Possibility of Photoinduced Superconductivity

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(Received 9 May 1968)

The possibility of interband electron pairing induced by the virtual exchange of quanta of two boson fields (photons and phonons) is theoretically examined. The system in question comprises two conduction bands which are, in general, separated by an energy gap E_{bc}. Radiative transitions between states of these bands are assumed to be allowed. Through appropriate canonical transformations, effective interband electron-electron interaction terms are obtained which involve, respectively, virtual exchange of (i) one photon (one-boson process), and (ii) one photon and one phonon (two-boson process). These are found to be attractive under certain conditions. The one-photon process is, however, rendered inconsequential owing to the very small volume of momentum space spanned by "interaction quantum" (photon) exchanged. The two-boson process, on the other hand, does not suffer from the above momentum-space restriction. Moreover, this latter process contains terms linear in the photon and the phonon occupation-number densities, and the reduction in the matrix elements can be compensated by boosting the boson occupation-number densities artificially. Estimates show that the photon-occupation-number term will be significant for a value of $n_{\lambda_0}/V \sim 10^{16}/\text{cm}^3$, where n_{λ_0} is the photon occupation number and V is the volume. The corresponding phonon occupation-number density has to be of the order of $10^{20}/\text{cm}^3$. In that case, the interaction term is found to be anisotropic with respect to the direction of injected phonons. Expressions for the superconducting energy gaps at $T=0°\text{K}$ are derived by a Green's-function technique. The transition temperature for $n_{\lambda_0}/V \sim 10^{17}/\text{cm}^3$ is found to be of the order of 10^2 °K.

1. INTRODUCTION

In recent years, there has been considerable interest, both theoretical and experimental, in the search for materials exhibiting high superconducting transition temperatures.[1,2] Among the possible physical factors investigated are the effects of pressure, impurities[3,4] (magnetic as well as nonmagnetic), and polarizability of inner-core electrons on the transition temperature of superconductors.[5,6] On the other hand, systems such as organic polymers having long conjugated hydrocarbon chains with polarizable side groups[2] have been suggested as possible one-dimensional systems that might show a high transition temperature T_c.

Some attempts have been directed towards the possible enhancement of T_c by increasing the strength of phonon-induced attractive electron-electron interaction and exploiting its energy dependence.[7] Others have invoked additional attractive mechanisms, purely electronic in nature, in overlapping multiband systems.[4,8,9] Further, it has been shown by Cohen that intervalley phonon-induced processes lead to a strong attractive electron-electron interaction in degenerate many-valley semiconductors resulting in superconductivity.[10] Since then superconductivity has been observed in a number of semiconductors.[11,12] However, the observed transition temperatures are rather low, i.e., $<1°\text{K}$.

So far, all these formulations have involved virtual dynamical interactions which are internal to the system. One can, however, envisage augmentation of some of these processes by external agencies. One possible mode of enhancement that has been suggested is through the injection of phonon pulses (the corresponding phonon density being $\sim 10^{15}/\text{cm}^3$) which can change the energy gap at an angle perpendicular to the direction of phonon pulses.[13] The concomitant increase in T_c was, however, small because of the fact that the artifice led only to renormalization of certain gross parameters.

In most of the above theoretical treatments, formulated within the framework of the Bardeen-Cooper-Schrieffer[14] (BCS) theory, the role of "real" phonons is essentially eliminated. This, of course, is true of any one-boson virtual process. Furthermore, in all these cases only the pairing of electrons in the same band was taken seriously. For the situation where the bands do not overlap, these were, of course, guided by the fact that one of the bands will necessarily be unoccupied in view of Fermi statistics.

Recent advances in laser and maser technology place at our disposal means of populating the otherwise unoccupied electron band and also ensure a macroscopic occupation of a single-photon state.[15] In view of the degenerate nature of the Bose statistics, the role of

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the latter is expected to be significant for two-boson processes in electron pairing mechanisms. It seems, therefore, worthwhile to investigate the theoretical possibilities of interband pairing between electrons of two bands (in general nonoverlapping) owing to virtual exchange of photons and phonons simultaneously.

For the development of the theoretical formulation, we consider an idealized model that is adequately described by two conduction bands (nonoverlapping in general) such that radiative electric dipole transitions are allowed between these two. Of course, there will be several filled bands below them separated by appropriate energy gaps.

In Sec. 2, after setting out the model Hamiltonian for the system, we shall derive pure phonon (intraband) and pure photon (interband) induced electron-electron interactions following the method of canonical transformation. Subsequently, after another canonical transformation, the induced interaction involving simultaneous exchange of two bosons (one photon and one phonon) between electrons of the two bands will be derived. It will be shown that these two-boson interaction terms are linear in the occupation numbers of phonons and photons. This leads to the possibility of increasing the strength of these higher-order processes artificially by augmenting the occupation-number density.

We do not emphasize the purely one-boson processes that involve interband exchange of virtual photons inasmuch as the volume of the momentum space spanned by the "interaction quantum" is necessarily small.

The two-boson processes, however, which involve intermediate phonon states as well, do not suffer from these limitations. Moreover, the reduction in the matrix element in these higher-order processes can be compensated by the realization of a large occupation number of the appropriate boson modes. In Sec. 3, the gap equation at $T=0°\mathrm{K}$ is obtained by the Green's function technique. Finally, based on order-of-magnitude estimates of the parameters involved, the feasibility of obtaining photoinduced superconductivity in some systems is discussed in Sec. 4.

2. FORMULATION OF INTERACTION HAMILTONIAN

As explained in the previous section, we consider a solid consisting of two conduction bands b and c, nonoverlapping in general. In this case, the energy gap is denoted by E_{bc}. If the bands do not overlap, an appreciable carrier concentration is assumed to have been realized in the otherwise empty band c by artificial optical pumping from the underlying filled valence bands. When the bands overlap, i.e., $E_{bc}=0$, the two will, of course, contain an adequate number of carriers.

