Model Dependence of Corrections to the Fermi Function and ft Values of Some $0^+ \rightarrow 0^+$ Super-Allowed β Decays, and the Value of the Vector Coupling Constant

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This paper presents a consistent field-theoretic perturbative calculation of the Fermi function and the analytical model dependences of corrections to it. The screening, finite nuclear size, and radiative corrections are considered in a unified treatment. We are able to separate out the point-nucleus Fermi function F_0 from the corrections, and thus only treat the corrections in a power-series expansion. The proper way of incorporating the radiative correction into F_0 is discussed. Detailed analytical results for the model dependence of the finite-nuclear-size and radiative corrections are presented. The most realistic model chosen employs the nuclear charge distributions as given by Hofstadter and the single-particle harmonic-oscillator model for the nuclear matrix elements. The model-dependent results are used to calculate the ft values of the super-allowed decays of O^{14}, Al^{26}, Cl^{34}, Sc^{42}, V^{46}, Mn^{50}, and Co^{54}. The values of the vector coupling constant G_V are calculated for the above nuclei using the most realistic model, and the variation of the G_V is discussed in the light of the predictions of the conserved-vector-current theory of weak interactions.

INTRODUCTION

THE importance of the value of the vector coupling constant G_V in weak interactions has been discussed in the literature.[1–5] Since the ft values of $0^+ \rightarrow 0^+$ nuclear β decays are used to obtain the most precise values of G_V,[6–8] the determination of the Fermi function[9,10] $F(Z,p)$ for allowed decays plays a central role. Precise calculations of F are also needed to investigate apparent deviations in the ft values of some super-allowed $0^+ \rightarrow 0^+$ decays.[6,11]

The purpose of this paper is to present a consistent field-theoretic perturbative treatment of the Fermi function for a point nucleus, $F_0(Z,p)$, and all the usual corrections[10] to it. Since the corrections to F_0 are usually treated in piecemeal fashion in the literature, it becomes difficult to see each correction in perspective. Our treatment will bring out the relative importance of the corrections and enable us to examine the corrections made previously in the literature for consistency with each other. We will be able to separate out F_0 from the corrections and thus effectively only treat the corrections in a perturbative expansion. In a previous paper, Chern *et al.*[7] gave the results of a calculation for a simple model where the nuclear structure was ignored in the Coulomb interaction but was taken into account in the β decay. We will show how the results in that paper were obtained and extend the formalism to take into account the nuclear structure during the Coulomb interaction. Thus we will obtain analytical model-dependent results for the finite-nuclear-size, screening, and radiative corrections. We then make use of these results to calculate the best values of G_V for some accurately measured $0^+ \rightarrow 0^+$ decays using realistic nuclear models. The field-theoretic method used in this paper has several advantages. It explicitly displays the approximations which are implicit in the usual treatment of the Fermi function, and markedly simplifies the model-dependence calculations. This is so since we can treat all static nuclear model dependences analytically in one calculation, whereas each new model dependence necessitates a new calculation for the electron's entire wave function in the nonperturbative treatment. In addition, our method considers the effects of the emitted electron back on the nucleus, i.e., where the quantum-mechanical aspects of nuclear structure play a role. In Sec. II we discuss the size of such corrections in comparison to the three mentioned above. We shall also be able to comment on the effect of possible isotopic impurities on the accepted values for the β-decay matrix elements.

The Fermi function arises naturally in the calculation of the transition probability per unit time for β decay from the expansion of the electron field operator in terms of positive energy Coulomb wave functions rather than plane waves. Thus F modifies the usual statistical distribution which occurs if the outgoing electron is regarded as a free particle. We review briefly the various corrections to F_0 and the models which have been implicitly assumed in their calculation.

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In Fermi's original treatment,[9] only the β-decay interaction was treated in perturbation theory and he used the Coulomb wave functions for an electron in the field of the daughter nucleus. The nucleus was regarded as a point charge as far as the Coulomb interaction was concerned. Because the Coulomb wave functions which are square integrable at the origin for $J=\frac{1}{2}$ possess a weak singularity at $r=0$, he introduced a configuration space cutoff R in the divergent parts. Letting $r\rightarrow 0$ everywhere (except the divergent parts where $R\sim$nuclear radius), one obtains $F_0(Z,p)$. If one retains the small terms containing r, one has $F^D(Z,p)$, which in the literature is said to include the finite de Broglie wavelength effects.[7,10,12] One can write[12]

$$F^D(Z,p)=F_0(Z,p)\times[1+(\text{terms proportional to powers of } r)].$$

Thus F^D differs from F_0 in containing some finite-nuclear-size corrections. The work in the literature on obtaining accurate values of the Fermi function can be interpreted in terms of obtaining corrections to F_0. The usual corrections considered in the literature are due to finite-nuclear-size,[13–16] screening of the Coluomb potential seen by the outgoing electron due to the presence of atomic electrons,[17–19] and the radiative corrections.[20–22]

Rose[17] was the first to consider the effects of screening on F_0. He used a modified WKB approximation to solve the Dirac equation and gave a prescription for modifying F_0. The method is lucidly discussed by Durand.[18] For our purposes we need only note here that the model used is that of a screened Coulomb potential for a point nucleus. Durand,[18] using particular forms of a screened point charge Coulomb potential (Hulthén potential), obtained exact solutions to the Schrödinger and Klein-Gordon equations. We will compare our results with his and Rose's in Sec. III. If the Coulomb potential is modified to take the finite nuclear size into account, in general we cannot solve the Dirac equation analytically. The models that have been assumed for the static nuclear charge density are those of a uniformly charged spherical shell[16] (UCS), and a uniformly charged spherical ball (UCB).[14–16] Some authors also consider the screening,[8,16] whereas others neglect it.[14,15] In order to use the wave functions obtained from these models to calculate the transition probability per unit time for β decay, one must perform the integration of the electron wave functions over the nuclear matrix elements. Some authors evaluate the numerical solutions for the electron wave functions at the nuclear surface,[8,9,14,15] while others average them over the nuclear matrix elements.[15,16] The first is equivalent to assuming a model in which the transforming nucleons are restricted to a spherical shell as far as the β-decay integrations are concerned, while the second depends on the choice of nuclear wave function. It should be pointed out here that the point charge nucleus Coulomb wave functions must also be evaluated at the nuclear surface or averaged. Thus there exist many possible combinations of model dependences which gave rise to "finite-nuclear-size corrections." Huffaker and Laird,[15] for the model of a UCB, have succeeded in approximating the electron radial functions analytically and thus provide a basis for comparison with our results.

The radiative corrections to β decay have been calculated treating the β interaction and electromagnetic interaction in perturbation theory. In the results most quoted in the literature[20,21] the model consists of treating the nucleus as a point charge in both the β decay and the Coulomb interaction.[23] The results for the radiative correction were simply added to the Fermi function as another correction.[16] However, the radiative corrections involve an ultraviolet cutoff in momentum space contrasting with the configuration space cutoff in F_0. Chern *et al.*[7] have discussed the incorporation of the results for the radiative correction into the ft values for nuclear β decays. In Sec. I we return to this point in more detail.

This paper is divided into four major sections. Section I treats the Fermi function for the model of a point nucleus. Section II considers the nuclear structure (model dependences) in both the β and Coulomb interactions. Section III applies the model-dependent results to $0^+\rightarrow 0^+$ decays and calculates the ft values. The results and their discussion are considered in Sec. IV.

I. FERMI FUNCTION FOR A POINT NUCLEUS

In this section we consider a perturbation treatment of the Coulomb correction to β decay which will give us to order $Z\alpha$ the Fermi function and field-theoretic corrections to it. The model considered in this section is the one used in Chern *et al.*[7] It ignores the nuclear structure in the Coulomb interaction (the Coulomb potential is assumed to be that due to a point nucleus), but takes the nuclear structure into account in the β interaction. As discussed in the Introduction, such a model will give rise to a Fermi function including the screening and finite de Broglie wavelength effects. Due to the simplicity of this model it will become clear how to connect up the perturbation treatment with the usual treatments in the literature and how to incorporate the screening, finite-nuclear-size, and the radiative cor-

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rection into the Fermi function. This clarifies the procedure to be used in Sec. II when the more complicated models are considered.

Since we treat both the β and Coulomb interactions in perturbation theory, the interaction Hamiltonian is $H_\beta+H_c$, where for positron emission[10,24]

$$H_\beta=\tfrac{1}{2}\sqrt{2}\int d^2x\,\bar{\psi}_n(\mathbf{x})\gamma_\mu(G_V-G_A\gamma_5)\psi_p(\mathbf{x})\times\bar{\psi}_\nu(\mathbf{x})\gamma_\mu(1+\gamma_5)\psi_e(\mathbf{x})\,,\quad(1.1)$$

and the Coulomb interaction between the positron and the nucleus of charge Z is given by[25] (units are $\hbar=c=m_e=1$)

$$H_c=-Z\alpha\int d^3x\frac{\bar{\psi}_e(\mathbf{x})\beta\psi_e(\mathbf{x})}{|\mathbf{x}|}\exp(-\Delta|\mathbf{x}|)\,.\quad(1.2)$$

We have modified the usual Coulomb potential to take into account the screening due to the atomic electrons, and Δ is a parameter which will eventually be determined by a comparison of the value of the atomic electron potential of the parent atom[24] at the origin with that given by a Hartree-Fock calculation. Note that in H_c the nucleon field operators do not appear.

The complete matrix element to lowest order in the Coulomb and β interactions is described by the diagrams[21,23] shown in Fig. 1. The electron-positron momentum and energy in the intermediate (final) state are denoted by $\mathbf{k}$ and e_k (p,e_p), respectively, and reference to spins is suppressed. Here the initial state describes a parent nucleus having $Z+1$ protons and $A-(Z+1)$ neutrons and no leptons present. The final state describes a daughter nucleus having Z protons and $A-Z$ neutrons and a neutrino and positron each described by its appropriate free-particle quantum numbers. The combined matrix element M_{fi} for diagrams $a+b+c$ can be obtained by the use of ordinary perturbation theory, and ignoring the nuclear recoil energies (the nucleons are regarded as the source of the external Coulomb field) one obtains[23]

$$\begin{aligned}M_{fi}=\tfrac{1}{2}\sqrt{2}\int d^{3A}x\sum_{r=1}^{A}\Psi_f^*[\beta\gamma_\mu(G_V-G_A\gamma_5)\tau^-]_r\Psi_i\\\times\exp(-i\mathbf{q}\cdot\mathbf{x}_r)V_+^\dagger(\mathbf{q})\beta\gamma_\mu(1+\gamma_5)\bigg[\exp(-i\mathbf{p}\cdot\mathbf{x}_r)\\+Z\alpha\frac{4\pi}{(2\pi)^3}\int\frac{d^3k(e_p+\boldsymbol{\alpha}\cdot\mathbf{k}-\beta)\exp(-i\mathbf{k}\cdot\mathbf{x}_r)}{[|\mathbf{p}-\mathbf{k}|^2+\Delta^2][e_p^2-e_k^2+i\epsilon]}\\+\alpha\frac{4\pi}{(2\pi)^3}\int d^3k\frac{\lambda^+(-\mathbf{k})\exp(-i\mathbf{k}\cdot\mathbf{x}_r)}{(|\mathbf{p}-\mathbf{k}|^2+\Delta^2)(e_p+e_k-i\epsilon)}\bigg]U_-(-\mathbf{p})\,,\end{aligned}\quad(1.3)$$

where Ψ_i and Ψ_f are the initial and final nuclear wave functions, $\lambda^\pm(\mathbf{k})=(2e_k)^{-1}(e_k\pm\boldsymbol{\alpha}\cdot\mathbf{k}\pm\beta)$, and $V_\pm(\mathbf{q})$ and $U_\pm(\mathbf{p})$ are the free-particle spinors for the neutrino and electron, respectively, of energy and momentum $\pm q$, $\mathbf{q}$ and $\pm e_p,\mathbf{p}$. One notes that the first two terms in the square brackets are the expansion of the wave function for an electron in an external Coulomb field due to the daughter nucleus[26] in plane waves to order $Z\alpha$. As first pointed out by Berman,[21] the last term arises from the fact that the nuclear charge is changed by the β decay and thus is not obtainable in the usual treatment of the Fermi function. If we neglect screening $(\Delta\to 0)$ and finite nuclear size even in the β decay $(x_r\to 0)$, M_{fi} reduces to that discussed by Berman. It is the third term in the brackets of (1.3) which is charge-independent and gives rise to the radiative correction[27] to order $Z\alpha$. In the limit $x_r\to 0$ this last term is logarithmically divergent, but in our treatment it has a configuration-space cutoff which corresponds to that used in the point-nucleus Fermi function.

