Light Scattering from a Plasma in a Magnetic Field

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The scattering of light from plasmas embedded in a homogeneous magnetic field B_0 is analyzed. In the geometry where the momentum (wave vector) transferred to the system is perpendicular to B_0, we discuss the influence of the Bernstein and upper hybrid modes on the scattering. The analysis is done in some detail for a wide range of plasma parameters. Particular emphasis is placed on possible interesting experimentally observable phenomena in both gaseous and solid-state plasmas.

INTRODUCTION

It is well known that the inelastic or Raman scattering of electromagnetic radiation from plasmas provides useful information about the spectrum and nature of its elementary excitations.[1] Such scattering has been intensively studied during the past decade.[2,3] Originally the technique was applied to gaseous plasmas[2] but more recently has been used to study plasmas in semiconductors as well.[3] In an ideal scattering experiment, a well-collimated beam of monochromatic radiation is incident on an almost transparent medium. A small fraction of the radiation is scattered. The spectrum of the radiation that comes off in a fixed direction is then analyzed. This spectral distribution, along with an associated angular distribution, provides the basic information contained in such a Raman scattering experiment.

The Raman scattering cross section is completely characterized (see Fig. 1) by the wave number ($\mathbf{k}=\mathbf{k}_{\text{in}}-\mathbf{k}_{\text{out}}$) and the frequency transferred ($\omega=\omega_{\text{in}}-\omega_{\text{out}}$) in the scattering event. Here $\mathbf{k}_{\text{in}}$ and ω_{in} ($\mathbf{k}_{\text{out}}$ and ω_{out}) are the incoming (outgoing) wave number and frequency, respectively. It is useful to classify the scattering according to the magnitude of $\mathbf{k}$ (fixed scattering angle θ) and then for this fixed θ to analyze the intensity of the scattered radiation as a function of ω (spectrum analysis). The wave number transferred to the system determines the spatial resolution that we looked at the system. If the wave number k is small relative to the Debye (or Fermi-Thomas) wave number ($k/k_D<1$) the scattering is from many electrons coherently. The spectrum in the small-k regime is directly related to the spectrum of collective excitations in the plasma. On the other hand, if $k/k_D\gg 1$, the scattering takes place from individual electrons. In this case, the spectrum measures single-particle properties of the plasma. For semiconductors the densities, temperatures, and effective masses of the carriers are such that k is small compared to the Debye or Fermi-Thomas wave vector, i.e., typically $k/k_D\sim 0.1-0.01$.[3] For gaseous plasmas it is possible to go from small k/k_D to k/k_D of the order of 10.[2]

In this paper, we consider the scattering of light from plasmas embedded in a homogeneous static magnetic field $\mathbf{B}_0$. The plasmas may either be the collection of mobile electrons in a semiconductor, treated in the one-band isotropic effective-mass approximation or the high-temperature gas plasmas found in a discharge. This problem has previously been treated by Salpeter[4] and by Farley *et al.*[5] These authors, however, were particularly interested in the scattering of microwaves from the ionospheric plasma. The parameters characterizing the ionospheric plasma, with its associated magnetic field, are quite different from those we will consider here.

The magnetic field adds a new dimension to the variety of physically interesting phenomenon that may be observed in the scattering. As the ratio of the electrons cyclotron frequency $\omega_c=eB_0/m^*c$ approaches other characteristic frequencies of the system, for example, the plasma frequency $\omega_p=(4\pi ne^2/m^*)^{1/2}$, the spectrum of fluctuations will be significantly modified in the plasma. This will, correspondingly, change the spectrum of scattered light. Since the magnetic field is easily varied (in a real experiment), an analysis of the spectrum for a range of magnetic fields will give new, unambiguous, and interesting information about the spectrum of fluctuations in the plasma. Although fluctuations can be excited at any angle relative to the field $\mathbf{B}_0$, we will only consider the particular geometry where the wave vector $\mathbf{k}$ is perpendicular to $\mathbf{B}_0$. This

[1] E. E. Salpeter, Phys. Rev. **120**, 1528 (1960); M. N. Rosenbluth and N. Rostoker, Phys. Fluids **5**, 776 (1962); P. M. Platzman, Phys. Rev. **139**, A379 (1965); A. L. McWhorter, in *Physics of Quantum Electronics*, edited by P. L. Kelley, B. Lax, and P. E. Tannenwald (McGraw-Hill Book Co., New York, 1966), p. 111.

[2] K. W. Bowles, Phys. Rev. Letters **1**, 454 (1958); S. A. Ramsden and W. E. R. Davies, *ibid.* **16**, 303 (1966); H. J. Kunze, E. Füner, B. Kronast, and W. H. Kegel, Phys. Letters **11**, 42 (1964).

[3] A. Mooradian and G. B. Wright, Phys. Rev. Letters **16**, 999 (1966); R. E. Slusher, C. K. N. Patel, and P. A. Fleury, *ibid.* **18**, 530 (1967); A. Mooradian and A. L. McWhorter, *ibid.* **19**, 850 (1967).