The Hamiltonian for the material system in the presence of the photon field can be written as

$$H=H_e^{(b)}+H_e^{(c)}+H_f+H_p+H_{ep}+H_{ef}+H_c, \quad (2.1)$$

where

$$H_e^{(b)}=\sum_{\mathbf{k},\sigma}\epsilon_k^{(b)}b_{\mathbf{k}\sigma}{}^{\dagger}b_{\mathbf{k}\sigma}, \quad (2.2)$$

$$H_e^{(c)}=\sum_{\mathbf{k}\sigma}\epsilon_{\mathbf{k}}^{(c)}c_{\mathbf{k}\sigma}{}^{\dagger}c_{\mathbf{k}\sigma}, \quad (2.3)$$

$$H_f=\sum_{\lambda}\hbar\Omega_{\lambda}(\alpha_{\lambda}{}^{\dagger}\alpha_{\lambda}+\tfrac{1}{2}), \quad (2.4)$$

$$H_p=\sum_{\mathbf{q}}\hbar\omega_{\mathbf{q}}(\beta_{\mathbf{q}}{}^{\dagger}\beta_{\mathbf{q}}+\tfrac{1}{2}). \quad (2.5)$$

Here the various symbols have the following significance: $(b_{\mathbf{k}\sigma}{}^{\dagger}, b_{\mathbf{k}\sigma})$ $(c_{\mathbf{k}\sigma}{}^{\dagger}, c_{\mathbf{k}\sigma})$ are the fermion creation and annihilation operators for the two bands b and c, respectively; $\mathbf{k}$ is the propagation vector and σ the spin index. $\epsilon_{\mathbf{k}}^{(b)}$ and $\epsilon_{\mathbf{k}}^{(c)}$ are the respective single-particle energies. $(\alpha_{\lambda}{}^{\dagger}, \alpha_{\lambda})$ are the photon creation and annihilation operators for wave vector $\boldsymbol{\lambda}$; $\boldsymbol{\lambda}$ includes the polarization index as well. Ω_{λ} is the photon mode-branch frequency. $\beta_{\mathbf{q}}{}^{\dagger}$, $\beta_{\mathbf{q}}$, $\omega_{\mathbf{q}}$ are the corresponding symbols for the phonon case.

Further, the electron-phonon interaction is given by

$$H_{ep}=i\sum_{\mathbf{k},\mathbf{q}}D_{\mathbf{q}}(c_{\mathbf{k}+\mathbf{q},\sigma}{}^{\dagger}c_{\mathbf{k}\sigma}\beta_{\mathbf{q}}-\text{H.c.}) +i\sum_{\mathbf{k},\mathbf{q}}D_{\mathbf{q}}(b_{\mathbf{k}+\mathbf{q},\sigma}{}^{\dagger}b_{\mathbf{k}\sigma}\beta_{\mathbf{q}}-\text{H.c.}), \quad (2.6)$$

where $D_{\mathbf{q}}$ is the electron-phonon coupling coefficient. The electron-photon interaction term H_{ef} can be expressed in the first quantization as

$$H_{ef}=-c^{-1}\int\mathbf{j}(\mathbf{r},t)\cdot\mathbf{A}(\mathbf{r},t)\,d^3r, \quad (2.7)$$

where $\mathbf{j}(\mathbf{r},t)$ and $\mathbf{A}(\mathbf{r},t)$ are, respectively, the electron current density and the vector magnetic potential associated with the radiation field. We consider the situation described by the Coulomb gauge in which the transverse (retarded) and the longitudinal (instantaneous) components of the field can be clearly distinguished. H_c is the interaction of the electrons with the longitudinal component of the electromagnetic field, i.e., screened Coulomb interaction between electrons. It will be explicitly included later on. The electron-photon interaction term (2.7) can be recast in the second quantized form by the following substitution:

$$\mathbf{j}(\mathbf{r},t)=\sum_i e\dot{\mathbf{r}}_i\delta(\mathbf{r}-\mathbf{r}_i), \quad (2.8)$$

where $\dot{\mathbf{r}}_i$ is the velocity of the ith electron and $\delta(\mathbf{r}-\mathbf{r}_i)$ is the dirac δ function.

$$\mathbf{A}(\mathbf{r},t)=\sum_{\lambda}(2\pi c\hbar/V)^{1/2}\boldsymbol{\varepsilon}_{\lambda}[\alpha_{\lambda}\exp(i\boldsymbol{\lambda}\cdot\mathbf{r}) +\alpha_{\lambda}{}^{\dagger}\exp(-i\boldsymbol{\lambda}\cdot\mathbf{r})], \quad (2.9)$$

where V is the normalization volume and $\boldsymbol{\varepsilon}_{\lambda}$ is the unit polarization vector of the photon. Here and above, e, $\hbar$, and c are the usual universal constants.

To go over to second-quantized representation we choose as our basis functions the set of Bloch states designated by $|\mathbf{k}\rangle_b$ and $|\mathbf{k}\rangle_c$ for the two bands in question. It will be convenient to re-express these in

the Wannier representation, namely,

$$|\mathbf{k}\rangle_b = N^{-1/2}\sum_l \phi_b(\mathbf{r}-\mathbf{R}_l)\exp(i\mathbf{k}\cdot\mathbf{R}_l), \quad (2.10a)$$

$$|\mathbf{k}\rangle_c = N^{-1/2}\sum_m \phi_c(\mathbf{r}-\mathbf{R}_m)\exp(i\mathbf{k}\cdot\mathbf{R}_m), \quad (2.10b)$$

where $\phi_b(\mathbf{r}-\mathbf{R}_l)$ and $\phi_c(\mathbf{r}-\mathbf{R}_m)$ are the Wannier functions of the respective bands localized around sites $\mathbf{R}_l$ and $\mathbf{R}_m$.

Taking into account (2.8), (2.9), (2.10), we can write (2.7) as

$$H_{ef} = i\sum_{\lambda,\mathbf{k}\sigma}\Gamma_\lambda(c_{\mathbf{k}+\lambda,\sigma}{}^\dagger b_{\mathbf{k}\sigma}\alpha_\lambda - \text{H.c.}), \quad (2.11)$$

$$\Gamma_\lambda = \gamma_{bc}(\hbar\Omega_\lambda/V)^{1/2}, \quad (2.12)$$

where

$$\gamma_{bc} = 2\pi\langle \mathfrak{y}_{bc}\cdot\boldsymbol{\varepsilon}_\lambda\rangle, \quad (2.13)$$

$$\mathfrak{y}_{bc} = \langle\phi_b \mid e\mathbf{r} \mid \phi_c\rangle, \quad (2.14)$$