The way the screening and finite nuclear size (here the finite de Broglie wavelength) enter the perturbation calculation is explicitly displayed. We note that in addition to the usual finite-nuclear-size effects quoted in the literature, which arise from the effect on the electron's wave function, the charge-independent term also gives small finite-size corrections. In this section we let $x_r\to 0$ in this term except in the part which would otherwise be logarithmically divergent. Using some manipulation, we can rewrite the square brackets of Eq. (1.3) as

$$[\cdots]=\exp(-i\mathbf{p}\cdot\mathbf{x}_r)+[4\pi/(2\pi)^3]\times[Z\alpha I(\mathbf{x}_r,\mathbf{p})+\alpha G(\mathbf{x}_r,\mathbf{p})]\,,$$

$$I(\mathbf{x}_r,\mathbf{p})=\int d^3k\frac{(e_p+\mathbf{a}\cdot\mathbf{k}-\beta)\exp(-i\mathbf{k}\cdot\mathbf{x}_r)}{(|\mathbf{p}-\mathbf{k}|^2+\Delta^2)(e_p^2-e_k^2+i\epsilon)},$$

$$\begin{aligned}G(\mathbf{x}_r,\mathbf{p})=\tfrac{1}{2}I(\mathbf{x}_r,\mathbf{p})-\frac{1}{2}\int d^3k\frac{e_k\exp(-i\mathbf{k}\cdot\mathbf{x}_r)}{(|\mathbf{p}-\mathbf{k}|^2+\Delta^2)(e_p^2-e_k^2+i\epsilon)}\\-\tfrac{1}{2}e_p\int\frac{d^3k(\boldsymbol{\alpha}\cdot\mathbf{k}+\beta)\exp(-i\mathbf{k}\cdot\mathbf{x}_r)}{(|\mathbf{p}-\mathbf{k}|^2+\Delta^2)(e_p^2-e_k^2+i\epsilon)}\,.\end{aligned}\quad(1.3')$$

I and G are given in Appendix A. The results for I are expressed in terms of a power-series expansion in px_r and $\mathbf{p}\cdot\mathbf{x}_r$. The three different expansion parameters which occur in the Fermi function are $Z\alpha$, px, and Δ/p, In a perturbation treatment of the Fermi function to order $Z\alpha$, since we will keep only terms in the transition probability per unit time which are at most proportional to the square of the expansion parameters and $Z\alpha$ multiplies I, consistency dictates we only retain terms in I at most linear in px and Δ. The dominant contribu-

[24] We have here ignored the fact that the Coulomb potential due to the daughter nucleus after the β decay is not fully screened. Actually $Z\alpha e^{-\Delta x'}/x'$ should be replaced by $(Z+1)\alpha e^{-\Delta x'}/x'-\alpha/x'$. This causes a change in the screening by $1/Z$ and is unimportant for the decays considered here. This is discussed further in Sec. III.

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[26] J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill Book Co., New York, 1964).

[27] This gives the static vertex part of the radiative correction.

tion to G comes from the first two terms in Eq. (1.3′). It is the second term in G which in the limit $x_r \to 0$ is logarithmically divergent and is opposite in sign to $\frac{1}{2}I$.

At this point we restrict ourselves to $0^+ \to 0^+$ superallowed decays. Thus there is no contribution from the axial-vector interaction in the β-decay matrix elements. Only those terms survive in the vector interaction which combine to form scalars. One then proceeds to expand the lepton wave function in a power series in x keeping terms at most to order $(px)^2$ in M_{fi}. Forming the transition probability per unit time for the emission of a positron with momentum between p and $p+dp$ in the usual way and performing the spin sums and angular integration for the leptons in the final state gives

$$P(p)dp = P_0(p)dp[1-\pi Z\alpha(e_p/p)+Z\alpha\Delta(2e_p^2-1)/p^2e_p + Z\alpha((5/3)e_p+1/3e_p+\tfrac{1}{3}q)R_0-\tfrac{1}{3}(p^2\{1+\tfrac{2}{3}q/e_p\}+q^2)R_0^2 + (\alpha/2\pi)(-\pi^2(e_p/p)+10-4C-4\ln(pR_0))], \quad (1.4)$$

where $q=e_m-e_p$ and $C=0.5772\cdots$ is Euler's constant. Here the position-dependent terms have been evaluated at the nuclear surface as is done in the usual treatment of F_0 and

$$P_0(p)=(1/2\pi^3)|G_V|^2|M_F|^2p^2(e_m-e_p)^2.$$

Here $|M_F|$ designates the Fermi matrix element.

All terms in the square brackets of Eq. (1.4) which are proportional to $(Z\alpha)^0$ and $Z\alpha$ should be the usual Fermi function for a point nucleus to order $Z\alpha$ and the screening and finite-nuclear-size corrections to it. Since Δ and R_0 identify the various corrections, it is easy to see the origins of the various terms. Thus ignoring terms proportional to α in Eq. (1.4) one sees that the first two terms in the square brackets are the expansion of $F_0(Z,p)$ to $Z\alpha$ for a positron decay. The third term in Eq. (1.4) embodies the screening correction to F_0 and the other terms represent the finite-nuclear-size corrections. Note that in the limit $\Delta\to 0$, $R_0\to 0$ only F_0 to order $Z\alpha$ thus survives.

In the usual nonperturbation calculations of the Fermi function the screening and finite-nuclear-size effects are obtained as corrections to the electromagnetic effects (point Coulomb interaction) which is itself a correction to the β decay. The screening and finite nuclear size only affect the electron wave function inside a sphere of essentially nuclear dimension, while the electron's wave function outside this sphere is that due to a point charge Coulomb interaction. Thus one can write the Fermi function as F_0 times a power series in R_0 and Δ. Since R_0 and Δ are independent parameters, in the limiting case $R_0 \Rightarrow 0$ and $\Delta\to 0$ we must obtain F_0. For simplicity consider only the finite de Broglie wavelength correction; one can then write[28]

$$F(Z,p)=F_0(Z,p)[1+f(Z\alpha)R_0+g(Z\alpha)R_0^2+\cdots], \quad (1.5)$$

where f and g are, respectively, odd and even functions

[28] M. E. Rose, *Relativistic Electron Theory* (John Wiley & Sons, Inc., New York, 1961).

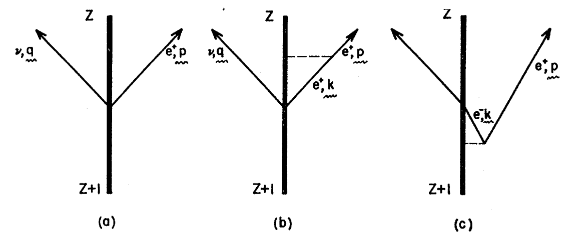

FIG. 1. Diagrams for positron decay with lowest-order Coulomb corrections. The nuclear structure is taken into account in the β decay but ignored in the Coulomb interaction.

of $Z\alpha$. Since the finite nuclear size (and also screening) are small corrections to the Coulomb interaction, such a representation is useful. To connect this with the perturbation calculation of F we note that if for the decays of interest we consider $Z\alpha$ as an expansion parameter in the correction terms to F_0, then keeping terms at most quadratic in the expansion parameter Eq. (1.5) becomes[12,19]

$$F(Z,p)=F_0(Z,p)(1+Z\alpha AR_0+BR_0^2),$$
$$A=(5/3)e_p+1/3e_p+\tfrac{1}{3}q, \quad (1.6)$$
$$B=-\tfrac{1}{3}[p^2(1+\tfrac{2}{3}q/e_p)+q^2].$$

Here the next contributions from f and g contribute to third and second order in $Z\alpha$, respectively, thus coming in over all to fourth order. This corresponds to approximating the electron's wave function inside the nucleus by a power-series expansion in $Z\alpha$ and pR_0 (also Δ when screening is considered). It can be done even for more complicated nuclear models.[15] Here we have not expanded F_0 in $Z\alpha$ since we wish to treat in this way only the corrections to F_0 which are small; i.e., the electron's wave function outside the sphere of radius R_0 is not to be expanded in $Z\alpha$. However, the perturbation treatment of the electron's wave function gives a consistent expansion in $Z\alpha$ everywhere. Therefore, to compare with the perturbation calculation we should replace F_0 in Eq. (1.6) by $(F_0)_{Z\alpha}=1-\pi Z\alpha e/p$ and keep in (1.6) only terms to $Z\alpha$. This gives exactly Eq. (1.5). Thus we see that to extract the results in the literature from the perturbative calculation to order $Z\alpha$ we need only factor out $(F_0)_{Z\alpha}$ from the perturbative results and replace it by F_0. It must be emphasized that the power-series expansion of the screening and finite-nuclear-size corrections to F_0 is a convenient approximation because we know that their effects on the electron's wave function are small. From a theoretical point of view the expansion is meaningful because we can think of px and Δ going to zero and still obtain F_0.

We have neglected the radiative correction in the discussion above since, as previously noted, it is not obtainable from the solution of a Dirac equation with a static potential. Equation (1.4) is to be written as

$$P(p)dp=P_0(p)dp \times[F_0(Z,p)(1+\text{f.n.s.}+\text{s.})+(\text{R.C.})_\alpha], \quad (1.7)$$

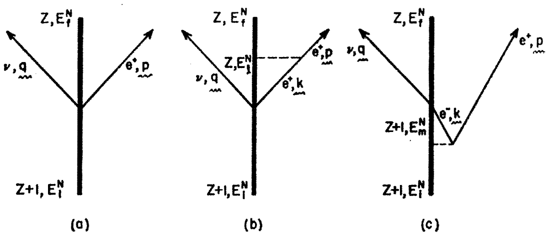

FIG. 2. Positron decay with lowest-order Coulomb corrections. **p** and **q** are the momenta of the outgoing positron and neutrino, respectively, **k** is the momentum of the lepton in the intermediate state, and E_l^N and E_m^N are the energies of the intermediate nuclear states in (b) and (c), respectively.

where f.n.s.+s. stand for the finite-nuclear-size and screening terms in the square bracket of Eq. (1.4), and $(\mathrm{R.C.})_\alpha$ is the radiative correction obtained from the perturbative calculation to order α. To incorporate the radiative correction into the Fermi function we rewrite Eq. (1.7) as

$$P(p)dp=P_0(p)dpF_0(Z,p)\times[1+\text{f.n.s.}+\text{s.}+(\mathrm{R.C.})_\alpha/F_0]. \quad (1.8)$$

Suppose we had performed the perturbation calculation to $Z^2\alpha^2$; what then would be the changes in Eq. (1.4)? First we would develop F_0 to $Z^2\alpha^2$, but in addition we would obtain the finite-nuclear-size and screening corrections to higher orders. If we keep only terms quadratic in the expansion parametes and at most first order in $Z\alpha$, then the screening and f.n.s. corrections are identical to what was obtained in the perturbation calculation to $Z\alpha$, and these are identical to the results in the literature. However, we would also generate the radiative correction to α, α^2, and $Z\alpha^2$. In general the radiative correction to any order in $Z\alpha$ ($\Delta\rightarrow 0$ and $\mathbf{x}\rightarrow 0$) is defined as $F_{\text{pert}}-F_0$ to that order. Thus in effect we again can write

$$[1-\pi Z\alpha e_p/p+(Z\alpha)^2(\cdots)][1+\text{f.n.s.}+\text{s.}]+(\mathrm{R.C.})_{\alpha^2}.$$

Now we see that we are generating F_0 always with the same finite-nuclear-size and screening corrections (to second order) but with higher-order radiative corrections. This can be written as

$$F_0(Z,p)[1+\text{f.n.s.}+\text{s.}+(\mathrm{R.C.})_{\alpha^2}/F_0]. \quad (1.9)$$

If we were to generate the perturbation expansion to all orders in $Z\alpha$, the only change in Eq. (1.9) would be that the radiative correction would be generated to all orders. We emphasize that this is the correct way to incorporate the radiative correction into the Fermi function.