[4] E. E. Salpeter, Phys. Rev. **122**, 1663 (1961).

[5] D. T. Farley, J. P. Dougherty, and D. W. Barron, Proc. Roy. Soc. (London) **A263**, 238 (1961).

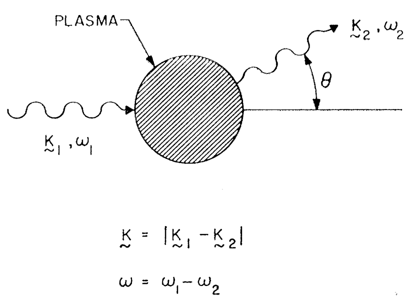

FIG. 1. Diagramatical representation of the incoherent scattering process. The dashed region represents the plasma, and the two wiggley lines the incoming and outgoing photon, respectively.

geometry is a particularly interesting one because it allows us to study the collective effects over an extremely wide range of k. In fact, in this geometry, the boundary in k space between the collective- and single-particle regime is not well defined.

In the general geometry where k has a component parallel to the magnetic field $\mathbf{B}_0$, the collective modes are strongly damped for $k_{||}\sim k_D$. This damping, Landau damping,[6] is due to the coupling of the collective mode to the single-particle continuum. The character of the scattered light spectrum completely changes as the quantity k/k_D approaches unity and for $k_{||}\geq k_D$ light scattering from collective modes is no longer observed. On the other hand, for $k\perp B_0$, we will show that it is possible to follow what is clearly a collective resonance for small k, out to large k where it merges continuously into a single-particle resonance at a multiple of the electron cyclotron frequency ω_c.

In the long-wavelength regime $k/k_D<1$, the light scattering cross section can be analyzed analytically. For $\mathbf{k}\perp\mathbf{B}_0$ the spectrum consists of a set of sharp lines. For arbitrary values of the parameter ω_p/ω_c most of the scattered light intensity is to be found in the upper hybrid mode near $\omega=(\omega_p^2+\omega_c^2)^{1/2}$.[7,8] The remaining light intensity appears in the set of modes known as Bernstein modes, which are located near the cyclotron harmonics $n\omega_c$ $(n>2)$.[4] The intensity of the nth-order Bernstein mode is smaller than that of the upper hybrid by a factor $(k/k_D)^{2n}$. However, when the upper hybrid mode becomes degenerate with one of the Bernstein modes [i.e., when $\omega_p=(n^2-1)^{1/2}\omega_c$, $n\geq 2$], the modes couple to one another. The coupling results in a splitting of the degeneracy,[4,7] and a sharing of the light intensity between the two modes for a small range of magnetic fields. In this range it may be possible to use light scattering as technique for studying the dispersion of the Bernstein modes.

For the case of large k/k_D, we have not found a simple closed-form analytic solution for the scattering cross section. We will, however, present numerical results that show how the spectrum of scattered radiation changes with increasing k. Initially, as k increases, the scattered radiation in the hybrid mode decreases while the intensity in the Bernstein modes increases. For large k the spectrum of light merges continuously with the single-particle excitations at ω_c, $2\omega_c\cdots$ etc. The intensity for large kR_c, where $R_c\equiv V_{\text{th}}/\omega_c$ is the cyclotron radius, is all contained in the first kR_c harmonics. The intensity in each line is roughly proportional to $1/kR_c$.

CALCULATION OF THE CROSS SECTION

To calculate the light scattering cross section we consider an interaction single-component electron gas immersed in a smeared neutralizing background of positive charge. This hypothetical system is a reasonably realistic model for many semiconductor plasmas and some gaseous plasmas whenever the vibrational modes of the ions (phonons, ion-acoustic waves, etc.) have frequencies that are very different from those of the electronic degrees of freedom.

For almost transparent plasmas the radiation field interacts weakly with the system, and the cross section for scattering may be computed in Born approximation. In this case we find[1,9]

$$d\sigma/d\omega d\Omega=V(d\sigma/d\Omega)_{\text{Th}}(1/2\pi)\times\int_{-\infty}^{+\infty}dt\, e^{i\omega t}\langle n_{\mathbf{k}}(t)n_{-\mathbf{k}}\rangle. \quad (1)$$

Here Ω is the solid angle, and

$$[(d\sigma/d\Omega)_{\text{Th}}\equiv(e^4/m^{*2}c^4)(\boldsymbol{\varepsilon}_1\cdot\boldsymbol{\varepsilon}_1)^2]$$

is the Thompson cross section, $\langle\ \rangle$ is the usual thermodynamic average over the operators appearing inside the brackets, and $n_{\mathbf{k}}$ is the electron density operator. In arriving at Eq. (1) we have neglected the $\mathbf{p}\cdot\mathbf{A}$ terms in the coupling of the electromagnetic field to the carriers.[9,10] This is a good approximation in nonrelativistic gaseous plasma, and is also valid in semiconductors whenever the effective-mass approximation applies.[11]

