## Effects of a Tilted Magnetic Field on a Two-Dimensional Electron Gas\*

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n-type inverted silicon surfaces were studied to examine properties of a two-dimensional electron gas as well as those properties of silicon itself. The effect of a magnetic field tilted up to  $\pm 90^{\circ}$  from the normal to the (100) Si surfaces was measured between 1.3 and 4.2°K and up to 90 kOe. The electric field associated with the *n*-type inversion layer quantizes the surface electrons and they behave as a two-dimensional electron gas. It was found that the oscillatory magnetoconductance, due to magnetic quantization into Landau levels, depends solely upon the normal component of the magnetic field with respect to the surface. The observed splitting due to spin appears to be determined by the total magnetic field. The parallel component of the magnetic field was found to have a negligible effect on the surface quantization. The cyclotron mass, measured from the temperature dependence of the oscillation amplitude, is equal to the transverse mass of the electron in silicon and is independent of the tilt angle. The spin splitting can be quantitatively identified at certain angles where the Landau splitting is twice that of the spin splitting. The Landé g factor determined in this way was found to be substantially larger than 2 and decreases with the increasing surface electron density. The g factor has values between 3.25 and 2.47 for a surface electron density ranging from  $1 \times 10^{12}$  to  $6 \times 10^{12}$  cm<sup>-2</sup>.

**P**REVIOUS experiments<sup>1</sup> of oscillatory magnetoconductance in silicon surfaces indicate that under the appropriate conditions n-type surfaces behave as a two-dimensional electron gas (2DEG) due to the normal electric field quantization at the surface. We have studied such a system to examine the properties of a 2DEG as well as to further characterize its properties by themselves. In particular, we have studied the effects of a magnetic field when it is not applied perpendicular to a 2DEG. Although the case of an inverted surface has been treated in a general way,<sup>2</sup> we shall first describe a simple model of a 2DEG in which the essential results are more readily understood. We shall then consider the results of the specific treatment for the inverted *n*-type silicon surface.

For our 2DEG system, which exists in a threedimensional world, we choose a system which is isotropic with mass m and free to move in x and y, and whose z position is given by a Lorentzian wave function centered about z=0 with a half-width chosen such that  $\hbar/2m(\Delta z)^2$  is finite. The Hamiltonian (ignoring spin) is written,

$$H = \frac{1}{2m} [(p_x - qA_x/c)^2 + (p_y - qA_y/c)^2],$$

and with  $B_z$  as the normal field and  $B_x$  as the parallel field, we choose a vector potential  $A = (-B_z y, -B_x z, 0)$ . The point that the Hamiltonian is separable, and a product wave function can be written, is the key to understanding the results. In this case the y dependence of the Hamiltonian is a simple harmonic oscillator type dependence, and the associated eigenvalues are well known:  $E_n = (n + \frac{1}{2})qB_z\hbar/mc$ . The fact that the cyclo-

tron frequency is determined by  $B_z$  only can be viewed as just Bohr-Sommerfield-type quantization of the flux threading the real space orbit. There is left in the Hamiltonian a term proportional to  $z^2 B_x^2$ . Since we have already described the dependence on z of our wave functions as a Lorentzian, the expectation value of this term just depends on  $B_x$  and  $\Delta z$ , and is a shift in the ground-state energy.

While the Landau splitting depends only on the normal field, one expects the energy due to the electron spin to depend on the total field. Now one usually speaks of spin splitting of Landau levels; however, we can alternatively talk about two electron bands in the presence of a field separated in energy by the spin splitting and possibly further split by Landau levels. This point of view of two spin bands is best illustrated in the case where the magnetic field is parallel to the 2DEG and the Landau splitting is identically zero. In this case when both bands are occupied, the spin-down band always has more electrons (equal to the spin splitting times the density of states) than the spin-up band. The relative spacing of Landau and spin levels as a function of angle is illustrated later.