Rigorously, then, one should use the radiative correction to all orders in α when one incorporates it into the Fermi function, since as one generates F_0 we generate the radiative correction to the same order in α. We cannot use the argument that α is to be considered an independent parameter like px and Δ and which, when allowed to go to zero, $F\rightarrow F_0$. The radiative correction to all orders contains an infinite number of terms which are logarithmically divergent as $x\rightarrow 0$. We do not *know* that this sum is small compared to F_0. Since the radiative correction is only known to order α, we have replcced R.C. by its value to order α to obtain corrections to the ft values. We are now investigating a field-theoretic perturbative treatment of the Fermi function to $Z^2\alpha^2$ and the radiative correction to this order. The results of the calculation and their effect on the accepted value of the lowest-order radiative correction will be communicated as soon as completed.

II. NUCLEAR STRUCTURE IN BOTH THE β AND COULOMB INTERACTIONS

The preceding section on the simple model of a point charged nucleus has shown how the corrections to the point-nucleus Fermi function may be extracted from a perturbation treatment of the Coulomb corrections to β decay. These ideas will now be extended to the more general case in which the nuclear structure plays a role in the Coulomb interaction as well as in the β decay. This will involve specific nuclear models for the β and Coulomb interactions. The corrections to the Fermi function may be expressed in terms of these models, thus allowing a study to be made of their model dependence.

The effect of the nuclear structure is taken into account by quantizing the nucleon fields in the β decay and in the Coulomb interaction between the nuclear protons and the emitted electron (positron). As in Sec. I, the $V-A$ theory of weak interactions is assumed; thus the β-decay Hamiltonian is given by Eq. (1.1). The Coulomb interaction between the nuclear protons and the emitted electron (positron) is represented by the "screened" Coulomb Hamiltonian

$$H_c=-\alpha\int d^3x'\int d^3x\frac{\bar{\psi}_p(\mathbf{x}')\beta\psi_p(\mathbf{x}')\bar{\psi}_e(\mathbf{x})\beta\psi_e(\mathbf{x})}{|\mathbf{x}-\mathbf{x}'|}\times\exp(-\Delta|\mathbf{x}-\mathbf{x}'|). \quad (2.1)$$

As in the β-decay Hamiltonian, we need not know the specific form of the nucleon spinors here, since they will be absorbed in the specification of the nuclear wave functions. The factor $\exp(-\Delta|\mathbf{x}-\mathbf{x}'|)$ has been inserted in (2.1) to account for the screening of the nuclear charge by the atomic electrons. The parameter Δ may be related to the atomic electron potential at the nucleus as mentioned in Sec. I. For the decays of interest in this paper, the screening correction to the ft value is very insensitive to the detailed form of the screening factor.

In the rest of this section, the transition amplitude for nuclear β decay with lowest-order Coulomb corrections will be presented. Various approximations which may be made to the amplitude are discussed. Of particular interest will be the one which leads to the formalism ordinarily employed in the study of β decay. Since we will eventually be interested in calculations to the vector coupling constant G_V, only the details for positron decay will be given. The extension to electron decay is straightforward and will be indicated at the end of this discussion.

In Fig. 2 are shown the diagrams for the positron decay of a nucleus of initial charge $Z+1$ and energy E_i^N into a nucleus of final charge Z and energy E_f^N. Diagram (a) gives the β decay with no Coulomb interaction. Diagrams (b) and (c) represent the first-order Coulomb corrections to the decay. These two must be summed over all intermediate states of the positron (electron) and nucleus. Because of the large nuclear rest mass energy compared to the decay energies involved, the nuclear recoil energy will again be neglected and the energies E_l^N and E_m^N taken to mean the internal energies of the nuclear states. The transition amplitude for the diagrams of Fig. 2 was obtained by Chern[23] using ordinary second-order perturbation theory, in which the intermediate states are on the energy shell. For diagram (a), we have

$$M_{fi}{}^a=\tfrac{1}{2}\sqrt{2}\int dx^{3A}\sum_{r=1}^{A}\Psi_f{}^*[\beta\gamma_\mu(G_V-G_A\gamma_5)\tau^-]_r\Psi_i \times\exp[-i(\mathbf{p}+\mathbf{q})\cdot\mathbf{x}_r]V_+{}^\dagger(\mathbf{q})\beta\gamma_\mu(1+\gamma_5)U_-(-\mathbf{p}); \quad (2.2)$$

for diagram (b),

$$M_{fi}{}^b=\alpha\frac{4\pi}{(2\pi)^3}\frac{1}{\sqrt{2}}\bigg[\sum_l\int d^{3A}x'\sum_{s=1}^{A}\Psi_f{}^*P_s\Psi_l\exp(-i\mathbf{p}\cdot\mathbf{x}_s{}')\int d^{3A}x\sum_{r=1}^{A}\Psi_l{}^*[\beta\gamma_\mu(G_V-G_A\gamma_5)\tau^-]_r\Psi_i\exp(-i\mathbf{q}\cdot\mathbf{x}_r)$$
$$\times V_+{}^\dagger(\mathbf{q})\beta\gamma_\mu(1+\gamma_5)\int d^3k\frac{\lambda^-(-\mathbf{k})\exp[-i(\mathbf{x}_r-\mathbf{x}_s{}')\cdot\mathbf{k}]}{(|\mathbf{p}-\mathbf{k}|^2+\Delta^2)(E_f{}^N-E_l{}^N+e_p-e_k+i\epsilon)}U_-(-\mathbf{p})\bigg]; \quad (2.3)$$

and for diagram (c)

$$M_{fi}{}^c=\alpha\frac{4\pi}{(2\pi)^3}\frac{1}{\sqrt{2}}\bigg[\sum_m\int d^{3A}x\sum_{r=1}^{A}\Psi_f{}^*[\beta\gamma_\mu(G_V-G_A\gamma_5)\tau^-]_r\Psi_m\exp(-i\mathbf{q}\cdot\mathbf{x}_r)\int d^{3A}x'\sum_{s=1}^{A}\Psi_m{}^*P_s\Psi_i$$
$$\times\exp(-i\mathbf{p}\cdot\mathbf{x}_s{}')V_+{}^\dagger(\mathbf{q})\beta\gamma_\mu(1+\gamma_5)\int d^3k\frac{\lambda^+(-\mathbf{k})\exp[-i(\mathbf{x}_r-\mathbf{x}_s{}')\cdot\mathbf{k}]}{(|\mathbf{p}-\mathbf{k}|^2+\Delta^2)(E_m{}^N-E_i{}^N+e_p+e_k-i\epsilon)}U_-(-\mathbf{p})\bigg], \quad (2.4)$$

where $P_s\equiv\frac{1}{2}+(\tau^3)_s$ is the isospin operator which gives 1 when it operates on a proton state and 0 when it operates on a neutron state. The total amplitude for the decay is then

$$M_{fi}=M_{fi}{}^a+M_{fi}{}^b+M_{fi}{}^c. \quad (2.5)$$

Because the present understanding of nuclear physics allows only an approximate knowledge of the nuclear wave functions, the sums over the intermediate nuclear states in $M_{fi}{}^b$ and $M_{fi}{}^c$, given by

$$S^b=\sum_l\frac{\Psi_l(\mathbf{x}_1{}',\mathbf{x}_2{}',\cdots,\mathbf{x}_A{}')\Psi_l{}^*(\mathbf{x}_1,\mathbf{x}_2,\cdots,\mathbf{x}_A)}{(E_f{}^N-E_l{}^N+e_p-e_k+i\epsilon)} \quad (2.6)$$

and

$$S^c=\sum_m\frac{\Psi_m(\mathbf{x}_1,\mathbf{x}_2,\cdots,\mathbf{x}_A)\Psi_m{}^*(\mathbf{x}_1{}',\mathbf{x}_2{}',\cdots,\mathbf{x}_A{}')}{(E_m{}^N-E_i{}^N+e_p+e_k-i\epsilon)}, \quad (2.7)$$

cannot be evaluated exactly. However, it is possible to make some reasonable approximations to them by making specific assumptions about the relative importance of the nuclear and lepton energies which appear in the denominators. An examination of the actual distribution of the contributing nuclear states of the parent and daughter nuclei for a particular decay should help in making an appropriate assumption. Two opposite approximations which do not require knowledge of the details of a particular decay have been discussed previously by Gell-Mann and Berman[29] and subsequently by Chern.[23] The closure approximation consists of the assumption that the important nuclear excitation energies in the denominators are negligible compared to the lepton energies e_p and e_k. Neglecting the nuclear energies altogether, the sums over the nuclear states may be performed by closure (Cl) to yield[23]

$$(M_{fi})_{\mathrm{Cl}}=\tfrac{1}{2}\sqrt{2}\int d^{3A}x\sum_{r=1}^{A}\Psi_f{}^*[\beta\gamma_\mu(G_V-G_A\gamma_5)\tau^-]_r\Psi_i\exp[-i(\mathbf{p}+\mathbf{q})\cdot\mathbf{x}_r]V_+{}^\dagger(\mathbf{q})\beta\gamma_\mu(1+\gamma_5)U_-(-\mathbf{p})$$
$$+\alpha\frac{4\pi}{(2\pi)^2}\frac{1}{\sqrt{2}}\bigg[\int d^{3A}x\sum_{r,s=1}^{A}\Psi_f{}^*P_s[\beta\gamma_\mu(G_V-G_A\gamma_5)\tau^-]_r\Psi_i\exp[-i(\mathbf{p}\cdot\mathbf{x}_s+\mathbf{q}\cdot\mathbf{x}_r)]$$
$$\times V_+{}^\dagger(\mathbf{q})\beta\gamma_\mu(1+\gamma_5)\int d^3k\frac{\lambda^-(-\mathbf{k})\exp[-i(\mathbf{x}_r-\mathbf{x}_s)\cdot\mathbf{k}]}{(|\mathbf{p}-\mathbf{k}|^2+\Delta^2)(e_p-e_k+i\epsilon)}U_-(-\mathbf{p})+\int d^{3A}x\sum_{r,s=1}^{A}\Psi_f{}^*[\beta\gamma_\mu(G_V-G_A\gamma_5)\tau^-]_rP_s\Psi_i$$
$$\times\exp[-i(\mathbf{p}\cdot\mathbf{x}_s+\mathbf{q}\cdot\mathbf{x}_r)]V_+{}^\dagger(\mathbf{q})\beta\gamma_\mu(1+\gamma_5)\int d^3k\frac{\lambda^+(-\mathbf{k})\exp[-i(\mathbf{x}_r-\mathbf{x}_s)\cdot\mathbf{k}]}{(|\mathbf{p}-\mathbf{k}|^2+\Delta^2)(e_p+e_k-i\epsilon)}U_-(-\mathbf{p})\bigg]. \quad (2.8)$$

[29] M. Gell-Mann and S. M. Berman, Phys. Rev. Letters 3, 99 (1959).

