ranges. One way to build such a link would be to base the NLS potential on a one-boson-exchange model and use the type of NLS potential shape suggested by Mitra[12] in which the range parameter may be related directly to the mass of the exchanged particle.

Finally we note that the kind of effect discussed here is present in double hypernuclei[13] (e.g., ${}_{\Lambda\Lambda}\mathrm{He}^{6}$, ${}_{\Lambda\Lambda}\mathrm{Be}^{10}$) not only in the form of $\Lambda N \leftrightarrow \Sigma N$ but also via $\Lambda\Lambda \leftrightarrow \Xi N$.[14] In fact, because the threshold for the latter process is only 25 MeV above the $\Lambda\Lambda$ elastic scattering threshold, it may play a much more important role than Λ-Σ conversion.

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Short-Range Dynamical Correlations in ^{6}Li from Electron Elastic Scattering*

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The charge form factor of the ^{6}Li nucleus was calculated in Born approximation, introducing in the ground-state density Jastrow-type correlations. Good agreement with the experimental data was obtained with different oscillator lengths for s and p nucleons. A diffraction minimum was predicted at a value of the momentum transfer $q \approx 2.8$ fm^{-1}.

I. INTRODUCTION

It is well known that among the lightest nuclei the ^{6}Li nucleus behaves anomalously as regards electron scattering, in the sense that the usual independent-particle shell model (IPSM) cannot fit the elastic scattering data.

Many other models have been used but they have not improved the situation substantially. After the recent measurements of Suelzle *et al.*[1] covering a range of momentum transfer $0.7 \lesssim q \lesssim 2.62$ fm^{-1}, we can summarize the situation concerning agreement between experimental data and theoretical calculations as follows:

(1) The IPSM with a common well for s and p nucleons cannot fit the data, either in the low-momentum or high-momentum part.

(2) By taking different oscillator lengths as suggested by Elton,[2] the low-momentum part ($q \lesssim 1.7$ fm^{-1}) can be fitted[1] ($\chi^2 = 19$ for 15 degrees of freedom) with $a_s = 1.632$ fm and $a_p = 1.980$ fm $[a = \hbar c/(M\hbar w)^{1/2}]$.

(3) The inclusion of configuration mixing of high-lying components at $2\hbar w$ and $4\hbar w$ excitations does not improve the fit.[3]

(4) Among cluster models, only the ^{4}He$+d$ one, with proper antisymmetrization between like particles (Hubbard model quoted in Ref. 1) can give a good fit to the data, though limited to the values of $q \leq 2$ fm^{-1}.

(5) The projected Hartree-Fock (PHF) wave functions of Bouten *et al.*[4] obtained by a variational calculation of the ground-state energy, based on a semi-realistic soft-core potential (Volkov potential[5]), give a considerable improvement in the low-momentum part with respect to the simple IPSM. Though these wave functions are essentially L-S coupling wave functions with the same oscillator lengths for the inner and outer particles, they contain considerable mixing of excited harmonic-oscillator states so that, as a consequence of this higher configuration expansion and deformation, the outer particles move, in fact, in a more extended well than the innermost ones and this probably simulates the difference of the oscillator lengths found in the IPSM analysis.[2]

In all these calculations the quadrupole scattering was not taken into account because of the small value of the quadrupole moment of the ^{6}Li nucleus ($Q = -0.08$ fm^2). In Ref. 6, the quadrupole scattering was properly taken into account and various intermediate coupling wave functions were used. The results of Ref. 6 showed that the intermediate-coupling wave functions (if they gave the proper value of the quadrupole moment)

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could not fit the data, and that the effect of the various L-S components is seen only in the quadrupole part of the form factor; this quadrupole contribution (if the wave functions give the proper quadrupole moment) is so small that it can be neglected without any consequences. As a matter of fact, it was pointed out several years ago[7] that the ^{6}Li form factor could be fitted if the quadrupole moment were ten times larger.

A more realistic model than the IPSM might be obtained by taking into account the effect of two-body correlations arising from repulsive cores of the nucleon-nucleon interaction. Such correlations are expected to have their greatest effect on the high-momentum components of the single-particle wave functions and consequently it is reasonable to assume that the high-momentum part of the elastic form factor will be very sensitive to these correlations. We have already presented[8] preliminary results of a calculation of the charge form factor of the ^{12}C nucleus, performed in Born approximation with a model in which Jastrow-type[9] correlations and different wells for the s and p protons were introduced in the ground-state density.

The same kind of calculation is now presented for ^{6}Li.

