Λ-Σ Conversion and Λnp Isospin-Zero Bound States

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The Faddeev equations with nonlocal separable Yamaguchi potentials have been used to calculate the binding energy B_Λ of the Λ hyperon in the spin-doublet and spin-quartet Λnp isospin-zero systems. The calculations have been performed with explicit consideration of Λ-Σ virtual conversion in neither, one, and both of the hyperon-nucleon (YN) spin channels. Relative to the case when it is neglected, Λ-Σ conversion changes the ΛN t matrix and introduces a ΛNN force. Results have been obtained when only the change in the ΛN t matrix was considered, as well as when both this change and the ΛNN force were used. The effect of the change in the ΛN t matrix is always to reduce B_Λ. The effect of the ΛNN force depends upon the YN spin channel in which it is used as well as the total spin of the Λnp system.

I. INTRODUCTION

In a previous paper[1] we calculated the effect of virtual Λ-Σ conversion on the binding energy B_Λ of the Λ hyperon in ${}_\Lambda H^3$. The hypertriton was taken to be an isospin-zero, spin-$\frac{1}{2}$ ($J=\frac{1}{2}$) system. We calculated B_Λ by finding the energy at which the Fredholm determinant of a set of coupled integral equations vanished and then subtracting the deuteron binding energy. The integral equations were derived from the Faddeev[2] equations for Λ-d elastic scattering using nonlocal separable (NLS) potentials. The introduction of Λ-Σ conversion into a given hyperon-nucleon (YN) spin channel was facilitated by the use of a 2×2 YN potential matrix. This led to both a modification of the amplitude for elastic ΛN scattering and the introduction of a ΛNN force into the three-body problem. No attempt was made in I to calculate the effect of each of these changes individually on B_Λ.

In this work we calculate the change in B_Λ due to explicit use of Λ-Σ conversion both with and without the ΛNN force. In the latter instance the ΛN scattering amplitude that took virtual Λ-Σ conversion into account was used, but the other YN amplitudes ($\Lambda N\leftrightarrow\Sigma N$ and $\Sigma N\leftrightarrow\Sigma N$) were set equal to zero. In addition, we have extended our previous calculation to the isospin-zero Λnp $J=\frac{3}{2}$ system.

There were several motivations for the present work. First, we hoped for a further understanding of the results of I. Second, we wanted to obtain an estimate of the magnitude of the contribution of the ΛNN force. If this force can be neglected in the problem, the number of coupled integral equations needed is the same as it is when Λ-Σ conversion is not explicitly taken into account. This would substantially reduce the amount of computer memory needed for the calculation and would open the way for a more realistic representation of the ΛN force; e.g., a repulsive core might be included. Finally, we wished to determine the effect of Λ-Σ conversion on a possible Λd $J=\frac{3}{2}$ bound state. The existence of such a state or a Λd resonance would make itself felt in measurements[3] of the ${}_\Lambda H^3$ lifetime. The recent work of Herndon and Tang[4] yields values for the ΛN low-energy scattering parameters that indicate the (predicted) nonexistence of such a state is not as clear cut as was previously thought.

II. CALCULATION

Details of the two- and three-body calculations are given in I. The isospin formalism was used with the Λ hyperon taken as an isospin singlet and the Σ as an isospin triplet. Only the charge symmetric (CS) part of the ΛN interaction entered into the calculation, since the third component of the total isospin of the three-particle system is zero.

An S-wave, central, spin-dependent potential was used to represent each two-particle interaction. Each of these was taken to be an NLS potential of the Yamaguchi form[5]; i.e., in a given spin channel the kernel of the np potential in a relative-momentum space representation has the form

$$V_N(p,p')=\lambda_N v_N(p)v_N(p')\,, \qquad (1)$$

with

$$v_N(p)=(p^2+\beta_N^2)^{-1}. \qquad (2)$$

The AB element ($A,B=\Lambda$ or Σ) of the 2×2 YN potential matrix is

$$V_{AB}(p,p')=\lambda_{AB}v_A(p)v_B(p')\,, \qquad (3)$$

with $v_A(p)=v_N(p)$ with $\beta_N\rightarrow\beta_A$. The symbol V_{AB} denotes the potential that describes the process $A+N\leftrightarrow B+N$, again with A, $B=\Lambda$ or Σ. We quickly drop the notation λ_{AB} in favor of

$$\lambda_\Lambda\equiv\lambda_{\Lambda\Lambda}\,,\quad \lambda_\Sigma\equiv\lambda_{\Sigma\Sigma}\,,\quad \lambda_X=\lambda_{\Sigma\Lambda}=\lambda_{\Lambda\Sigma}\,. \qquad (4)$$