the matrix element of the electric dipole operator connecting the Wannier states ϕ_b and ϕ_c on the same site. This will be the dominant term (strictly so in the tight-binding limit) when allowed by selection rule for the electric dipole transition. In deriving this, we have made use of the usual dipolar approximation, i.e., $|\mathfrak{a}|a_0 \ll 1$, where a_0 is the Bohr radius. The form of (2.11) is similar to that of Fröhlich Hamiltonian[16] for the case of electron-phonon interaction [cf. Eq. (2.6)] with $D_q = g(\hbar\omega_q/2V)^{1/2}$ as written by Bogoliubov.[17] Their coupling constant g corresponds to γ_{bc} in the photon case and has the same dimension, namely (energy×volume)$^{1/2}$. At this stage, it will be appropriate to make a rough estimate of the two coupling constants. As given by Fröhlich, $g = C_1/(\rho v_s^2)^{1/2}$, where C_1 is energy of the order of 10 eV, ρ is a mass density and v_s is the velocity of sound. With $\rho = 1$ g/cm^3 and $v_s \sim 5\times10^5$ cm/sec, we get $g \sim 10^{-17}$ (erg cm^3)$^{1/2}$. We make a conservative estimate of γ_{bc} assuming $\phi_b \equiv s$ orbital, $\phi_c \equiv p$ orbital. Thus

$$\begin{aligned}\gamma_{bc} &= 2\pi\langle s \mid e\mathbf{r}\cdot\boldsymbol{\epsilon}_\lambda \mid p\rangle \\ &= 2\pi\times 4.8\times10^{-10}\times 2\times10^{-8}\ (\text{erg cm}^3)^{1/2} \\ &= 10^{-17}\ (\text{erg cm}^3)^{1/2}.\end{aligned}$$

This corresponds to an oscillator strength of the order of unity for the intra-atomic allowed electric dipole transition.

This compares favorably with the electron-phonon interaction strength. *However, the reason why pure photon-induced processes are weaker than the phonon processes is that the volume of the momentum space in which the photon energy is comparable with other energies of the problem in question is extremely small, i.e., smaller by a factor* $(v_s/c)^2 \sim 10^{-9}$.

This renders the one-boson process involving virtual exchange of photons inconsequential.

We shall show that two-boson processes involving a photon and a phonon will not suffer from this limitation. Although, there will be reduction in the magnitude of the matrix element, the occurrence of occupation-number density, which can be boosted artificially, will restore the value.

In order to get the effective electron-electron interaction, we eliminate the electron-phonon and electron-photon interaction terms in the first order of the respective coupling coefficients by means of a suitable canonical transformation. For this purpose the full Hamiltonian is rewritten as

$$H = H_0 + H_{\text{int}}, \quad (2.15)$$

where

$$H_0 = H_e{}^{(b)} + H_e{}^{(c)} + H_p + H_f + H_c, \quad (2.16)$$

$$H_{\text{int}} = H_{ep} + H_{ef}. \quad (2.17)$$

The transformed Hamiltonian is given by

$$H_T = H_0 + H_{\text{int}} + [H_0, S_1] + [H_{\text{int}}, S_1] + \tfrac{1}{2}[(H_0, S_1), S_1] + \cdots, \quad (2.18)$$

where S_1 is determined from the condition

$$H_{\text{int}} + [H_0, S_1] = 0. \quad (2.19)$$

We get

$$\begin{aligned}S_1 = i\sum_{\lambda,\mathbf{k}\sigma}\Gamma_\lambda\left(\frac{c_{\mathbf{k}+\lambda,\sigma}{}^\dagger b_{\mathbf{k}\sigma}\alpha_\lambda}{\epsilon_{\mathbf{k}}{}^b+\hbar\Omega_\lambda-\epsilon_{\mathbf{k}+\lambda}{}^c} - \frac{b_{\mathbf{k}\sigma}{}^\dagger c_{\mathbf{k}+\lambda,\sigma}\alpha_\lambda{}^\dagger}{\epsilon_{\mathbf{k}+\lambda}{}^c-\hbar\Omega_\lambda-\epsilon_{\mathbf{k}}{}^b}\right) \\ + i\sum_{\mathbf{q},\mathbf{k}\sigma} D_\mathbf{q}\left(\frac{c_{\mathbf{k}+\mathbf{q},\sigma}{}^\dagger c_{\mathbf{k}\sigma}\beta_\mathbf{q}}{\epsilon_{\mathbf{k}}{}^c+\hbar\omega_\mathbf{q}-\epsilon_{\mathbf{k}+\mathbf{q}}{}^c} - \frac{c_{\mathbf{k}\sigma}{}^\dagger c_{\mathbf{k}+\mathbf{q},\sigma}\beta_\mathbf{q}{}^\dagger}{\epsilon_{\mathbf{k}+\mathbf{q}}{}^c-\epsilon_{\mathbf{k}}{}^c-\hbar\omega_\mathbf{q}}\right) \\ + i\sum_{\mathbf{q},\mathbf{k}\sigma} D_\mathbf{q}\left(\frac{b_{\mathbf{k}+\mathbf{q}\sigma}{}^\dagger b_{\mathbf{k}\sigma}\beta_\mathbf{q}}{\epsilon_{\mathbf{k}}{}^b+\hbar\omega_q-\epsilon_{\mathbf{k}+\mathbf{q}}{}^b} - \frac{b_{\mathbf{k}\sigma}{}^\dagger b_{\mathbf{k}+\mathbf{q},\sigma}\beta_\mathbf{q}{}^\dagger}{\epsilon_{\mathbf{k}+\mathbf{q}}{}^b-\hbar\omega_\mathbf{q}-\epsilon_{\mathbf{k}}{}^b}\right).\end{aligned} \quad (2.20)$$

The transformed Hamiltonian then becomes

$$H_{T1} = H_{T0} + H_T(\text{int}), \quad (2.21)$$

where

$$\begin{aligned}H_{T0} = H_0 + \sum_{\mathbf{q},\mathbf{k},\mathbf{k}'\sigma}\frac{D_\mathbf{q}{}^2\hbar\omega_\mathbf{q}}{(\epsilon_{\mathbf{k}}{}^b-\epsilon_{\mathbf{k}-\mathbf{q}}{}^b)^2-(\hbar\omega_q)^2} \\ \times b_{\mathbf{k}'+\mathbf{q},\sigma}{}^\dagger b_{\mathbf{k}-\mathbf{q},-\sigma}{}^\dagger b_{\mathbf{k},-\sigma} b_{\mathbf{k}'\sigma} \\ + \sum_{\mathbf{q},\mathbf{k},\mathbf{k}'\sigma}\frac{D_\mathbf{q}{}^2\hbar\omega_\mathbf{q}}{(\epsilon_{\mathbf{k}}{}^c-\epsilon_{\mathbf{k}-\mathbf{q}}{}^c)-(\hbar\omega_\mathbf{q})^2} c_{\mathbf{k}'+\mathbf{q},\sigma}{}^\dagger c_{\mathbf{k}-\mathbf{q},-\sigma}{}^\dagger c_{\mathbf{k},-\sigma} c_{\mathbf{k}'\sigma} \\ + \sum_{\lambda,\mathbf{k},\mathbf{k}'\sigma}\frac{\Gamma_\lambda{}^2\hbar\Omega_\lambda}{(\epsilon_{\mathbf{k}}{}^c-\epsilon_{\mathbf{k}-\lambda}{}^b)^2-(\hbar\Omega_\lambda)^2} b_{\mathbf{k}'+\lambda,\sigma}{}^\dagger c_{\mathbf{k}-\lambda,-\sigma}{}^\dagger b_{\mathbf{k},-\sigma} c_{\mathbf{k}'\sigma},\end{aligned} \quad (2.22)$$