For systems in thermal equilibrium the Fourier transform of the density correlation function in Eq. (1) is simply related to the response function for the system and indirectly, through Maxwell's equations, to the retarded frequency and wave number dielectric tensor for the medium.[1] In order to simplify the results somewhat we note that in most light scattering experiments the quantity $\omega/kc\ll 1$, since the "velocity" ω/k of fluctuations in the plasma is typically of the order of the kinetic velocity of the particles in the plasmas. in this limit $(\omega/kc\ll 1)$ there is very little coupling between transverse and longitudinal density fluctuations in the plasma and Eq. (1) may be written, to the order $(\omega/kc)^2$, in terms of the longitudinal dielectric

[6] L. D. Landau, J. Phys. U.S.S.R. **10**, 25 (1946).
[7] I. B. Bernstein, Phys. Rev. **109**, 10 (1958).
[8] T. Stix, *Theory of Plasma Waves* (McGraw-Hill Book Co., New York, 1962), p. 32–43.
[9] D. Dubois and V. Gilinski, Phys. Rev. **133**, A1308 (1964).
[10] P. M. Platzman and N. Tzoar, Phys. Rev. **136**, A11 (1964).
[11] P. A. Wolff, Phys. Rev. **171**, 436 (1968).

function:

$$\epsilon(\mathbf{k},\omega)=\hat{k}\cdot\boldsymbol{\varepsilon}(\mathbf{k},\omega)\cdot\hat{k}.$$

One finds

$$\frac{d\sigma}{d\omega d\Omega}=V\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Th}}\frac{1}{\pi}\frac{\hbar}{1-\exp(-\beta\hbar\omega)}\frac{k^2}{4\pi e^2}\operatorname{Im}\frac{1}{\epsilon(\mathbf{k},\omega)}. \quad (2)$$

Equation (2) is the analog of the well-known expression for light scattering from a single-component plasma in the absence of a field.[1,4] The effects of the magnetic field are buried in the behavior of the dielectric function. In the random-phase approximation, the analytic form for the longitudinal dielectric function is well known both for degenerate and nondegenerate plasmas.[6,12] In our discussion we will use the semiclassical expression for ϵ, which is valid when $\hbar\omega_c\ll k_BT$ and $\hbar\omega_c\ll\epsilon_F$. This approximation neglects the oscillatory behavior associated with the quantization of the electrons' energy levels in the magnetic field.

To calculate the spectrum of scattered radiation from a nondegenerate plasma, we let $\hbar\to0$ and $kT/\epsilon_F\to\infty$ in Eq. (2). The expression for the scattering cross section becomes

$$\frac{d\sigma}{d\omega d\Omega}=N\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Th}}\frac{1}{\pi}\left(\frac{k}{k_D}\right)^2\frac{1}{\omega}\operatorname{Im}\frac{1}{\epsilon(\mathbf{k},\omega)}, \quad (3)$$

where $k_D{}^2=4\pi ne^2\beta\equiv\omega_p{}^2/V_{\mathrm{th}}{}^2$ is the Debye wave number squared and N is the total number of particles. The dielectric function $\epsilon(\mathbf{k},\omega)$ has previously been calculated in the random-phase approximation, and is given (for the geometry $\mathbf{k}\perp\mathbf{B}_0$) by[6]

$$\epsilon(\mathbf{k},\omega)=1-\frac{\omega_p{}^2}{\omega_c{}^2}\frac{1}{\lambda}\left\{[e^{-\lambda}I_0(\lambda)-1]+2\left(\frac{\omega}{\omega_c}\right)^2\sum_{n=1}^{\infty}\frac{e^{-\lambda}I_n(\lambda)}{(\omega/\omega_c)^2-n^2}\right\}. \quad (4)$$

Here ω_p and ω_c are the electron plasma and cyclotron frequencies, respectively, $\lambda=k^2V_{\mathrm{th}}{}^2/\omega_c{}^2$ and the I_n's are Bessel functions of the second kind. For $\mathbf{k}\perp\mathbf{B}$, the dielectric function has simple poles at $\omega=n\omega_c$, and Eq. (4) is real and finite for $\omega\neq n\omega_c$.

The component of the electrons motion along $\mathbf{B}_0$ is unaffected by the magnetic field. When $\mathbf{k}\cdot\mathbf{B}_0\neq0$ there may, in general, be some electrons whose velocity along B_0 will match the phase velocity (ω/k) of the wave. This leads to resonant or Landau damping of the wave. When $\mathbf{k}\cdot\mathbf{B}_0=0$ no such absorption is possible, i.e., $\epsilon(k,\omega)$ is real except for a set of discrete poles. For realistic systems we must consider the existence of other absorption mechanisms, such as electron-ion collisions or, in a solid, electron-lattice collisions. These introducing a phenomenological collision time τ. The effects will be included in our dielectric function by collision time is simply incorporated in Eq. (4) by writing

$$\epsilon(\mathbf{k},\omega)=1+(4\pi i/\omega)\sigma(\mathbf{k},\omega)$$

and replacing $\sigma(\mathbf{k},\omega)$ by $\sigma(\mathbf{k},\omega+i/\tau)$.