II. CHARGE FORM FACTOR AND DENSITY

We use a Jastrow-type[9] ground-state density of the form

$$\rho(1\cdots A)=|\psi(1\cdots A)|^2\prod_{i<j}F(r_{ij})$$
$$=|\psi(1\cdots A)|^2(1-\sum f+\sum ff-\cdots), \quad (1)$$

where ψ is the properly antisymmetrized wave function of the ground state, and the correlation function, $f(r_{ij})=1-F(r_{ij})$, takes into account correlations between nucleons at short relative distances $r_{ij}=|\mathbf{r}_i-\mathbf{r}_j|$, since it satisfies the conditions

$$\text{(a)}\ \lim_{r_{ij}\to 0}f(r_{ij})=1, \quad \text{(b)}\ f(r_{ij})\approx 0 \quad \text{if} \quad r_{ij}\gtrsim r_h,$$

where r_h is the "healing distance" of the wave function describing the relative motion of two nucleons.

The expression (1) was previously used in Refs. 8 and 10 to compute the charge form factor of the ^{12}C and ^{4}He nuclei, respectively.

We choose the ground-state wave function Ψ as a Slater determinant

$$\psi(1\cdots A)=(A!)^{-1/2}\det\{\varphi_i\},$$

where the radial part of the single-particle state $\varphi_i\equiv|i\rangle$ is given by the well-known harmonic-oscillator wave functions. Neglecting the quadrupole contribution to the scattering and taking into account only the single-pair terms of the cluster expansion (1), i.e.,[11]

$$(1-\sum f+\sum ff-\sum fff+\cdots)\to(1-\sum f),$$

we easily obtain for the charge form factor

$$F(q)=N\{\sum_i{}_1^A\langle i|\epsilon_1 j_0(qr_1)|i\rangle$$
$$-\sum_{ij}{}_1^A\langle ij|\epsilon_1 j_0(qr_1)f(r_{12})|ij-ji\rangle$$
$$-\sum_i{}_1^A\langle i|\epsilon_1 j_0(qr_1)|i\rangle$$
$$\times\sum_{kl}{}_1^{A-1}\langle kl|f(r_{12})|kl-lk\rangle\}, \quad (2)$$

where N is the normalizing factor such that $F(0)=1$, $\epsilon_1=\frac{1}{2}[1+\tau_3(1)]$, and i, j, k, and l are shell-model single-particle states.

Using the Talmi-Moshinsky method, the standard Racah algebra, the Wigner-Eckart theorem, and some properties of the spherical harmonics, the matrix elements which appear in (2) can be evaluated. We have done this in L-S coupling, choosing explicity $f(r_{ij})=\exp(-r_{ij}^2/b^2)$ and taking into account correlations only in the s states of relative motion. The derivation of the final expression of the charge form factor is quite cumbersome; it will be presented in a forthcoming and detailed paper where results for ^{4}He, ^{12}C, and ^{16}O will also be given.

Since correlations affect only the intrinsic part of the wave function, the usual center-of-mass motion correction, $C_{\text{c.m.}}=\exp(q^2a^2/4A)$, can be still used.

The proton sizes were taken into account both with the simple Gaussian form factor $F_{pG}=\exp(-\frac{1}{4}q^2a_P^2)$ with $a_P=0.653$ fm, and the more realistic form factor of the form[12]

$$F_{pR}=0.5\left(\frac{2.5}{1+q^2/15.7}-\frac{1.6}{1+q^2/26.7}+\frac{1.16}{1+q^2/8.19}\right)-0.03.$$

Since, as we shall see, F_{pG} and F_{pR} do not give substantially different results, when we calculate the charge density performing the Fourier transform of the form factor

$$\rho_{\text{ch}}(r)=\frac{1}{2\pi^2}\int j_0(qr)F_{\text{ch}}(q)q^2dq,$$

we use, for sake of simplicity, the Gaussian proton form factor F_{pG}.

III. RESULTS AND DISCUSSION

The fit to the experimental data is presented in Fig. 1. All the curves are best-fitting curves obtained by varying the oscillator lengths a_s and a_p and the correla-

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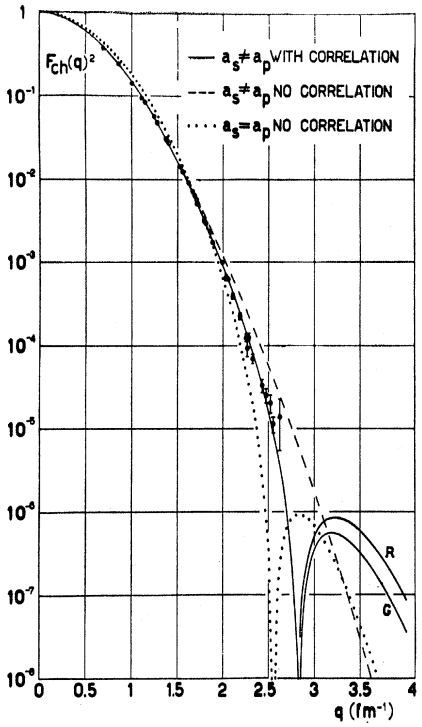