In an isotropic two dimensional system at T=0, the density of states per unit area in k space and per unit area in real space is given by  $(2\pi^2)^{-1}$ , including a factor 2 for spin. The density of electrons  $n_{2\text{DEG}}$  occupies an area  $\pi k^2$ , where  $k^2/2\pi = n_{2DEG}$ . Bohr-Sommerfield quantization of the quasimomentum  $\hbar k - qA/c$  over an orbit in real space and scaling the real space orbit to the orbit in k space by  $(qH_1/c)^2$  leads to  $\hbar k_n^2 = 2(n+\frac{1}{2})$  $\times qH_{\perp}/c$ , where  $H_{\perp}$  is the field normal to the 2DEG. (It is of course well known that electrons in crystals experience B and not H. We are dealing with materials in which the magnetization is negligibly small, and in the system of units employed here, B = H. Therefore, we follow the customary procedure of using B and Hinterchangeably, favoring the use of H as is the usual procedure, unless the use of B is more illustrative.) The

<sup>\*</sup> Some of the results of this work were presented in Bull. Am.

 <sup>&</sup>lt;sup>1</sup> A. B. Fowler, F. F. Fang, W. E. Howard, and P. J. Stiles, Phys. Rev. Letters 16, 901 (1966); J. Phys. Soc. Japan Suppl. 21, 331 (1966).
 <sup>2</sup> F. Stern and W. E. Howard, Phys. Rev. 163, 816 (1967).



FIG. 1. Angular dependence of the period of the gate voltage ( $\propto$  surface electron density) in the surface-oscillatory magnetoconductance at 42 kOe for a sample of the gate oxide 1150 Å thick.

allowed momentum states are discrete levels (Landau levels and not Landau subbands as in three dimensions) and must be highly degenerate, in order to preserve the average density  $(2\pi^2)^{-1}$ . In fact, each level must contain the number of states  $qH_1/\pi\hbar c$  which existed between levels before the magnetic field was applied. Notice that this is independent of energy, contrary to the case in three dimensions. Therefore, both the energy and the degeneracy of the levels are proportional to  $H_{\perp}$ . When  $q\hbar H_1/mc > kT, \hbar/\tau$ , where  $\tau$  is the relaxation time, one would expect that the conductance of the electron gas should oscillate with a constant periodicity in the electron density at constant magnetic field, or in  $1/H_1$  with constant electron density.

The physical system that was studied was an inverted (100) surface of a p-type silicon single crystal. The six conduction-band minima are not equivalent in the presence of the normal electric field associated with the inversion layer. The quantization by the electric field removes the degeneracy3 because the mass perpen-



FIG. 2. The oscillatory behavior of the transconductance (or field-effect mobility) as a function of  $1/\cos\theta$ , where  $\theta$  is the angle between the magnetic field and the normal to the surface for H = 42 kOe and  $V_q = 10$  V. Uniform  $(1/\cos\theta)$  periodicity is shown.

<sup>3</sup> F. F. Fang and W. E. Howard, Phys. Rev. Letters 16, 797 (1966).

dicular to the surface is the longitudinal mass for two of the minima, and the transverse mass for the other four. Because the longitudinal mass is heavier than the transverse mass, the two-fold subband lies at lower energy than the fourfold subband. Since we work only where this lowest subband is populated, we are effectively working with a simple 2DEG which has an additional degeneracy because there are two valleys present. The effective mass of the electrons parallel to the surface,  $m_{11}$ , is expected to be the transverse mass of the conduction band of silicon. In the presence of magnetic field, Stern and Howard<sup>2</sup> have previously shown that the expression for the energy, neglecting spin, is given by

$$E = E'' + (n + \frac{1}{2}) \frac{qB_{\perp}}{m_{||}} \frac{h}{c} + \frac{1}{2} \frac{qB_{||}^{2}}{c} \frac{(\bar{z})^{2} - (\bar{z}^{2})}{m_{||}},$$

where E'' is the ground-state energy due to the electric field quantization in the z direction. They estimate the corrections due to the third term (which involves only averages of the quantized position) to be negligible under the present experimental conditions.

In what follows, we describe the effects of a magnetic field at an angle  $\theta$  (from  $\theta = 0$  to  $\theta = 90^{\circ}$ ) to the (100) surface in a temperature range 1.35-4.2°K. Most of the samples used in this experiment are *n*-channel circular insulated gate field-effect transistors (FET)<sup>4</sup> on (100) Si surfaces. The samples were prepared by the usual photolithographic technique. The substrates are p-type Si with room-temperature resistivity about 130  $\Omega$  cm. The diffused  $n^+$  surface contacts (source and drain) are coaxial with distance  $L = 10 \mu$ . The circumference of the conducting channel  $W = 500 \mu$ . The gate insulator is silicon dioxide having a thickness  $\delta$  (ranging from 1000 to 5500 Å for different samples).