Thus, $\lambda_X\neq 0$ ($\lambda_X=0$) denotes the case for which Λ-Σ conversion is (is not) explicitly taken into account.

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[2] L. D. Faddeev, *Mathematical Aspects of the Three-Body Problem in Quantum Scattering Theory* (Daniel Davey & Co., Inc., New York, 1965).

[3] G. Keyes, M. Derrick, T. Fields, L. G. Hyman, J. B. Fetkovich, J. McKenzie, B. Riley, and I-T. Wang, Phys. Rev. Letters **20**, 819 (1968).

[4] R. C. Herndon and Y. C. Tang, Phys. Rev. **153**, 1091 (1967); **159**, 853 (1967); **165**, 1093 (1968).

[5] Y. Yamaguchi, Phys. Rev. **95**, 1628 (1954).

TABLE I. YN potential parameters.

a(F)	r_0(F)	β^{-1}(F)	$10^2\eta$	λ [MeV²/(20)³]	λ_X [MeV²/(20)³]
−2.46	3.87	0.8750	0.00	−0.6659	0.0000
		0.8765	0.17	−0.6526	0.2178
		0.8800	0.57	−0.6215	0.3971
		0.8850	1.14	−0.5780	0.5583
		0.9000	2.85	−0.4527	0.8663
		0.9150	4.56	−0.3349	1.0754
−2.07	4.50	0.9359	0.00	−0.4892	0.0000
		0.9450	0.97	−0.4274	0.4522
		0.9700	3.64	−0.2700	0.8456
		1.0201	9.00	0.0000	1.2450
		1.0850	15.92	+0.2758	1.5268

The $NN\,{}^3S_1$ potential parameters were fixed by fitting the np triplet scattering length ($a_t=5.37$ F) and the deuteron binding energy ($\epsilon=2.225$ MeV). The ${}^1S_0\,np$ potential enters into the problem only when the ΛNN force is explicitly included (i.e., it occurs only in the ΣNN channel) and even then only for the three-body $J=\frac{1}{2}$ state. It was shown in I that the effect of np singlet forces was small and therefore we have set that interaction equal to zero in the following calculations.

The YN potential contains five parameters in each spin channel: λ_Λ, λ_Σ, λ_X, β_Λ, and β_Σ. As in I the number of independent parameters was reduced to three by setting $\beta_\Sigma=\beta_\Lambda\equiv\beta$ and using an "equal-strength" model, $\lambda_\Sigma=\lambda_\Lambda\equiv\lambda$. No attempt was made to relate the β's to range parameters arising from more fundamental theories (e.g., the one-boson-exchange model, etc.). Since purely attractive potentials were used in our calculations, such an identification would be meaningless. Furthermore, our results in I showed that the shifts in B_Λ were insensitive to the choice of models used to relate λ_Λ and λ_Σ, and therefore the equal-strength model was assumed to be adequate for the present calculations.

For fixed values of β, the parameters λ and λ_X were fit to the low-energy data for Λp elastic scattering in the singlet and triplet spin channels given by Alexander *et al.*[6] (AEA):

$$a_s=-2.46\text{ F},\quad r_s=3.87\text{ F},\qquad (5)$$
$$a_t=-2.07\text{ F},\quad r_t=4.50\text{ F}.$$

TABLE II. Effect of explicit use of $\Lambda\Sigma$ conversion in the spin-zero YN channel on B_Λ for $J=\frac{1}{2}$.