FIG. 1. The charge form factor of ^{6}Li nucleus. The wave function and correlation parameters corresponding to the various cases are the following:

full line G:
$a_s = 1.547 \pm 0.011$ fm, $a_p = 1.962 \pm 0.044$ fm, $b = 0.753 \pm 0.017$ fm;
full line R:
$a_s = 1.544 \pm 0.011$ fm, $a_p = 1.942 \pm 0.041$ fm, $b = 0.755 \pm 0.016$ fm;
dotted line:
$a_s = 1.635 \pm 0.009$ fm, $a_p = 2.008 \pm 0.059$ fm, $b = 0$.

The broken line was obtained in the same way as the full line but with $b = 0$. The full lines G and R have been obtained using the Gaussian proton and realistic form factors, respectively. The errors of the parameters were found varying by $\sqrt{e}$ the minimum value of the χ^2.

tion parameter b. Using the Gaussian proton form factor, we have first tested the IPSM with both equal and different oscillator lengths and found practically the same results as those in Ref. 1 where the "realistic" proton's form factor was used. In fact the low-momentum part ($q \leqslant 1.715$ fm^{-1}) can be fitted quite well ($\chi^2 = 15.5$ for 15 degrees of freedom) with $a_s = 1.635 \pm 0.009$ fm and $a_p = 2.008 \pm 0.059$ fm. Taking $a_s = a_p$ and $b = 0$, practically the same results (dotted line of Fig. 1) are obtained as those labelled BGT-7 MeV and HJ-5 MeV of Ref. 6, which indicates that the quadrupole scattering is negligible.

Then we have introduced correlations and have made a search to the best fit both with the proton Gaussian form factor (curve G of Fig. 1) and the realistic one (curve R of Fig. 1). In both cases we obtained practically the same results with the same set of best-fit parameters, namely $a_s = 1.547 \pm 0.011$ fm, $a_p = 1.962 \pm 0.044$ fm, and $b = 0.753 \pm 0.017$ fm, in the case G, and $a_s = 1.544 \pm 0.011$ fm, $a_p = 1.942 \pm 0.041$ fm, and $b = 0.755 \pm 0.016$ fm, in the case R. A sensible difference is seen only at very high momentum transfer where the experimental points are lacking. For these reasons we shall refer, in the following, only to the set of parameters found with the Gaussian form factor.

As can be seen from Fig. 1 the introduction of correlations corrects in a very satisfactory way the high-momentum part of the form factor, thus giving a quite good agreement in the whole range of momentum transfer. The value of the χ^2 is really not very impressive ($\chi^2 = 48$ for 32 degrees of freedom). This fact, however, does not worry us very much, since the main contribution to the χ^2 comes from the low-momentum experimental points (the errors of these points are actually less than the full points in the graph). The χ^2 given by the $q \leqslant 1.71$-fm^{-1} points only was, in fact, 30 for 15 degrees of freedom. On the other side, the low-momentum part of the form factor is certainly sensitive to the long-range behavior of the single-particle wave function in coordinate space, so that a small improvement in the asymptotic part of the single-particle wave function would probably result in a better fit at low-momentum transfer.

In Ref. 1 the experimental data were fitted very well ($\chi^2 = 26$ for 32 degrees of freedom) with a 3-parameter form factor of the type

$$F(q) = \exp(-a^2q^2) - c^2q^2 \exp(-b^2q^2), \qquad (3)$$

with $a^2 = 0.87$ fm^2, $b^2 = 1.7$ fm^2, and $c^2 = 0.205$ fm^2.

In Fig. 2 we have compared the charge densities given by the Fourier transform of (3) (dotted line), the shell model (dashed line), and the model with correlations (full line). We call these three densities ρ_F, ρ_{SM}, and ρ_C, respectively. The quantity $r^2\Delta\rho = r^2(\rho_C - \rho_{SM})$