$$\begin{aligned}H_T(\text{int}) = [\tfrac{1}{2}\sum_{\mathbf{k}\sigma,\lambda\mathbf{q}}\Gamma_\lambda D_\mathbf{q}\{(\epsilon_{\mathbf{k}+\lambda}{}^c-\hbar\omega_\mathbf{q}-\epsilon_{\mathbf{k}+\mathbf{q}+\lambda}{}^c)^{-1} \\ -(\epsilon_{\mathbf{k}}{}^b+\hbar\omega_\mathbf{q}-\epsilon_{\mathbf{k}+\mathbf{q}}{}^b)^{-1}+(\epsilon_{\mathbf{k}+\lambda}{}^c-\hbar\Omega_\lambda-\epsilon_{\mathbf{k}}{}^b)^{-1} \\ -(\epsilon_{\mathbf{k}+\mathbf{q}+\lambda}{}^c-\hbar\Omega_\lambda-\epsilon_{\mathbf{k}+\mathbf{q}}{}^b)^{-1}\} c_{\mathbf{k}+\mathbf{q}+\lambda,\sigma}{}^\dagger b_{\mathbf{k}\sigma}\alpha_\lambda\beta_\mathbf{q} \\ +\tfrac{1}{2}\sum_{\mathbf{k}\sigma,\lambda\mathbf{q}}\Gamma_\lambda D_\mathbf{q}\{(\epsilon_{\mathbf{k}}{}^c+\hbar\omega_\mathbf{q}-\epsilon_{\mathbf{k}+\mathbf{q}}{}^c)^{-1} \\ -(\epsilon_{\mathbf{k}-\lambda}{}^b+\hbar\omega_\mathbf{q}-\epsilon_{\mathbf{k}+\mathbf{q}-\lambda}{}^b)^{-1}+(\epsilon_{\mathbf{k}-\lambda+\mathbf{q}}{}^b+\hbar\Omega_\lambda-\epsilon_{\mathbf{k}+\mathbf{q}}{}^c)^{-1} \\ -(\epsilon_{\mathbf{k}-\lambda}{}^b+\hbar\Omega_\lambda-\epsilon_{\mathbf{k}}{}^c)^{-1}\} c_{\mathbf{k}\sigma}{}^\dagger b_{\mathbf{k}+\mathbf{q}-\lambda,\sigma}\alpha_\lambda\beta_\mathbf{q}{}^\dagger + \text{H.c.}].\end{aligned} \quad (2.23)$$

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The second and the third terms in (2.22) are the intraband phonon-induced electron-electron interactions for bands b and c, respectively. The third term is the purely photon-induced process and involves interband pairing. $H_T(\text{int})$ is the two-boson (i.e., one-phonon and one-photon) interband scattering process. In addition, there will be two-phonon and two-photon scattering processes that have not been written.

As already remarked λ spans a very small fraction of the available momentum space and thus the purely one-photon–induced process [cf. 4th term of (2.22)] will give negligible contribution towards pairing. However, we need to investigate the role of $H_T(\text{int})$, which in higher order should give rise to interband pairing. It may be noted that (2.23) will survive only if the two bands differ in terms of dispersion relations, i.e., the electrons in the two bands have different effective masses. In the following, we shall assume the two bands to differ only to the extent that (2.23) is nonvanishing.

Assuming

$$\epsilon_k{}^b=(\hbar^2/2m_b)k^2$$

and

$$\epsilon_k{}^c=E_{bc}+(\hbar^2/2m_c)k^2,$$

$H_T(\text{int})$ can be written as

$$H_T(\text{int})=\sum_{k\sigma,\lambda q}(\Gamma_\lambda D_q/\Delta E_R)(c_{k+\lambda+q,\sigma}{}^\dagger b_{k\sigma}\alpha_\lambda\beta_q + c_{k+\lambda-q,\sigma}{}^\dagger b_{k\sigma}\alpha_\lambda\beta_q{}^\dagger+\text{H.c.}), \quad (2.24)$$

where

$$1/\Delta E_R=1/\Delta E_D+1/\Delta W, \quad (2.25)$$

with

$$1/\Delta E_D=(\Delta m/\bar m)\,1/2\bar m v_s{}^2, \quad (2.26)$$

$$|\Delta m|=|m_a-m_b|,$$

$$\bar m=\tfrac12(m_a+m_b),$$

$$1/\Delta W=\tfrac12\hbar^2(\Delta m/\bar m^2)q_D{}^2/w_a{}^2; \quad (2.27)$$

q_D is the Debye wave vector magnitude and $w_a=(\hbar\Omega_\lambda-E_{bc})$, the width of the absorption band. For $\Delta m/\bar m\sim1$ and $v_s\sim5\times10^5$ cm/sec, $q_D\sim10^3$/cm and $w_a\sim0.1$ eV, we get $\Delta E_R\sim10^{-16}$ erg.

We shall now perform a second canonical transformation such that

$$H_{T2}=H_{T0}+\tfrac12[H_T(\text{int}),\,S_2]+\cdots, \quad (2.28)$$

where

$$S_2=\sum_{k\sigma,\lambda q}\frac{\Gamma_\lambda D_q}{\Delta E_R}\left\{\frac{c_{k+q+\lambda,\sigma}{}^\dagger b_{k\sigma}\alpha_\lambda\beta_q-b_{k\sigma}{}^\dagger c_{k+q+\lambda,\sigma}\alpha_\lambda{}^\dagger\beta_q{}^\dagger}{\epsilon_k{}^b+\hbar\Omega_\lambda+\hbar\omega_q-\epsilon_{k+\lambda+q}{}^c}\right.$$
$$\left.+\frac{c_{k+\lambda-q,\sigma}{}^\dagger b_{k\sigma}\alpha_\lambda\beta_q{}^\dagger-b_{k\sigma}{}^\dagger c_{k+\lambda-q\sigma}\alpha_\lambda{}^\dagger\beta_q}{\epsilon_k{}^b+\hbar\Omega_\lambda-\hbar\omega_q-\epsilon_{k+\lambda-q}{}^c}\right\}. \quad (2.29)$$