$10^2\eta$	B_Λ (MeV) Without ΛNN	With ΛNN
0.00	0.21	
0.17	0.21	0.19
0.57	0.19	0.17
1.14	0.18	0.14
2.85	0.15	0.06
4.56	0.08	0.02

[6] G. Alexander, O. Benary, U. Karshon, A. Shapiro, G. Yekutieli, R. Englemann, H. Filthuth, A. Fridman, and B. Schilby, Phys. Letters **19**, 715 (1966).

There are more recent values available from Λp scattering experiments[7] that differ from these, and there are values available from analysis of the binding energies of the light hypernuclei[4] that differ from these. Furthermore, if there is a charge-symmetry-breaking (CSB)ΛN force, the AEA values are not the CS parameters that we should use in our three-body calculations. However, it was shown in I that: (a) the percentage change in B_Λ due to explicit use of Λ-Σ conversion was not sensitive to the values of these ΛN parameters provided they were close enough to the correct values, and (b) the values quoted in Eqs. (5) *are* close enough to the correct values because without the explicit use of Λ-Σ conversion these values give a B_Λ of 0.21 MeV, in excellent agreement with the experimental value of 0.20±0.12 MeV.[8] (*Note added in proof.* More recent measurements give a much smaller value for B_Λ.)

The YN potential parameters used in our calculations are given in Table I. The parameter η is defined by

$$\eta=(\beta_0/\beta)-1\,,\qquad (6)$$

where β_0 is the value of β when $\lambda_X=0$, is a convenient measure of the coupling between the ΛN and ΣN channels. It may be seen from Table I that the effect of the closed ΣN channel is to increase the range of the ΛN potential and decrease the magnitude of the strength parameter λ. That this result is to be expected can be seen in the following way. In a general coupled-channel scattering problem, the presence of a strong coupling between channels or an attractive interaction in one channel will increase the phase shift in the other channel and can lead to a resonant state in this channel below the threshold of the first mentioned channel.[9] This occurs when the *potential parameters* in each channel are held constant and the coupling between the two channels is increased. In our work, we have effectively held the *phase shift* fixed and varied the potential

TABLE III. Effect of explicit use of $\Lambda\Sigma$ conversion in the spin-one YN channel on B_Λ for $J=\frac{1}{2}$.

$10^2\eta$	B_Λ (MeV) Without ΛNN	With ΛNN
0.00	0.21	
0.97	0.21	0.22
3.64	0.17	0.24
9.00	0.13	0.26
15.92	0.09	0.32

[7] G. Alexander and U. Karshon, in *High-Energy Physics and Nuclear Structure*, edited by G. Alexander (North-Holland Publishing Co., Amsterdam, 1967).

[8] W. Gajewski, C. Mayeur, J. Sacton, P. Vilain, G. Wilquet, D. Harmsen, R. Levi-Setti, M. Raymund, J. Zakrzewski, D. Stanley, D. H. Davis, E. R. Fletcher, J. E. Allen, V. A. Bull, A. P. Conway, and P. V. March, Nucl. Phys. **B1**, 105 (1967).

[9] Marc H. Ross and Gordon L. Shaw, Ann. Phys. (N. Y.) **9**, 391 (1960).

parameters in each channel. In this case the potentials in the open ΛN channel will necessarily become less attractive as the coupling to the ΣN channel is increased in order to give the same effective range and scattering length.

III. RESULTS

The results of our calculations of B_Λ for the $J=\frac{1}{2}$ Λnp system are given in Tables II, III, and IV.[10] The parameter η is given in the left-hand column of each of these tables. In the right-hand columns are listed the corresponding values of B_Λ for the case when $\lambda_X \neq 0$ and the effect of the ΛNN force was included. Each middle column lists the values obtained for B_Λ when $\lambda_X \neq 0$, but the ΛNN force was not included. This latter approximation was obtained by writing the YN t matrix as

$$\begin{pmatrix} t_\Lambda & t_X \\ t_X & t_\Sigma \end{pmatrix} \rightarrow \begin{pmatrix} t_\Lambda & 0 \\ 0 & 0 \end{pmatrix}.$$

In this case t_Λ contains terms depending upon λ_X and λ which correspond to the given value of η.