The oxides were grown in dry oxygen. The oxide charges in these samples are about  $4.4 \times 10^{11}$  cm<sup>-2</sup>. The surface states are negligible beyond threshold, i.e., after the onset of surface channel conduction.<sup>5</sup> Thus, the surface electron density can be accurately determined by  $n_s = K_{ox}(V_g - V_0)/4\pi q\delta$ , where  $K_{ox}$  is the dielectric constant of the oxide,  $V_g$  is the gate voltage,  $V_0$  is the threshold voltage, q is the electronic charge, and  $\delta$  is the thickness of the oxide. It should be noted here that the threshold voltage  $V_0$  is measured at 77°K. since at low temperature  $(<4.2^{\circ}K)$  the formation of bond states<sup>2</sup> near the threshold and subsequent screening effect beyond threshold make the  $V_0$  determination difficult. From the extrapolation of the observed Landau level versus magnetic field<sup>1</sup> spectra to zero magnetic field, the threshold at low temperature is indeed identical to that at 77°K. We also employed a linear rather than a circular sample to check on azimuthial orientation effects. A rotating sample holder was employed to tilt

<sup>&</sup>lt;sup>4</sup> For the operation of these devices see, e.g., S. R. Holstein and F. P. Heiman, Proc. IEEE **51**, 1190 (1963). <sup>5</sup> F. F. Fang and A. B. Fowler, Phys. Rev. **169**, 619 (1968).

the sample surface with respect to an axial magnetic field. The calibration was accurate to 0.8%, and uncontrollable backlash was less than  $0.5^{\circ}$ .

At a moderate magnetic field where the spin splitting was sufficiently smaller than splitting of Landau levels and was not resolved, it was verified that the period of the surface oscillatory magnetoconductance depends solely upon the normal component of the magnetic field with respect to the surface. The 2DEG electron density  $n_s$  is proportional to  $V_g - V_0$ , where  $V_g$  is the gate voltage and  $V_0$  is the gate voltage threshold. Since the Landau-level degeneracy is proportional to  $H_{\perp}$  $=H\cos\theta$ , the difference in gate voltage between oscillations,  $\Delta V_{\theta}$ , is proportional to  $\cos\theta$  for fixed *H*. Figure 1 shows the angular dependence of the period of the surface-oscillatory magnetoconductance at 42 kOe. The solid line is a cosine curve normalized at  $\theta = 0^{\circ}$ . The fact that  $\Delta V_q \propto H_1$  is observed verifies that the surface quantization due to the normal electric field is unperturbed by the parallel component of the magnetic fields.

Knowing the angular dependence of the Landau splitting, one may perform a Shubnikov-de Haas type of experiment by fixing both the magnetic and the surface electric fields and rotating the magnetic field with respect to the surface. Since in this way the surface electron density  $n_s$  remains constant and the Landau splittings depend only on  $H_1$ , one expects periodicity in  $1/H_1$  in Shubnikov-de Haas oscillations with period  $\Delta(1/H_1) = g_v q / \pi \hbar c n_s$ , where  $g_v = 2$  is the valley degeneracy. Therefore, there is an angular periodicity with period  $\Delta(1/\cos\theta) \propto 1/n_s$ . Figure 2 is a plot of the transconductance, or field effect mobility, at  $V_q = 10$  V and 42 kOe as a function of  $1/\cos\theta$  for a 1150 Å-thick gate-oxide sample. The constant period in this measurement clearly establishes the dependence of the quantization on  $H_1$  only. We have established that there is no variation within experimental accuracy in the above dependence when the magnetic field is rotated in different crystalline planes by experiments on a linear sample.