We next calculate the cross section, for the $\mathbf{k}\perp\mathbf{B}_0$ geometry, in the limit $k/k_D\ll1$. In this limit

$$\epsilon(\mathbf{k},\omega)\simeq1-\tilde{\omega}\frac{\omega_p{}^2}{\omega}\left[\frac{(1-\lambda)}{\tilde{\omega}^2-\omega_c{}^2}+\frac{\lambda}{\tilde{\omega}^2-4\omega_c{}^2}\right], \quad (5)$$

where $\tilde{\omega}=\omega+i/\tau$. It is clear from Eq. (3) that for large $\omega_c\tau$, the scattering cross section has peaks at the points

$$\epsilon(\mathbf{k},\omega)_{\tau\to\infty}=0. \quad (6)$$

In this long-wavelength approximation Eq. (5) has two roots which are solutions of a biquadratic equation, i.e.,

$$\omega_{1,2}{}^2=\tfrac{1}{2}\{5\omega_c{}^2+\omega_p{}^2\mp[(3\omega_c{}^2-\omega_p{}^2)^2+12k^2V_{\mathrm{th}}{}^2\omega_p{}^2]^{1/2}\}. \quad (7)$$

The mode at frequency ω_1 is called the upper-hybrid mode. The one at ω_2 is one of the infinite set of Bernstein modes.

In the limit of long wavelengths it is possible to calculate the integrated intensity in the sharp spectral distribution. Assuming $\omega\tau\gg1$, i.e., that $\operatorname{Im}(1/\epsilon)$ is essentially a δ function, we obtain

$$\left.\frac{d\sigma}{d\Omega}\right|_{\omega=\omega_{1,2}}\cong N\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Th}}\left(\frac{k}{k_D}\right)^2\frac{1}{2\omega_{1,2}{}^2}\left(\frac{\partial\epsilon(k,\omega)}{\partial\omega^2}\right)^{-1}_{\omega=\omega_{1,2}}. \quad (8)$$

The quantities $\omega_{1,2}$ are defined in Eq. (7) and $\partial\epsilon(k,\omega)/\partial\omega^2$ is easily calculated utilizing Eq. (5). When the two roots $\omega_{1,2}$ are nondegenerate, i.e., $\omega_p\neq\sqrt{3}\omega_c$ then we find that all of the scattered intensity, to leading order in λ, is contained in the root ω_1, i.e., the upper hybrid. The intensity in the scattered light near ω_1 is

$$\left.\frac{d\sigma}{d\Omega}\right|_{\omega=\omega_1}=N\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Th}}\left(\frac{k}{k_D}\right)^2\frac{\omega_p{}^2}{2(\omega_p{}^2+\omega_c{}^2)}. \quad (9)$$

The intensity in the Bernstein mode in this case is

$$\left.\frac{d\sigma}{d\omega}\right|_{\omega=\omega_2}\simeq N\lambda\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Th}}\left(\frac{k}{k_D}\right)^2\frac{9}{8}\left(\frac{\omega_c}{\omega_p}\right)^2\left(\frac{\omega_p{}^2}{3\omega_c{}^2-\omega_p{}^2}\right). \quad (10)$$

When $\omega_p\to\sqrt{3}\omega_c$ and $(\omega_p-\sqrt{3}\omega_c)/\omega_c\gg\lambda$ the intensity in the upper hybrid is approximately given by

$$\left.\frac{d\sigma}{d\omega}\right|_{\omega=\omega_1}=N\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Th}}\left(\frac{k}{k_D}\right)^2\times\frac{3}{8}. \quad (11)$$

As ω_p approaches within λ of ω_c the upper hybrid begins to couple to the Bernstein mode near the second cyclotron harmonic. This peak begins to grow in intensity and the upper hybrid begins to lose some of its intensity, the sum remaining roughly constant.

Very near the crossover point, when $\omega_p=\sqrt{3}\omega_c$, the intensity in the Bernstein mode is given by

$$\left.\frac{d\sigma}{d\Omega}\right|_{\omega=\omega_2}\cong N\left.\frac{d\sigma}{d\Omega}\right|_{\mathrm{Th}}\left(\frac{k}{k_D}\right)^2\frac{1}{8\omega_c{}^2}\left(\frac{1}{3\omega_c{}^2}+\frac{3\omega_c{}^2\lambda}{(\omega_2{}^2-4\omega_c{}^2)^2}\right)^{-1}. \quad (12)$$

[12] S. J. Buchsbaum and P. M. Platzman, Phys. Rev. **154**, 395 (1967).

