Microscopic Theory of Effective Operators for Electromagnetic Interactions in Nuclei*

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(Received 24 June 1968)

Effective operators for valence (open-shell) nucleons are constructed for electromagnetic interactions in nuclei. Such operators are calculated in terms of excitations and deexcitations of particle-hole pairs of the core nucleons. The nuclear force involved here is a realistic nucleon-nucleon potential (Yale-Shakin). The theory is applied to the tin isotopes which are described in terms of five valence neutron subshells and of all the core and empty subshells of both the neutrons and protons between the magic numbers 8 and 126. The effective quadrupole charge matrix, with all its elements of about the same order of magnitude, gives the first partial support to the concept of a constant neutron effective charge. Calculations of observables are performed with the states of the even tin isotopes described in terms of the two- and four-quasiparticle Tamm-Dancoff theories involving explicitly only the valence neutrons. Results are presented for the $B(E2, 2_1^+ \rightarrow 0_1^+)$, the quadrupole moment $Q(2_1^+)$, the gyromagnetic factor $g_{5_1^-}$, and the inelastic electron scattering form factors for the 2_1^+ and 3_1^- states of Sn^{116}. Except for the last (3_1^-) form factor, good semiquantitative agreement with all the corresponding experimental data is obtained. The reported calculations involve no *ad hoc* adjustable parameters.

I. INTRODUCTION

THE enormous complexity and the prohibitively large energy matrices which arise in the many-body problem of a finite nucleus force us, in the shell-model description of the nuclear spectra and of the nuclear ground state, to eliminate from our explicit treatment the so-called core. The core, which constitutes the main bulk of the nucleus, is thought to be the inert part of the nucleus: usually it is the ensemble of all the protons and neutrons of all the closed subshells (possibly of all the closed major shells) in the ground state. These subshells lie deep inside the Fermi sea and are less important for the properties of all the low-lying states than the open (valence) shell nucleons. This picture is, however, a fair approximation only when one works with effective phenomenological nuclear forces which are supposed to contain implicitly all the effects of the excited configurations of the core nucleons. In fact, it has been shown[1–6] that the effects of such configurations, i.e., the core polarization effects, are extremely important in describing the low-lying states in terms of realistic nucleon-nucleon potentials. These have to be drastically renormalized if they are to be used for mixing configurations of the valence nucleons only.

Similarly, in a phenomenological description of electromagnetic interactions of nuclei, the nucleonic charges of the valence protons and neutrons are supposed to be renormalized for all the contributions of the core nucleons. This is the concept of the effective charge which is, in a phenomenological theory, an adjustable parameter. This leads to an uncomfortable freedom, since the effective charge is different for the neutrons and the protons and for various multipoles. In fact, in addition to the philosophical difficulty of mixing purely phenomenological and microscopic concepts, one also usually has too many adjustable parameters in the theory. Since the electromagnetic interactions with nucleons are well known, we have even less excuse for a phenomenological approach to these interactions than for such an approach to the nucleon-nucleon interactions between the valence protons and neutrons. It is clear that a fully microscopic theory of nuclear properties and of nuclear spectra in terms of realistic nucleon-nucleon potentials should be free of the concept of a phenomenological adjustable parameter.

* Supported in part by Istituto Nazionale di Fisica Nucleare.
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Several authors[7–9] have attempted microscopic derivations and estimates of the effective charge using the picture of virtual excitations of core nucleons. For example, a neutron-effective charge could arise from second-order processes in which a virtual or a real photon is absorbed by a core proton, creating a particle-hole pair which is subsequently annihilated (de-excited) in a collision with a valence neutron. Unfortunately, the description of Refs. 7–9 has been based only on phenomenological nuclear forces and involved many crude, schematic approximations, thus giving only qualitative or, at best, semiquantitative estimates.

It is clear that only realistic nucleon-nucleon potentials, avoiding the introduction of new adjustable parameters, are to be used in these calculations when a quantitative comparison with experimental data is wanted.[10]

In this paper we study the problem in detail and in a quantitative way in relation to a realistic nucleon-nucleon potential. Our numerical analysis is performed on the example of the even tin isotopes, which are representative of the so-called vibrational nuclei. We derive formulas for the effective electric (or magnetic) multipole operator $\hat{O}_{\lambda\mu}{}^{(\mathrm{eff})}$, which may then be treated with all the retardation effects (no approximation for the radial integrals involved) or in the long-wavelength approximation.

In particular, we examine the question: To what extent can be effective multipole operator $\hat{O}_{\lambda\mu}{}^{(\mathrm{eff})}$ be replaced by $e_{\mathrm{eff}}{}^{(\lambda)}\hat{O}_{\lambda\mu}$ when $e_{(\mathrm{eff})}{}^{(\lambda)}$ is a unique over-all constant effective charge (independent of the transition configuration)?

We apply our computed $\hat{O}_{\lambda\mu}{}^{(\mathrm{eff})}$ to the study of some of the reduced $E2$ transition probabilties $B(E2,I_i\rightarrow I_f)$; the quadrupole moment of the first excited $2_1{}^+$ state, $Q(2_1{}^+)$; the gyromagnetic factor $g_{5_1}{}^-$ of the $5_1{}^-$ state; and the inelastic electron scattering form factors corresponding to the final states $2_1{}^+$ and $3_1{}^-$. We compare our results with the recent experimental data, in particular for the nucleus Sn^{116}.

II. EFFECTIVE SINGLE-PARTICLE OPERATORS

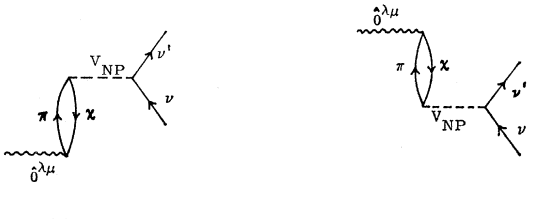
In the case of a realistic nucleon-nucleon potential containing a strongly repulsive part at small separation distances (hard core or at least a "soft" core for a local, static potential) the effective nuclear interaction Hamiltonian is defined in the sense of the Brueckner theory. This means that the two-body potential $V(i,j)$ is replaced by the appropriate Brueckner reaction matrix $G(i,j)$ in the original expression for the potential energy operator. Any perturbation-theory calculation is based on such an effective Hamiltonian [an expansion in terms of $G(i,j)$ (e.g., Refs. 1–3)]. Only if one uses a strongly nonlocal or velocity-dependent potential, e.g., Tabakin's[11] potential, can one use the standard perturbation theory in terms of $V(i,j)$ itself.

To be specific, let us now consider the problem of a nucleus with a doubly-magic core possessing neutrons only in the open (valence) shells, like a tin isotope with the 50-50 core. We can then calculate the effective operator of a multipole $\hat{O}_{\lambda\mu}$ for an extra-core single neutron. Clearly, here the neutron-proton two-body potential V_{NP} (or K_{NP}) is responsible for the transmission of the electromagnetic interaction from the core protons to the neutrons. To lowest order, this effective interaction can be represented by the two diagrams of Fig. 1.

In fact, since there is no first-order contribution (except for $\hat{O}_{\lambda\mu}$ being a magnetic interaction with the neutron spin), we obtain in this case the following expression for the single-neutron matrix element:

$$\langle\nu'|\hat{O}_{\lambda\mu}{}^{(\mathrm{eff})}|\nu\rangle\cong-\sum_{\pi\chi}[\langle\nu'\chi|V_{NP}|\nu\pi\rangle(q/e_-)\langle\pi|\hat{O}_{\lambda\mu}|\chi\rangle + \langle\chi|\hat{O}_{\lambda\mu}|\pi\rangle(q/e_+)\langle\nu'\pi|V_{NP}|\nu\chi\rangle], \quad (1)$$

where the first term in the sum on the right-hand side of Eq. (1) corresponds to diagram (a) of Fig. 1 and the second to diagram (b). Here $q=+1$ if π is a (proton) particle and χ a (proton) hole, and $q=0$ otherwise; and the energy denominators are $e_\pm=E_\pi{}^0-E_\chi{}^0\pm(E_{\nu'}{}^0-E_\nu{}^0)$. In the following we shall limit ourselves to spherical symmetry with degeneracy in the magnetic substates. Consequently, we utilize the Wigner-Eckart theorem, and only reduced matrix elements of the operators are involved. Our notation uses Latin subscripts for the single-particle states, except for the j-projection quantum number m. [a corresponds to $\alpha=(a,m_a)$, etc.] It is convenient to introduce the particle-hole coupled reduced matrix elements $F_{NP}(abcd,J')$ of the neutron-



(a) (b)

FIG. 1. Lowest-order diagrams for processes contributing to matrix elements $\langle n'\|\hat{O}_{\mathrm{eff}}{}^\lambda\|n\rangle$ in the case of a nucleus with neutrons only in the valence shells.

