasize that this approximation does not affect the asymptotic behavior of the integrands so that the divergent contributions are produced with the proper coefficients in the transition probability. It will be seen that these divergent terms dominate the static β-vertex correction to this order.

Finally, we have the α^2 contributions from the second-order diagrams:

$$\begin{aligned}(h+i)=\frac{(4\pi)^2}{(2\pi)^6}\Bigg\{&\int d^3k''\frac{(E_{k''}-\boldsymbol{\alpha}\cdot\mathbf{k}''-\beta\mu)}{(|\mathbf{p}-\mathbf{k}''|^2+\Delta^2)2E_{k''}(E+E_{k''})}\int d^3k'\frac{E_{k'}(E_{k'}-\boldsymbol{\alpha}\cdot\mathbf{k}'-\beta\mu)}{(E+E_{k'})(|\mathbf{k}''-\mathbf{k}'|^2+\Delta^2)2E_{k'}(E_{k'}+E_{k''})}\\&+\int d^3k''\frac{(\boldsymbol{\alpha}\cdot\mathbf{k}''+\beta\mu)}{(|\mathbf{p}-\mathbf{k}''|^2+\Delta^2)(E+E_{k''})}\int d^3k'\frac{(E_{k'}-\boldsymbol{\alpha}\cdot\mathbf{k}'-\beta\mu)}{(E+E_{k'})(|\mathbf{k}''-\mathbf{k}'|^2+\Delta^2)2E_{k'}(E_{k'}+E_{k''})}\\&-\int d^3k''\frac{(E_{k''}+\boldsymbol{\alpha}\cdot\mathbf{k}''+\beta\mu)}{(|\mathbf{p}-\mathbf{k}''|^2+\Delta^2)2E_{k''}(E+E_{k''})}\int d^3k'\frac{E(E_{k'}-\boldsymbol{\alpha}\cdot\mathbf{k}'-\beta\mu)}{(E+E_{k'})(|\mathbf{k}''-\mathbf{k}'|^2+\Delta^2)2E_{k'}(E_{k'}+E_{k''})}\Bigg\}.\end{aligned} \tag{3.15}$$

In this case the integrands do not have the sharp peaks of the previous expressions. However, we find that the first term dominates over the others since both the k' and k'' integrations are divergent. Since even this term is small compared with the α and $Z\alpha^2$ terms, we keep only the first term and again make use of the decoupling approximation

$$(h+i)\approx(c)(c). \tag{3.16}$$

Substituting Eqs. (3.12), (3.14), and (3.16) into Eq. (3.3) for T then gives

$$\begin{aligned}T\approx 2g'^2(4\pi)^2|M_F|^2\,\mathrm{Tr}(1+\gamma_5)[[&\Lambda_+(\mathbf{p})+\{\Lambda_+(\mathbf{p})(b+c)+(b+c)^\dagger\Lambda_+(\mathbf{p})\}Z\alpha\\&+\{\Lambda_+(\mathbf{p})[(b+c)^2+(J_{\alpha\alpha})_\infty]+(b+c)^\dagger\Lambda_+(\mathbf{p})(b+c)+[(b+c)^2+(J_{\alpha\alpha})_\infty]^\dagger\Lambda_+(\mathbf{p})\}Z^2\alpha^2]\\&+[-\{\Lambda_+(\mathbf{p})c+c^\dagger\Lambda_+(\mathbf{p})\}\alpha+\{\Lambda_+(\mathbf{p})cc+c^\dagger c^\dagger\Lambda_+(\mathbf{p})+c^\dagger\Lambda_+(\mathbf{p})c\}\alpha^2\\&-\{\Lambda_+(\mathbf{p})(b+c)c+(b+c)^\dagger c^\dagger\Lambda_+(\mathbf{p})+(b+c)^\dagger\Lambda_+(\mathbf{p})c+c^\dagger\Lambda_+(\mathbf{p})(b+c)\}Z\alpha^2]]\Lambda_-(-\mathbf{q}).\end{aligned} \tag{3.17}$$