TABLE IV. Effect of explicit use of $\Lambda\Sigma$ conversion in both YN spin channels on B_Λ for $J=\frac{1}{2}$. In the $S_{YN}=0$ channel $\eta=0.0114$.

$10^2\eta$	B_Λ (MeV) Without ΛNN	With ΛNN
0.00	0.18	0.14
0.97	0.17	0.08
3.64	0.15	0.05
9.00	0.09	0.03
15.92	0.07	0.02

In Table II we have used $\lambda_X \neq 0$ only in the $S=0$ YN channel. In Table III, $\lambda_X \neq 0$ only in the $S=1$ YN channel. The first value listed for B_Λ in each of these tables is just that for no explicit Λ-Σ conversion. Finally, in Table IV, a fixed value for $\eta \neq 0$ was chosen in the $S=0$ YN channel and η was again varied in the $S=1$ YN channel. It should be noted that the middle column in Table IV corresponds to the ΛNN force being omitted in *both* YN spin channels, and the right-hand column results from the inclusion of the ΛNN force in *both* of these channels. Some of the results in the right-hand columns of Tables II–IV were published previously in I.

Table V is similar to Tables II–IV but the results here correspond to the Λnp $J=\frac{3}{2}$ system where only the $S=1$ YN interaction enters into the calculations.

In order to make the results listed in Tables II–IV more meaningful, we have listed in Table V the values obtained for B_Λ when $\lambda_X=0$ and the scattering lengths are varied away from the values given in Eq. (5). In these calculations the effective ranges were kept fixed at the values given in Eq. (5).

[10] All numerical work was performed on the Honeywell H-800 Computer at the University of Southern California Computer Science Laboratory.

TABLE V. Effect of explicit use of $\Lambda\Sigma$ conversion on B_Λ for $J=\frac{3}{2}$.

$10^2\eta$	B_Λ (MeV) Without ΛNN	With ΛNN
0.00	0.06	
0.97	0.03	<0
3.64	0.01	<0
9.00	<0	<0

It is evident from the results in Tables II–IV that the contributions to B_Λ from the ΛNN force is complicated. When used in the $S=1$ YN channel this force increases B_Λ, when used in the $S=0$ channel it decreases B_Λ, and when used in both channels it decreases B_Λ even more than when used in the $S=0$ channel alone. As pointed out in I, this behavior is possible because the total spin of neither YN pair is conserved. From Tables III and V it is also clear that the over-all effect of the ΛNN force in a given spin channel depends on how the three particles are coupled together. In the Λnp $J=\frac{1}{2}$ state, $S=1$ Λ-Σ conversion increases B_Λ while in the Λnp $J=\frac{3}{2}$ state it decreases B_Λ. In any case there is no striking correlation between the change in B_Λ without using the ΛNN force and the change in B_Λ when this force is included. It does appear in most cases, however, that when the former effect is large so is the latter.

On the basis of the one- and two-pion exchange model of de Swart and Iddings,[11] Λ-Σ conversion effects would be more important in the $S=1$ YN channel than in the $S=0$ channel. This is a result of tensor forces. The results in Tables IV and V would indicate that a small coupling to the ΣN channel would be enough to destroy a Λnp $J=\frac{3}{2}$ bound state without appreciably affecting the binding energy in the $J=\frac{1}{2}$ state. (See Table VI).

One important factor missing from the present calculations is a repulsive core in each two-body potential. Repulsive cores are being used in a calculation that one of us (AJT) now has in progress.

A second major missing ingredient is a link between the YN potential parameters used here and the more fundamental YN interaction coupling constants and

TABLE VI. Variation in B_Λ with variation in the ΛN scattering length.

Λnp spin state	a(F) for $S_{\Lambda N}=0$	a(F) for $S_{\Lambda N}=1$	B_Λ (MeV)
$\frac{1}{2}$	−2.46	−1.45	0.14
	−2.46	−1.66	0.17
	−2.46	−2.07	0.21
	−2.46	−2.69	0.26
	−1.72	−2.07	0.03
	−1.97	−2.07	0.09
	−2.21	−2.07	0.15
$\frac{3}{2}$	···	−2.07	0.06
	···	−1.86	0.02
	···	−1.66	<0

[11] J. J. de Swart and C. K. Iddings, Phys. Rev. **128**, 2810 (1962).

ranges. One way to build such a link would be to base the NLS potential on a one-boson-exchange model and use the type of NLS potential shape suggested by Mitra[12] in which the range parameter may be related directly to the mass of the exchanged particle.