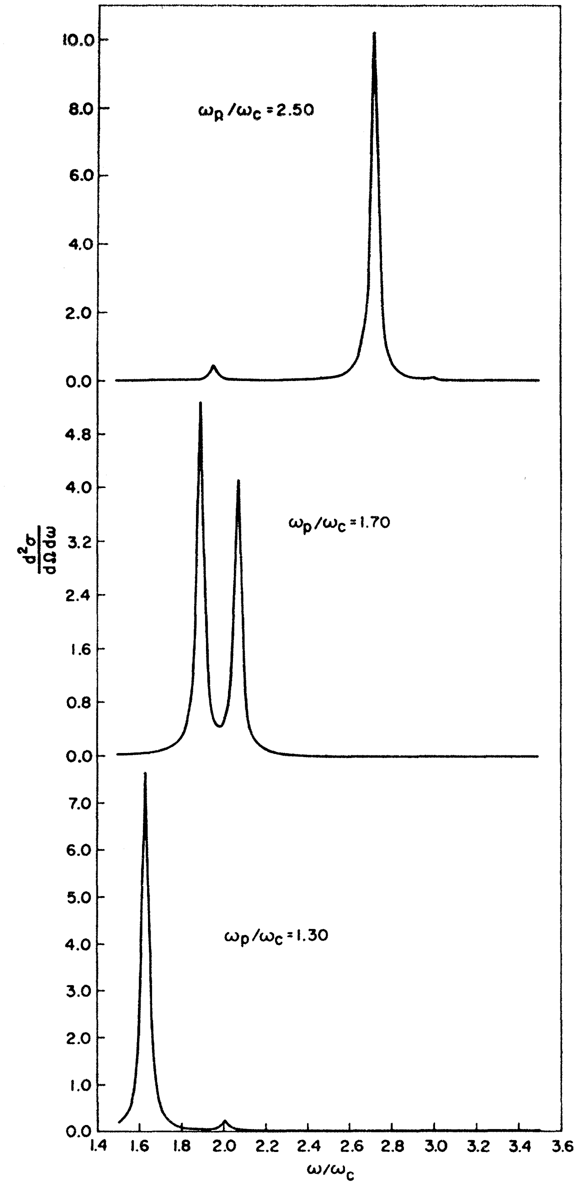

FIG. 2. The scattering cross section per particle per unit solid angle in units of one-tenth the Thompson cross section is plotted versus the frequency shift of the scattered light. Here $(k/k_D)^2=0.005$ and $\omega_c\tau=50$. When $\omega_p/\omega_c=1.3$ (top picture) most of the scattered intensity is in the upperhybrid mode at $\omega_p/\omega_c=1.65$ with small intensity at the cyclotron harmonica mode. For $\omega_p/\omega_c=1.7$ (middle picture) the two modes have similar intensities. For $\omega_p/\omega_c=2.5$ (lower picture) the intensity is in the upperhybrid mode at $\omega/\omega_c=2.7$.

When $\omega_p=\sqrt{3}\omega_c$, then the solution of Eq. (6) for ω_2 is given by

$$\omega_2^2=4\omega_c^2-\sqrt{3}kV_{\text{th}}\omega_p. \tag{13}$$

Substituting Eq. (13) into Eq. (12) we obtain the line intensity in the Bernstein mode when the ratio of ω_p/ω_c is near the crossover value the square root of three, i.e.,

$$\left.\frac{d\sigma}{d\Omega}\right|_{\omega_2}=\tfrac{3}{16}N\left(\frac{d\sigma}{d\Omega}\right)_{\text{Th}}\left(\frac{k}{k_D}\right)^2. \tag{14}$$

Equation (14) states that near the crossover point about one-half of the intensity has been transferred from the upper-hybrid mode to the nearby Bernstein mode at $\omega\cong2\omega_c$. It is easy to show, in this long-wavelength regime, that the transference of intensity from the upper hybrid occurs for all the Bernstein modes as the upper hybrid approaches the values $\omega=3\omega_c$, $4\omega_c$, $\cdots$ etc. From an experimental point of view this enhancement of the line intensity makes it possible to observe the Bernstein modes.

We can summarize our findings to this point in the following way: At long wavelengths when $\omega=2\omega_c$, $\text{Im}(1/\epsilon)$ has a pole with a residue proportional to λ. If we had considered higher powers of λ in our expansion of $\epsilon(k,\omega)$ we would have found that the higher Bernstein modes at $\omega=3\omega_c$, $4\omega_c$, $\cdots$ would have had intensities proportional to $(k/k_D)^2\lambda^2$, $(k/k_D)^2\lambda^3$, $\cdots$ etc. In the long-wavelength limit (as for most semiconducting plasmas) it will in general be difficult to observe scattering from the Bernstein modes.[13] However, if we