Previous measurements<sup>1</sup> of the effective mass of the inverted surface electrons with the magnetic field normal to the surface gave results which were from 5 to 10% higher than that reported from bulk measurements of the appropriate mass. The mass is measured from the temperature dependence of the amplitude of the oscillation of the transconductance  $G_m$ . The theoretical expression with which the experimental data were compared is of the form<sup>6</sup> amplitude  $\propto T/\sinh(2\pi^2kT/\hbar\omega_c)$ , where  $\omega_c = qH_1/m_{11}c$ , which is appropriate to the magnetoconductance variation for a three-dimensional gas. Howard<sup>7</sup> has shown for two dimensions that for a small sinusoidal variation on a constant density of



FIG. 3. Temperature dependence of the oscillation amplitude at H=84 kOe and  $\theta=57^{\circ}$  for  $n_s\simeq 2\times 10^{12}$  cm<sup>-2</sup>. The circled dots are the experimental points. The lines are the calculated curves for different mass values and g values.

states the same expression is obeyed. We have measured the mass at  $\theta = 57^{\circ}$  and  $71.5^{\circ}$  for  $n_s \simeq 2 \times 10^{12}$  cm<sup>-2</sup>, where the spacing of Landau and spin levels is uniform and obtain  $m_{11}=0.185\pm0.005m_0$ . Figure 3 is a plot of the experimentally measured amplitude as a function of the temperature, measured at  $\theta = 57^{\circ}$ . Normalized



FIG. 4. Transconductance oscillation in  $1/\cos\theta$  similar to Fig. 2 at 84 kOe. The fundamental and the second harmonic of the oscillation are due to Landau levels and their resolved spin splitting.

<sup>&</sup>lt;sup>6</sup> E. N. Adams and T. D. Holstein, J. Phys. Chem. Solids 10, 254 (1956).

 $<sup>^{7}</sup>$  We are indebted to W. E. Howard for communicating to us his unpublished results on this point.



FIG. 5. Magnetic spectrum showing the Landau-level separations and the spin splitting as a function of the tilt angle  $\theta$ .

theoretical dependences for  $m_{11}=0.185m_0$  and  $0.190m_0$ are shown for comparison. Our value agrees with values measured on bulk material within experimental error. When we measured the temperature dependence at other angles (including the case with *H* normal to the surface) and even though the oscillations are nearly sinusoidal we obtain values for  $m_{11}$  which are higher than  $0.185m_0$  by amounts up to 10%.

We believe that there is no real variation of the effective mass, and that its value is  $m_{II}=0.185m_0$ , which is essentially the same as the appropriate bulk effective mass, and that the previously reported results were obtained when the density of states was not a single, small, periodic variation of the density of states.

At higher fields where the spin splitting can be resolved, the angular dependence of the oscillation appears to show as expected that the spin splitting is determined by the total field, i.e., independent of the angle of the field. A plot of the field-effect mobility as a function of  $1/\cos\theta$  at 84 kOe and 1.38°K is shown in Fig. 4. The thickness of the oxide gate is 1150 Å and the gate voltage is 10.25 V. The fine structure at small angles is due to spin splitting. The fundamental of the oscillation has the period  $g_{v}H/\pi\hbar cn_{s}$ . The straight lines are the plots of the quantum numbers versus  $1/\cos\theta$  at which the corresponding peaks or valley of the oscillation occur. It is seen that the fundamental (or first harmonic) oscillation disappears near 60° and reappears again somewhat beyond this angle with the phase reversed. In the neighborhood of this angle only secondharmonic oscillations exist. We interpret this as the indication that near this angle the energy separations between neighboring levels are approximately equal. Furthermore, the separations are half that of the fundamental separation, the Landau levels at that angle. The general behavior as shown in Fig. 4 is observed for any given gate voltage at any magnetic field strength sufficient to resolve the spin splitting.

It was shown in Fig. 4 that at angles larger than about  $60^{\circ}$  we observed a phase reversal in the fundamental oscillations. This can be understood by examining the magnetic spectrum in Fig. 5. The pairs of parallel