Finally, we substitute for $(b+c)$ and c, noting that the terms in the first square bracket give the first three terms in the expansion of $F_0(Z,E)$, and perform the indicated traces:

$$\begin{aligned}T\approx 2g'^2(4\pi)^2|M_F|^2\Bigg\{&(F_0)_{Z^2\alpha^2}-\left[\frac{\pi}{2}\left(\frac{E}{p}\right)-\frac{5}{\pi}+\frac{2}{\pi}(\gamma+\ln(pR))\right]\alpha\\&-\left[\frac{\pi^2}{2}\left(\frac{E}{p}\right)^2-5\left(\frac{E}{p}\right)+2\left(\frac{E}{p}\right)(\gamma+\ln(pR))+\frac{\pi^2}{4}\left(1-\frac{E^2}{p^2}\right)+\frac{\pi^2}{8}\left(\frac{\mu}{p}\right)^2\left(\frac{E}{p}+1\right)\right]Z\alpha^2\\&+\left[\frac{3}{64\pi^2}(\pi^4+16)\left(\frac{E}{p}\right)^2+\frac{3(17)}{64}\left(\frac{\mu}{p}\right)^2-\frac{3}{4}\left(\frac{3E^2+2p^2+16\mu^2}{Ep}\right)+3\left(\frac{6}{\pi^2}+\frac{3\pi^2}{64}+\frac{1}{16}\right)\right.\\&\left.+3\left(\frac{E^2+p^2+8\mu^2}{4Ep}-\frac{5}{\pi^2}\right)(\gamma-\ln(pR))+\frac{3}{\pi^2}(\gamma+\ln(pR))^2\right]\alpha^2\Bigg\}.\end{aligned} \tag{3.18}$$

The corresponding expression for β^+ decay can be obtained simply by replacing Z by $-Z$ everywhere in Eq. (3.18); this affects only the sign of the $Z\alpha$ and $Z\alpha^2$ terms. For the decays used in the determination of G_v, the end-point energies are quite large so that $E/p\approx1$ over most of the spectrum. Furthermore, the statistical factor suppresses the decay at low momenta so that $E/p=1$ is a good approximation for these decays. Thus we see that the nonlogarithmic parts of each term in the static β-vertex correction tend to cancel, so that the highest powers of $\ln(pR)$ that appear in each coefficient dominate that coefficient.

This use of this extreme relativistic limit also allows us to obtain an estimate of the error incurred in the decoupling approximation. In this limit the nondivergent contribution to $Z^2\alpha^2$ obtained by using Eq. (3.12) is

$\sim\frac{3}{4}$ of the corresponding term in the expansion of $F_0(Z,E)$. This gives us considerable confidence in the approximation of the finite parts of the $Z\alpha^2$ and α^2 terms since the terms themselves are quite small.

4. RESULTS AND DISCUSSION

We have seen in Sec. 2 that the simple model used produces the logarithmically divergent $Z^2\alpha^2$ term which occurs in the expansion of the point-nucleus Fermi function. In addition, making use of the decoupling approximation described in Sec. 3, we obtain a good approximation to the nonlogarithmic $Z^2\alpha^2$ terms. Thus, we have isolated $(F_0)_{Z^2\alpha^2}$ in the transition probability so that the remainder of the transition probability represents the static β-vertex correction to order α^2 as given by this model.

One of the more significant results of this calculation is the relation of the divergence in the static β-vertex correction to that in the Fermi function. These two divergences are seen to arise from the same source so that the cutoff procedure must be the same in both cases. This result also raises the possibility that the divergence in the static β-vertex correction may be isolated in the same manner as that in $F_0(Z,E)$. This approach is contrasted with that of Källén,[22] Sirlin,[23] and others, which treats the divergence in the radiative correction from an elementary-particle viewpoint, but does not relate it to the Fermi function. Since, as we have emphasized, $F_0(Z,E)$ contains the major part of the electromagnetic correction, the treatment of the radiative correction must be consistent with it.