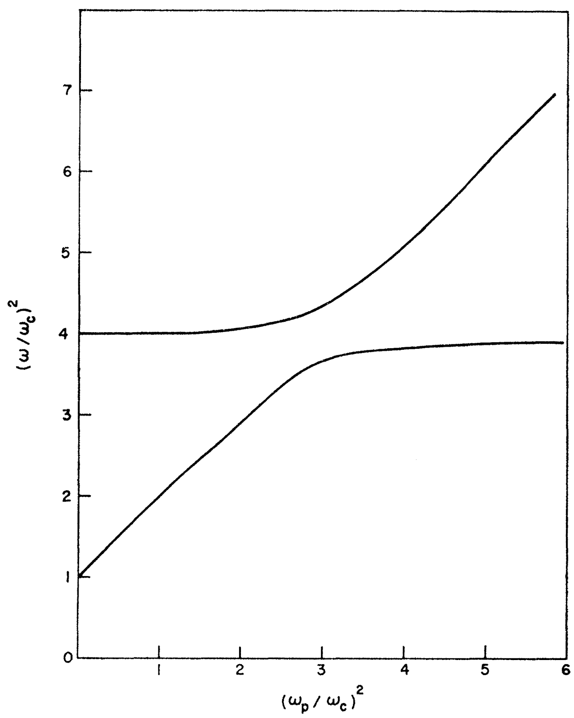

FIG. 3. The dispersion curve for the coupled upperhybrid and first Bernstein mode is plotted as a function of ω for $(k/k_D)^2\cong0.005$. The splitting at the crossover point $\omega_p/\omega_c=\sqrt{3}$, is $\Delta\omega/\omega_c=0.18$.

[13] This statement is only true in classical plasmas, i.e., those in which the electrons have an energy-momentum relation of the form $p^2/2m^*$. If band nonparabolicity is important light couples to fluctuations (Ref. 11) other than density fluctuations, and the Bernstein mode has a finite strength as $k/k_D\to0$.

choose our magnetic field so that one of the Bernstein modes becomes degenerate with the upper hybrid, i.e., $\omega_p = (n^2-1)^{1/2}\omega_c$, $n \geq 2$, then the two modes will couple with one another and the intensity will be fed from the upper hybrid into the degenerate Bernstein mode. This technique enables one to study the Bernstein modes in the long-wavelength regime.

Figure 2 illustrates these points. We have numerically evaluated Eq. (3) utilizing Eq. (4) and choosing $\omega_c\tau = 50$ $(k/k_D)^2 = 0.005$. The scattering cross section per particle per unit volume per unit solid angle in units of one tenth the Thompson cross section is plotted versus the frequency shift of the scattered light. When

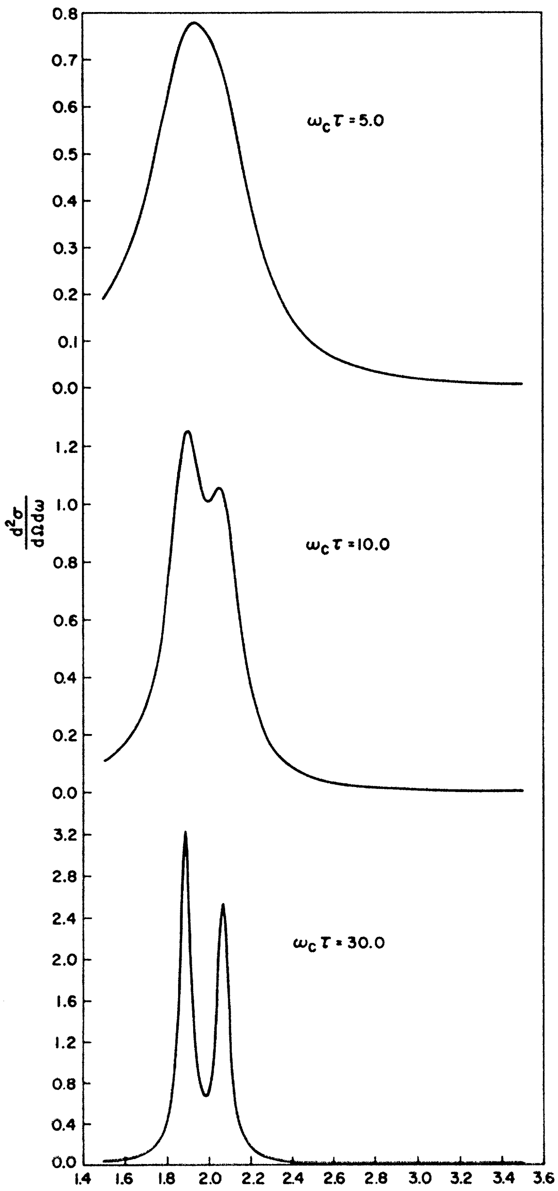

FIG. 4. The spectrum of scattered light is plotted as a function of ω with $(k/k_D)^2 = 0.005$ and $\omega_p/\omega_c = \sqrt{3}$ for $\omega_c\tau = 50$, 10, and 5.