[7] A. De-Shalit, Phys. Rev. **113**, 547 (1959); S. Fallieros and R. A. Ferrell, *ibid.* **116**, 660 (1959).

[8] B. Mottelson, in *International School of Physics "Enrico Fermi," Course XV*, edited by G. Racah (Academic Press Inc., New York, 1962), p. 44.

[9] Historically, the earliest considerations in terms of configuration mixing are R. D. Amado and R. J. Blin-Stoyle, Proc. Phys. Soc. (London) **A70**, 532 (1957); A. Arima and H. Hović, Progr. Theoret. Phys. (Kyoto) **11**, 509 (1954); **12**, 622 (1954); R. Blin-Stoyle, Proc. Phys. Soc. (London) **A66**, 729 (1953); R. J. Blin-Stoyle and M. A. Perks, *ibid.* **A67**, 885 (1954).

[10] M. Gmitro, A. Rimini, J. Sawicki, and T. Weber, Phys. Rev. Letters **20**, 1185 (1968); because of a misprint, the definitions of the symbols e_1 and e_2 of Eq. (1) of Ref. 10 are interchanged. However, all the numerical results are correct.

[11] F. Tabakin, Ann. Phys. (N. Y.) **30**, 51 (1964).

proton potential V_{NP} as defined by

$$F_{NP}(abcd,J')=-\tfrac{1}{2}\sum_m (j_a j_b; m_\alpha - m_\beta | J'M') \times (j_c j_d; m_\gamma - m_\delta | J'M') \times (-)^{j_b-m_\beta+j_d-m_\delta}\langle \alpha\delta | V_{NP} | \beta\gamma\rangle. \quad (2)$$

The Wigner-Eckart theorem for the element $(\sigma'|\hat{O}_{\lambda\mu}|\sigma)$ can be written as

$$\langle \sigma'|\hat{O}_{\lambda\mu}|\sigma\rangle \equiv \langle s'\|\hat{O}_\lambda\|s\rangle \hat{\lambda}^{-1}(-)^{j_s-m_\sigma} \times (j_{s'} j_s; m_{\sigma'} - m_\sigma | \lambda\mu), \quad (3)$$
$$\hat{\lambda}\equiv(2\lambda+1)^{1/2}.$$

It follows then immediately from Eq. (1) that for an electric 2^λ pole

$$\langle n'\|\hat{O}_\lambda{}^{(\mathrm{eff})}\|n\rangle = 2\sum_{\mathrm{ph}} [F_{NP}(n'nph,\lambda)e_-^{-1}\langle p\|\hat{O}_\lambda\|h\rangle + \langle h\|\hat{O}_\lambda\|p\rangle e_+^{-1} F_{NP}(n'nhp,\lambda)]. \quad (4)$$

In Ref. 5 reduced F elements in the isotopic spin formalism are employed. Our $F_{NP}(n'npp',J)$ in Eq. (4) are connected with these F elements through the formula

$$F_{NP}(abcd,\lambda)\equiv\tfrac{1}{2}[F(abcd,\lambda T=0)-F(abcd,\lambda T=1)], \quad (5)$$

where T is the isospin of a pair (ab and cd) and the corresponding matrix element $\langle\alpha\delta|V(1-P_{12})|\beta\gamma\rangle$ is antisymmetrized in the two interacting nucleons.

One can also consider higher-order terms as corrections to $\langle\nu'|\hat{O}_{\lambda\mu}{}^{(\mathrm{eff})}|\nu\rangle$ of Eq. (1). For example, one can easily include iterations of the bubble diagram of Fig. 1, which is the simplest correction (cf. Fig. 2). One can easily calculate the correspondingly corrected $\langle n'\|\hat{O}_\lambda{}^{(\mathrm{eff})}+\Delta\hat{O}_\lambda{}^{(\mathrm{eff})}\|n\rangle$, even including all the appropriate exchange diagrams (not indicated in Fig. 2). In fact, one then obtains Eq. (4) with $F(abcd,\lambda)$ merely replaced by appropriately renormalized $\mathfrak{F}$ elements of Ref. 5. However the corresponding corrections are, in general, quite small. In Sec. IV we shall discuss these corrections in connection with our numerical examples.

Here, however, we shall consider the case when we have valence protons in addition to valence neutrons in a given nucleus. The following formulas will also be applicable to the magnetic multipoles of interaction via the core neutron spin. We calculate the effective $\hat{O}_\lambda$ operator for a valence proton. This contains the direct interaction besides diagrams similar to those of Fig. 1

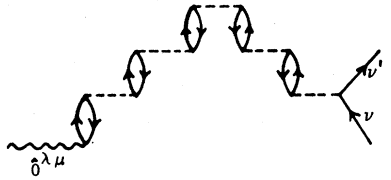


FIG. 2. Simplest diagram contributing to the higher-order corrections to matrix elements $\langle n'\|\hat{O}_\lambda{}^{\mathrm{eff}}\|n\rangle$ of Eq. (1).

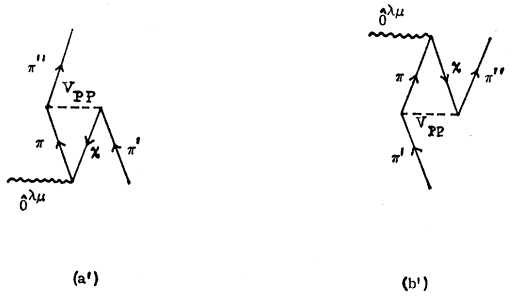


FIG. 3. Diagrams contributing to the exchange terms of F_{PP} in Eq. (6).

with V_{NP} replaced by $V_{PP}(1-P_{12})$, where P_{12} is the exchange operator of the two protons involved. For the reduced matrix element we find

$$\langle p''\|\hat{O}_\lambda{}^{(\mathrm{eff})}\|p'\rangle = \langle p''\|\hat{O}_\lambda\|p'\rangle + 2\sum_{\mathrm{ph}}^{(\mathrm{protons})} [F_{PP}(p''p'ph,\lambda)(q_\pi/e_-)\langle p\|\hat{O}_\lambda\|h\rangle + \langle h\|\hat{O}_\lambda\|p\rangle(q_\pi/e_+)F_{PP}(p''p'hp,\lambda)] + 2\sum_{\mathrm{ph}}^{(\mathrm{neutrons})} [F_{NP}(p''p'ph,\lambda)(q_\nu/e_-)\langle p\|\hat{O}_\lambda\|h\rangle + \langle h\|\hat{O}_\lambda\|p\rangle(q_\nu/e_+)F_{NP}(p''p'hp,\lambda)], \quad (6)$$

where q_π and q_ν are appropriate projectors of the proton and neutron particle-hole (ph) pairs, respectively. The last sum in Eq. (6), which runs over all the neutron particle-hole pairs, is present only if $\hat{O}_\lambda$ is a magnetic multipole operator of interaction with the neutron spin. The quantity $F_{PP}(abcd,\lambda)$ is expressed in terms of F with definite isotopic spin T, i.e., we have

$$F_{PP}(abcd,\lambda)=\tfrac{1}{2}[F(abcd,\lambda T=0)+F(abcd,\lambda T=1)]. \quad (7)$$

The reduced matrix elements F_{PP} correspond to antisymmetrized elements of the potential V_{PP}, i.e., to elements of $U_{PP}\equiv V_{PP}(1-P_{12})$. The exchange terms of F_{PP} in Eq. (6) are connected with the elementary processes represented by the diagrams of Fig. 3. Obviously, Eq. (6), rather than Eq. (4), holds for neutrons when $\hat{Q}_\lambda$ has nonvanishing single-neutron matrix elements (e.g., for $M1$ transitions); here F_{PP} is replaced by F_{NN}, again of the form of Eq. (7).

Equations (4) and (6) are essentially similar to the formulas given in Ref. 7 in terms of Slater integrals. References 8 and 9 omit terms corresponding to our diagrams (b) and (b') of Figs. 1 and 3, respectively.

In the present paper we do not attempt to analyze any more complicated diagrams (microscopic processes) which may give additional contributions to $\hat{O}_{\lambda\mu}{}^{(\mathrm{eff})}$. However, the diagrams of Figs. 1 and 3 are dominant.

III. ELECTROMAGNETIC EFFECTIVE TRANSITION MATRIX ELEMENTS

We use the nuclear many-body states which are explicitly constructed with only valence nucleons.

