Rare-Gas-Induced g_J Shifts in the Ground States of Alkali Atoms*

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Based on the formal similarity between perturbations governing gyromagnetic ratio shifting and spin relaxation of alkali atoms in collisions with rare-gas atoms, a relatively direct semiempirical estimate of the g_J shift as a function of buffer-gas density can be achieved. The expression $\Delta g_J/g_J = -(4/3\pi)^{1/2}\, n\, \sigma_{\rm dis}^{1/2}\sigma_{\rm kin}$ is obtained for the fractional shift in g_J, where n is the rare-gas number density, $\sigma_{\rm kin}$ the alkali-rare-gas kinetic cross section, and $\sigma_{\rm dis}$ the cross section for alkali spin disorientation. With this relationship, we predict a fractional g_J shift for Rb by He equal to -5.3×10^{-11} Torr^{-1}, as compared with a recent value, $-(5\pm3)\times10^{-11}$ Torr^{-1}, obtained through measurement by Hayne, Robinson, and their co-workers.

The properties of alkali atoms in their ground state traditionally have been a subject of intense study in optical pumping experiments. Interest in rare-gas-induced changes in these properties (e.g., hyperfine state populations and frequency separations) has been motivated, in part, by the widespread use of rare-gas buffers in reducing the diffusion of optically oriented atoms. As an example,[1] rare-gas-induced hyperfine frequency shifts of alkali atoms in their ground state have been accessible to experiment and have been theoretically understood for a number of years. Until recently, however, rare-gas-induced shifts in the gyromagnetic ratio have eluded detection in optical pumping experiments. Within the past year, though, Hayne, Robinson, White, and Hughes[2] have measured a g_J shift in both Rb^{85} and Rb^{87}, with He as the buffer gas. Accordingly, in the present communication, we develop a method for the semiempirical calculation of g_J shifts in terms of the cross section for rare-gas-induced alkali-atom electronic-spin disorientation and the alkali-rare-gas kinetic cross section. The results are then compared with experiment for the Rb-He system.

The Hamiltonian which governs a colliding Rb-noble-gas binary pair in the presence of an externally applied uniform magnetic field $\vec{H}$ can be represented, for our purposes, as

$$\mathcal{H}=\mathcal{H}_0(t)+(g_J{}^0\beta/\hbar)\vec{H}\cdot\vec{S}+(\beta/\hbar)\vec{H}\cdot\vec{L}+\mathcal{H}_{\rm so}\,, \qquad (1)$$

where $\mathcal{H}_0(t)$ represents the full nonmagnetic Hamiltonian for the colliding pair, (assuming classical paths during collisions), $g_J{}^0$ ($\cong 2$) is the electronic gyromagnetic ratio for an isolated Rb atom in the ground state, β is the Bohr magneton, $\vec{S}$ is the total spin operator, $\vec{L}$ is the total electronic orbital angular momentum (relative to the center of mass of the binary pair) operator, regarded as having zero expectation for separated binary pairs, and $\mathcal{H}_{\rm so}$ is the total spin-orbit energy operator. In a previous paper,[3] the author concerned himself with the solution to a mathematically similar problem (rare-gas-induced Rb spin disorientation), in which the total Hamiltonian was

$$\mathcal{H}=\mathcal{H}_0(t)-\vec{K}\cdot\vec{L}/I(t)+\mathcal{H}_{\rm so}\,, \qquad (2)$$

$\vec{K}$ being the (classical) angular momentum of translation for the alkali-atom–rare-gas pair, and $I(t)$ the time-dependent instantaneous value of the moment of inertia for the pair. In I, it was shown that the last two terms in Eq. (2) have the same effect as an equivalent (second-order) coupling between $\vec{K}$ and $\vec{S}$ of the form

$$\mathcal{H}_{\rm eff}{}'=\gamma(t)(\vec{K}\cdot\vec{S}-K_zS_z)/I(t), \qquad (3)$$

where γ and I depend on t only through the magnitude of the instantaneous atomic separation, the z axis taken to be instantaneously parallel to the interatomic line. In the present problem, therefore, we can write, instead of Eq. (1),

$$\mathcal{H}_{\rm eff}=\mathcal{H}_0(t)+(g_J{}^0\beta/\hbar)\vec{H}\cdot\vec{S}-(\beta/\hbar)\gamma(t)(\vec{H}\cdot\vec{S}-\mathcal{H}_zS_z). \qquad (4)$$

Since we are interested only in first-order applications[4] of the last term in Eq. (4), the magnetic interactions are equal in their effect to

$$\mathcal{H}_{\rm eff}{}'=(\beta/\hbar)[g_J{}^0-\tfrac{2}{3}\langle\gamma(t)\rangle_t]\vec{H}\cdot\vec{S}\,, \qquad (5)$$