It should be noted that the matrix elements between the initial and final nuclear states in the second and third terms of (2.8) are not just that of the β-decay interaction, but include the Coulomb interaction with the positron as well. This is to be compared with the opposite approximation in which the assumption is made that the energies of the contributing excited nuclear states are much larger than the lepton energies. In this case it is assumed that the nucleus is not excited in the intermediate state and only the terms in $S^{(b)}$ and $S^{(c)}$ for which $l=f$ and $m=i$, respectively, are retained. The "no-excitation" (NE) approximation thus yields[23]

$$(M_{fi})_{\mathrm{NE}}=\tfrac{1}{2}\sqrt{2}\int d^{3A}x \sum_{r=1}^{A} \Psi_f^*[\beta\gamma_\mu(G_V-G_A\gamma_5)\tau^-]_r\Psi_i \exp[-i\mathbf{q}\cdot\mathbf{x}_r]V_+^\dagger(\mathbf{q})\beta\gamma_\mu(1+\gamma_5)$$
$$\times\left\{\exp(-i\mathbf{p}\cdot\mathbf{x}_r)+\alpha\frac{4\pi}{(2\pi)^3}\left[\int d^{3A}x' \sum_{s=1}^{A}\Psi_f^*P_s\Psi_f\exp(-i\mathbf{p}\cdot\mathbf{x}_s')\int d^3k\frac{\lambda^-(-\mathbf{k})\exp[-i(\mathbf{x}_r-\mathbf{x}_s')\cdot\mathbf{k}]}{(|\mathbf{p}-\mathbf{k}|^2+\Delta^2)(e_p-e_k+i\epsilon)}\right.\right.$$
$$\left.\left.+\int d^{3A}x' \sum_{s=1}^{A}\Psi_i^*P_s\Psi_i\exp(-i\mathbf{p}\cdot\mathbf{x}_s')\int d^3k\frac{\lambda^+(-\mathbf{k})\exp[-i(\mathbf{x}_r-\mathbf{x}_s')\cdot\mathbf{k}]}{(|\mathbf{p}-\mathbf{k}|^2+\Delta^2)(e_p+e_k-i\epsilon)}\right]\right\}U_-(-\mathbf{p}). \quad (2.9)$$

It should be noted that in this case it has been possible to factor out the β interaction from the Coulomb interaction. In effect, this approximation "decouples" the two from each other. The positron wave function in $(M_{fi})_{\mathrm{NE}}$ depends only upon the Coulomb interaction of the positron with the nuclear protons, but does not depend upon the particular details of the β-decay interaction. This feature is true only for the NE approximation. The inclusion of excitated intermediate nuclear states will specifically introduce the particular nuclear states playing a role in the β decay into the calculation of the positron wave function. It may be recalled that in the usual studies of β decay one tacitly assumes that the positron wave function has been separately calculated for an assumed nuclear charge distribution and is then inserted into the β-decay matrix element. From inspection of $(M_{fi})_{\mathrm{NE}}$, it can be seen that the approximation of no nuclear excitation is precisely what is assumed in the usual treatments of β decay. The excited intermediate nuclear states, which occur as the result of the emitted positron (or electron) acting back on the nucleus[30] via a "virtual Coulomb excitation process," are thus neglected in these treatments.

The no-nuclear-excitation approximation is probably the more realistic of the two discussed above. This may be inferred from a close look at the general amplitude in Eq. (2.5). If, as far as the Coulomb interaction is concerned, the nucleus was strictly a point charge, then the Coulomb matrix elements appearing in M_{fi}^b and M_{fi}^c would vanish for all excited nuclear states. This is due to the orthogonality of these states with the initial or final nuclear states. Only the NE terms would contribute in the point charge nucleus limit. When the nuclear electromagnetic size is considered, the excited states are brought in as f.n.s. corrections along with the electromagnetic size corrections arising in the NE terms. The latter corrections would be expected to be more important, since they would be calculated with the nuclear charge density $\rho(\mathbf{x}')$, with $\int d^3x'\rho_c(\mathbf{x}')=1$. In contrast, the f.n.s. corrections from the excited states would be calculated from a "transition charge density" $\rho_c^t(\mathbf{x}')$, with $\int d^3x'\rho_c^t(\mathbf{x}')=0$. Furthermore, the values of $\mathbf{k}$ which contribute most to the integration over the intermediate lepton momentum would be expected to be near the value of $\mathbf{p}$, the observed positron momentum. Now in allowed decays the observed lepton energies are generally comparable to typical nuclear excitation energies, while in forbidden decays the latter may be expected to dominate. Since the excited nuclear states can contribute to $S^{(b)}$ and $S^{(c)}$ only as small f.n.s. effects, the no-nuclear-excitation approximation would be more appropriate. The characteristic feature of the closure approximation is that it overemphasizes the high values of momentum $\mathbf{k}$ of the lepton in the intermediate state.[23] This becomes important in the part of M_{fi} which is usually referred to as the radiative correction,[31] namely, that part in which the same nucleon participates in both the β and Coulomb interactions. Because of this overemphasis of high values of $\mathbf{k}$, the closure approximation leads to a radiative correction that is logarithmically divergent as $\mathbf{k}\rightarrow\infty$ even in the presence of an extended nuclear charge distribution.

Since our main purpose in this paper is to study the model dependence of the usual f.n.s. and radiative corrections to β decay, and, as will be shown, these corrections are in exact agreement with the results obtained by the use of the no-nuclear-excitation approximation, the remainder of this paper will be restricted to use of this approximation to treat the sum over intermediate nuclear states. Work is presently in progress on determining the contributions of the excited nuclear states to these corrections, and the results will be communicated at a future date.

Before applying $(M_{fi})_{\mathrm{NE}}$ to a calculation of the corrections mentioned, another simplifying approximation may be made in the brackets for the positron's wave

[30] In addition to the effects of the nuclear recoil.

[31] Only the static vertex part of this correction has been considered here. See Ref. 7 for reasons for including only this part.

function. This is the assumption that the proton distribution in the daughter nucleus is essentially the same as that in the parent nucleus, with the exception of the proton which β decays. This will be particularly good for decays for which the parent and daughter nuclei are at or near the ground state. In this case, the protons which may take part in the β decay may be thought of as moving outside a nuclear core which distorts the positron's wave function but does not contribute to the β decay. With this approximation, $(M_{fi})_{\mathrm{NE}}$ becomes

$$(M_{fi})_{\mathrm{NE}}=\tfrac{1}{2}\sqrt{2}\int d^{3A}x\sum_{r=1}^{A}\Psi_f^*[\beta\gamma_\mu(G_V-G_A\gamma_5)\tau^-]\Psi_i\exp(-i\mathbf{q}\cdot\mathbf{x}_r)V_+^\dagger(\mathbf{q})\beta\gamma_\mu(1+\gamma_5)\Big[\exp(-i\mathbf{p}\cdot\mathbf{x}_r)$$
$$+Z\alpha\frac{4\pi}{(2\pi)^3}\int d^3x'\rho_c(\mathbf{x}')\exp(-i\mathbf{p}\cdot\mathbf{x}')I(\mathbf{x}_r-\mathbf{x}')+\alpha\frac{4\pi}{(2\pi)^3}\int d^3x'\rho_\beta(\mathbf{x}')\exp(-i\mathbf{p}\cdot\mathbf{x}')G(\mathbf{x}_r-\mathbf{x}')\Big]U_-(-\mathbf{p}),\quad(2.10)$$

where $\rho_c(\mathbf{x}')$ is the charge density of the daughter nucleus (normalized to unity) of charge Z and $\rho_\beta(\mathbf{x}')$ is the probability density of the β-decaying proton. The functions $I(\mathbf{x}_r-\mathbf{x}')$ and $G(\mathbf{x}_r-\mathbf{x}')$ are the same functions as given in Sec. I but with $\mathbf{x}_r$ replaced by $(\mathbf{x}_r-\mathbf{x}')$ in I and $\mathbf{x}_r$ by $(\mathbf{x}_r-\mathbf{x}')$ in G. Again, we note that the first and second terms in the brackets in Eq. (2.10) are just the Neumann expansion to first order in $Z\alpha$ of the integral form of the Dirac equation[26] for a positron moving in the potential

$$V(\mathbf{x})=Ze\int d^3x'\frac{\rho_c(x')}{|\mathbf{x}-\mathbf{x}'|}\exp(-\Delta|\mathbf{x}-\mathbf{x}'|).\quad(2.11)$$

If it were not for the third term in the brackets, which is the static vertex part of the radiative correction, all higher terms in the Neumann expansion would just correspond to multiple scatterings of the positron in the potential $V(\mathbf{x})$. The presence of the radiative correction serves to introduce additional terms in the expansion.

The extension of the above formalism to electron decay is obtained by the substitutions

$$Z\to -Z,\quad V_+^\dagger(\mathbf{q})\beta\gamma_\mu(1+\gamma_5)\to U_+^\dagger(\mathbf{p}),$$
$$\beta\to-\beta,\qquad U_-(-\mathbf{p})\to\beta\gamma_\mu(1+\gamma_5)V_-(-\mathbf{x}).\quad(2.12)$$

In the radiative correction, $\rho_\beta(\mathbf{x}')$ is taken to mean the probability density for the proton created in the decay.

III. APPLICATION TO $0^+\to0^+$ DECAYS AND CALCULATION OF ft VALUES

The results of Sec. II are now applied to a study of the effects of the nuclear model dependence of the f.n.s. and radiative corrections to the ft values of some $0^+\to0^+$ super-allowed positron decays. The atomic electron screening correction is also calculated. The particular decays that will be considered here are those of ${}_8\mathrm{O}^{14}$, ${}_{13}\mathrm{Al}^{26m}$, ${}_{17}\mathrm{Cl}^{34}$, ${}_{21}\mathrm{Sc}^{42}$, ${}_{23}\mathrm{V}^{46}$, ${}_{25}\mathrm{M}^{50}$, and ${}_{27}\mathrm{Co}^{54}$, for which the endpoint energies and half-lives are accurately known.[6] The model dependences are obtained by assuming various charge distributions for the daughter nucleus and various probability densities for the β-decaying proton. The results are expressed in terms of a correction factor which multiplies the exact point charge nucleus Fermi function. Terms up to order $(pR)^2$, $Z\alpha pR$, $|\mathbf{v}/c|_N(pR)$, $|\mathbf{v}/c|_N Z\alpha$, and $Z\alpha\Delta$ have been retained in the correction factor. In the above, R denotes the nuclear radius and $(\mathbf{v}/c)_N$ denotes a typical velocity of a nucleon in the nucleus. For decays of interest in this paper, pR, $Z\alpha$, and $|\mathbf{v}/c|_N$ are each ~0.1, while $\Delta\sim0.05$; thus higher-order terms in the correction factor, which would go as $(pR)^4$, $Z\alpha(pR)^3$, $(Z\alpha)^2(pR)^4$, etc., are negligible for these decays. The ft values are obtained from the integration of the corrected Fermi function over the decay spectrum. An approximation described in Appendix B allows this integration to be performed analytically. The percentage error in the ft values due to this approximation is at most 0.02% for the above decays.

For $0^+\to0^+$ decays, the amplitude $(M_{fi})_{\mathrm{NE}}$ in Eq. (2.10) becomes

$$(M_{fi})_{\mathrm{NE}}=\frac{G_V}{\sqrt2}\int d^{3A}x\sum_{r=1}^{A}\Psi_f^*\tau_r^-\Psi_i\exp(-i\mathbf{q}\cdot\mathbf{x}_r)V_+^\dagger(\mathbf{q})(1+\gamma_5)\Big\{\exp(-i\mathbf{p}\cdot\mathbf{x}_r)+Z\alpha\frac{4\pi}{(2\pi)^3}\int d^3x'\rho_c(\mathbf{x}')\exp(-i\mathbf{p}\cdot\mathbf{x}_r')$$
$$\times I(\mathbf{x}_r-\mathbf{x}')+\alpha\frac{4\pi}{(2\pi)^3}\int d^3x'\rho_\beta(\mathbf{x}')\exp(-i\mathbf{p}\cdot\mathbf{x}')G(\mathbf{x}_r-\mathbf{x}')\Big\}U_-(-\mathbf{p})-\frac{G_V}{\sqrt2}\int d^{3A}x\sum_{r=1}^{A}\Psi_f^*(\alpha^N\tau^-)_r\Psi_i$$
$$\times\exp(-i\mathbf{q}\cdot\mathbf{x}_r)V_+^\dagger(\mathbf{q})\boldsymbol{\alpha}(1+\gamma_5)\Big[\exp(-i\mathbf{p}\cdot\mathbf{x}_r)+Z\alpha\frac{4\pi}{(2\pi)^3}\int d^3x'\rho_c(\mathbf{x}')\exp(-i\mathbf{p}\cdot\mathbf{x}')I(\mathbf{x}_r-\mathbf{x}')$$
$$+\alpha\frac{4\pi}{(2\pi)^3}\int d^3x'\rho_\beta(\mathbf{x}')\exp(-i\mathbf{p}\cdot\mathbf{x}')G(\mathbf{x}_r-\mathbf{x}')\Big]U_-(-\mathbf{p}).\quad(3.1)$$

The first term in (3.1) is the usual nonrelativistic part of the Fermi matrix element and the second is the relativistic contribution. For $0^+\to 0^+$ decays, the charge density $\rho_c(\mathbf{x}')$ may be taken to be spherically symmetric. Furthermore, for super-allowed decays the nuclear wave functions may be taken to be states of definite isospin to a good first approximation. Calculations of the isospin mixing[32–37] for the decay of $_8O^{14}$ have shown it to be small, and this is assumed to be the case for the other decays. On the other hand, one could assume the conserved-vector-current hypothesis to be true and then look for evidence of isospin mixing in these decays by examining the variation of their ft values. Some comments about this approach will be made after the results for the ft values have been presented.