Since such states are usually given in the second quantized form, it is useful to express the operators $\hat{O}_{\lambda\mu}{}^{(\mathrm{eff})}$ in terms of the creation ($c_\nu{}^\dagger$) and annihilation (c_ν) operators. Using the Wigner-Eckart theorem one can write

$$\hat{O}_{\lambda\mu}{}^{(\mathrm{eff})} \equiv -\sum_{nn'} \frac{\langle n'\|\hat{O}_\lambda{}^{(\mathrm{eff})}\|n\rangle}{\hat{\lambda}} X(n'n,\lambda\mu), \tag{8}$$

where $\langle n'\|\hat{O}_\lambda{}^{(\mathrm{eff})}\|n\rangle$ is a reduced matrix element, and

$$X(n'n,\lambda\mu) \equiv \sum_{m_\nu m_{\nu'}} (-)^{j_n+m_\nu}(j_{n'}j_n; m_{\nu'}-m_\nu|\lambda\mu) c_{\nu'}{}^\dagger c_\nu. \tag{9}$$

In practice, in medium-heavy nuclei one deals with many quasiparticle states, and it is useful to perform a quasiparticle transformation of Eq. (9). An explicit expression for this form of X is given in Eq. (11) of Ref. 12.

The matrix element for a 2^λ transition from the nuclear state $|\Psi_{JM}{}^E\rangle$ to the state $|\Psi_{J'M'}{}^{E'}\rangle$ can now be expressed as

$$\langle\psi_{J'M'}{}^{E'}|\hat{O}_{\lambda\mu}{}^{(\mathrm{eff})}|\psi_{JM}{}^E\rangle \equiv \hat{J}'^{-1}\langle E'J'\|\hat{O}_\lambda{}^{(\mathrm{eff})}\|EJ\rangle(J\lambda;M\mu|J'M'),$$

$$\langle E'J'\|\hat{O}_\lambda{}^{(\mathrm{eff})}\|EJ\rangle = -\hat{\lambda}^{-1}\sum_{nn'}\langle n'\|\hat{O}_\lambda{}^{(\mathrm{eff})}\|n\rangle\langle E'J'\|X(n'n\lambda)\|EJ\rangle, \tag{10}$$

where $\langle E'J'\|X\|EJ\rangle$ is a reduced matrix element of X between the two many-body states in question. If one uses the expressions of Eqs. (4) or (6) for $\langle a\|\hat{O}_\lambda{}^{(\mathrm{eff})}\|a'\rangle$ in Eq. (10), one introduces an approximation which is not identical with the standard many-body perturbation theory procedure used to obtain the nuclear many-body states with which to calculate the matrix elements of $\hat{O}_{\lambda\mu}$. In our numerical analysis we shall examine in detail the goodness of this approximation. Strictly speaking, the formulas of Eqs. (4) and (6) are directly applicable to the independent-particle model, e.g., in describing creations of pure single particle-hole pairs in a shell model without residual interactions.

It is interesting to compare the above prescription, that of using Eqs. (4) and (6), with the corresponding formula obtained from standard perturbation theory. Let us consider the Hamiltonian H_{NP} of the interaction of the valence neutrons with the core protons. The perturbation field for the many-body system is described by $\{|\psi_{JM}{}^E\rangle|0\rangle_\pi\}$, where $|\psi_{JM}{}^E\rangle$ refers to the valence neutrons only and $|0\rangle_\pi$ is the spin-zero ground state of the proton core. By perturbation theory we calculate the perturbed set involving the proton (ph) pair excited configurations $\{(|\psi_J{}^{\bar{E}}\rangle\otimes|\mathrm{ph},\bar{\lambda}\rangle_\pi)_{JM}\}$ (a vector product). On the perturbed set we now calculate the matrix elements of $\hat{O}_{\lambda\mu}{}^{(P)}$, an operator which acts only on the proton coordinates. Again one obtains the formula of Eq. (10); only the single-particle (sp) reduced matrix element $\langle n\|\hat{O}_\lambda{}^{(\mathrm{eff})}\|n'\rangle$ has to be replaced by (cf. Appendix A)

$$\langle n'\|\hat{O}_\lambda{}^{(\mathrm{eff})}(E,E')\|n\rangle \equiv 2\sum_{\mathrm{ph}}[F_{NP}(n'nph,\lambda)(E_p{}^0-E_h{}^0-(E'-E))^{-1} \times\langle p\|\hat{O}_\lambda{}^{(P)}\|h\rangle+\langle h\|\hat{O}_\lambda{}^{(P)}\|p\rangle \times(E_p{}^0-E_h{}^0+(E'-E))^{-1}F_{NP}(n'nhp,\lambda)]. \tag{11}$$

The only difference between Eqs. (11) and (4) is the dependence on the energies E and E' of the two states of the valence neutron system involved instead of the unperturbed sp neutron energies $E_n{}^0$ and $E_{n'}{}^0$, respectively. In operator form, $\hat{O}_{\lambda\mu}{}^{(\mathrm{eff})}$ of Eq. (11) can be written as

$$\hat{O}_{\lambda\mu}{}^{(\mathrm{eff})}(E) \equiv -\sum_{nn'}\hat{\lambda}^{-1}\langle n'\|\hat{O}_\lambda{}^{(\mathrm{eff})}(E,H_N)\|n\rangle \times X(n'n,\lambda\mu). \tag{12}$$

This differs from Eq. (8) by its dependence on E and on the effective total Hamiltonian of the valence neutrons, H_N. Like $\hat{O}_{\lambda\mu}{}^{(\mathrm{eff})}$ of Eq. (8), $\hat{O}_{\lambda\mu}{}^{(\mathrm{eff})}(E)$ of Eq. (12) is a nonlinear neutron many-body operator. The formulas of Eqs. (11) and (12) are quite general; i.e., they are valid for any many-body theory of the valence nuclear system.

Similar formulas are obtained for the case involving core nucleons that have the same nucleonic charge as the valence nucleons. For example, if p' and p'' are two single-valence proton states, we obtain $\langle p''\|\hat{O}_\lambda{}^{(\mathrm{eff})}(E,H_{\mathrm{val}})\|p'\rangle$ in exactly the same form as Eq. (6), where the term $E_{p''}{}^0-E_{p'}{}^0$ in e_+ and e_- is replaced by $H_{\mathrm{val}}-E$. Here H_{val} is the effective total Hamiltonian of the valence nucleon system and E is its initial-state eigenvalue. In our application we use a similar formula for the valence neutrons in calculating magnetic moments of excited states of the even tin isotopes.

Explicit formulas for the reduced matrix elements $\langle E'J'\|X(n'n,\lambda)\|EJ\rangle$ where $\{|\psi_{JM}{}^E\rangle\}$ are zero-, two-, or four-quasiparticle eigenvectors are given in Refs. 12–14.

The Feynman diagrams appropriate to the processes of Eq. (11) are those of Fig. 1, where the lines ν and ν' should be replaced by the "phonon" lines of $|\psi_{JM}{}^E\rangle$ and $|\psi_{J'M'}{}^{E'}\rangle$, respectively.

Because of its dependence on E and E', the use of

$$\langle\|\hat{O}_\lambda{}^{(\mathrm{eff})}(E,E')\|\rangle$$

of Eq. (11) is more complicated than the use of $\langle\|\hat{O}_\lambda{}^{(\mathrm{eff})}\|\rangle$ of Eqs. (4), (6), and (8). It is clear that the latter version of our theory has, in practical calculations, the advantage of universality, i.e., of the independence of the many-body eigenstates involved in the applications. Still another complication arises from the following difference between the symmetry properties of $\hat{O}_{\lambda\mu}{}^{(\mathrm{eff})}$ and $\hat{O}_{\lambda\mu}{}^{(\mathrm{eff})}(E,E')$ of Eq. (11): With

[12] J. Hendeković, P. L. Ottaviani, M. Savoia, and J. Sawicki, Nuovo Cimento **54B**, 80 (1968).

the usual phase conventions[12–14] which we use in the present work it is immediately verified that, while $\langle a'\|\hat{O}_\lambda\|a\rangle \equiv (-)^{j_a-j_{a'}}\langle a\|\hat{O}_\lambda\|a'\rangle$, the same symmetry relation holds for the effective operator of Eqs. (4), (6), and (8):

$$\langle n'\|\hat{O}_\lambda^{(\mathrm{eff})}\|n\rangle = (-)^{j_n-j_{n'}}\langle n\|\hat{O}_\lambda^{(\mathrm{eff})}\|n'\rangle. \quad (13)$$