lines are spin-split Landau levels. In this schematic diagram the Landé g factor is assumed to be independent of angle. The energy spacing between Landau levels decreases with  $\cos\theta$ . At the angle of tilt  $\theta_1$ , the energy spacing between Landau levels is twice that of the spin splitting. When the angle of tilt is smaller than  $\theta_1$ , the spin splitting is well separated by the Landau levels. The predominant parts of the fundamental oscillation are at the Landau levels. When the field is tilted at an angle larger than  $\theta_1$ , it is seen that spin levels of adjacent Landau states are closer than spin levels of a given Landau level and the former forms the fundamental oscillations. Thus the phases of the oscillations are reversed for the two situations. The phase reversal can be best demonstrated when the spin splitting is not resolved. In Fig. 6 we illustrate the magneto-oscillations in gate voltage for a sample of  $\delta = 5330$  Å. At H = 24kOe and  $\theta = 0^{\circ}$ , the spin splittings are too small to be resolved. One observes the usual oscillations due just to Landau levels. At H=72 kOe and  $\theta=70.5^{\circ}$ , where  $H_{\perp}$  is 24 kOe, one observes the expected oscillations with the period identical to that of the former conditions (H=24 kOe,  $\theta=0^{\circ}$ ). Since the energy separation of the spin levels of the adjacent Landau states  $(\hbar\omega_c - g\beta H)$ , where  $\beta$  is the Bohr magneton, is smaller than the spin splitting and cannot be resolved in this case, the phase of the oscillations is seen to be reversed. Since the scale for  $G_m$  is the same for both cases, and with the normal magnetic fields the same but the parallel fields being 0 and 58 kOe, we see that the parallel field has at most a small effect on the amplitude of the oscillations. We have also compared the cases for zero normal field, zero and 90 kOe parallel fields and find the results, although not identically the same, being reasonably so.

To examine the energy-level structures in detail near the angle, where the first-harmonic component is minimal, we measure the field-effect mobility as a function of the gate voltage (proportional to the surface electron density). A set of these oscillations for a sample of 5330 Å gate oxide near this angle is shown in Fig. 7.



FIG. 6. Transconductance oscillation with the gate voltage  $V_{\rho}(\sim n_s)$  for H=72 kOe at  $\theta=70.5^{\circ}$  and H=24 kOe at  $\theta=0$ .  $H_{\perp}$  is the same for both cases. The phase reversal of the two cases are shown.



FIG. 7. Transconductance oscillations with the surface-electron concentrations at 90 kOe for different tilt angles  $\theta$ . The arrows identify the surface-electron concentration at which the energy level separations are equal, i.e.,  $2g\beta H = \hbar\omega_e = (\hbar q H \cos\theta)/m_{11}c$ .

In each curve we mark a range of surface electron density within which the strength of oscillation appears to be uniform, i.e., the content of the fundamental component of the oscillation is minimal. Within these ranges we have identified the energy levels as being equally spaced. This corresponds to the condition that the energy between Landau levels is twice that of the spin splitting  $g\beta H$ .

The energy-level structure in a tilted field can be simply illustrated in an idealized magnetic spectrum shown in Fig. 5. At an angle  $\theta_1$ , where the energy spacings between the states are equal, one has  $\hbar\omega_c = 2g\beta H$ . Therefore g may be determined by  $(m_0/m_{11})\cos\theta_1$ . The g factors found in this way for the cases in Fig. 7 are the circular dots in Fig. 8. Since this method requires at least a few periods to identify  $\theta_1$ , the resolution for the corresponding surface electron density  $n_s$  is rather poor.

Previous experimental results indicate that the magnitude of the conductance oscillation depends on the magnitude of k (or the magnetic quantum number n) but not very much on the spin states. Therefore even when  $2g\beta H = \hbar \omega_c$ , the conductance will still have a residual first-harmonic component due to the dependence on quantum number n. When one works at sufficiently high fields so that level broadening is much less than the separation, this residual first-harmonic component is a small part on top of a large secondharmonic component. Such is the case in Fig. 7. However, to obtain data at lower  $n_s$ , we were forced to work at low fields to improve resolution, such that now the second-harmonic component is slightly less than the residual first-harmonic component. We identify the points which correspond to the condition  $2g\beta H = \hbar\omega_c$  as that surface density  $n_s$  where the phase relation between the second-harmonic and residual first-harmonic component is half-way between its phase when  $2g\beta H < \hbar\omega_c$ and its phase relation when  $2g\beta H > \hbar\omega_c$ . We have taken data at 30 kOe under such conditions and these data are plotted as square dots in Fig. 8. The variation of g value with  $n_s$  is clearly shown (assuming g does not depend on  $\theta$ ).