We now apply the results of Sec. 3 to the question of an approximate factorization of the transition probability. The dominant contributions to ω, to order α^2, are taken from Eq. (3.18), giving

$$\omega\approx F_0(Z,E)_{Z^2\alpha^2}-(2/\pi)(\gamma+\ln(pR))\alpha-(2E/p)\times(\gamma+\ln(pR))Z\alpha^2+(3/\pi^2)(\gamma+\ln(pR))^2\alpha^2. \quad (4.1)$$

As in the first order, it is possible to factor this expression to second order. The result, analogous to Eq. (1.4), is

$$\omega\approx F_0(Z,E)\{1-(2/\pi)(\gamma+\ln(pR))\alpha+(3/\pi^2)(\gamma+\ln(pR))^2\alpha^2\}, \quad (4.2)$$

while the expression analogous to Eq. (1.5) is

$$\omega\approx F_0(Z,E)\{1-[(2/\pi)(\gamma+\ln(pR))\alpha+2(E/p)(\gamma+\ln(pR))Z\alpha^2-(3/\pi^2)(\gamma+\ln(pR))^2\alpha^2]/F_0(Z,E)\}. \quad (4.3)$$

Here, we have dropped the terms of order $Z^2\alpha^3$, $Z\alpha^3$, α^3 and higher, as well as the small nonlogarithmic terms in Eq. (3.18). It must be pointed out that rapid convergence of the series for $F_0(Z,E)$ depends greatly on the parameter $Z\alpha$, which ranges from ~0.05 for O^{14} to ~0.2 for Co^{54}. The approximate factorization in Eq. (4.2) also seems to rest on the assumption that $Z\alpha$ and α can be formally treated as independent expansion parameters. It is difficult to provide a physical interpretation of this assumption. If we formally let $\alpha\rightarrow0$, then we find $\omega\rightarrow1$ rather than $\omega\rightarrow F_0(Z,E)$, which we would expect in the treatment of a small correction in perturbation theory. We emphasize that the convergence of the static β-vertex correction portion of the expansion is not well established. We have seen that the higher-order terms continue to diverge with the order of the divergence increasing with the order of the expansion. There are of course an infinite number of such terms and it is not clear that they can be summed. The existence of significant Z-dependent terms in the radiative correction is also somewhat disturbing in view of the nature of the source of the radiative correction as described in Sec. 2.

Concerning the rapid convergence of the static β-vertex correction, we note that the $Z\alpha^2$ term in the square bracket of Eq. (4.3) is not necessarily negligible when compared with the α term. The ratio of these terms ranges from ~0.16 for O^{14} to ~0.59 for Co^{54}. The α^2 term is down by an order of magnitude, being ~0.07 times the α term. For heavy elements, inclusion of the $Z\alpha^2$ term would result in a significant reduction of the radiative correction to the ft values. Thus for Co^{54}, the first-order value of $\sim1.5\%$[3] would be reduced to $\sim0.6\%$, whereas application of Eq. (4.2) would result in a negligible change in the corrections given in the literature. We point out that the $Z\alpha^2$ term is of the same order of magnitude as the screening correction so that this ambiguity must be resolved if these small corrections are to be reliably applied. For light elements such as C^{10} or O^{14} the difference between using Eq. (4.2) or Eq. (4.3) is so small that it would be masked by the experimental errors so that the approximate factorization can be used with considerable confidence. For the intermediate case of Al^{26}, for which the nuclear matrix element is felt to be well known, the difference between the two cases is comparable to the experimental error. The Al^{26} decay then stands on the borderline between what we call light and heavy nuclei in this sense. Thus, one can only unambiguously incorporate the static β-vertex correction into $F_0(Z,E)$ to $Z^2\alpha^2$ for elements with Z less than that of Al and for precise determinations of G_v one should restrict oneself to such decays. Such a conclusion has also been reached by Brene *et al.*[16] based on a discussion concerning the factorization of the radiative corrections to order α. Our results confirm the approximate validity of the factorization to order α^2.