Instead, for the operator of Eq. (11) we find

$$\langle n'\|\hat{O}_\lambda^{(\mathrm{eff})}(E,E')\|n\rangle = (-)^{j_n-j_{n'}}\langle n\|\hat{O}_\lambda^{(\mathrm{eff})}(E',E)\|n'\rangle. \quad (14)$$

The interchange of E and E' and of n and n' in the latter case renders the calculation of $\langle E'J'\|\hat{O}_\lambda^{(\mathrm{eff})}(E,E')\|EJ\rangle$ more complicated than that of $\langle E'J'\|\hat{O}_\lambda^{(\mathrm{eff})}\|EJ\rangle$ for a given $EJ \to E'J'$ transition. Only for diagonal ($E=E'$) matrix elements does no such extra trouble arise [as in calculations of the quadrupole moments, magnetic moments, etc.; in this case calculations with $\hat{O}_{\lambda\mu}^{(\mathrm{eff})}$ ($E,E'\equiv E$) are equivalent to those with $\hat{O}_{\lambda\mu}^{(\mathrm{eff})}$ of Eqs. (4) and (6), neglecting $E_{n'}{}^0-E_n{}^0$ in $e_\pm$]. It follows from our numerical results, presented below, that the two different definitions of the effective electromagnetic operators lead to very small differences in the values of the calculated observables. This seems to justify extensive use of the simpler version $\hat{O}_{\lambda\mu}^{(\mathrm{eff})}$, which is independent of E and E'.

As for a critical evaluation of the usefulness of the extremely crude effective-charge approach, we calculate the "effective-charge matrix" (ECM), defined as

$$e_\lambda(n',n) \equiv \langle n'\|\hat{O}_\lambda^{(\mathrm{eff})}\|n\rangle / \langle n'\|\hat{O}_\lambda\|n\rangle_{\mathrm{ref}}, \quad (15)$$

where $\langle n'\|\hat{O}_\lambda\|n\rangle_{\mathrm{ref}}$ is the "reference matrix" defined in the usual way for "direct" $n \to n'$ transitions, and $e_{\mathrm{eff}}{}^{(\lambda)}=1$. ECM gives the actual *theoretical* effective charge for each individual $n \to n'$ transition.

IV. APPLICATION TO ELECTROMAGNETIC PROPERTIES AND TO INELASTIC SCATTERING OF ELECTRONS FROM THE EVEN ISOTOPES OF TIN

The spectra of the even isotopes of tin have been generally successfully described by two- and four-quasiparticle random-phase-approximation (RPA) and Tamm-Dancoff theories.[5,6,13–16] In particular, the realistic nucleon-nucleon potentials of Tabakin[11] and of Yale-Shakin[17,18] including the core polarization renormalizaton have been applied in the quasiparticle Tamm-Dancoff (QTD) and second Tamm-Dancoff (QSTD) theories.[5,6] It is our aim here to calculate the effective electromagnetic operators appropriate to our equations of Sec. III, corresponding to one such realistic effective nuclear force, and then to use them to calculate some electromagnetic properties of the even tin isotopes. The five valence neutron subshells are $2d_{5/2}$, $1g_{7/2}$, $3s_{1/2}$, $2d_{3/2}$, and $1h_{11/2}$. For the particle-hole excitations of the core nucleons we consider all subshells between the magic numbers 8 and 126.

Unfortunately, the lack of direct experimental information on the shell-model sp energies on the one hand, and of any self-consistent Hartree-Fock (HF) or Hartree-Fock-Bogolubov (HFB) calculations of such energies on the other, forces us to use other methods for determining them which are much less satisfactory. In addition, many observables and even some energy levels not of a collective character are well known to be sensitive to the input values of the sp energies. A procedure which seems to be appropriate in the circumstances is that of deriving the sp energies from the observed energy levels of the odd isotopes by the inverse-gap-equation (IGE) method.[19,20] This procedure is currently being applied by the present authors in collaboration with Alzetta and Gambhir.[21] Since the validity of the main conclusions of the present paper does not hinge on a detailed quantitative fit to any particular experimental data, we are in the present numerical applications limiting ourselves to a less well-justified choice of the set of the sp energies involved. To do so, we have employed the values of the sp binding energies obtained by the Bonn group[22] with a reasonable Woods-Saxon potential. The energies (in MeV) for the five valence subshells (nlj) are $-10.52(2d_{\frac{5}{2}})$, $-9.36(1g_{\frac{7}{2}})$, $-8.45(3s_{\frac{1}{2}})$, $-7.78(2d_{\frac{3}{2}})$, $-7.16(1h_{\frac{11}{2}})$; for the eight important proton core (hole) subshells we have $-30.09(1d_{\frac{5}{2}})$, $-27.93(1d_{\frac{3}{2}})$, $-27.07(2s_{\frac{1}{2}})$, $-22.91(1f_{\frac{7}{2}})$, $-19.07(1f_{\frac{5}{2}})$, $-18.82(2p_{\frac{3}{2}})$, $-17.28(2p_{\frac{1}{2}})$, $-15.24(1g_{\frac{9}{2}})$; in addition to the five valence subshells we consider six higher proton particle subshells: $-2.26(2f_{\frac{7}{2}})$, $-1.14(3p_{\frac{3}{2}})$, $-0.23(3p_{\frac{1}{2}})$, $+1.01(2f_{\frac{5}{2}})$, $+1.04(1i_{\frac{13}{2}})$, $+1.07(1h_{\frac{9}{2}})$. These energies are most appropriate to the isotope Sn^{116}. Any other particle or hole subshells give negligibile contributions. The Woods-Saxon radial wave functions are reasonably approximated with those of the harmonic oscillator with $b^{-1}=\sqrt{\nu}=0.46\mathrm{F}^{-1}$. It appears that the $(e,e'p)$ data on some light nuclei and other similar information suggest, in general, considerably larger binding energies for the deep-lying subshells than those corresponding to a "reasonable" Woods-Saxon potential. Interpreted as a correction for many-body effects, this discrepancy

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[14] A. Rimini, J. Sawicki, and T. Weber, Phys. Rev. **168**, 1401 (1968).
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[20] Y. K. Gambhir, International Center for Theoretical Physics, Trieste, Report No. IC/68/32 (unpublished); Phys. Letters **26B**, 695 (1968).
[21] R. Alzetta *et al.*, International Centre for Theoretical Physics, Trieste, Report No. 1C/68/83 (unpublished).
[22] K. Bleuler, M. Beiner, and R. De Toureil, Nuovo Cimento **52B**, 45 (1967); **52B**, 149 (1967); and private communication from M. Beiner.

TABLE I. Matrix of the effective quadrupole charge of Eq. (15) for $E2$ transitions for the five valence-neutron subshells in Sn. The numbers without parenthesis refer to $\hat{O}_{E2}^{(\mathrm{eff})}$ of Eq. (4), and those in parenthesis to the $\hat{O}_{E2}^{(\mathrm{eff})}$ $(E=E')$ of Eq. (11). The sp and other parameter values are explained in the text.

	$3s_{\frac{1}{2}}$	$2d_{\frac{3}{2}}$	$2d_{\frac{5}{2}}$	$1g_{\frac{7}{2}}$	$1h_{\frac{11}{2}}$
$3s_{\frac{1}{2}}$	...	0.6143 (0.6154)	0.6757 (0.6096)	...	...
$2d_{\frac{3}{2}}$		0.6459	0.6989 (0.6398)	1.1636 (1.0516)	...
$2d_{\frac{5}{2}}$			0.6521	1.1132 (1.1054)	...
$1g_{\frac{7}{2}}$				1.0844	...
$1h_{\frac{11}{2}}$					0.6535

would imply more spread-out sp HF energies than in our present calculation (a smaller sp level density). This could lead to a reduction of the contributions to $\langle\|\hat{O}_\lambda^{(\mathrm{eff})}\|\rangle$ of the deep-lying core nucleons. On the other hand, the most important contributions (such as that of the $1g_{\frac{9}{2}}$ subshell) may be even greater with an actual HF basis.