The final result, after transformation, is

$$H_{T2}=H_{T0}-\sum_{\lambda,k,k'}\left[\frac{|\Lambda_{bc}|^2(\hbar\Omega_\lambda-\hbar\omega_{k'-k-\lambda})(N_{k'-k-\lambda}-n_\lambda)}{(\hbar\Omega_\lambda-\hbar\omega_{k'-k-\lambda})^2-(\epsilon_{k'}{}^b-\epsilon_k{}^c)^2}\right.$$
$$\left.+\frac{|\Lambda_{bc}|^2(\hbar\Omega_\lambda+\hbar\omega_{k'-k-\lambda})(1+N_{k'-k-\lambda}+n_\lambda)}{(\hbar\Omega_\lambda+\hbar\omega_{k'-k-\lambda})^2-(\epsilon_{k'}{}^b-\epsilon_k{}^c)^2}\right]$$
$$\times b_{k'\sigma}{}^\dagger c_{-k'-\sigma}{}^\dagger b_{-k-\sigma}c_{k\sigma}, \quad (2.30)$$

where we have used the interference condition $\mathbf{k}'-\mathbf{k}=\mathbf{q}+\boldsymbol{\lambda}$; N_q and n_λ are the phonon and the photon occupation numbers, respectively, and $|\Lambda_{bc}(\boldsymbol{\lambda}\mathbf{q})|^2=\Gamma_\lambda{}^2D_{k'-k-\lambda}{}^2/\Delta E_R{}^2$. In (2.30) the fermion operators correspond to the BCS-type interband pairing.

As pointed out earlier, the reduction in matrix elements is to be compensated by the occupation-number enhancement. The latter can be taken into account by writing $n_\lambda=n_{\lambda_0}\delta_{\lambda\lambda_0}$, where $(n_{\lambda0}/V)$ is the macroscopic occupation number density of injected photons of wave vector λ_0, say of the order of 10^{18}/cm^3 obtained by appropriate pumping from a coherent source with negligible dispersion.

For the situation

$$\hbar\Omega_\lambda\sim\hbar\Omega_{\lambda_0}\gg\hbar\omega_{q_D}$$

and

$$(\hbar\Omega_{\lambda_0}-E_{bc})\sim w_a$$

(the width of the absorption band), the two terms in the square bracket of (2.30) can be combined by ignoring $\hbar\omega_q$ in the denominator. We get

$$H_{T2}=H_{T0}-\left[\sum_{\lambda k,k'\sigma}\frac{2|\Lambda_{bc}|^2}{(\hbar\Omega_\lambda)^2-(\epsilon_{k'}{}^b-\epsilon_k{}^c)^2}\hbar\Omega_\lambda(N_{k'-k-\lambda}+\tfrac12)\right.$$
$$\left.+\sum_{k,k',\sigma}\frac{2|\Lambda_{bc}|^2\hbar\omega_{k'-k-\lambda_0}(n_{\lambda_0}+\frac12)}{(\hbar\Omega_{\lambda_0})^2-(\epsilon_{k'}{}^b-\epsilon_k{}^c)^2}\right]b_{k'\sigma}{}^\dagger c_{-k'-\sigma}{}^\dagger b_{-k-\sigma}c_{k\sigma}. \quad (2.31)$$

Thus, the effective interband electron-electron interaction is attractive for $\hbar\Omega_\lambda\sim\hbar\Omega_{\lambda_0}>|\epsilon_{k'}{}^b-\epsilon_k{}^c|$, i.e., in an energy shell of width $2|\hbar\Omega_{\lambda_0}-E_{bc}|$ about the Fermi surface.

The first term in the curly bracket of (2.31) depends on the phonon occupation number. Approximating $(\epsilon_k{}^c-\epsilon_{k'}{}^b)\sim E_{bc}$ and noting that $|\boldsymbol{\lambda}|\ll|\mathbf{k}'-\mathbf{k}|$, this term can be written, after integration over $\boldsymbol{\lambda}$, as

$$\left(\frac{2|\Lambda_{bc}|^2\hbar\Omega_{\lambda_0}}{(\hbar\Omega_{\lambda_0})^2-E_{bc}{}^2}\right)\frac{V\lambda_0{}^3}{3\pi^2}(N_{k'-k}+\tfrac12). \quad (2.32)$$

Independent $\boldsymbol{\lambda}$ summation is justified since $\boldsymbol{\lambda}$ spans a small volume in the momentum space. Estimates show that (2.32) will be comparable to the pure phonon-induced matrix element (BCS) provided $(N_{k'-k}/V)\sim$

$10^6/\text{cm}^3$. This amounts to injecting coherent phonon pulses, necessarily of a specific wave vector. Accordingly, the matrix element becomes extremely anisotropic with respect to the direction of injected phonon wave vector. In the final analysis, the energy gap will also become anisotropic. However, this process also restricts the volume of the interaction momentum space. Furthermore, it is much more difficult to pump phonons than photons.

The second term of the curly bracket of (2.31) does not suffer from these limitations. This can be written as

$$\frac{2\,|\,\Lambda_{bc}\,|^2\hbar\omega_D}{(\hbar\Omega_{\lambda_0})^2-E_{bc}{}^2}\,(n_{\lambda_0}+\tfrac{1}{2}). \tag{2.33}$$

This will be comparable to the phonon-induced (BCS) process if the factor

$$\frac{2\gamma_{bc}{}^2\hbar\Omega_{\lambda_0}(\hbar\omega_D)^2(n_{\lambda_0}/V)}{\Delta E_R{}^2[(\hbar\Omega_{\lambda_0})^2-E_{bc}{}^2]}$$

is of the order of unity. This implies that n_{λ_0}/V should be of the order of $10^{16}/\text{cm}^3$. This is within the range of feasibility in view of recent developments in modern laser and maser technology. For further calculations, we shall reduce the effective Hamiltonian so as to retain terms relevant to interband pairing only, in the presence of photon pumping.

$$H_{\text{red}}=H_e{}^{(b)}+H_e{}^{(c)}-\Lambda\sum_{\mathbf{k}\mathbf{k}'\sigma}b_{\mathbf{k}'\sigma}{}^\dagger c_{-\mathbf{k}'-\sigma}{}^\dagger b_{-\mathbf{k},-\sigma}c_{\mathbf{k}\sigma}, \tag{2.34}$$

where Λ is a parameter defined as

$$\Lambda=\Big\langle\frac{2(\hbar\Omega_{\lambda_0}/V)\,|\,\gamma_{bc}\,|^2g^2(\hbar\omega_D)^2(n_{\lambda_0}/V)}{\Delta E_R{}^2[(\hbar\Omega_{\lambda_0})^2-E_{bc}{}^2]} - \frac{4\pi e^2/V}{|\,\mathbf{k}-\mathbf{k}'\,|^2+K_{bc}{}^2}\Big\rangle. \tag{2.35}$$