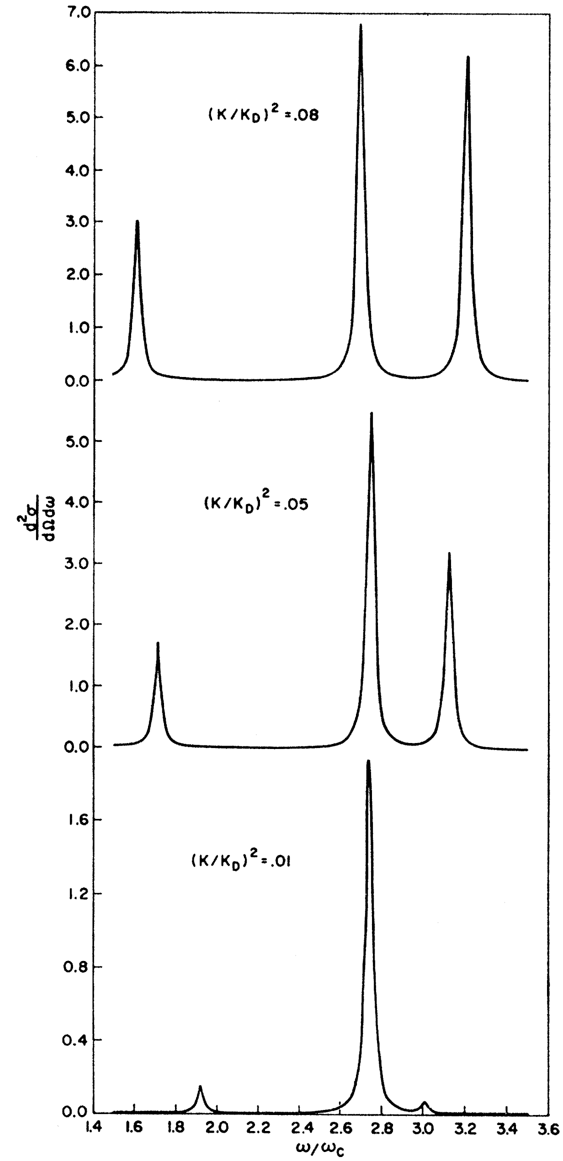

FIG. 5. The scattered spectrum in the neighborhood of the first two cyclotron harmonics for $\omega_p/\omega_c = 2.5$ and $\omega_c\tau = 50$, as a function of k/k_D. When k/k_D increases the mode at the first cyclotron harmonic moves toward ω_c, the upper-hybrid mode at $\omega/\omega_c = 2.9$ (for $k \approx 0$) moves toward $2\omega_c$.

$\omega_p/\omega_c = 1.3$ most of the intensity is in the upper hybrid at $\omega/\omega_c = 1.65$ with a small amount at $\omega = 2\omega_c$. As ω_p/ω_c approaches the $\sqrt{3}$ the intensity in the mode at $\omega = 2\omega_c$ grows until near the resonance, Fig. 2(b), the intensities in the two modes are approximately equal. When $\omega_p/\omega_c = 2.5$ the modes have crossed over Fig. 2(c) and the intensity is again in the upper hybrid at $\omega/\omega_c = 2.9$.

In the neighborhood of the crossover point $(\omega_p/\omega_c \cong \sqrt{3})$ the two modes "repel" one another and Eq. (7) gives a splitting

$$\Delta\omega/\omega_c \equiv (\omega_1 - \omega_2)/\omega_c = \tfrac{3}{2}\sqrt{3}(k/k_D) \qquad (15)$$

The interesting feature of Eq. (15) is that the splitting

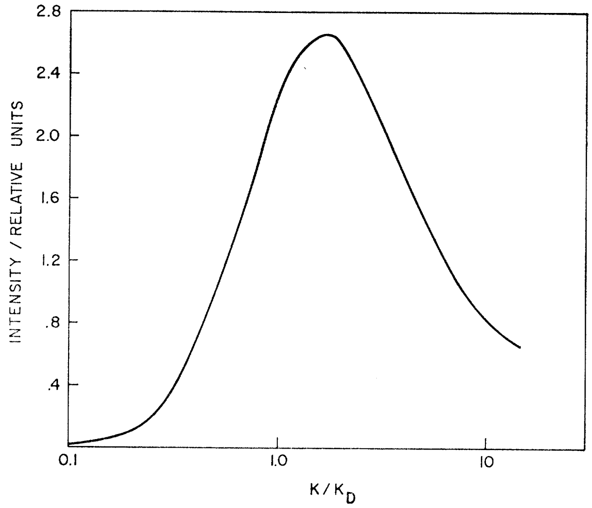

FIG. 6. The intensity of scattered light in the first Bernstein mode ($\omega=2\omega_c$ at $k/k_D\approx0$) is plotted against k/k_D for $\omega_p/\omega_c=2.5$. As we increase k/k_D the intensity increases until $k/k_D\sim2$, i.e., $kR_c\sim5$ and then decreases.