The present status of the theory of weak interactions and the importance of the value of the vector coupling constant for nuclear β decay make it imperative that a definitive treatment of the radiative corrections be achieved. Such a treatment must regard the Fermi function as fundamental and provide a method of unambigu-

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ously extracting both the Z dependence and the divergence treated in this paper from the radiative correction.

If the approximate factorization expressed by Eq. (4.2) is indeed valid, it would be important to be able to sum the series in the curly bracket in a manner similar to that used in the Fermi function. For a typical decay of interest, the quantity $2\alpha(\gamma+\ln(pR))/\pi$ is on the order of 0.025 so that it would be a useful expansion parameter. We could then, to a good approximation write

$$\omega \approx F_0(Z,E)\exp[-(2\alpha/\pi)(\gamma+\ln(pR))] \\ =F_0(Z,E)(pR)^{-2\alpha/\pi}\exp[-2\gamma\alpha/\pi]. \quad (4.4)$$

This expression has the advantage that it explicitly displays the relationship between the divergence in the Fermi function and that in the static β-vertex correction. It might appear that the same effect could be accomplished by the introduction of an effective charge Z' in place of Z in the Fermi function. This cannot be done however because S_0 is an even function of α, while the correction term is odd. This is inherent in Eq. (4.2) since it implies directly that the static β-vertex correction is independent of the total charge of the nucleus. We therefore re-emphasize the importance of isolating the divergence of the radiative correction in a form analogous to Eq. (4.4).

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APPENDIX

We must also make some comment about the sensitivity of the cutoff dependence of the quantities that appear in the transition probability. First, we consider the effect on the first-order static β-vertex correction. We have

$$d(\mathrm{R.C.})_\alpha/dR=-2\alpha/\pi R, \quad (A1)$$

so that,

$$\delta(\mathrm{R.C.})_\alpha=-(2\alpha/\pi)\delta R/R. \quad (A2)$$

Then a 10% change in the cutoff R would give a change in the static β-vertex correction of $\delta(\mathrm{R.C.})_\alpha\simeq-4.7\times10^{-4}$. The static β-vertex correction itself is on the order of 2×10^{-2} so that such a change in R gives an entirely negligible change in the first-order static β-vertex correction.

However, the Fermi function itself is much more sensitive to variations in R. We can write $F_0(Z,E)$ as

$$F_0(Z,E)=C(Z,E)(2pR)^{2(S_0-1)}, \quad (A3)$$

where $C(Z,E)$ is independent of R. Then

$$dF_0/dR=C(Z,E)2(S_0-1)(2p)(2pR)^{[2(S_0-1)-1]}, \quad (A4)$$

so that,

$$\delta F_0/F_0=2(S_0-1)\delta R/R. \quad (A5)$$

Now,

$$S_0=(1-Z^2\alpha^2)^{1/2}=1-\tfrac{1}{2}Z^2\alpha^2+\cdots. \quad (A6)$$

This is a good expansion even for relatively large values Z since only even powers of $Z\alpha$ appear. Thus we have

$$\delta F_0/F_0\approx-Z^2\alpha^2\delta R/R. \quad (A7)$$

Again using Co^{54} as an example, $Z^2\alpha^2\simeq4\times10^{-2}$ so that a 10% change in R gives $\delta F_0/F_0\simeq-0.4\%$. This is a change on the order of the electron screening correction, and is in fact larger than the screening for this particular decay. The effect is of course much smaller for small values of Z, the figure for O^{14} being $\delta F_0/F_0\simeq-0.025\%$. The choice of the cutoff R depends upon the choice of a nuclear model used in determining the Fermi function. That is, it represents the radius outside which the potential can be considered as pure Coulomb, again emphasizing the need for using the most realistic nuclear model available. The strong Z dependence of the uncertainty in $F_0(Z,E)$ serves as another reason to restrict the calculation of G_v to nuclei with low Z.