A retardation expansion of the lepton wave functions in the amplitude (3.1) is now made in powers of px, px', and qx, keeping terms to second order. The parts of the expansion which contribute to $0^+\to 0^+$ decays are those which combine with 1 or $\boldsymbol{\alpha}^N$ to give scalars. In the nonrelativistic part of (3.1), we have neglected all position dependence in the radiative correction except in that part which diverges as $\ln|\mathbf{x}-\mathbf{x}'|$ as $|\mathbf{x}-\mathbf{x}'|\to 0$. Most of the model dependence of the radiative correction comes from this log term, since the $|\ln|\mathbf{x}-\mathbf{x}'||\gg|\mathbf{x}-\mathbf{x}'|$ for values of $|\mathbf{x}-\mathbf{x}'|$ which are important in (3.1) (typically, $|\mathbf{x}-\mathbf{x}'|\sim 0.01$). Furthermore, a rough estimate of the contribution of the nonlogarithmic position-dependent parts of the radiative correction to the f.n.s. correction is at most $\sim 0.08\%$ of f_0t. We have also completely omitted the radiative correction to the relativistic part of (3.1), since the latter is itself small compared to the nonrelativistic part.

The corrected Fermi function is calculated in the usual manner by inserting (3.1) into the transition probability per unit time and summing over the unobserved lepton spins and the neutrino momentum. Retaining at most those terms arising from the interference of the Coulomb, finite size, radiative, and screening corrections with the zeroth-order amplitude, the transition probability per unit time for the emission of a positron of momentum p is (see note added in proof)

$$\begin{aligned}P(p)dp=P_0(p)dp\{&1-\pi Z\alpha e_p/p+Z\alpha\Delta(2e_p^2-1)/p^2e_p\\&-\tfrac{1}{3}[p^2(1+\tfrac{2}{3}q/e_p)+q^2]A_1+Z\alpha[e_p(\tfrac{4}{3}A_2+A_3)+qA_3\\&+(1/e_p)(\tfrac{2}{3}A_2-A_3)]+[\tfrac{1}{3}(e_m-1/e_p)A_1-\tfrac{1}{2}Z\alpha A_2]\mathfrak{R}\\&+(\alpha/2\pi)(-\pi^2e_p/p+10-4C-4\ln p-4A_4)\}\,,\end{aligned}\qquad(3.2)$$

where p is in units of m_ec, e_m is the decay endpoint energy, $q=e_m-e_p$, $C=0.5772\cdots$ is Euler's constant, the factor[38] $\mathfrak{R}=E_i^N-E_f^N+M_{\text{neutron}}-M_{\text{proton}}-\lambda Z\alpha/2R_0$ results from the estimation of the relativistic nuclear-matrix element, and

$$P_0(p)=(1/2\pi^3)|G_V|^2|M_p|^2P^2(e_m-e_p)^2.$$

λ in $\mathfrak{R}$ is a variable parameter which will be discussed later. The quantities A_1, A_2, A_3, and A_4 appearing in (3.2) contain the details of the nuclear models and are given by

$$A_1=\int d^3x\,\rho_\beta(\mathbf{x})|\mathbf{x}|^2,\qquad(3.3)$$

$$A_2=\int d^3x\rho_\beta(\mathbf{x})\int d^3x'\rho_c(\mathbf{x})|\mathbf{x}-\mathbf{x}'|\,,\qquad(3.4)$$

$$\mathbf{a}\cdot\mathbf{b}A_3=\int d^3x\rho_\beta(\mathbf{x})\int d^3x'\rho_c(\mathbf{x}')\frac{\mathbf{a}\cdot\mathbf{x}\mathbf{b}\cdot(\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|},\qquad(3.5)$$

$$A_4=\int d^3x\rho_\beta(\mathbf{x})\int d^3x'\rho_\beta(\mathbf{x}')\ln|\mathbf{x}-\mathbf{x}'|\,,\qquad(3.6)$$

where $\rho_c(\mathbf{x}')$ is the charge density of the daughter nucleus and $\rho_\beta(\mathbf{x})$ is the probability density for the proton which β decays.

The first two terms in the angular brackets of (3.2) may be recognized as the expansion to first order in $Z\alpha$ of the point charge Fermi function[39] for positron decay:

$$F_0^\pm(Z,p)=4\left(\frac{1+\gamma}{2}\right)(2pR_0)^{2(\gamma-1)}\exp\left[\mp\pi Z\alpha\frac{e_p}{p}\right]\times\frac{|\Gamma(\gamma+iZ\alpha(e_p/p))|^2}{|\Gamma(2\gamma+1)|^2},\qquad(3.7)$$
$$\gamma=(1-Z^2\alpha^2)^{1/2},$$

and R_0 is a suitably chosen nuclear radius. Here and subsequently in the paper the upper sign applies to positron emission and the lower sign to negatron emission. The third term in the atomic electron screening correction to first order in Δ and $Z\alpha$. To this order, it agrees precisely with the WKB approximation of Rose[18] as applied to the Dirac equation. This consists of the prescription that the screened Fermi function may be obtained from (3.7) by the relation

$$F_S^\pm(Z,p)=(e_p'p'/e_pp)F_0^\pm(Z,p')\,,\qquad(3.8)$$

where $e_p'=e_p\pm V_0$, $p'=(e_p^2-1)^{1/2}$, and V_0 is the potential at the nucleus due to the atomic electrons. In this paper, $V_0=Z\alpha\Delta$. For the decays of interest here, the only significant part of (3.8) which contributes to the screening is the factor $p'e_p'/pe_p$. Expanding this to

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[39] Note that this definition of the point-nucleus Fermi function differs by the factor $\frac{1}{2}(1+\gamma)$ from that used by some other authors.

first order in Δ gives

$$F_S^{\pm}(Z,p)\simeq\left[1\pm Z\alpha\Delta\left(\frac{2e_p^2-1}{p^2e_p}\right)\right]F_0^{\pm}(Z,p). \quad (3.9)$$

The correction factor in (3.9) is exactly that which appears in (3.2) and, to $Z\alpha$, is the screening correction. The validity of the WKB approximation as applied to the Schrödinger and Klein-Gordon equations has been demonstrated by Durand.[18] Using exact solutions of these equations for a Hulthén potential, it was shown that, in the limit $p/\Delta\gg1$, the exact screened Fermi functions coincided with those that would be obtained from Rose's WKB approximation. That this is also true for the Dirac Fermi function is shown by our calculation of the screening correction as given in Eq. (3.2). It is interesting to see the effects of including the electron spin in the screening correction to $F_0(Z,p)$. Spin is neglected in Durand's calculation, but is included in ours. For decays with high endpoint energies (i.e., super-allowed) and for values of $p/\Delta\gg1$, our screening correction (which agrees with that of Rose) is about twice that obtained by Durand using the Klein-Gordon equation. Hence, when accurate calculations of the screening for these decays is desired, one must include the effects due to the electron spin. It should be noted here that the odd charge of the daughter atom has been neglected in the screening correction in (3.2). Inclusion of this effect may be made by replacing $Z\alpha\Delta$ by $(Z\pm1)\alpha\Delta$ (for $e^{\pm}$ decay), where Δ is related to the parent nucleus atomic electron charge distribution. The fractional change in the screening correction due to this effect is $1/Z$, and is negligible for the decays considered here. The fourth and fifth terms in the angular brackets of (3.2) are the usual f.n.s. corrections, including contributions from the neutrino wave function. It is interesting to note that the screening does not appear in the first-order f.n.s. correction for these decays. This may be understood from the fact that the lowest-order screening correction occurs as $Z\alpha\Delta$; hence a term like $(Z\alpha pR)(Z\alpha\Delta)$ would not appear in a perturbation treatment of the Coulomb interaction to order $Z\alpha$. In view of the fact that the screening correction to the ft values of the decays considered here is so small ($<0.3\%$), the screening correction to the finite-size correction would be very small indeed. The maximum correction for ${}_{27}Co^{54}$ is estimated at less than 0.03% of f_0t. Thus these two corrections may be treated separately with negligible error. The sixth term in (3.2) comes from the relativistic part of the nuclear matrix element. The form of $\mathcal{R}$ is from Ahrens and Feenberg.[38] Various theoretical estimates of the parameter have been made.[38,40–42] The most recent value,[42] $\lambda=2.4$, has been used here and is based on the hypothesis of conserved vector currents (CVC). The other values of λ are somewhat smaller.[40,41] For the decays considered here, the relativistic contribution to the ft values was found to be very small. The largest value of this correction for Co^{54} was $\simeq0.10\%$ of f_0t. The last term in (3.2) is the static vertex part of the radiative correction to lowest order in α. As described in Appendix A, some small energy-dependent terms were neglected in the calculation of $G(\mathbf{x}_r-\mathbf{x})$. A rough estimate of the resulting error in ft may be put at $\simeq0.1\%$ for ${}_8O^{14}$, the decay for which the error would be the largest.

Neglecting for the moment the radiative correction, the terms in the brackets of the decay rate (3.2) constitute an expansion of the total Fermi function in powers of $Z\alpha$, pR, and Δ. In the limit of a point nucleus for both the β decay and Coulomb interactions, the only surviving terms are the expansion in $Z\alpha$ of the point-nucleus Fermi function and the lowest-order screening correction. However, we want to treat the point charge Fermi function exactly, while treating just the f.n.s. and screening corrections in perturbation theory. Now, the positron's wave function in Eq. (3.1) was expanded in a power series in px and px' as well as in $Z\alpha$. What is desired is that the $Z\alpha$ expansion be used only in the f.n.s. and screening corrections, but not in the parts corresponding to the point Fermi function. When this is done, as discussed in Sec. I, the brackets in the transition rate become

$$F_0(Z,p)C(Z,p)+(\text{R.C.}), \quad (3.10)$$

where the correction factor $C(Z,p)$ is

$$\begin{aligned}C(Z,p)=1&+Z\alpha\Delta(2e_p^2-1)/p^2e_p-\tfrac{1}{3}[p^2(1+\tfrac{2}{3}q/e_p)+q^2]A_1\\&+Z\alpha[e_p(\tfrac{4}{3}A_2+A_3)+qA_3+(1/e_p)(\tfrac{2}{3}A_2-A_3)]\\&+[\tfrac{1}{3}(e_m-1/e_p)A_1-\tfrac{1}{2}Z\alpha A_2]\mathcal{R}. \quad (3.11)\end{aligned}$$

The R.C. cannot be considered as part of the usual f.n.s. correction because it has a different origin, namely, the change of the nuclear charge in the β decay. Even in the limit of a point nucleus for both the β and Coulomb interactions, it is present and diverges as $\ln|\mathbf{x}|$ as $|\mathbf{x}|\to0$. In an exact treatment of this effect, the parentheses in Eq. (3.10) would mean all of the terms associated with the radiative correction. The assumption has been made here and elsewhere[7,20,22] that only the part to lowest order in α is the most important. Because of the divergent nature of the first-order R.C., it may turn out that higher-order terms could significantly alter this correction. These terms are presently under study. The calculation of the R.C. in this paper employs, in essence, a position space cutoff in the logarithmically divergent term in the same way as used in the point-nucleus Fermi function. This is in contrast to other treatments[20,22] which employ an ultraviolet cutoff λ in the integration over the intermediate lepton momentum **k**. The cutoff is estimated from the form factors of the β-decaying nucleon.[22] In our treatment of the static

[40] D. L. Pursey, Phil. Mag. **42**, 1193 (1951).
[41] M. E. Rose and R. K. Osborn, Phys. Rev. **93**, 1315 (1954).
[42] J. Fujita, Phys. Rev. **126**, 202 (1962).

vertex part of this correction, the charge form factor arising from the translational motion of the β-decaying nucleon in the nucleus appears in $\rho_\beta(\mathbf{x})$ along with the charge form factor associated with the nucleon itself. Since the first form factor contributes the major characteristics of the nuclear charge distribution due to the β decaying proton (or final proton for electron decay), the second has been neglected in this paper.