$\langle\ \rangle_t$ denoting the ensemble time average, so that the fractional g_J shift is simply

$$\Delta g_J/g_J=-\tfrac{1}{3}\langle\gamma(t)\rangle_t\,. \qquad (6)$$

Now $\gamma(t)$ is significant only during short-range interactions,[3] in which case, for single collisions, we have

$$\int_{\rm coll}\gamma(t)dt\cong0, \qquad b>b_0\,,$$
$$\cong(b_0/\overline{V})\gamma(b_0),\quad b\le b_0 \qquad (7)$$

$\overline{V}$ being the mean impact velocity, b the impact parameter, and b_0 an effective distance inside which repulsive short-range interactions dominate, approximately equal to the Lennard-Jones radius. The frequency of hard impacts for each Rb atom is $n\overline{V}\sigma_{\rm kin}$, n being the number density of rare-gas atoms and $\sigma_{\rm kin}$ the gas-kinetic cross section. Therefore

$$\langle\gamma(t)\rangle_t\cong n\overline{V}\sigma_{\rm kin}\int_{\rm coll}\gamma(t)dt$$
$$\cong n\sigma_{\rm kin}b_0\gamma(b_0) \qquad (8)$$

follows. Finally, in I it was shown that

$$\sigma_{\rm dis}\cong[\gamma(b_0)^2/12]\sigma_{\rm kin}. \qquad (9)$$

From Eqs. (6), (8), and (9), and the fact that $\sigma_{\text{kin}} \cong \pi b_0^2$, we obtain, for the fractional shift,

$$\Delta g_J/g_J = -(4/3\pi)^{1/2} n\sigma_{\text{dis}}^{1/2}\sigma_{\text{kin}}. \tag{10}$$

(While $\Delta g_J/g_J$ is proportional to phase shifts induced during hard collisions, σ_{dis} depends on the *squares* of similar phase shifts; hence $\Delta g_J/g_J \propto \sigma_{\text{dis}}^{1/2}$.)

The necessity for calculating $\gamma(R)$ from first principles, which could have involved significant errors, has been averted by our use of the disorientation cross section, which is known empirically in many cases. Errors in Eq. (10) arise only from assumptions regarding trajectories, kinetic cross sections, etc., so that Eq. (10) may be relatively accurate as a semiempirical relation. Numerical evaluation for the ground-state Rb-He system, using the value[5] $\sigma_{\text{dis}} = 3.3\times10^{-25}\,\text{cm}^2$ and[3] $\sigma_{\text{kin}} = 4.0\times10^{-15}\,\text{cm}^2$, yields

$$\Delta g_J/g_J = -5.3\times10^{-11}/\text{Torr (He)}.$$

which shows good agreement with the experimental value,[2] equal to $-(5\pm3)\times10^{-11}$/Torr (He).

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[2] G. S. Hayne and H. G. Robinson, private communication.

[3] R. M. Herman, Phys. Rev. 136, A1576 (1964), referred to as I.

[4] This represents the impact theory of line shifts, inasmuch as collisionally induced phase shifts responsible for the broadening and shifting of transitions between hyperfine magnetic states are much less than unity.

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Polarization Effects in Induced Emission of Bremsstrahlung

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The effect of induced bremsstrahlung is studied with respect to photon intensity and polarization. It is shown that the polarization effect is of importance when net induced emission occurs.

1. INDUCED EMISSION

It is well known from field theory that any particle reaction involving the emission of a photon may be enhanced by the mere presence of other photons in the same state as the one emitted. We consider here the process where an electron of energy E and momentum $\vec{p}$ is deflected by a nucleus in the presence of a radiation field consisting of $n_{\vec{k}}$ identical photons with frequency ω, wave vector $\vec{k}$, and polarization vector $\vec{\epsilon}$. The electron can emit or absorb photons of this kind, provided $\hbar\omega < E - mc^2$, by free-free transitions in the Coulomb field of the nucleus. The probabilities per unit time for emission and absorption are

$$d^2w_e = \frac{2\pi}{\hbar}(n_k+1)|M_e|^2\frac{p'E'}{(2\pi)^3}d\Omega' ,$$

$$d^2w_a = \frac{2\pi}{\hbar}n_k|M_a|^2\frac{p'E'}{(2\pi)^3}d\Omega', \tag{1}$$

where M_e and M_a are the matrix elements for emission and absorption respectively. The final electron kinetic energy E' and momentum $\vec{p}'$ are given by

$$E' = E \mp \hbar\omega ,$$

$$\vec{p}' = \vec{p} \mp \hbar\vec{k} - \vec{q} , \tag{2}$$

where the upper sign refers to emission, and $\vec{q}$ is the momentum transferred to the nucleus.