Let us consider first the most important $E2$ transitions. With all the above-mentioned sp states there are 29 nonvanishing ($E2$ allowed transitions) proton matrix elements. The nine nonvanishing distinct $(n\geq n')$ elements $e_2(n,n')$ of Eq. (15) for the valence neutrons are given in Table I. $\hat{O}_{\lambda\mu}^{(\mathrm{eff})}$ is given by Eq. (4). The nuclear force (F_{NP} elements) is the "bare" Yale-Shakin reaction matrix. The corresponding values of $e_2^{(E'=E)}(n,n')$ with $\langle n\|\hat{O}_\lambda^{(\mathrm{eff})}(E,E'=E)\|n'\rangle$ of Eq. (11) are given in parenthesis for comparison (clearly the diagonal elements of the two variants of our theory are identical for $E'=E$). We observe only very small differences between the two table entries for each off-diagonal $n'\rightarrow n$ element in Table I. The differences between the elements $\langle n\|\hat{O}_\lambda^{(\mathrm{eff})}\|n'\rangle$ of Eq. (9) and the elements $\langle n\|\hat{O}_\lambda^{(\mathrm{eff})}(E,E'=E+1.29\ \mathrm{MeV})\|n'\rangle$ are precisely of the same order. The values of $e_2(n,n')$ are of the desired sign and the same order of magnitude. They are actually grouped in two clusters: those higher than unity and those somewhat smaller than 0.7. The composition of each one of the nine elements of Table I in terms of the partial contributions of the subshells of the (ph) pairs involved is given in Table II. The entire $1f2p$ major shell, plus $1g_{\frac{9}{2}}$ of the core with all their transitions to the five lowest-lying particle subshells, contribute, on the average, slightly more than about 50% of all the $e_2(n,n')$. Transitions from the same to the six sp levels of the upper major shell ($2f,3p$, $1i_{\frac{13}{2}}$, $1h_{\frac{9}{2}}$) contribute the surprisingly large amount of 30–40% of all the $e_2(n,n')$. The $1d2s$ major shell of the core is of little importance. The numbers of Table II refer to $\hat{O}_2^{(\mathrm{eff})}$ as defined in Eq. (4). A similar distribution of the (ph) contribution obtains for $\hat{O}_2^{(\mathrm{eff})}(E,E'=E)$. The two terms on the right-hand side of Eq. (4) [or of Eq. (11)] each gives contributions of the same order of magnitude (equal for $n=n'$ and for $E'=E$). Some authors[8,9] suggested schematic models in which they were considering only one of the two terms. This is clearly not suitable for a quantitative analysis in view of the symmetry properties of these terms leading to Eqs. (13) or (14). In order to examine the relative importance of our individual $e_2(n,n')$ we compute the observables $B(E2,2_1^+\rightarrow 0_1^+)$ and quadrupole moment of the 2_1^+ state, $Q(2_1^+)$, using numbers of Table I. The corresponding eigenvectors $|0_1^+\rangle$ and $|2_1^+\rangle$ of Sn^{116} are those of Ref. 23 computed in the QTD and QSTD approximation with the Yale-Shakin force renormalized for the core polarization. The core-polarization renormalization of all the proton and neutron subshells mentioned is taken to second order which is a good approximation[5,6]; the valence neutron subshells are assumed to be, on the average, exactly half occupied; no other approximation of the propagators of the core-polarization terms is made. The sp energies and wave functions are exactly those of our $e_2(n,n')$ calculation. In the QTD approximation, $|0_1^+\rangle$ is quasiparticle (qp) vacuum itself and $|2_1^+\rangle$ is a nine-component vector. The corresponding QSTD vectors of Ref. 23 have 56 and 94 components, respectively. These are free of all the basic spurions due to the nucleon-number nonconservation (such kets are projected out). The QSTD 0_1^+ eigenvalue lies -0.363 MeV lower than the qp vacuum, and the QSTD 2_1^+ eigenvalue is 1.153 MeV; the QTD 2_1^+ energy lies at 1.259 MeV; the observed 2_1^+ energy is 1.291 MeV. In QSTD we distinguish for $|0_1^+\rangle$ between the case (I), in which we define the four-qp spurious kets $|\psi'_{\mathrm{sp4}}\rangle$ due to the nucleon number nonconservation so that they have no vacuum component ($\langle 0|\psi'_{\mathrm{sp4}}\rangle=0$), and the case (II) in which $\langle 0|\psi_{\mathrm{sp4}}\rangle\neq 0$.[23] (In the latter case we project out exactly the fluctuation of $N^2-N_0^2$, where N is the nucleon number operator and N_0 its correct eigenvalue.)

In Table III we compare the calculated (QTD and

TABLE II. Partial contributions to the elements $e_2(n,n')$ of Table I coming from the four groups of the $h\leftrightarrow p$ transitions: $A\equiv(2p,1f_{\frac{5}{2}},1g_{\frac{9}{2}})\leftrightarrow(3s,2d,1g_{\frac{7}{2}},1h_{\frac{11}{2}})\equiv(a)$, $B\equiv 1f_{\frac{7}{2}}\leftrightarrow(a)$, $C\equiv(2s1d)\leftrightarrow(a)$ $D=(2p1f)\leftrightarrow(2f,3p,1i_{\frac{13}{2}},1h_{\frac{9}{2}})=$(upper major shell).

(ph) \ (n,n')	$(3s_{\frac{1}{2}}2d_{\frac{3}{2}})$	$(2d_{\frac{3}{2}}2d_{\frac{3}{2}})$	$(3s_{\frac{1}{2}}2d_{\frac{5}{2}})$	$(2d_{\frac{3}{2}}2d_{\frac{5}{2}})$	$(2d_{\frac{5}{2}}2d_{\frac{5}{2}})$	$(2d_{\frac{3}{2}}1g_{\frac{7}{2}})$	$(2d_{\frac{5}{2}}1g_{\frac{7}{2}})$	$(1g_{\frac{7}{2}}1g_{\frac{7}{2}})$	$(1h_{\frac{11}{2}}1h_{\frac{11}{2}})$
A	0.1840	0.1465	0.2254	0.1761	0.1760	0.5914	0.5813	0.3476	0.1837
B	0.1016	0.1314	0.0966	0.1221	0.0946	0.1021	0.0630	0.2451	0.0847
C	0.0319	0.0417	0.0497	0.0647	0.0772	0.1050	0.1308	0.0845	0.1054
D	0.2968	0.3264	0.3040	0.3359	0.3042	0.3651	0.3382	0.4072	0.2797

[23] M. Gmitro, A. Rimini, J. Sawicki, and T. Weber, Phys. Rev. 173, 964 (1968).

TABLE III. Values of the reduced transition rate $B(E2,2_1^+ \to 0_1^+)$ and of $Q(2_1^+)$ of Sn^{116} calculated with $e_2^{(n)}=1$ and with $e_2(n,n')$ of Table I. The QDT and QSTD eigenvectors (I and II, explained in the text) refer to the renormalized Yale-Shakin force of Ref. 23.

Observable \ Theory	$e_2^{(n)}=1$		$e_2(n,n')$ of Table I				
				of Eq. (4)		of Eq. (11)	
	QTD	QSTD(II)	QTD	QSTD(I)	QSTD(II)	QTD	QSTD
$B(E2,2_1^+ \to 0_1^+)$ (units of e^2F^4)	317.0	273.7	232.3	259.4	202.2	229.5	···
$Q(2_1^+)$ (b)	+0.036	+0.125	+0.021	+0.094		+0.022	+0.091

QSTD) values of $B(E2,2_1^+ \to 0_1^+)$ and $Q(2_1^+)$, both theoretical [computed with the $e_2(n,n')$ of Table I] and computed with the neutron effective charge, $e_2^{(n)}=1$. The reported observed value of $B(E2,2_1^+ \to 0_1^+)$ varies between 200 and 500 e^2F^4. In comparing this with our theoretical values of Table III we should keep in mind that our predicted results were obtained without any adjustable parameter. It is clear that by varying the sp parameters a better agreement with experiment could be obtained. The observed value of $Q(2_1^+)$ of Sn^{116} is[24] $+0.4\pm0.3$ b. Our QSTD values lie around the lower limit of the experimental error. $Q(2_1^+)$ is generally a "delicate" quantity sensitive to the detailed structure of the $|2_1^+\rangle$ vector. The QSTD predictions are much better than those of QTD because of the very important enhancement due to the large two-qp–four-qp interference terms even in the case of quite small four-qp components. It should be stressed that our theory is based on the purely spherical shell model; we feel therefore that the assumption of a stable deformation in the 2_1^+ state in Sn is probably premature.