Here the second term in (2.35) represents screened Coulomb potential for interband Coulomb scattering. K_{bc} is the corresponding screening parameter. The first term in (2.35) is positive inasmuch as $\hbar\Omega_{\lambda_0}>E_{bc}$, and for sufficiently high degeneracy of photons ($n_{\lambda_0}/V>10^{16}/\text{cm}^3$) it will dominate the repulsive screened Coulomb term. It may be noted, in passing, that the interaction term (2.34) can be derived by the path integral method by eliminating photon and phonon coordinates, as has been shown recently for the pure phonon case.[18]

3. DETERMINATION OF ENERGY GAP AT $T=0$°K AND TRANSITION TEMPERATURE

We shall follow a method similar to that adopted by Gor'kov.[19] We define the following normal (G) and anomalous (F) Green's functions since the present interest is interband pairing:

$$G_{c_{\mathbf{k}\sigma}c_{\mathbf{k}\sigma}}(t-t')=(-i)\,\langle Tc_{\mathbf{k}\sigma}(t)\,c_{\mathbf{k}\sigma}{}^\dagger(t')\,\rangle, \tag{3.1}$$

$$\exp[i(2\mu+E_{bc})t/\hbar]\,F_{b_{\mathbf{k}\uparrow}c_{-\mathbf{k}\downarrow}}{}^\dagger(t-t') =\langle N+2\,|\,Tb_{\mathbf{k}\uparrow}{}^\dagger(t)\,c_{-\mathbf{k}\downarrow}{}^\dagger(t')\,|\,N\rangle, \tag{3.2}$$

$$\exp[-i(2\mu+E_{bc})t/\hbar]\,F_{b_{-\mathbf{k}\downarrow}c_{\mathbf{k}\uparrow}}(t-t') =\langle N\,|\,Tb_{-\mathbf{k}\downarrow}(t)\,c_{\mathbf{k}\uparrow}(t')\,|\,N+2\rangle. \tag{3.3}$$

The bracket $\langle N\,|\,T\cdots\,|\,N'\rangle$ refers to time-ordered contraction between appropriate states. The equations of motion for these functions are derived following the usual method.[20] We get

$$[i\hbar(\partial/\partial t)-\epsilon_{\mathbf{k}}{}^{(c)}]G_{c_{-\mathbf{k}\downarrow}c_{-\mathbf{k}\downarrow}}(t-t') +i\Lambda\Big(\sum_{\mathbf{k}'}F_{b_{-\mathbf{k}'\downarrow}c_{\mathbf{k}'\uparrow}}(0)\Big)F_{b_{\mathbf{k}\uparrow}c_{-\mathbf{k}\downarrow}}{}^\dagger(t-t')=\hbar\delta(t-t') \tag{3.4}$$

and

$$[i\hbar(\partial/\partial t)+\epsilon_{\mathbf{k}}{}^{(b)}-(2\mu+E_{bc})]F_{b_{\mathbf{k}\uparrow}c_{-\mathbf{k}\downarrow}}{}^\dagger(t-t') -i\Lambda\Big(\sum_{\mathbf{k}'}F_{c_{\mathbf{k}'\uparrow}b_{-\mathbf{k}'\downarrow}}{}^\dagger(0)\Big)G_{c_{-\mathbf{k}\downarrow}c_{-\mathbf{k}\downarrow}}(t-t')=0, \tag{3.5}$$

where we have made use of Gor'kov[19] factorization appropriate to interband pairing. The inherent time dependence of the quantities on the right-hand side of (3.2) and (3.3) has been shown explicitly by introducing the factor $\exp[\pm i(2\mu+E_{bc})t/\hbar]$ on the left-hand side, where μ is the chemical potential for each band measured from the bottom of respective bands. This follows from the general quantum-mechanical equation of motion of any operator in the Heisenberg representation. Here we have assumed equal population of the two bands b and c for simplicity; E_{bc} is the band gap. Next we carry out the energy Fourier transformation of the two equations.

We obtain, in the matrix form,

$$\begin{pmatrix} E-\epsilon_{\mathbf{k}}{}^{(c)} & i\Lambda\sum_{\mathbf{k}'}F_{b_{-\mathbf{k}'\downarrow}c_{\mathbf{k}'\uparrow}}(0) \\ -i\Lambda\sum_{\mathbf{k}'}F_{c_{\mathbf{k}'\uparrow}b_{-\mathbf{k}'\downarrow}}{}^\dagger & E+\epsilon_{\mathbf{k}}{}^{(b)}-(2\mu+E_{bc}) \end{pmatrix} \times\begin{pmatrix} G_{c_{-\mathbf{k}\downarrow}c_{-\mathbf{k}\downarrow}}(E) \\ F_{b_{\mathbf{k}\uparrow}c_{-\mathbf{k}\downarrow}}{}^\dagger(E) \end{pmatrix}=\begin{pmatrix}\hbar\\0\end{pmatrix}. \tag{3.6}$$

Displacing all energy levels by μ and then solving for $G_{c_{-\mathbf{k}\downarrow}c_{-\mathbf{k}\downarrow}}(E)$ and $F_{b_{\mathbf{k}\uparrow}c_{-\mathbf{k}\downarrow}}{}^\dagger(E)$, we get

$$G_{c_{-\mathbf{k}\downarrow}c_{-\mathbf{k}\downarrow}}(E)=\frac{\hbar(E+\epsilon_{\mathbf{k}}{}^{(b)}-E_{bc})}{D} \tag{3.7}$$

$$F_{b_{\mathbf{k}\uparrow}c_{-\mathbf{k}\downarrow}}{}^\dagger(E)=\frac{i\Lambda\hbar\,\sum_{\mathbf{k}'}F_{c_{\mathbf{k}'\uparrow}b_{-\mathbf{k}'\downarrow}}{}^\dagger}{D}, \tag{3.8}$$

where D is the determinant of the coefficient matrix on

[18] N. Kumar and K. P. Sinha, Nuovo Cimento **51B**, 547 (1967).
[19] L. P. Gor'kov, Zh. Eksperim. i Teor. Fiz. **34**, 735 (1958) [English transl.: Soviet Phys.—JETP **7**, 505 (1958)].
[20] A. A. Abrikosov, L. P. Gor'kov, and I. E. Dzyaloshinski, *Methods of Quantum Field Theory in Statistical Physics* (Prentice-Hall, Inc., Englewood Cliffs, N.J., 1963).