depends on nonlocal effects, i.e., finite k and is linearly proportional to k/k_D. Because of the small values of k/k_D achieved in light scattering experiments from semiconducting plasmas, there are (to date) no quantitative experimental results on these interesting nonlocal effects. In the absence of a magnetic field one is forced to look for the dispersion or collisionless damping of the plasmon to get information about finite k values. The dispersion of the collective mode [see Eq. (7)] is quadratic in (k/k_D) and thus exceedingly difficult to observe.[14] The effects are of the order of 1%. The Landau damping of the plasmon is exponentially small and gets mixed in with impurity effects.[13] The splitting of the Bernstein and upper hybrid should be an observable effect.

In Fig. 3 the dispersion relation for the coupled upper hybrid and $\omega=2\omega_c$ Bernstein modes for a fixed $(k/k_D)^2=0.005$ is plotted. The splitting at the crossover point is $\Delta\omega/\omega_c\cong0.18$, which agrees quite well with our approximate estimate Eq. (15). Since the splitting is of considerable interest from the experimental point of view, it is important to know how a finite collision time τ effects it. Such considerations tell us whether or not, with reasonable $\omega\tau$'s, one can experimentally resolve the splitting. In Fig. 4 we have plotted the scattered spectrum for fixed $(k/k_D)^2=0.005$ and a fixed $(\omega_p/\omega_c)\cong1.71$ for several values of $\omega_c\tau$. For an $\omega_c\tau\geq10$, the *two* lines are clearly resolvable. Since these $\omega_c\tau$'s can be achieved in both gas plasmas and semiconductors such as n-type GaAs, experiments similar to those we have proposed here seem feasible.

As k increases the problem of calculating the cross section and/or the dispersion relation of the modes is analytically intractable. In order to illuminate some of the features of the high-k portion of the spectrum, we have numerically evaluated the scattering cross section for a fixed $(\omega_p/\omega_c)=2.5$ and for several values of $(k/k_D)^2$. Figure 5 show the spectrum in the neighborhood of the first two cyclotron harmonics. As $(k/k_D)^2$ increases the upper-hybrid mode at $\omega/\omega_c=2.9$ for small $(k/k_D)^2$(see Fig. 2) moves over towards the second cyclotron harmonic. As it moves it loses intensity. The intensity of the Bernstein mode, which for small $(k/k_D)^2$ is located near the second cyclotron harmonic, increases and moves toward the point $\omega/\omega_c=1$. For large $(k/k_D)^2$, the peaks in the scattered spectrum occur near the points $\omega=n\omega_c(n\geq1)$. The intensity in each of the first kR_c of these modes is approximately the same and the sum of the intensities in all of the peaks remains nearly constant as k increases.

In Fig. 6 we have plotted the intensity in the Bernstein mode located at $\omega\simeq2\omega_c$ (when $k=0$) as a function of $(k/k_D)^2$, for a fixed $\omega_p/\omega_c=2.5$. There is an initial increase in intensity until a $kR_c\cong2$ is reached whereupon the intensity begins to fall off with increasing kR_c. This falloff can be understood as follows. At large kR_c there are more and more Bernstein modes which are being excited in the scattering. In the limit $k\rightarrow\infty$, it is possible[15] to show that the *total* integrated cross section (the intensity under all lines is a constant), i.e.,

$$d\sigma/d\Omega|_{k\rightarrow\infty}=N(d\sigma/d\Omega)_{\mathrm{Th}}. \quad (16)$$

Equation (16) states that as $k\rightarrow\infty$ the particles scattered as if they were a set of independent particles. Equation (16) is obviously true to lowest order in the plasma parameter (random-phase approximation) but is also valid for a gas of charged particles at arbitrary densities and temperatures.[14] Since the number of modes excited is roughly proportional to $\lambda^{1/2}$ we would expect the intensity in each mode to decrease.

CONCLUSIONS

We have shown that it is possible to study the Bernstein modes in plasmas even when $k/k_D<1$. This is accomplished by suitably picking the magnetic field so that the upper hybrid and Bernstein modes will couple. The dispersion of the Bernstein mode and its coupling to the hybrid is interesting because it depends linearly on nonlocal effects. The perpendicular geometery discussed in some detail here, allows one to follow the "sharp" resonances in the spectrum out to arbitrarily large k values. The behavior of these resonances will depend upon nonlocal effects in the plasma. The detailed observation, in semiconductors, of the phenomenon described here and comparison of it with the simple theory will be useful in determining the importance of band-structure effects. These effects have been completely neglected in the present treatment.

[14] B. Tell and R. Martin, Phys. Rev. **167**, 381 (1968).
[15] P. M. Platzman and N. Tzoar, Phys. Rev. **139**, 410 (1965).