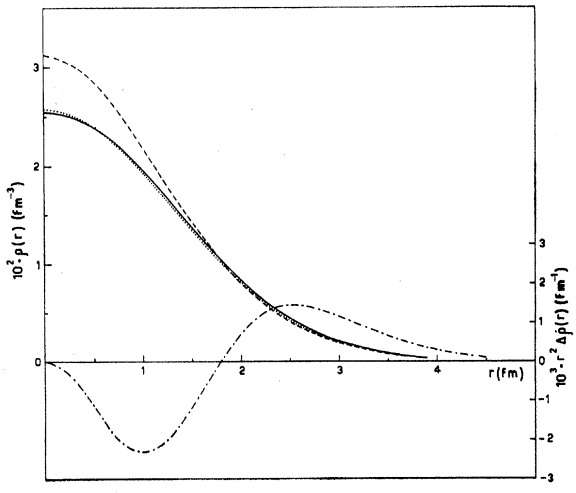

FIG. 2. The charge density of ^{6}Li nucleus. The full line represents the correlated density ρ_C, the dashed line represents the shell-model density ρ_{SM} (with different oscillator lengths a_s and a_p), and the dotted line represents the phenomenological density ρ_F used in Ref. 1. The quantity $r^2\Delta\rho = r^2(\rho_C - \rho_{SM})$ is also shown (dot-dashed line). The scale for this curve is given on the right-hand side.

TABLE I. Single-particle wave function and correlation parameters for ^{6}Li. a_s and a_p are the oscillator lengths, b is the parameter of the correlation function $f(r)=\exp(-r^2/b^2)$, and r_h is the "healing distance" determined as the average internucleon separation at which the ratio between the squares of the correlated and uncorrelated (normalized) wave functions equals approximately 1. $\langle r_s^2\rangle^{1/2}$, $\langle r_p^2\rangle^{1/2}$, and $\langle r_N^2\rangle^{1/2}$ are the s-shell, p-shell, and nuclear radii, respectively. The experimental rms radius was determined in Ref. 1 in a model-independent way.

	a_s	a_p	b	r_h	$\langle r_s^2\rangle^{1/2}$	$\langle r_p^2\rangle^{1/2}$	$\langle r_N^2\rangle^{1/2}$	$\langle r_N^2\rangle_{\text{expt}}^{1/2}$
Shell model ($a_s\neq a_p$, $b=0$)	1.635±0.009	2.008±0.059	0	0	1.994	3.115	2.427	2.5±0.1
Correlations ($a_s\neq a_p$, $b\neq 0$)	1.547±0.011	1.962±0.044	0.753±0.017	1.85	1.961	3.088	2.397	

is also shown (dot-dashed line). First of all it is worth noting that ρ_C and ρ_F do not differ appreciably. They can be distinguished, however, by means of a χ^2 test of the data, which indicates the very high degree of accuracy achieved by the elastic electron scattering experiments.

As can be seen from Fig. 2 the main effect of correlations is that of depressing the central part of the density, as a consequence of the strong repulsion between particles at short relative distances. In the last years, with the increasing accuracy of the electron scattering experiments, a systematic lowering of the central density was observed both for light[13] and heavy nuclei.[14] It was suggested[14] that this lowering could be explained by a nonlocal shell-model potential. We want to stress in this connection that the lowering of the central density is the main and systematic effect of the inclusion of correlations in the shell-model wave function.

The dot-dashed line in Fig. 2 shows that the effects of correlations can be simulated by an oscillating function which must be added to the uncorrelated density. This was already pointed out in Ref. 15.

In Table I we list the values of the oscillator lengths, the correlation parameter, the "healing distance" as defined in Sec. 2, and the rms radii. The value of the experimental rms radius written in the last column was found in Ref. 1 in a model-independent way. All the values in Table I are in fermis.

In closing it should be noted that:

(1) As it was shown in Ref. 1, the form factor of ^{6}Li can be reproduced only by means of a 3-parameter charge density. From our analysis it seems that while two of these parameters characterize the harmonic-oscillator potentials, the third additional parameter could be related to correlations of nucleons at small relative distances.

(2) Measurements at higher momentum transfer would give valuable information on the structure of the wave function. In fact, we predict a diffraction minimum at $q\approx 2.8$ fm^{-1}, while the form factor used in Ref. 1 does not give any minumum in a wide range of momentum transfer.

(3) Correlations of the type used in our analysis (i.e., modification of the wave function describing the relative motion of two nucleons) have been used both to calculate the ground-state energy of light nuclei[16] and to treat the two-nucleons emission following π-meson absorption[17] since this reaction seems to occur mainly on a correlated nucleon pair. In both these types of calculations, the oscillator lengths were taken from electron scattering and the correlation parameters were adjusted to fit either the experimental data on the cross sections or the value of the ground-state energy. On the other hand, if the relative part of the two-nucleons wave function is changed, the single-particle density

$$\rho(1)=\int \psi^*(1\cdots A)\psi(1\cdots A)d2\cdots dA$$

measured in electron scattering is also automatically changed so that the complete set of parameters characterizing the wave function (oscillator lengths and correlation parameters) can be fixed from electron scattering.

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