the left-hand side of (3.6); explicitly

$$D=(E-\epsilon_k^{(a)})(E+\epsilon_k^{(b)}-E_{bc})-\Delta_{bc}^2, \quad (3.9)$$

where

$$\Delta_{bc}^2\equiv\Lambda^2\sum_{k'k''}F_{b_{-k'\downarrow}c_{k'\uparrow}}(0)F_{c_{k''\uparrow}b_{-k''\downarrow}}^{\dagger}(0). \quad (3.10)$$

The roots of the determinant give the poles of the Green's functions and in turn determine the energy spectrum of the elementary excitations. The two roots are

$$E_{\pm}=E_{bc}\pm(\epsilon_k^{(b)2}+\Delta_{bc}^2)^{1/2}. \quad (3.11)$$

Thus the two functions of present interest can be written as

$$G_{c_{-k\downarrow}c_{-k\downarrow}}(E)=\frac{\hbar(E+\epsilon_k^{(b)}-E_{bc})}{(E-E_+)(E-E_-)}, \quad (3.12)$$

$$F_{b_{k\uparrow}c_{-k\downarrow}}^{\dagger}(E)=\frac{i\Lambda\hbar\sum_{k'}F_{c_{k'\uparrow}b_{-k'\downarrow}}^{\dagger}(0)}{(E-E_+)(E-E_-)}. \quad (3.13)$$

The poles of the Green's function [cf. (3.12)] are to be treated so as to satisfy the Landau condition.[21] In the present case, this amounts to the condition that the imaginary part of the Green's function should have sign opposite to that of $(E-E_{bc})$. It can be shown that

$$G_{c_{-k\downarrow}c_{-k\downarrow}}(E)=\frac{\frac{1}{2}\hbar[1+\epsilon_k^{(b)}/(\epsilon_k^{(b)2}+\Delta_{bc}^2)^{1/2}]}{(E-E_++i\delta)}+\frac{\frac{1}{2}\hbar[1-\epsilon_k^{(b)}/(\epsilon_k^{(b)2}+\Delta_{bc}^2)^{1/2}]}{(E-E_--i\delta)}, \quad (3.14)$$

$$F_{b_{k\uparrow}c_{-k\downarrow}}^{\dagger}=\frac{i\Lambda\hbar\sum_{k'}F_{c_{k'\uparrow}b_{-k'\downarrow}}^{\dagger}(0)}{(E-E_++i\delta)(E-E_--i\delta)}, \quad (3.15)$$

satisfy the above conditions.

In order to get a self-consistent equation for the energy gap we take the inverse Fourier transform of (3.15):

$$(2\pi\hbar)^{-1}\int_{-\infty}^{\infty}F_{b_{k\uparrow}c_{-k\downarrow}}^{\dagger}(E)\exp[-iE(t-t')/\hbar]\,dE=\frac{i\Lambda\hbar\sum_{k'}F_{c_{k'\uparrow}b_{-k'\downarrow}}^{\dagger}(0)}{2\pi\hbar}\times\int_{-\infty}^{\infty}\frac{dE\exp[-iE(t-t')/\hbar]}{(E-E_++i\delta)(E-E_--i\delta)}. \quad (3.16)$$

Putting $t=t'$, performing $\mathbf{k}$-space summation, and noting that

$$\sum_k F_{b_{k\uparrow}c_{-k\downarrow}}^{\dagger}(0)=\sum_{k'}F_{c_{k'\uparrow}b_{-k'\downarrow}}^{\dagger}(0),$$

we can write

$$1=(i\Lambda/2\pi)\sum_k\int_{-\infty}^{\infty}\frac{dE}{(E-E_++i\delta)(E-E_--i\delta)}. \quad (3.17)$$

[21] L. D. Landau, Zh. Eksperim. i Teor. Fiz. **34**, 261 (1958) [English transl.: Soviet Phys.—JETP **7**, 182 (1958)].

Performing the E integration, we get

$$1=(\Lambda V/16\pi^3)\int\frac{d^3\mathbf{k}}{(\epsilon_k^{(b)2}+\Delta_{bc}^2)^{1/2}}, \quad (3.18)$$

$$1=(\Lambda V/16\pi^3)\rho_{Av}\int_{-W_a}^{W_a}\frac{d\epsilon_k^{(b)}}{(\epsilon_k^{(b)2}+\Delta_{bc}^2)^{1/2}}, \quad (3.19)$$

where we have replaced summation over $\mathbf{k}$ by integration. It should be noted that the cutoff W in the present case should be chosen to be $W_a=(\hbar\Omega_0-E_{bc})<\mu$ inasmuch as the interaction is attractive in this range. Integration yields

$$\Delta_{bc}=W_a\,\mathrm{csch}(8\pi^3/\Lambda V\rho_{Av}), \quad (3.20)$$

where ρ_{Av} is the average density of states given by

$$\rho_{Av}=\mu^{-1}\int_0^{\mu}\rho(E)\,dE=\frac{(2\bar{m}^*/\hbar^2)^{3/2}}{3\pi^2}\mu^{1/2} \quad (3.21)$$

and $\bar{m}^*$ is the mean effective electron mass. Thus the superconducting energy gap at the absolute zero temperature,

$$\Delta_{bc}=W_a\,\mathrm{csch}(\chi/\mu^{1/2}), \quad (3.22)$$

where

$$\chi=6\sqrt{2}\hbar^3/\Lambda V\bar{m}^{*3/2}. \quad (3.23)$$

The superconducting transition temperature is related to this gap parameter; i.e.,

$$k_BT_C\approx(\gamma/\pi)\Delta_{bc}, \quad (3.24)$$

where $\gamma=0.577$ (Euler's constant).