An interesting observable is the static magnetic moment $\mu(5_1^-)$ of the 5_1^- state (observed at 2.35 MeV in Sn^{116} and at 2.29 MeV in Sn^{120}). This 5_1^- state may be almost degenerate with the 3_1^- state in Sn^{120} and thus any data for 5_1^- in Sn^{120} have to be taken with caution. For $\mu(5_1^-)$ Bodenstaedt *et al.*[25] give the following respective values of the g factors:

$$g_{5_1^-}(A=116)=-0.065\pm0.005,$$
$$g_{5_1^-}(A=120)=-0.058\pm0.007.$$

The g factor is defined as

$$g_J=\{J(J+1)(2J+1)\}^{-1/2}\langle J\|\hat{\mu}_{(1)}\|J\rangle \qquad (16)$$

in the usual notation, where the magnetic dipole operator is

$$\hat{\mu}_{(1)}=g_{(s)}s_z+g_{(l)}l_z. \qquad (17)$$

In Table IV we give the computed "effective magnetic reduction matrix" (EMRM) analogous to ECM of Table I. The name suggests that the "bare" sp matrix elements $\langle n\|\hat{\mu}_{(1)}\|n'\rangle_{\text{bare}}$ are generally greater than the computed theoretical values based on Eq. (6) (here specified for neutrons). This is indeed the case and it goes in the direction of a better agreement with experiment. The bare matrix corresponds in this case to pure neutronic matrix elements of the valence neutrons only. We choose here to use the neutron (ph) projector $q_\nu=\frac{1}{2}$ for all the (ph) pairs involved, which corresponds to an average occupation of the neutron valence subshells equal to $\frac{1}{2}$; indeed in Sn^{116} we have $N/32=\frac{1}{2}$. We find that the contributions of the (ph) pairs (both neutrons and protons) are of opposite sign to those of the valence neutron, and lead to an over-all reduction, i.e., $0<\mu(n,n')<1$ where $\mu(n,n')$ is our

$$\text{EMRM}\equiv\langle n\|\hat{\mu}_{(1)}{}^{(\text{eff})}\|n'\rangle/\langle n\|\hat{\mu}_{(1)}\|n'\rangle.$$

The allowed $M1$ transitions (ph pairs) are $1g_{\frac{9}{2}} \leftrightarrow 1g_{\frac{7}{2}}$ (both protons and neutrons) and $1h_{\frac{11}{2}} \leftrightarrow 1h_{\frac{9}{2}}$ (neutrons only). The inclusion of the latter transitions (with the upper major shell) leads to up to 40% reductions of $\mu(n,n')$. The off-diagonal $\mu(2d_{\frac{3}{2}}2d_{\frac{5}{2}})$ is given also for $\hat{\mu}_{(1)}{}^{(\text{eff})}(E,E'=E)$ of the type of Eq. (12) (the number in parenthesis). The symbol ∞ for $\mu(3s_{\frac{1}{2}}2d_{\frac{3}{2}})$ and $\mu(2d_{\frac{5}{2}}1g_{\frac{7}{2}})$ in Table IV indicates that the bare elements $\langle n\|\hat{\mu}_{(1)}\|n'\rangle_{\text{bare}}$ vanish (are l-forbidden) in these cases, while the corresponding theoretical "effective" elements $\langle n\|\hat{\mu}_{(1)}{}^{(\text{eff})}\|n'\rangle$ are $\neq 0$ [the l-selection rule is relaxed through Eq. (6)]. The most important reduction in $\mu(n,n')$ comes from the (ph)-neutron spin parts of $\langle n\|\hat{O}_\lambda{}^{(\text{eff})}\|n'\rangle$; less important negative contributions are those of the core-proton spin terms. The core-proton current (electric) terms are roughly smaller by an order of magnitude.

In Table V we give our computed QTD and QSTD bare and theoretical values of the g factor $g_{5_1^-}$ of Sn^{116}. The QTD and QSTD eigenvectors $|5_1^-\rangle$ refer to the

TABLE IV. Matrix of $\mu(n,n')\equiv\langle n\|\mu^{(\text{eff})}\|n'\rangle/\langle n\|\hat{\mu}_{(1)}\|n'\rangle_{\text{bare}}$ defined in the text for $M1$ transitions for the five valence-neutron subshells in Sn. The numbers without parentheses refer to $\mu_{(1)}{}^{(\text{eff})}$ of Eq. (6) and those in parentheses to $\mu_{(1)}{}^{(\text{eff})}(E'=E)$ of the analog of Eq. (11).

	$3s_{\frac{1}{2}}$	$2d_{\frac{3}{2}}$	$2d_{\frac{5}{2}}$	$1g_{\frac{7}{2}}$	$1h_{\frac{11}{2}}$
$3s_{\frac{1}{2}}$	0.6377	∞	···	···	···
$2d_{\frac{3}{2}}$		0.6972	0.3900 (0.5280)	···	···
$2d_{\frac{5}{2}}$			0.6333	∞	···
$1g_{\frac{7}{2}}$				0.4275	···
$1h_{\frac{11}{2}}$					0.5935

[24] J. De Boer, in Proceedings of the International Conference on Nuclear Structure, Tokyo, 1967, p. 203 (unpublished).

[25] E. Bodenstaedt *et al.*, Z. Physik. **168**, 370 (1962); Cooperation of the Angular Correlation Groups of Bonn and Hamburg, Nucl. Phys. **89**, 305 (1966).

TABLE V. The gyromagnetic factor $g_{5_1^-}$ of the 5_1^- state of Sn^{116} calculated with $\langle n\|\hat{\mu}_{(1)}\|n'\rangle_{\text{bare}}$ and with $\hat{\mu}_{(1)}^{(\text{eff})}$ given in the text. The QTD and QSTD eigenvectors refer to the renormalized Yale-Shakin force of Ref. 23.

	with $\langle n\|\hat{\mu}_{(1)}\|n'\rangle_{\text{bare}}$		with $\mu_{(1)}^{(\text{eff})}$ of Eq. (6)		with $\hat{\mu}_{(1)}^{(\text{eff})}$ $(E,E'=E)$	
Theory	QTD	QSTD	QTD	QSTD	QTD	QSTD
$g_{5_1^-}$	−0.1382	−0.1504	−0.0579	−0.0648	−0.0536	−0.0611

same Yale-Shakin force renormalized for core polarization which was used for our computations of Table III. We note that while our four theoretical values are very close to each other, they are by a factor of $\frac{1}{3}$ smaller than the bare values. The theoretical results compare very well with the observed value for Sn^{116}. We may mention that a calculation by Lombard[26] based on the $|5_1^-\rangle$ QTD eigenvector of Ref. 15 and QTD and QSTD calculations of Ref. 23 using $\langle n\|\hat{\mu}_{(1)}\|n'\rangle_{\text{bare}}$ led to a sharp disagreement with experiment, similar to what we find with the same $\langle n\|\hat{\mu}_{(1)}\|n'\rangle_{\text{bare}}$.

Recently, Barreau and Bellicard[27] published the first experimental data on the inelastic electron scattering from the even tin isotopes 116, 120, and 124 with excitation of the 2_1^+ and the 3_1^- states. The bombarding electron energy was 150 MeV, and the scattering angle varied between 45° and 80°. The electric quadrupole and octupole form factors squared, $|F_{\text{in}}(Q)|^2$, have been extracted from the differential cross sections as

$$|F_{\text{in}}|^2=\sigma(E_0\theta)/z^2\sigma_{\text{Mott}}(Z=1).$$

Both the absolute values of $|F_{\text{in}}|^2$ and their angular distributions should, particularly when combined with the static $E\lambda$ moments and the $B(E\lambda)$, serve as a good test of any microscopic or other nuclear wave functions of the excited states in question.

In a letter by three of us[28] the corresponding squares of theoretical form factors $|F_{\text{in}}|^2$ have been presented calculated with the QTD and QSTD $|2_1^+\rangle$ and $|3_1^-\rangle$ eigenvectors of Ref. 23, which correspond to the two-body nuclear potential of Tabakin[11] renormalized for core polarization. The concept of a constant effective charge $e_\lambda^{(\text{eff})}$ was applied, and the calculations were based on the "reference" sp matrix elements $\langle n\|\hat{O}_\lambda\|n'\rangle_{\text{ref}}$ of the appropriate $\hat{O}_\lambda$. The Coulomb and the transverse electric parts (spin and current terms) of $|F_{\text{in}}|^2$ were calculated according to Eq. (3.64) of de Forest and Walecka.[29] Although the calculations were done essentially in Born approximation, corrections for the distortion effects as proposed by Czyż and Gottfried[30] [cf. also Eq. (8.13) of Ref. 29] were included. The transverse electric terms are found to be negligible as compared with the Coulomb parts. With the numerical values of the effective constants $e_2^{(\text{eff})}=1.23$ and $e_3^{(\text{eff})}=2.19$, good fits to the data of Ref. 27 were obtained.

In the present work we have recalculated the results of Ref. 28 with our present (i.e., those of Ref. 23) QTD and QSTD eigenvectors $|2_1^+\rangle$ and $|3_1^-\rangle$ appropriate to the core-renormalized Yale-Shakin two-body force. The constant effective charge results we then compare with those obtained with the theoretical effective operators calculated according to Eqs. (4) and (6).