Finally we note that the kind of effect discussed here is present in double hypernuclei[13] (e.g., $_{\Lambda\Lambda}He^6$, $_{\Lambda\Lambda}Be^{10}$) not only in the form of $\Lambda N \leftrightarrow \Sigma N$ but also via $\Lambda\Lambda \leftrightarrow \Xi N$.[14] In fact, because the threshold for the latter process is only 25 MeV above the $\Lambda\Lambda$ elastic scattering threshold, it may play a much more important role than Λ-Σ conversion.

[12] A. N. Mitra, Phys. Rev. **123**, 1892 (1961).
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[14] J. N. Pappademos, Phys. Rev. **163**, 1788 (1967); **134**, B1132 (1964); N. Panchapakesan, *ibid.* **143**, 1166 (1966).

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Short-Range Dynamical Correlations in ^{6}Li from Electron Elastic Scattering*

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The charge form factor of the ^{6}Li nucleus was calculated in Born approximation, introducing in the ground-state density Jastrow-type correlations. Good agreement with the experimental data was obtained with different oscillator lengths for s and p nucleons. A diffraction minimum was predicted at a value of the momentum transfer $q \approx 2.8$ fm^{-1}.

I. INTRODUCTION

It is well known that among the lightest nuclei the ^{6}Li nucleus behaves anomalously as regards electron scattering, in the sense that the usual independent-particle shell model (IPSM) cannot fit the elastic scattering data.

Many other models have been used but they have not improved the situation substantially. After the recent measurements of Suelzle *et al.*[1] covering a range of momentum transfer $0.7 \lesssim q \lesssim 2.62$ fm^{-1}, we can summarize the situation concerning agreement between experimental data and theoretical calculations as follows:

(1) The IPSM with a common well for s and p nucleons cannot fit the data, either in the low-momentum or high-momentum part.

(2) By taking different oscillator lengths as suggested by Elton,[2] the low-momentum part ($q \lesssim 1.7$ fm^{-1}) can be fitted[1] ($\chi^2 = 19$ for 15 degrees of freedom) with $a_s = 1.632$ fm and $a_p = 1.980$ fm $[a = \hbar c/(M\hbar w)^{1/2}]$.

(3) The inclusion of configuration mixing of high-lying components at $2\hbar w$ and $4\hbar w$ excitations does not improve the fit.[3]

(4) Among cluster models, only the ^{4}He$+d$ one, with proper antisymmetrization between like particles (Hubbard model quoted in Ref. 1) can give a good fit to the data, though limited to the values of $q \lesssim 2$ fm^{-1}.

(5) The projected Hartree-Fock (PHF) wave functions of Bouten *et al.*[4] obtained by a variational calculation of the ground-state energy, based on a semi-realistic soft-core potential (Volkov potential[5]), give a considerable improvement in the low-momentum part with respect to the simple IPSM. Though these wave functions are essentially L-S coupling wave functions with the same oscillator lengths for the inner and outer particles, they contain considerable mixing of excited harmonic-oscillator states so that, as a consequence of this higher configuration expansion and deformation, the outer particles move, in fact, in a more extended well than the innermost ones and this probably simulates the difference of the oscillator lengths found in the IPSM analysis.[2]

In all these calculations the quadrupole scattering was not taken into account because of the small value of the quadrupole moment of the ^{6}Li nucleus ($Q = -0.08$ fm^2). In Ref. 6, the quadrupole scattering was properly taken into account and various intermediate coupling wave functions were used. The results of Ref. 6 showed that the intermediate-coupling wave functions (if they gave the proper value of the quadrupole moment)

* This work is a part of a research program within the framework of the activity of the "Sottosezione Sanità" of the Istituto Nazionale di Fisica Nucleare.
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