The corrected ft values may now be obtained from the integration of (3.10) over the decay spectrum. Using the approximation in Appendix B,

$$f(Z,p_m) = f_0(Z,p_m)\int_0^m dp\,C(Z,p)p^2(e_m-e_p)^2/f_0(Z=0,\,p_m) + \int_0^m dp(\text{R.C.})p^2(e_m-e_p)^{2p}, \quad (3.12)$$

where

$$f_0(Z,p_m)\equiv\int_0^{p_m} dp F_0(Z,p)p^2(e_m-e_p)^2.$$

The indicated integrals may be done analytically to yield an expression for the fractional correction to $f_0(Z,p_m)t$ in terms of the nuclear-model-dependent quantities A_1–A_4 and the β endpoint momentum p_m. From Eq. (3.12), the fractional correction to f_0t is

$$\frac{\delta(ft)}{f_0t}=\frac{f-f_0}{f_0}\equiv\left(\frac{\delta f}{f_0}\right)_S+\left(\frac{\delta f}{f_0}\right)_{\text{f.n.s.}}+\left(\frac{\delta f}{f_0}\right)_{\text{RAD}}, \quad (3.13)$$

where

$$\left(\frac{\delta f}{f_0}\right)_S = Z\alpha\Delta\left\langle\frac{2e_p^2-1}{p^2e_p}\right\rangle_{SF}, \quad (3.14)$$

$$\left(\frac{\delta f}{f_0}\right)_{\text{f.n.s.}} = Z\alpha(\tfrac{4}{3}A_2+A_3)\langle e_p\rangle_{SF}+Z\alpha A_3\langle q\rangle_{SF} + Z\alpha(\tfrac{2}{3}A_2-A_3)\langle 1/e_p\rangle_{SF} - \tfrac{1}{3}A_1\langle p^2(1+\tfrac{2}{3}q/e_p)+q^2\rangle_{SF} + [\tfrac{1}{3}(e_m-\langle 1/e_p\rangle_{SF})A_1-\tfrac{1}{2}Z\alpha A_2]\mathcal{R}, \quad (3.15)$$

$$\left(\frac{\delta f}{f_0}\right)_{\text{RAD}} = \frac{\alpha}{2\pi}\frac{f_0(Z=0,\,p_m)}{f_0(Z,p_m)} \times\left(10-4C-\pi^2\left\langle\frac{e_p}{p}\right\rangle_{SF}-4\langle\ln p\rangle_{SF}-4A_4\right). \quad (3.16)$$

The quantities $\langle X\rangle_{SF}$ are defined in Appendix B and are given by

$$\left\langle\frac{2e_p^2-1}{p^2e_p}\right\rangle_{SF}=\frac{5e_m}{p_m^2}g_m^{-1}\left[1-\frac{3}{2p_m^2}+\frac{3\ln h_m}{2e_mp_m^3}\right], \quad (3.17)$$

$$\langle e_p\rangle_{SF}=\tfrac{1}{2}e_mg_m^{-1}\times\left[1+\frac{45}{4p_m^4}-\frac{15(2e_m^2+1)\ln h_m}{4e_mp_m^5}\right], \quad (3.18)$$

$$\langle q\rangle_{SF}=e_m-\langle e_p\rangle_{SF}, \quad (3.19)$$

$$\left\langle\frac{1}{e_p}\right\rangle_{SF}=\frac{5}{2e_m}g_m^{-1}\left[1+\frac{17}{2p_m^2}+\frac{15}{4p_m^4}-\frac{3e_m(4e_m^2+1)\ln h_m}{2p_m^5}\right]. \quad (3.20)$$

$$\left\langle p^2\left(1+\frac{2}{3}\frac{q}{e_p}\right)+q^2\right\rangle_{SF} =\frac{5p_m^2}{7}g_m^{-1}\left[1-\frac{49}{10p_m^2}-\frac{91}{2p_m^4}-\frac{42}{p_m^6}+\frac{21e_m(3e_m^2+1)\ln h_m}{2p_m^7}\right], \quad (3.21)$$

$$\begin{aligned}\langle\ln p\rangle_{SF}=\frac{30}{p_m^5}g_m^{-1}\Bigg[&\left(\frac{8}{15}p_m^5+\tfrac{2}{3}p_m^3\right)\ln p_m -\left(\frac{34}{225}p_m^5+\frac{2}{9}p_m^3\right)-\frac{e_m}{32}\Bigg\{\left(\frac{h_m^8-1}{h_m^4}\right)\\ &\times\ln\left(\frac{h_m^2-1}{2h_m}\right)+\frac{1}{4h_m^4}+\frac{1}{h_m^2}-h_m^2-\frac{h_m^2}{4}\\ &-[1-2\ln h_m-4\ln 2+4\ln(h_m^2-1)]\Bigg\} +\tfrac{1}{8}e_mL(h_m^2)\Bigg],\end{aligned} \quad (3.22)$$

$$\left\langle\frac{e_p}{p}\right\rangle_{SF}=\left(\frac{e_m}{p_m}\right)^5g_m^{-1}\left[1-\frac{10}{e_m^3}+\frac{15}{e_m^4}-\frac{6}{e_m^5}\right], \quad (3.23)$$

where

$$L(x)=-\int_0^x\frac{dy\ln y}{y-1}$$

is the Spence function of (x), and

$$h_m=e_m+p_m,$$

$$g_m=\left(1-\frac{5}{2p_m^2}-\frac{15}{2p_m^4}+\frac{15e_m\ln h_m}{2p_m^5}\right).$$

IV. RESULTS AND DISCUSSION

The expressions just presented for the f.n.s., radiative, and screening corrections are now applied to the f_0t values of the $0^+\to 0^+$ positron decays specified at the beginning of Sec. III. The results are presented in

TABLE I. Model-dependent parameters.

	B-S[a]	B-B[b]	P-S[c]	P-B[d]	O^{14}	Hofstadter HO[e] Al^{26m}	Cl^{34}	Sc^{42}	V^{46}	Mn^{50}	Co^{54}
A_1	R_0^2	$\frac{3}{5}R_0^2$	R_0^2	$\frac{3}{5}R_0^2$	$1.66r^2$	$1.32r^2$	$1.23r^2$	$1.50r^2$	$1.45r^2$	$1.39r^2$	$1.36r^2$
A_2	$(6/5)R_0$	$(36/35)R_0$	R_0	$\frac{3}{4}R_0$	$1.36r$	$1.43r$	$1.41r$	$1.47r$	$1.47r$	$1.47r$	$1.44r$
A_3	$(4/15)R_0$	$(6/35)R_0$	$\frac{1}{3}R_0$	$\frac{1}{4}R_0$	$0.29r$	$0.28r$	$0.27r$	$0.30r$	$0.30r$	$0.29r$	$0.30r$
A_4	$\ln R_0+0.193$	$\ln R_0-1.140$	$\ln R_0$	$\ln R_0-0.333$	$\frac{1}{2}\ln A_1+\frac{1}{2}\ln(\frac{4}{5})+0.193$	$\frac{1}{2}\ln A_1+\frac{1}{2}\ln(6/7)+0.193$		$\frac{1}{2}\ln A_1+\frac{1}{2}\ln(8/9)+0.193$			

[a] Uniformly charged sphere of radius R_0 for charge density of daughter nucleus. Lepton wave function evaluated at the nuclear surface.
[b] Same charge density as in a. Lepton wave functions averaged over a uniform sphere of radius R_0.
[c] Point charged nucleus. Lepton wave functions evaluated on a shell of radius R_0.
[d] Same charge density as in c. Lepton wave functions averaged over uniform sphere of radius R_0.
[e] Hofstadter (Ref. 43) charge densities for the daughter nucleus, where r denotes the root-mean-square radius of the daughter nucleus charge distribution. The HO shell model was used for ${}_8O^{14}$. The charge distributions of the other nuclei were represented with a trapezoidal model with the same half-density radius and shin thickness as given by Hofstadter. The lepton wave functions were averaged over the position of the decaying proton with the probability density obtained from the single-particle HO shell model.

three tables. Table I gives the values of the model-dependent parameters A_1, A_2, A_3, and A_4 for five representative models of the nucleus. Above each column, the first word denotes the daughter nucleus charge distribution, while the second indicates the assumed probability density of the β-decaying proton. A legend below the table gives a description of the models. Table II gives the percentage corrections to the f_0t values from the f.n.s. and radiative corrections for the three most realistic models of Table I. These are the ball-shell (B-S), the ball-ball (B-B), and the Hofstadter harmonic-oscillator (H-HO) models. The screening correction as calculated from Eq. (3.17) is also presented. The f.n.s., screening, and radiative corrections usually quoted in the literature are included for comparison. In Table III, the H-HO f.n.s. and radiative corrections are combined with the screening correction of Eq. (3.17) to obtain the corrected ft values for the seven positron decays considered in this paper. For comparison, the same f.n.s. and screening corrections have been combined with the radiative correction as calculated by Källén.[22] The percent deviation of the average G_V^β from G^μ, the Cabibbo angle, and δ, an isospin mixing parameter, are given for both cases.

The purpose of this study is to examine the analytical model dependence of the f.n.s. and radiative corrections and to compare them to other corrections to the f_0t values. The first four models in Table I are those which have been used in the literature to calculate electron radial wave functions.[12–16] As a check on our model-dependent parameters, we have used the approximate analytical lepton wave functions obtained by Huffaker and Laird[15] to calculate the parameters A_1–A_3 for the models considered by them (B-B, B-S, and Point-Shell). The results are in exact agreement with ours. For a given charge density, the differences in these parameters resulting from the evaluation of the lepton wave functions at the nuclear surface or averaging them over a uniform sphere can be seen by comparing the B-S with the B-B, or comparing the P-S with P-B. Differences arising from the use of different charge densities (a uniform sphere or a point) but with a given treatment of the lepton wave functions are seen by comparing the B-S with the P-S, or comparing the B-B with P-B.

The major part of the f.n.s. percentage correction to f_0t comes from the first term in $(\Delta f/f)_{\text{f.n.s.}}$, namely, $Z\alpha(\frac{4}{3}A_2+A_3)\langle e_p\rangle_{SF}$. A look at Table I shows that this term varies by as much as 20% with the use of different charge distributions (the B-S versus the P-S or the B-B versus the P-B) or with the different treatment of the lepton wave functions (B-S versus the B-B or the P-S versus the P-B). Since the f.n.s. corrections for some of the heavier nuclei considered here (${}_{21}Sc^{42}$, ${}_{23}V^{46}$, ${}_{25}M^{50}$, and ${}_{27}Co^{54}$) are from 1.5 to 2.5% of f_0t, the

TABLE II. Corrections to f_0t. (See note added in proof.)

Decay	$(\delta f/f_0)_{\text{f.n.s.}}$, % a	b	c	d	e	$(\delta f/f_0)_{\text{RAD}}$, % a	b	c	f	g	$(\delta f/f_0)_S$, % h	i
$O^{14}\to N^{14*}$	0.20	0.16	0.18	0.20	0.34	1.64	2.37	1.69	1.75	2.52	0.12	0.16
$Al^{26m}\to Mg^{26}$	0.61	0.52	0.56	0.66	0.69	1.48	2.29	1.61	1.60	2.49	0.17	0.13
$Cl^{34}\to S^{34}$	1.10	0.95	1.02	1.13	1.05	1.40	2.27	1.58	1.53	2.60	0.17	0.16
$Sc^{42}\to Co^{42}$	1.68	1.49	1.60	2.01	1.74	1.38	2.30	1.46	1.52	2.70	0.19	...
$V^{46}\to Ti^{46}$	2.09	1.82	1.93	2.61	1.82	1.38	2.32	1.47	1.52	2.72	0.20	0.20
$Mn^{50}\to Cr^{50}$	2.55	2.18	2.41	3.00	2.21	1.37	2.36	1.48	1.52	2.75	0.22	...
$Co^{54}\to Fe^{54}$	3.00	2.62	2.84	4.08	3.05	1.37	2.40	1.49	1.52	2.77	0.23	0.24

[a] Ball-Shell $R_0=\sqrt{(5/3)}\,(r_{\text{rms}})$ Hofstadter.
[b] Ball-Ball.
[c] Hofstadter-HO.
[d] Previous f.n.s. corrections [taken from Freeman *et al.* (Ref. 6)].
[e] f.n.s. corrections of Blin-Stoyle and Nair (Ref. 8), after subtracting Rose's screening.
[f] Radiative correction of Ref. 7, using point-shell model $R_0=1.2A^{1/3}\times10^{-13}$ cm.
[g] Radiative correction of Källén [G. Källén, Ref. 22; *Elementary Particle Physics* (Addison-Wesley Publishing Co., Inc., Reading, Mass., (1964)] after dividing by $F_0(Z,P_m)$.
[h] Screening correction as obtained in this paper. Values of Δ taken from V_0 of Matese and Johnson (Ref. 19).
[i] Numerical calculation of Rose's screening. Taken from Matese and Johnson (Ref. 19).