In Fig. 4 we compare the data of Ref. 27 with our theoretical $|F_{\text{in}}|^2(0_1^+\rightarrow 2_1^+)$ calculated with the QTD, QSTD(I) and QSTD(II) eigenvectors $|0_1^+\rangle$ and $|2_1^+\rangle$. Except for large angles (large momentum transfer), agreement with the data is rather good both for the angular distribution and for the absolute values. The calculated angular distribution of $|F_{\text{in}}|^2(0_1^+\rightarrow 3_1^-)$ is consistent with the data of Ref. 27. Unfortunately, the absolute values of the same quantity are too small, as also are those of Ref. 28. If the 3_1^- cross-section data of Ref. 27 are indeed based on a sufficiently precise resolution discriminating between neighboring 5_1^- and other states, the explanation of the latter discrepancy is to be sought probably in the inadequacy of our treatment of some excited configurations of the core nucleons.

In Table VI we give the (e,e') angular distribution of the ratio of the theoretical effective form factor $|F_{\text{in}}(\text{theoret})|$ to the reference form factor $|F_{\text{in}}(\text{ref}, e_\lambda^{(\text{eff})}=1)|$, i.e., to the one computed with a constant effective charge $(=1)$. This ratio can be interpreted as an effective 2^λ-pole charge which depends on the (e,e') scattering angle θ. This dependence measures the inadequacy of a constant effective charge theory of the (e,e') cross sections. From Table VI we notice that the ratio $|F_{\text{in}}(\text{theoret})|/|F_{\text{in}}(\text{ref}, e_\lambda^{(\text{eff})}=1)|$ varies between 0.79 and 0.86 for the $0_1^+\rightarrow 3_1^-$ excitation and between 0.94 and 1.03 for $0_1^+\rightarrow 3_1^-$ in the region $45°\leq\theta\leq 65°$. Although the theoretical effective charge is larger in the 3_1^- case, it is still much too small (by a factor of about 3) to explain the experimental data on the absolute values of the (e,e') cross section of Ref. 27 (cf. the results of Ref. 28).

TABLE VI. Ratios $|F_{\text{in}}(\text{theor})|/|F_{\text{in}}(e_\lambda^{(\text{eff})}=1)|$ of the theoretical and the reference $(e_\lambda^{(\text{eff})}=1)$ inelastic electron form factors for the reactions $Sn^{116}(e,e')Sn^{116}(2_1^+,3_1^-)$. The theoretical effective $\hat{O}_\lambda^{(\text{eff})}$ operators are calculated as from Eqs. (4) and (6) for each (e,e') scattering angle θ. The QTD and QSTD (I and II) eigenvectors refer to the renormalized Yale-Shakin force of Ref. 23.

	Theory \ θ	45°	50°	55°	60°	65°
2^+	QTD	0.861	0.858	0.850	0.832	0.795
	QSTD(I)	0.860	0.856	0.848	0.831	0.793
	QSTD(II)	0.863	0.858	0.850	0.832	0.794
3^-	QTD	0.937	0.954	0.975	1.001	1.032
	QSTD(I)	0.935	0.953	0.974	1.000	1.032
	QSTD(II)	0.938	0.955	0.976	1.001	1.032

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[27] P. Barreau and J. B. Bellicard, Phys. Rev. Letters **19**, 1444 (1967).
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[29] T. de Forest and J. D. Walecka, Advan. Phys. **15**, 1 (1966).
[30] W. Czyż and K. Gottfried, Ann. Phys. (N. Y.) **21**, 47 (1963).

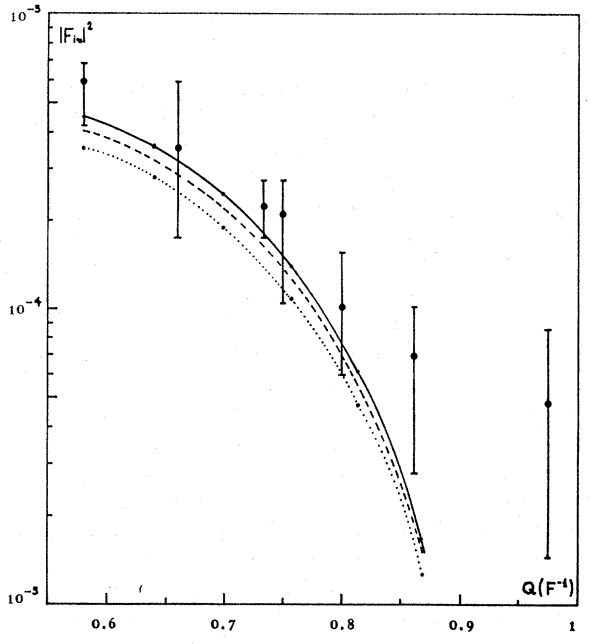

FIG. 4. Theoretical inelastic scattering form factor $|F_{\text{in}}(\text{theoret})|^2$ for the reaction $Sn^{116}(e,e')$ Sn^{116} (2_1^+) at 150 MeV (incoming electron energy). Q is the momentum transfer. The QTD (dashed line), QSTD(I) (solid line), and QSTD(II) (dotted line) results refer to the eigenvectors of Ref. 23 obtained for the renormalized Yale-Shakin force. The experimental data (bars) are those of Ref. 27.

In general, the differences between the respective QTD, QSTD(I), and QSTD(II) results (cf. Table VI) are quite small. By increasing the (e,e') scattering angle θ the theoretical-to-reference ratio of Table VI decreases slightly in the 2_1^+ case and increases slightly in the 3_1^- case.

V. CONCLUSIONS

Our calculations of the effective operators of electromagnetic interactions with nuclei are fully microscopic and they involve essentially no adjustable parameters, i.e., the only parameters of the theory are the same single-particle parameters which are involved in the corresponding shell-model spectra. The two-body nuclear force (reaction matrix) is derived from a realistic nucleon-nucleon potential. The theoretical construction is based on a perturbation theory treatment of the particle-hole excitations of the core nucleons. In this sense it is similar to the Kuo-Brown[1-4] core polarization of the two-body nuclear force (cf. also Refs. 5 and 6) and to the double (multiple) scattering terms of the Brueckner theory.

Our numerical calculations for even Sn isotopes provide a partial justification (for an otherwise completely arbitrary) concept and approximation of a constant effective charge. The computations are based on the Yale-Shakin realistic nucleon-nucleon force and on a set of sp parameters of a Woods-Saxon potential. The overall agreement with the observed values of $B(E2)$, $Q(2_1^+)$, and $\mu(5_1^-)$ and the inelastic electron scattering form factors (except for the $0_1^+ \rightarrow 3_1^-$ transition) is even surprisingly good, in view of the lack of any *ad hoc* adjustable parameter, and in view of the crudeness of some of the theoretical assumptions. The results seem to be encouraging, and future calculations for other nuclei and calculations with sp basis which are determined in a possibly self-consistent (HF) way are most desirable.

A direct proof of the validity of our perturbation-type procedure by performing direct calculations with shell-model wave functions explicitly involving configurations of the core nucleons treated in a more exact way is not possible as yet because of the enormous dimensions of such shell-model problems. Approximate treatments (cf. Appendix B) based on simple QTD and QRPA calculations seem to be quantitatively inconsistent (e.g., they leave out energetically equivalent four-qp or two-particle–two-hole excited configurations), and cannot provide a valid criterion for our theoretical $\hat{O}_\lambda^{(\text{eff})}$. We may point out that, in contrast to a suggestion by Bandō,[31] we find that the bare nuclear force is a sufficient approximation in the second-order calculation $[F(abcd,J)$ in Eqs. (4), (6), and (11)$]$ of the effective electromagnetic interaction $\hat{O}_\lambda^{(\text{eff})}$.

After the present work was completed an independent work by Hamamoto and Molinari[32] came to our attention. Their letter[32] is concerned with the quadrupole effective charge of nuclei around $Z=28$, and conclusions are reached similar to those of Ref. 10.

ACKNOWLEDGMENTS

We are indebted to Dr. M. Beiner for communicating to us details of the eigenstates of Ref. 22. All our computations were performed on the IBM 7044 computer of the University of Trieste. Two of us (M.G. and J.S.) express their thanks to Professor Abdus Salam and the IAEA for their kind hospitality at the International Centre for Theoretical Physics, Trieste. Financial support from UNESCO to one of us (M.G.) is gratefully acknowledged. We would like to thank Professor H. H. Aly for advice on correcting the English of our original manuscript.