As a verification of the value of g, independent of the angular measurements, we refer to the temperature dependence of the amplitude shown in Fig. 3. We have identified that at this setting the energy-level spacing is uniform, and therefore we may also interpret the results as the temperature variation of the oscillation amplitude for levels spaced in energy by  $g\beta H$ , i.e., the amplitude is proportional to  $T/\sinh(2\pi^2kT/g\beta H)$ . The amplitude of the oscillation was measured near the gate voltage corresponding to  $n_s \simeq 2 \times 10^{12}$  cm<sup>-2</sup>. It is seen in Fig. 3 that the fit for g=2.8 is excellent. It should be noted that in this measurement the actual angle does not enter the evaluation, but just a measurement of the total field.

It has been suggested by Janak<sup>8</sup> that the anomalous g value and its dependence on the surface electron density may be accounted for by electron-electron interaction.

We have examined the case where a magnetic field which is applied at an arbitrary angle to the surface of an *n*-type inverted (100) surface, and found that the results are what one would expect for a 2DEG. We



FIG. 8. Landé g factor as a function of the surfaceelectron concentration.

<sup>8</sup> J. Janak (private communication).

find that the quantizing component of the magnetic field is just that which threads the real space orbit, and that aside from a possible but unobserved small change in the ground state energy, the component of the field that is in the plane of the 2DEG does not significantly effect the behavior of the electrons. We have established that the effective mass for electrons moving parallel to the surface is the same as in the bulk, independent of the

direction of the applied magnetic field. The fact that the Landé g value is substantially larger than the bulk value and depends on the density of the surface electrons remains to be quantitatively accounted for.

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## New Evidence for the Existence of Exciton Effects at Hyperbolic Critical Points\*

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The derivative of the reflectivity spectrum of InSb near the  $E_1$  and  $E_1+\Delta_1$  peaks has been measured at 77°K with a double-beam wavelength-modulation method which emphasizes singularities in the dielectric constant,  $\epsilon_1(\omega) + i\epsilon_2(\omega)$ . The experimental line shapes of  $d\epsilon_1/d\omega$  and  $d\epsilon_2/d\omega$  cannot be explained as being due to the generally accepted  $M_1$  (hyperbolic) critical points, but are intermediate between the line shapes for  $M_1$  and  $M_2$  critical points. This mixture of line shapes is, in view of our knowledge of the one-electron band structure of InSb, evidence for the contribution of the electron-hole interaction (exciton effects) to the observed optical spectra.

'HE existence of a significant contribution of the electron-hole Coulomb interaction (i.e., exciton effects) to the experimental optical spectra in the vicinity of  $M_1$  interband critical points has been suggested by several authors.<sup>1,2</sup> However, the experimental evidence available so far is not conclusive, and the situation is further complicated by difficulties in the theoretical treatment of this exciton problem.<sup>3,4</sup> The purpose of this paper is to show that wavelength-derivative spectra of transitions near  $M_1$  critical points in zincblende-type semiconductors, coupled with the present knowledge of the band structure of these materials, constitutes strong evidence for the importance of the electron-hole Coulomb interaction in the vicinity of  $M_1$  critical points.

The usefulness of optical-modulation methods for studying the nature of interband critical points has been emphasized by many authors.<sup>5-7</sup> These methods extract the critical-point structure from the large, uniform background and thus yield a more detailed picture of the singular line shape. Techniques which modulate the optical properties of the sample (e.g., electroreflectance $^{5,7}$ ) are particulally simple from an experimental point of view, since any structure due to the spectral dependence of the light-source intensity, optical system, or detector response is easily eliminated. However, the interpretation of these sample modulation experiments requires a theoretical description of both the optical properties of the sample and the effect of the modulating perturbation on these properties. The theoretical treatment of the modulation is especially complicated in cases where the perturbation destroys the translational invariance of the crystal (such as electroreflectance<sup>8</sup>). These difficulties in interpretation are obviated by using a wavelength-derivative method at the expense of experimental simplicity; a suitable double-beam system must be used to eliminate the derivative structure due to the spectral dependence of the source, optical system, and detector.

We have developed a sensitive double-beam wavelength-modulation spectrometer which eliminates most of the difficulties usually encountered in this technique.<sup>9</sup> The modulation of the wavelength is produced by a rotationally vibrating quartz plate<sup>10</sup> placed in the entrance

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