TABLE III. Corrected ft values and vector coupling constants.

Decay	e_m (MeV)[a]	$t_{\frac{1}{2}}$ (sec)[a]	f_0t (sec)[a]	ft (sec) b	ft (sec) c	$G_V^\beta\times10^{+49}$ (erg cm^3) b	$G_V^\beta\times10^{+49}$ (erg cm^3) c	$G^\mu-G_V^\beta/G^\mu$ (%) d	$G^\mu-G_V^\beta/G^\mu$ (%) e	θ (Cabbibo) d	θ (Cabbibo) e	$\delta\times100$ b	$\delta\times100$ e
$O^{14}\rightarrow N^{14*}$	2.3236 ±0.0014	71.360 ±0.090	3066±10	3128±10	3153±10	1.4025 ±0.0022	1.3967 ±0.0022					1.40 ±0.32	1.32 ±0.32
$Al^{26m}\rightarrow Mg^{26}$	3.7190 ±0.0019	6.376 ±0.006	3015±7	3085±7	3112±7	1.4117 ±0.0016	1.4059 ±0.0016					0.00	0.00
$Cl^{34}\rightarrow S^{34}$	4.9707 ±0.0040	1.565 ±0.007	3055±20	3140±20	3172±20	1.3992 ±0.0044	1.3926 ±0.0044					1.78 ±0.64	1.92 ±0.64
$Sc^{42}\rightarrow Ca^{42}$	5.9200 ±0.0023	0.6830 ±0.0015	3009±10	3106±10	3145±10	1.4065 ±0.0023	1.3985 ±0.0023	2.08 ±0.16	2.58 ±0.16	0.205 ±0.003	0.226 ±0.003	0.71 ±0.32	1.09 ±0.32
$V^{46}\rightarrow Ti^{46}$	6.5431 ±0.0022	0.4259 ±0.0008	3004±8	3112±8	3151±8	1.4057 ±0.0018	1.3974 ±0.0018					0.91 ±0.30	1.26 ±0.30
$Mn^{50}\rightarrow Cr^{50}$	7.1200 ±0.0026	0.2857 ±0.0006	2989±9	3111±9	3151±9	1.4057 ±0.0020	1.3971 ±0.0020					0.91 ±0.30	1.29 ±0.30
$Co^{54}\rightarrow Fe^{54}$	7.7387 ±0.0038	0.1937 ±0.0010	2966±22	3103±22	3140±22	1.4080 ±0.0052	1.3996 ±0.0052					0.65 ±0.71	0.93 ±0.71

[a] Values of e_m and half-lives $t_{1/2}$ from Freeman *et al.* (Ref. 6). f_0t values from N.B.S. tables. All errors shown in this table are experimental.
[b] H-HO f.n.s. and radiative correction with screening of Eq. (3.14).
[c] Same f.n.s. and screening as b, but with radiative correction of Källén divided by $F_0(Z,p_m)$.
[d] Results of column b with $G^\mu=1.4350\pm0.0011\times10^{-49}$ erg cm^3.
[e] Results of column c with $G^\mu=1.4350\pm0.0011\times10^{-49}$ erg cm^3.

variation of $Z\alpha(\frac{4}{3}A_2+A_3)\langle e_p\rangle_{SF}$ by 20% would result in a variation of the f.n.s. corrections on the order of 0.3–0.5% of f_0t. But this is larger than the screening correction for these decays (0.20% of f_0t for $_{21}Sc^{42}$ to 0.25% of f_0t for $_{27}Co^{54}$). Recently, several detailed calculations of the screening effect[8,16,19] have been made for the purpose of obtaining precise ft values for the $0^+\rightarrow0^+$ positron decays. The efforts put into these calculations will have been in vain, however, if one combines the screening corrections with f.n.s. corrections whose uncertainty is as large as or larger than the screening itself. For this reason, an effort has been made in this paper to calculate a more precise value of the f.n.s. correction to the f_0t values. This has been done by using the charge distributions of Hofstadter,[43] as determined from high-energy electron scattering experiments, for the charge density of the daughter nucleus and the single-particle HO shell model for the probability density of the β-decaying proton. The oscillator constants were adjusted so that the nuclear charge density that would be obtained with the HO model matched that given by Hofstadter. The parameters A_1, A_2, and A_3 as calculated for this model are given in Table I. The radiative correction was also treated with this model. In this case, the HO model was replaced by a shell of radius r_{max}, the value of r for which the HO radial probability density for the β-decaying proton is a maximum. This is a reasonable approximation since the HO radial probability density peaks rather well at this value of $r=r_{max}$.

The results for the f.n.s. correction for the B-S, B-B, and H-HO models are given in Table II. The estimated percentage error in the ft values due to the approximation used in performing the integration of these corrections over the decay spectrum ranges from ~0.00% of f_0t for $_8O^{14}$ to ~0.02% of f_0t for $_{27}Co^{54}$. The relativistic matrix elements were found to contribute very little to the f.n.s. corrections for the value of $\lambda=2.4$. Even for the Ahrens *et al.* value ($\lambda=1.0$), which is not in accord with the CVC, the maximum contribution to the ft of $_{27}Co^{54}$ would be only 0.40%. There is good agreement of the f.n.s. corrections as calculated here for $_8O^{14}$, $_{13}Al^{26m}$, and $_{17}Cl^{34}$ with those given in the literature[6,13] for the same models (column d, Table II). On the other hand, the f.n.s. corrections for the other nuclei considered here are, for a given model, lower than those usually quoted in the literature by amounts ranging from ~0.3% for $_{21}Sc^{42}$ to ~1.1% for $_{27}Co^{54}$. This substantiates the findings of several recent papers[8,15,16] which have pointed out an inconsistency in the phase conventions adopted in Ref. 13 for the numerical calculation of the positron's wave function for the B-S model. The f.n.s. corrections calculated in this paper are in good agreement with this finding.

The model dependence of the f.n.s. correction for the decays of $_8O^{14}$, $_{13}Al^{26m}$, and $_{17}Cl^{34}$ was found to be very

[43] R. Hofstadter, Ann. Rev. Nucl. Sci. 7, 231 (1957).

small. One would have expected this, since this correction is itself small for these decays. The variation of the f.n.s. correction for ${}_{21}Sc^{42}$, ${}_{23}V^{46}$, ${}_{25}Mn^{50}$, and ${}_{27}Co^{54}$ due to the use of different nuclear models was found to be comparable to the screening correction for these decays. The difference between the H-HO and B-S f.n.s. correction ranged from about $\sim$0.5 of the screening for ${}_{21}Sc^{42}$ to about $\sim$0.7 of the screening for ${}_{27}Co^{54}$. It should be remarked that the B-S model is most commonly used in the literature. The difference between the H-HO and B-B f.n.s. correction is approximately the same size as the screening correction, while the difference between the B-S and B-B f.n.s. correction ranges from $\sim$1.2 times the screening for ${}_{21}Sc^{47}$ to $\sim$1.7 times the screening for ${}_{27}Co^{54}$. This would indicate that the inclusion of such a small effect as the screening correction in the calculation of ft values requires, for consistency's sake, a more precise treatment of the f.n.s. correction than has been previously done. The use of the H-HO model in this paper represents such a calculation. For further comparison, the recent f.n.s. corrections of Blin-Stoyle and Nair[8] have been included in column e of Table II. The B-S model was used in Ref. 8 in a numerical calculation which combined both the screening and f.n.s. corrections. To compare their f.n.s. correction with ours, the screening correction, assumed to be that given by Rose or Bühring, was subtracted from their results to get the values in column e. Within the 0.3% error quoted by Ref. 8, the values of column e are in agreement with those of column a which we have calculated for the same model (B-S). There is, however, considerable scatter between the two sets of numbers, especially for ${}_{8}O^{14}$, ${}_{21}Sc^{42}$, ${}_{23}V^{46}$, and ${}_{25}M^{50}$. We were not able to determine the reason for this. No neutrino f.n.s. or relativistic effects were considered in their work, but this would not account for the erratic difference between the two sets of corrections.

The radiative correction for the B-S, B-B, and H-HO models is given in the middle of Table II. Aside from the neglect of some small energy-dependent terms, the f.n.s. correction to the radiative correction was also ignored. This was based on the fact that the major variation in the radiative correction due to the nuclear models used will come from the logarithmically divergent part. While the treatment of this effect is not quite as precise as the f.n.s. calculation, the main purpose here is to examine the general nuclear model dependence of the radiative correction. A look at Table II shows that there are considerable differences in this correction among the models given. The largest occurs between the B-S and the B-B models and is $\sim$1% of f_0t for ${}_{27}Co^{54}$. On the other hand, the B-S and finite de Broglie wavelength models give a radiative correction close to that given by the H-HO model. These results show the radiative correction to be somewhat more sensitive to the nuclear models than the usual f.n.s. correction. This would answer, in part, a query posed by Freeman *et al.*,[6] who questioned whether nuclear structure effects could alter the radiative correction by amounts of the same order as the correction itself. For the most realistic models for this correction, namely, the H-HO and the B-S, the variation is about on the order of the screening.

The corrected ft values of the seven $0^+\to 0^+$ positron decays considered here are given in Table III for the H-HO radiative correction and for the radiative correction of Källén.[22] The H-HO finite nuclear size and the screening as given by Eq. (3.14) were used in both cases. The ft values are fairly consistent among each other, with an rms deviation from the unweighted mean of 0.5% of f_0t. This is especially true for the heavier elements ${}_{21}Sc^{42}$, ${}_{23}V^{46}$, ${}_{25}M^{50}$, and ${}_{27}C0^{54}$, with an rms deviation of their ft values from their mean of about 0.2% of f_0t. The over-all uniformity of the ft values is slightly better using the radiative correction of Källén. With the exception of the ft value of ${}_{13}Al^{26m}$, which is about 1% lower than the rest, the other ft values may be said to be equal to within the experimental errors. The vector coupling constants $G_V{}^\beta$, the percent deviation of the average $G_V{}^\beta$ from G^μ, and the average Cabibbo angle are also given in Table III for both radiative corrections. The Cabibbo angles for both cases are in fair agreement with $\theta_c=0.21$ as obtained from the $\Delta S=1$ semileptonic decays.

We comment now about the anomalously low ft value of ${}_{13}Al^{26m}$ in the light of the CVC theory. If the CVC is valid, then the variation in $G_V{}^\beta$ calculated under the assumption of isospin conservation should be due to the presence of Coulomb and other charge-dependent nuclear interactions. If we knew with confidence the magnitude of the isotopic spin mixing corrections to the Fermi matrix elements for all of the above decays, a good check on the CVC could be made. Unfortunately, the current status of the calculations[32–36] of this mixing are still uncertain. Most of the work has been done only for the ${}_{8}O^{14}\to{}_{7}N^{14*}$ decay, giving values of $\delta(O^{14})$, the fractional change in the square of the Fermi matrix element due to isotopic spin mixing, ranging from a few hundreths of a percent[36] to slightly over 1%.[37]

One could work backwards to get an estimate of what the relative mixing should be in order that the CVC be valid. By assuming the CVC to be true, the isospin mixing parameters δ_j and δ_k for the decays j and k are related by

$$\delta_j=1-\frac{(ft)_k}{(ft)_j}(1-\delta_k). \tag{4.1}$$

According to Blin-Stoyle and Nair,[8] the isospin mixing is expected to decrease the Fermi matrix element. Since $(ft)_{Al}$ is the lowest of all the $0^+\to 0^+$ decays, one may assume the mixing to be least for ${}_{13}Al^{26m}$. If one takes this to zero, the prediction of the isospin mixing in the other decays can be made from

$$\delta_j=((ft)_j-(ft)_k)/(ft)_j, \tag{4.2}$$

where j can be any of the other $0^+\to0^+$ decays. Values of δ_j using this relation are given at the end of Table III. For O^{14}, this gives $\delta^b(O^{14})=1.40\pm0.32\%$ and $\delta^c(O^{14})=1.32\pm0.33\%$. These are just at the edge of the range of values calculated in the literature. More work on this is needed in order to determine whether or not the variation of $G_V{}^\beta$ among the nuclei can be accounted for solely on the basis of isotopic spin mixing. What is needed is a careful calculation of the relative isospin mixing among $0^+\to0^+$ decays.