APPENDIX A: DERIVATION OF THE FORMULA OF EQ. (11)

The interaction Hamiltonian of the valence neutrons and the core protons, H_{NP}, generates proton particle-hole pairs (ph) from a given unperturbed state $|\psi_{JM}{}^E\rangle|0\rangle_\pi$, where $|\psi_{JM}{}^E\rangle$ is the eigenvector of the valence neutron system and $|0\rangle_\pi$ is the ground state of the closed-shell proton core. The many-body perturbation theory then gives for the state perturbed to first order the expression

$$|\tilde{\psi}_{JM}{}^E\rangle \cong |\psi_{JM}{}^E\rangle|0\rangle_\pi + \sum_{\lambda''J''} \sum_{\text{ph},E''} \alpha_{(\lambda''J'')J}(E'',\text{ph}) \times (|\psi_{J''}{}^{E''}\rangle \otimes |(\text{ph})\lambda''\rangle)_{JM}, \quad \text{(A1)}$$

[31] H. Bandō, Progr. Theoret. Phys. (Kyoto) 38, 1285 (1967).
[32] I. Hamamoto and A. Molinari, Phys. Letters 26B, 649 (1968).

where

$$\alpha_{(\lambda''J'')J}(E'',\mathrm{ph}) \equiv \frac{(\langle\psi_{J''}{}^{E''}|\otimes\langle(\mathrm{ph})\lambda''|)_{JM}H_{NP}|0\rangle_\pi|\psi_{JM}{}^{E}\rangle}{E_p{}^0-E_h{}^0+E''-E}, \quad \text{(A2)}$$

and where $(|\psi_{J''}{}^{E''}\rangle\otimes|(\mathrm{ph})\lambda''\rangle)_{JM}$ is a vector-coupled product of the unperturbed valence neutron eigenvector of energy E'', spin J'' and a proton (ph) pair of spin λ''.

The Hamiltonian H_{NP} can be put in the form

$$H_{NP}=-2\sum F_{NP}(n'np'p,J')X(n'n,J'M')(-)^{J'-M'}\bar{C}_{J'-M'}(p'p), \quad \text{(A3)}$$

where $X(nn',\lambda\mu)$ is defined in Eq. (9) and

$$\bar{C}_{JM}(aa')\equiv\sum(-)^{j_\alpha-m_\alpha}\times(j_\alpha j_{\alpha'};-m_\alpha m_{\alpha'}|JM)b_{\alpha'}{}^\dagger b_\alpha \quad \text{(A4)}$$

is a proton (aa') particle-hole creation operator. Straightforward algebra gives for the expansion coefficient (A2) the result

$$\alpha_{(\lambda''J'')J}(E'',\mathrm{ph})=2\sum_{nn'}F_{NP}(n'nhp,\lambda'')(-)^{j_p-j_h+J-J''}\hat{J}^{-1}\times\frac{\langle\psi_{J''}{}^{E''}\|X(n'n,\lambda)\|\psi_J{}^E\rangle}{E_p{}^0-E_h{}^0+E''-E}. \quad \text{(A5)}$$

We now proceed to calculate the lowest-order nonvanishing contributions to a matrix element of the electric 2^λ pole, $\hat{O}_{\lambda\mu}$, $\langle\tilde{\Psi}_{J'M'}{}^{E'}|\hat{O}_{\lambda\mu}|\tilde{\Psi}_{JM}{}^E\rangle$. There are two first-order terms; the first one is

$$\langle\hat{O}_{\lambda\mu}\rangle_{(1)}\equiv\sum_{\mathrm{ph},E''}\sum_{\lambda''J''}\alpha_{(\lambda''J'')J}(E'',\mathrm{ph})\times\langle\psi_{J'M'}{}^{E'}|{}_\pi\langle0|\hat{O}_{\lambda\mu}(|\psi_{J''}{}^{E''}\rangle\otimes|(\mathrm{ph})\lambda''\rangle)_{JM}. \quad \text{(A6)}$$

A simple evaluation of this expression leads to

$$\langle\hat{O}_{\lambda\mu}\rangle_{(1)}=-\sum_{nn'}\langle n'\|\hat{O}_\lambda{}^{(\mathrm{eff})}(E,E')\|n\rangle_{(1)}\hat{\lambda}^{-1}\hat{J}^{-1}\times\langle\psi_{J'}{}^{E'}\|X(n'n,\lambda)\|\psi_J{}^E\rangle(J\lambda;M\mu|J'M'), \quad \text{(A7)}$$

where

$$\langle n'\|\hat{O}_\lambda{}^{(\mathrm{eff})}(E,E')\|n\rangle_{(1)}\equiv2\sum_{\mathrm{ph}}\frac{\langle h\|\hat{O}_\lambda\|p\rangle}{\hat{\lambda}}\times(E_p{}^0-E_h{}^0+E'-E)^{-1}F_{NP}(n'nhp,\lambda). \quad \text{(A8)}$$

A similar formula is easily obtained for the second term. Combining the two terms, we finally obtain Eq. (11).

Let us now take a 2^λ-pole magnetic operator $\hat{O}_{\lambda\mu}$. In addition to the nonvanishing zero-order (bare) matrix elements $\langle\psi_{J'M'}{}^{E'}|\hat{O}_{\lambda\mu}|\psi_{JM}{}^E\rangle$, we find in this case the first-order terms of the virtual excitation of the core neutrons corresponding to the terms of Eq. (11) with F_{NP} replaced by the antisymmetrized elements F_{NN} of the valence-neutron–core-neutron interaction. In deriving the latter formula, in the same way as for Eq. (6), all the contractions between the neutron creation and annihilation operators ($c^\dagger$ and c) are to be made in the matrix elements

$$\langle\psi_{J''M''}{}^{E''}|\sum(-)^{j_h-m_\chi}(\mathrm{ph};\pi-\chi|\lambda''\mu'')\times c_\chi{}^\dagger c_\pi c_\nu{}^\dagger c_{\bar{\nu}}{}^\dagger c_{\bar{\nu}'}c_{\nu'}|\psi_{JM}{}^E\rangle,$$

i.e., π is contracted with ν and $\bar{\nu}$, while χ is contracted with ν' and $\bar{\nu}'$ (χ and π are distinct as a hole and a particle, respectively).

APPENDIX B: REMARKS ON EXPLICIT TREATMENT OF CORE CONFIGURATIONS IN QTD

In the QTD approximation it is still possible to include explicitly at least some of the excited configurations of the core nucleons. Even the limitation to a relatively small number of extra subshells in Sn (in addition to those of the valence neutrons) does not render such a model very reasonable.

First, an explicit treatment of the core nucleons means that, to avoid double-counting, no corresponding core-polarization corrections must be included. It is readily found that for bare realistic two-body forces (K matrix), one finds generally only a weak Bardeen-Cooper-Schrieffer (BCS) pairing effect even for the subshells close to the Fermi level, i.e., the corresponding energy gaps and single-qp energies are small.[5,6] It is then clear that, in the circumstances, the quasiparticle approach is bound to fail and an exact shell-model approach is necessary. On the other hand, we know that the Cooper pair-elements of the core-polarization corrections to the effective nuclear force are large. Consequently, they must and in fact can be treated on the same footing with the corresponding bare elements, i.e., by the BCS method. The remaining (residual) parts of the qp-transformed Hamiltonian are then relatively weak and can be treated by a Tamm-Dancoff-type approximation.

As an illustration we give here the numerical prediction of $B(E2, 2_1{}^+\rightarrow 0_1{}^+)$ in such a simple extended QTD. We have considered 10 neutron and 10 proton subshells in Sn, i.e., the subshell $1g_{\frac{9}{2}}$ and the entire major shell $(2p,1f)$ in addition to the five valence neutron subshells. The numerical values of all the sp parameters involved were exactly those of the main text of the present paper, i.e., those of Ref. 23, and the same Yale-Shakin nuclear force was employed.

The calculated energy spectrum of Sn^{116} is in marked disagreement with experiment. For $B(E2, 2_1{}^+\rightarrow 0_1{}^+)$ we find only the very small value of $5.03e^2F^4$ (with the neutron effective charge exactly=0). One extra reason why the model fails to reproduce even the right order of magnitude of the observed value is that the six subshells of the upper major shell should be the most important for a collective effect of all the single-proton transitions in $B(E2, 2_1{}^+\rightarrow 0_1{}^+)$, and the $(2s,1d)$ major shell should be included. In fact, one must not compare the last result with the results of Table III but, if at all, rather with the corresponding model where only the five core subshells ($1g_{\frac{9}{2}}$ and pf) are included both in calculating the QTD eigenvectors and the effective charge matrix (ECM). In this case we obtain

$$B(E2, 2_1{}^+\rightarrow 0_1{}^+)=50.19e^2F^4,$$

which is about five times smaller than the value calculated in our theory with all the sp levels between the magic numbers 8 and 126.