We shall now examine the energy gap arising from the term that depends on phonon occupation number [cf. first part of Eq. (2.31)], the coefficient being reduced as (2.32). Accordingly, we put $N_{k'-k}=\delta_{k'-k,q_0}N_0$ in (2.32), where N_0/V is the phonon occupation-number density per unit volume for one specific mode $\mathbf{q}_0$. Equation (2.35) then takes the form

$$\Lambda_a=\left\langle\frac{2\gamma_{bc}^2(\hbar\Omega_{\lambda_0})^2g^2(\hbar\omega_D/V)V\lambda_0^3(N_0/V)}{[(\hbar\Omega_{\lambda_0})^2-E_{bc}^2]}-\frac{4\pi e^2/V}{|\mathbf{q}_0|^2+K_{bc}^2}\right\rangle_{av}\quad\text{for }\mathbf{k}'-\mathbf{k}=\mathbf{q}_0. \quad (3.25)$$

Here, we have assumed that the dispersion in the injected phonon wave vector is sufficiently large compared with λ to justify the integration over λ, as in (2.32). It is to be noted that it is anisotropic with respect to $\mathbf{q}_0$. Rest of the calculations are similar from (3.1) to (3.15). However, we have to recalculate the subsequent steps. Equation (3.18) gets replaced by

$$F_{b_{k\uparrow}c_{-k\downarrow}}^{\dagger}(0)=2\Lambda\,F_{c_{k-q_0\uparrow}b_{-(k-q_0)\downarrow}}^{\dagger}(0)[\epsilon_k^{b2}+\Delta_{bc}^{a2}(\mathbf{k})]^{-1/2}. \quad (3.26)$$

This can be readily solved for certain specific conditions. For example, if we take $\mathbf{k}$ normal to $\mathbf{q}_0$, we can

approximate

$$|\mathbf{k}-\mathbf{q}_0| \sim |\mathbf{k}| \quad \text{for} \quad |\mathbf{q}_0| \ll |\mathbf{k}| \sim k_\mu.$$

Then we get

$$1=2\Lambda_a(\epsilon_k{}^{b2}+\Delta_{bc}{}^{a2})^{-1/2}. \tag{3.27}$$

This gives

$$\Delta_{bc}{}^a(k)=(4\Lambda_a{}^2-\epsilon_k{}^{b2})^{1/2}. \tag{3.28}$$

In this case the anisotropic gap is a function of the electron wave vector, and, of course, the anisotropic coupling coefficient. The maximum value of this is $\Delta_{bc}{}^a(\max)\sim 2\Lambda_a$.

4. ESTIMATES AND DISCUSSION

We shall now make estimates of the various parameters involved in the expressions of the energy gaps derived in the preceding section. For this purpose, it will be appropriate to compare the magnitudes of the attractive two-boson process derived here with those of pure phonon (BCS) process. Thus, disregarding screened Coulomb part, we get for Λ given by (2.35),

$$\frac{\Lambda(\text{2-boson})}{V_{\text{phonon}}(\text{BCS})}=\frac{\gamma_{bc}{}^2(\hbar\omega_D)^2}{\Delta E_R{}^2[\hbar\Omega_{\lambda_0}-E_{bc}]}\left(\frac{n_{\lambda_0}}{V}\right). \tag{4.1}$$

For the typical choice of parameters

$$(\hbar\Omega_{\lambda_0}-E_{bc})=0.05\times 1.6\times 10^{-12} \text{ erg};$$

$$\gamma_{bc}=10^{-17} \text{ (erg cm}^3)^{1/2},$$

$$\hbar\omega_D\approx 0.02\times 1.6\times 10^{-12} \text{ erg},$$

$$\Delta E_R=10^{-16} \text{ erg},$$

we get the ratio as $10^{-16}\times(n_{\lambda_0}/V)$. Thus for this process to be comparable or larger than the phonon process, we must have $(n_{\lambda_0}/V)\geq 10^{16}/\text{cm}^3$. Assuming that the system is bathed in a photon field with $(n_{\lambda_0}/V)\sim 10^{17}/\text{cm}^3$, we estimate the transition temperature from Eqs. (3.24) and (3.25). This will involve the magnitude of a few more parameters. The chemical potential μ is chosen to be of the order of 10^{-13} erg, which corresponds to a population $10^{20}/\text{cm}^3$ in either band b or c and $\chi\sim 4\times 10^{-7}$ erg$^{1/2}$. With these values the transition temperature is of the order of 10^2 °K. It is to be noted that for larger effective mass, higher carrier concentration, and higher photon density the estimate of T_c goes up considerably.

For completeness we next estimate the gap parameter for the term that depends on the phonon occupation number. Here we find that the gap parameter [cf. Eq. (3.28)] for highly anisotropic situations has the maximum values $2\Lambda_a$. For the same value of the parameters chosen above we get

$$\Delta_{bc}{}^a(\max)\sim 10^{-24}\times(N_0/V) \text{ eV}.$$

For this to be comparable with the usual (BCS) energy gap (i.e., 10^{-4} eV) the occupation-number density (N_0/V) must be of the order of $10^{20}/\text{cm}^3$. Present ultrasonic techniques for the generation of coherent phonons go up to $10^{15}/\text{cm}^3$.[13] Perhaps future developments may lead to sources that can give phonon densities as high as $10^{20}/\text{cm}^3$.

In the foregoing sections, we have treated the case of interband pairing between two conduction bands separated in general by a gap E_{bc}. For $E_{bc}\neq 0$ one has to resort to optical pumping to create carriers in the otherwise empty band c up to a concentration $10^{20}/\text{cm}^3$. If the two bands overlap, i.e., $E_{bc}=0$, then the optical pumping can be dispensed with; there will be a sufficient number of carriers in each band. We shall, however, require in both the situations a strong coherence source of photon or phonons to boost the respective boson occupation numbers. As shown above, the photon case seems much more favorable and feasible.

In view of the higher reflectivity in the superconducting state, one has to use only thin films of the materials since the mechanism is likely to be confined to surface layers only. This restriction does not apply to the case of phonon pumping.

Further, in an indirect bandgap system (i.e., one in which the conduction-band minima do not coincide in the momentum space) there is always a nonzero momentum transfer required by momentum conservation in the two-boson processes (one photon and one phonon, normal or unklapp). The large momentum transfer has the merit of screening the Coulomb repulsion more effectively. This leads one to the conclusion that the phonon-assisted photon exchange mechanism is the one most favorable for realizing photoinduced superconductivity, providing the boson occupation-number density is sufficiently high. To eliminate the heating effect associated with the incident radiation flux an effective heat sink has to be provided by suitable cryogenic techniques.

In order to detect the interband pairing effect predicted here, it may be preferable to use coherent radiation pulses of duration shorter than thermalization time (time required by the coherent radiation to be recognizable as blackbody radiation at the temperature in question).

In conclusion, we would like to remark that we have considered some optimal conditions favorable to the realization of photoinduced superconductivity. Estimates show it to be a physical possibility and it is desirable to carry out appropriate experiments to test the occurrence of induced superconductivity in degenerate semiconductor systems. Such two-conduction-band systems can be obtained by heavily doping a semiconductor so as to create an impurity band exhibiting almost metallike conductivity. The fact that the physical realization of such systems will place at our disposal controlled superconductivity (with high enough T_c) is bound to open up numerous avenues to practical applications.