On the basis of the analytical model-dependent results presented in this section the following comments may be made:

1. For the heavier nuclei considered ($A>34$), the f.n.s. corrections as calculated in this paper are as much as 1% of f_0t lower than those that have been previously quoted in the literature.[6,13] This is in agreement with the conclusions of several recent papers[8,15,16] which have calculated electron radial wave functions.
2. The nuclear model dependence of the f.n.s. correction is negligible for O^{14}, Al^{26m}, Cl^{34}.
3. For Sc^{42}, V^{46}, Mn^{50}, and Co^{54}, the variation of the f.n.s. correction due to the use of different models is comparable to the screening corrections.
4. In view of this last finding, we have made a more precise calculation of f.n.s. correction in which the Hofstadter charge densities have been used for the daughter nuclei, and the single-particle HO has been used for the wave function of the decaying proton.

For all of the decays considered in this paper, the variation of the radiative correction due to the use of different nuclear models was found to be as much as 1% of f_0t. However, the H-HO nuclear model gave results close to that given by Chern *et al.*[7] for the finite de Broglie wavelength model.

The details of the atomic electron screening correction calculation have been presented for the results given in Chern *et al.* The equivalence between this calculation of the screening and Rose's WKB approximation is shown.

APPENDIX A: EXPRESSIONS FOR $I(\mathbf{x})$ AND $G(\mathbf{x})$

The expression for $I(\mathbf{x})$ is

$$I(\mathbf{x})=\int d^3k\frac{(e_p+\boldsymbol{\alpha}\cdot\mathbf{k}-\beta)\exp(-i\mathbf{k}\cdot\mathbf{x})}{(|\mathbf{p}-\mathbf{k}|^2+\Delta^2)(e_p{}^2-e_k{}^2+i\epsilon)}, \tag{A1}$$

where the limit $\epsilon\to0$ is taken after the integration is performed. The integration is carried out by first expressing the denominators in the integrand of (A1) in terms of an auxiliary Feynman variable, then performing the $\mathbf{k}$ integration, and finally performing the integration over the auxiliary variable. The exact result is[23]

$$\begin{aligned}I(\mathbf{x})=-\frac{\pi^2}{p}\bigg[&(e_p+\sigma\boldsymbol{\alpha}\cdot\mathbf{p}-\beta)\bigg(i\int_{-\sigma}^{y_0-\sigma}dt\frac{\exp[ip|\mathbf{x}|(t^2+1-\sigma^2)^{1/2}-i\mathbf{p}\cdot\mathbf{x}(\sigma+t)]}{(t^2+1-\sigma^2)^{1/2}}\\&+\int_{y_0-\sigma}^{1-\sigma}dt\frac{\exp[-p|\mathbf{x}|(\sigma^2-1-t^2)^{1/2}-i\mathbf{p}\cdot\mathbf{x}(\sigma+t)]}{(\sigma^2-1-t^2)^{1/2}}\bigg)+\boldsymbol{\alpha}\cdot\mathbf{p}\bigg(i\int_{-\sigma}^{y_0-\sigma}dt\,t\frac{\exp[ip|\mathbf{x}|(t^2+1-\sigma^2)^{1/2}-i\mathbf{p}\cdot\mathbf{x}(\sigma+t)]}{(t^2+1-\sigma^2)^{1/2}}\\&+\int_{y_0-\sigma}^{1-\sigma}dt\,t\frac{\exp[-p|\mathbf{x}|(\sigma^2-1-t^2)^{1/2}-i\mathbf{p}\cdot\mathbf{x}(\sigma+t)]}{(\sigma^2-1-t^2)^{1/2}}\bigg)+p\frac{\boldsymbol{\alpha}\cdot\mathbf{x}}{|\mathbf{x}|}\bigg(i\int_{-\sigma}^{y_0-\sigma}dt\exp[ip|\mathbf{x}|(t^2+1-\sigma^2)^{1/2}-i\mathbf{p}\cdot\mathbf{x}(\sigma+t)]\\&+\int_{y_0-\sigma}^{1-\sigma}dt\exp[-p|\mathbf{x}|(\sigma^2-1-t^2)^{1/2}-i\mathbf{p}\cdot\mathbf{x}(\sigma+t)]\bigg)\bigg],\end{aligned} \tag{A2}$$

where $\sigma=1+\Delta^2/2p^2$ and $y_0=\sigma-(\sigma^2-1)^{1/2}$. The integration over the auxiliary variable may be performed by expanding the exponentials in (A2) in powers of px and $\mathbf{p}\cdot\mathbf{x}$. This is a good expansion for β decay, since px and $\mathbf{p}\cdot\mathbf{x}\sim0.1$ for x and $\mathbf{x}$ on the order of the nuclear dimensions. For the $0^+\to0^+$ decays considered in this paper, the contributing terms of the expansion of (A2) are those that will combine with 1 or $\boldsymbol{\alpha}^N$ to form scalars.

For the radiative correction, we have to evaluate the expression for $G(\mathbf{x})$:

$$G(\mathbf{x})=\frac{1}{2}\int d^3k\frac{(e_k-\boldsymbol{\alpha}\cdot\mathbf{k}+\beta)\exp(-i\mathbf{k}\cdot\mathbf{x})}{(|\mathbf{p}-\mathbf{k}|^2+\Delta^2)(e_k)(e_p+e_k-i\epsilon)}. \tag{A3}$$

This may be rewritten as

$$\begin{aligned}G(\mathbf{x})=\tfrac{1}{2}I(\mathbf{x})-\frac{1}{2}\int d^3k&\frac{e_k\exp(-i\mathbf{k}\cdot\mathbf{x})}{(|\mathbf{p}-\mathbf{k}|^2+\Delta^2)(e_p{}^2-e_k{}^2+i\epsilon)}\\-\tfrac{1}{2}e_p\int d^3k&\frac{(\boldsymbol{\alpha}\cdot\mathbf{k}+\beta)\exp(-i\mathbf{k}\cdot\mathbf{x})}{e_k(|\mathbf{p}-\mathbf{k}|^2+\Delta^2)(e_p{}^2-e_k{}^2+i\epsilon)}.\end{aligned} \tag{A4}$$

For the decays considered in this paper, $G(\mathbf{x})$ was approximated as follows: First, Δ and x were put equal to zero everywhere except in the part of $G(x)$ [Eq. (A4)] which diverges as $\ln|\mathbf{x}|$ as $\mathbf{x}\to0$. For values of $|\mathbf{x}|$ (typically $\simeq0.01$) which are important in $G(\mathbf{x})$, the

$\ln|\mathbf{x}|$ is much larger than the other f.n.s. terms in $G(\mathbf{x})$, since these other terms occur as powers of $\mathbf{x}$. Furthermore, the $\ln|\mathbf{x}|$ varies much more than do the powers of $\mathbf{x}$ for a given change in $|\mathbf{x}|$. Thus the major nuclear model dependence will come from the $\ln|\mathbf{x}|$ term. The other approximation to $G(\mathbf{x})$ was to set $e_k \simeq |\mathbf{k}|$ in (A4) which is valid if most of the contribution to the integrals come from $|\mathbf{k}|\gg 1$. This will be the case for decays with large endpoint energies, such as the super-allowed decays considered here.

The above two approximations give

$$G(\mathbf{x})\approx \tfrac{1}{2}I(0)+\pi[2(2-C)-2\ln(p|\mathbf{x}|)+e_p\boldsymbol{\alpha}\cdot\mathbf{p}/p^2],$$
$$C=0.57721\cdots.$$

APPENDIX B: APPROXIMATE ANALYTICAL INTEGRATION OF TRANSITION RATE OVER DECAY SPECTRUM

In the integration of $P(p)dp$ over the decay spectrum, we have to evaluate quantities defined by

$$\langle X\rangle_{F_0(SF)}\equiv \int dpF_0^+(Z,p)(SF)X \Big/ \int dpF_0^+(Z,p)(SF), \quad \text{(B1)}$$

where $F_0^+(Z,p)$ is the point-nucleus Fermi function [Eq. (3.7)], SF stands for $p^2(e_m-e_p)^2$, and X denotes the various quantities appearing in the correction factor $C(Z,p)$ [Eq. (3.11)]. An approximate but quite accurate analytical calculation of $\langle X\rangle_{F_0(SF)}$ may be made if one takes advantage of the fact that, for decays with high endpoint energies ($p_m\gg 1$), $F_0^+(Z,p)$ is essentially constant over the decay spectrum. In these cases, $F_0^+(Z,p)$ may be factored out of the integrals in (B1) to give

$$\langle X\rangle_{F_0(SF)}\approx\langle X\rangle_{SF}\equiv\int dp(SF)X\Big/\int dp(SF). \quad \text{(B2)}$$

The error in $\langle X\rangle_{F_0(SF)}$ due to the use of (B2) in place of (B1) may be estimated as follows: Expand $F_0^+(Z,p)$ in (B1) in powers of $Z\alpha$ to get

$$\langle X\rangle_{F_0(SF)}=\langle X\rangle_{SF}\left[\frac{1-\pi Z\alpha\langle e_pp^{-1}\rangle_{(SF)X}\cdots}{1-\pi Z\alpha\langle e_pp^{-1}\rangle_{SF}\cdots}\right]. \quad \text{(B3)}$$

For the decays of interest in this paper, the terms of the expansion in (B3) are less than unity, so the denominator may be expanded further to get

$$\langle X\rangle_{F_0(SF)}=\langle X\rangle_{SF}[1-\pi Z\alpha(\langle e_pp^{-1}\rangle_{(SF)X}-\langle e_pp^{-1}\rangle_{SF})\cdots]. \quad \text{(B4)}$$

When $P_m\gg 1$, both $\langle e_p/p\rangle_{(SF)X}$ and $\langle e^n/p\rangle_{SF}$ will be very close to 1, which gives $\langle X\rangle_{F_0(SF)}\simeq\langle X\rangle_{SF}$. As a specific example, consider the f.n.s. correction. Most of this correction is due to the first term in Eq. (3.15), which is proportional to $\langle e_p\rangle_{SF}$. Equation (B4) gives (keeping just the term to first order in $Z\alpha$)

$$\langle e_p\rangle_{F_0(SF)}\approx 1.0005\langle e_p\rangle_{SF}$$

for ${}_{27}\mathrm{Co}^{54}$, and

$$\langle e_p\rangle_{F_0(SF)}\approx 1.007\langle e_p\rangle_{SF}$$

for ${}_{8}\mathrm{O}^{14}$. This results in an error in the ft values for ${}_{27}\mathrm{Co}^{54}$ and ${}_{8}\mathrm{O}^{14}$ of 0.015% and 0.000%, respectively.

Note added in proof. In the transition probability Eq. (3.2), and in succeeding formulae, the contribution from the relativistic nuclear matrix elements appears as

$$\left[\frac{1}{3}\left\{e_m-\frac{1}{e_p}\right\}A_1-Z\alpha(A_2/2)\right]\mathcal{R}$$

The parameter $A_2/2$ should be replaced by A_2-A_5, where A_5 is the mean radius of the daughter nucleus charge distribution:

$$A_5=\int d^3x'\rho_c(x')|\mathbf{x}'|.$$

The results of this change are as follows:

(1) The finite-nuclear-size corrections for Cl, Sc, V, Mn, and Co in Table II, columns a, b, and c should all be decreased by an amount ranging from 0.03 f_0t for C to 0.01 f_0t for Cl.

(2) The effect is negligible for O and Al.

(3) The conclusions pertaining to the model dependence of the finite-nuclear-size correction are unchanged.

(4) These corrections have been incorporated into Table III.